Relativistic phenomenology of meson spectra with a covariant quark model in Minkowski space

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Outline

- Motivation
- Covariant Spectator Theory Bethe-Salpeter (CST-BS) Formalism
- Heavy and heavy-light mesons with CST-BS
- Numerical solution of CST-BS
- Results and the predictive power of covariant interaction kernels
- Summary and Outlook

For further details see: arXiv:1608.08065

Motivation

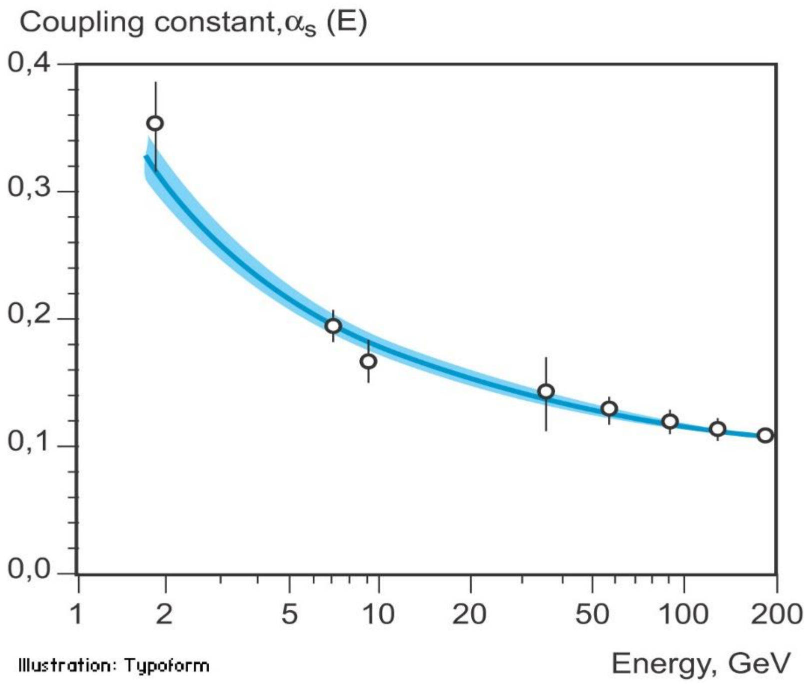
- The dynamical content of QCD as a *local* quantum field theory of quarks and gluons is described by its Lagrangian or, equivalently, its action.

- *But*, to unfold
  the *physical content* of QCD Lagrangian *towards* a *quantum-field theoretical* description of hadrons, is a *very* difficult task!

- @ large energies & small distances (ultraviolet region)
  interaction between quarks and gluons is weak and
  perturbative methods *can* be used;

  @ low energies & large distances (infrared region)
  coupling becomes strong and perturbation theory "fails"

  – dynamical chiral symmetry breaking
  – generation of large constituent quark masses from almost massless quarks
  – formation of hadrons (mesons, baryons)
  – confinement of quarks and gluons inside hadrons



Theoretical tools for QCD

- Effective field theories (EFTs)
  grew out of the operator-product expansion (OPE) and the formalism of phenomenological Lagrangians and, thus, provide a standard way to analyze physical systems with many different energy scales.

- Lattice gauge theory
  speedily progressing in what concerns systematic finite volume effects as well as increasingly small quark masses

- Other non-perturbative approaches
  among the most used techniques are: the limit of the large number of colors, generalizations of the original Shifman–Vainshtein–Zakharov sum rules, QCD vacuum models and effective string models, the AdS/CFT conjecture, and **Schwinger–Dyson equations**.

↓

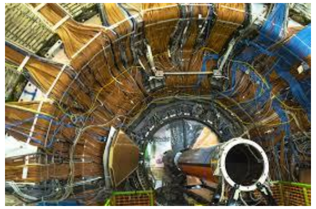
close in spirit, we aim a *self-consistent* quantum field theoretical approach, *but* in Minkowski space, designed for *all* $q\bar{q}$-type mesons and satisfying:

1. Poincaré covariance
   in general quarks require relativistic treatment
2. Confinement
   linear: suggested from nonrelativistic potential models and lattice QCD studies
3. Spontaneous chiral symmetry breaking
   existence of massless Goldstone pion and dynamical generation of constituent (dressed) quark mass from self-interactions
   Nambu-Jona-Lasinio-type mechanism

Covariant Spectator Theory Bethe-Salpeter (CST-BS)

Motivation

- The physics of mesons, in particular, is a very active area of research, especially due to the ample amount of new experimental data measured at facilities such as the LHC, BaBaR, Belle, CLEO, and more exciting results can also be expected from Jefferson Lab (GlueX) and FAIR (PANDA) in the near future.



  — *spectroscopy: classification of mesons (new states)*
  — *structure: form factors (*Minkowski can be more convenient than Euclidean formulations because form factors can be computed directly in the timelike region with no need for analytical continuations).

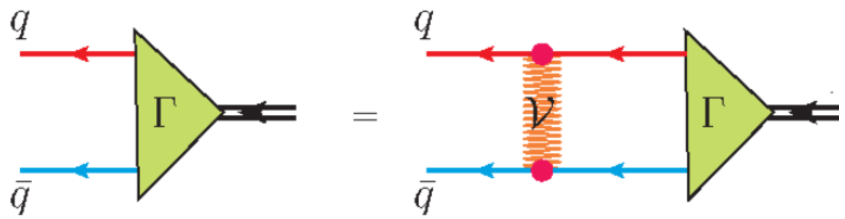
- Models with *testable* dynamics

Lattice QCD calculations

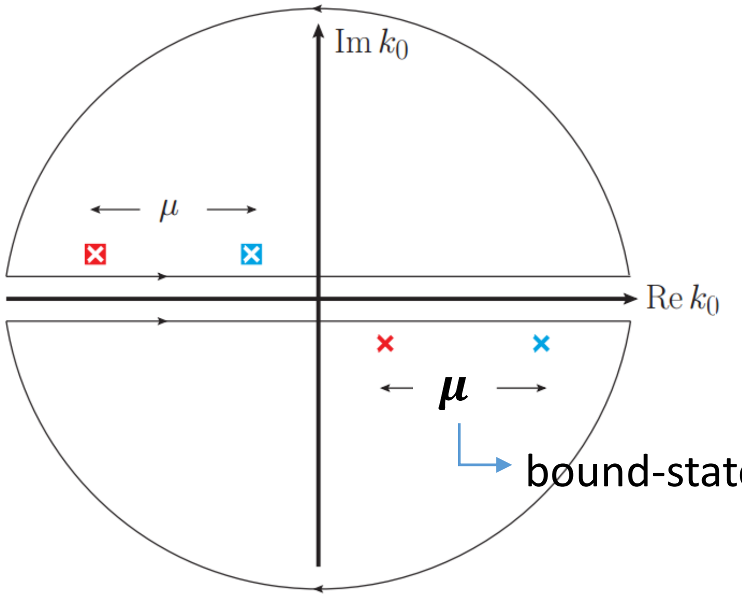
Experiment

CST-BS Formalism overview

- Bethe-Salpeter (BS) Equation

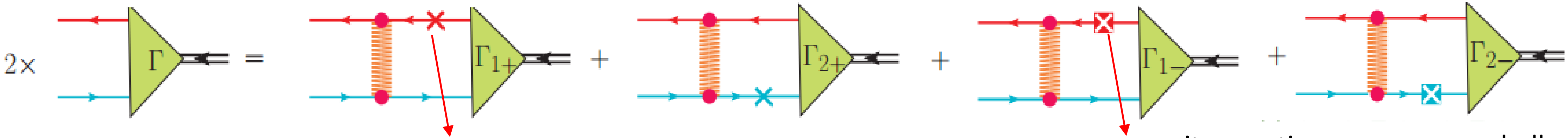


- Covariant Spectator Theory (CST): propagator pole contributions *approximate* sum of ladder and crossed ladders (to be seen later)



bound-state mass

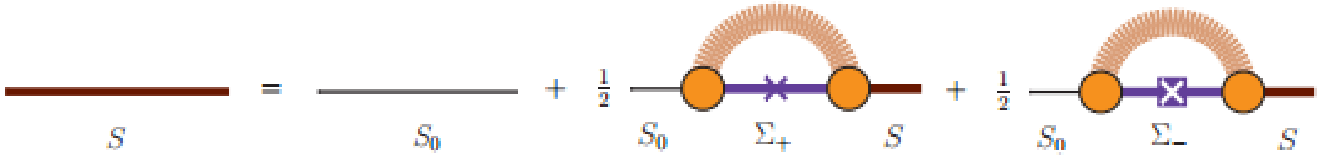
- light equal-mass quarks and deeply bound states ($\boldsymbol{\mu}$ small) like pion, require a charge-conjugation symmetric equation, the so-called four-channel (4CST-BS) equation:



q on its positive energy mass-shell

q on its negative energy mass-shell

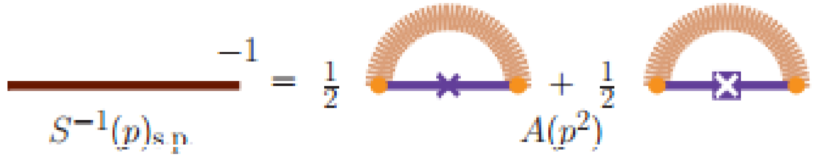
- calculate dynamical CST quark mass function $M(k^2)$ with one-body CST-Dyson (mass gap equation)

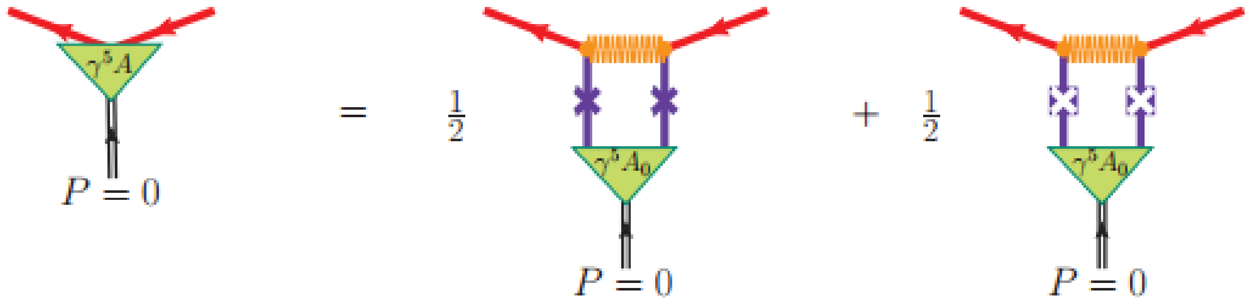


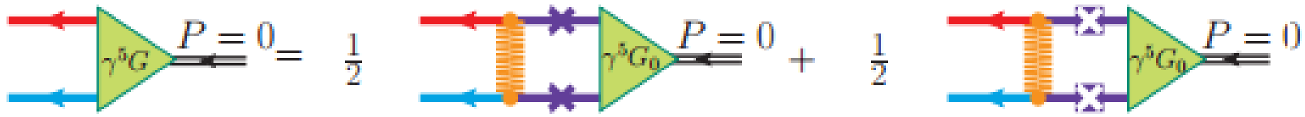
self-energy $\Sigma(p) = A(p^2) + \not{p}\, B(p^2)$

CST-BS Formalism overview

- chiral limit ($m_0 = 0$): scalar part (s. p.) of one-body equation for A and bound-state equation for a massless pion are identical







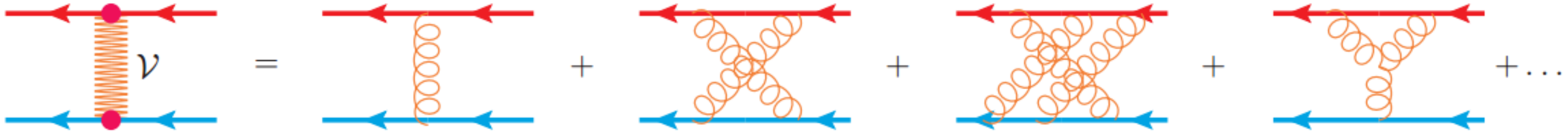
→ a massless pion state exists! Goldstone pion in chiral limit associated with spontaneous chiral symmetry breaking.

- CST-BS formalism has also been applied recently to compute π e.m. form factor and study of $\pi - \pi$ scattering

E. P. Biernat, M. T. Peña, A. Stadler, F. Gross: PRD **89**, 016005, 016006 (2014); PRD **92**, 076011 (2015);
& also with J. E. Ribeiro: PRD **90**, 096008 (2014).

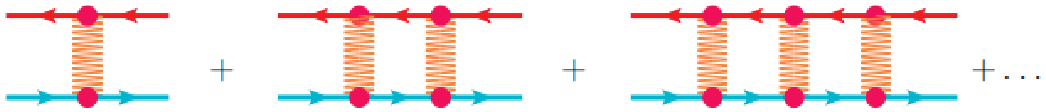
Interaction kernel truncation — *CST* key feature

- The kernel contains all two-body irreducible diagrams

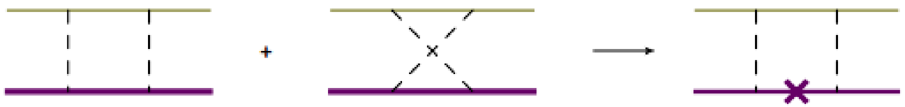


- In the BS equation the kernel is effectively iterated to all orders
  But the complete kernel is a sum of an infinite number of irreducible diagrams has to be truncated
  (most often: ladder approximation)

  However,
  - No one-body limit (missing crossed ladders)
  - Not best suited to describe bound states
    (crossed-ladder contributions are significant)
    see Nieuwenhuis and Tjon, PRL77, 814 (1996)



- In a ϕ^3-theory the sum of box and crossed box diagrams is approximated by heavy particle pole contribution of box diagram.



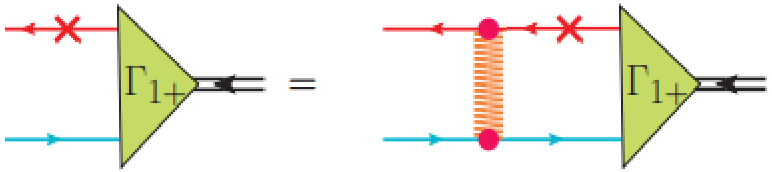
proof: Gross, Relativistic Quantum Mechanics and Field Theory, (2004)

cancellation in all orders and exact in heavy mass limit ⇒
one-boson-exchange kernel with heavy particle on-mass shell
produces exact sum of all ladder and crossed ladder diagrams!

- CST prescription of placing particles on their mass-shell, *effectively*, goes beyond rainbow-ladder approximation!

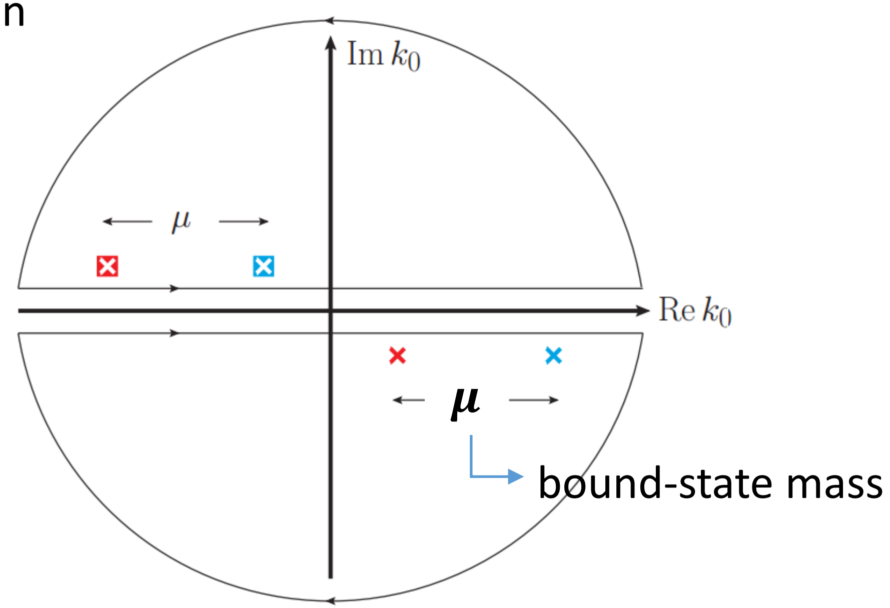
Heavy and heavy-light mesons with CST-BS

- If $\boldsymbol{\mu}$ is large, the one-channel (1CST-BS) equation is a good approximation



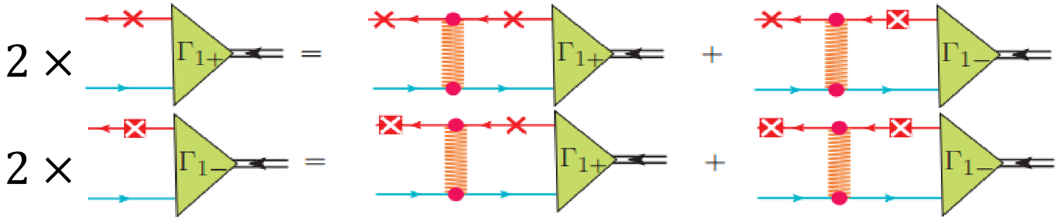
and possesses *important* features:

√ smooth nonrelativistic limit (to the Schrödinger equation),
√ correct one-body limit,
√ it is manifestly covariant (despite its loop integrations being 3-dimensional)!



- *However*, heavy quarkonium states calculated with the 1CST-BS equation have no definite C-parity.

This would be the correct system of equations:



- *Not a problem!* only the axial-vector mesons have both parities (separated *only* by $5-6\ MeV$ in $b\bar{b}$, $14\ MeV$ in $c\bar{c}$)

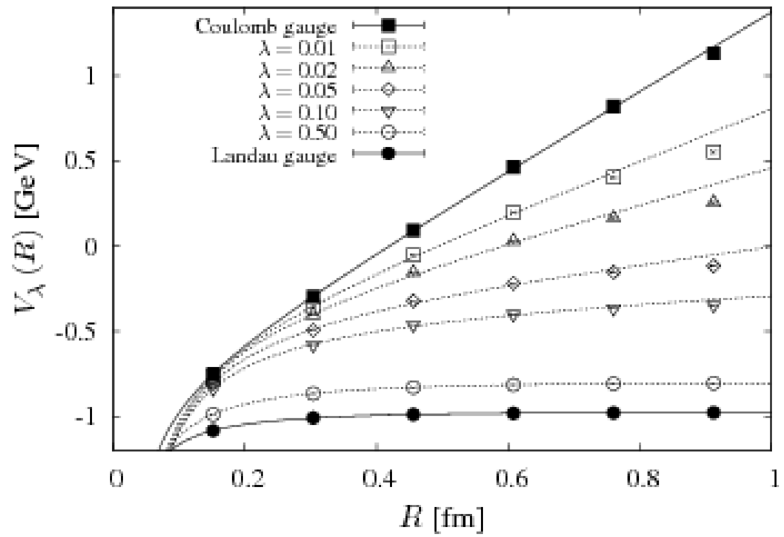
Confining potential in momentum space

- Phenomenological $q\bar{q}$ kernel is inspired by Cornell potential:

$$\boxed{V(r) = \sigma r - {}^{\alpha_s}/_{r} - C}$$

A. Laschka *et al.* PRD **83**, 094002 (2011)

Static QCD potential from lattice



- NR linear potential in momentum space: ?

Fourier transform of screened potential

Usually: $$\sigma r = \lim_{\epsilon\to 0} \sigma \frac{\partial^2}{\partial\epsilon^2}\frac{e^{-\epsilon r}}{r}$$

But simpler: $$\sigma r = \lim_{\epsilon\to 0} -\frac{\sigma}{\epsilon}(e^{-\epsilon r} - 1) \equiv \tilde{V}_A(r) - \tilde{V}_A(0)$$

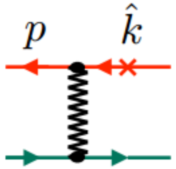
FT: $$V_L(\boldsymbol{q}) = V_A(\boldsymbol{q}) - (2\pi)^3\delta(\boldsymbol{q})\int \frac{d^3\boldsymbol{q}'}{(2\pi)^3} V_A(\boldsymbol{q}') \quad \text{with} \quad V_A(q) = -\frac{8\pi\sigma}{\boldsymbol{q}^4}$$

$$\langle V_L\phi\rangle(\boldsymbol{q}) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} V_L(\boldsymbol{p}-\boldsymbol{k})\phi(\boldsymbol{k}) = -8\pi\sigma \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{\phi(\boldsymbol{k})-\phi(\boldsymbol{p})}{(\boldsymbol{p}-\boldsymbol{k})^4}$$

only a Cauchy principal value singularity remains (and this one can also be explicitly removed)

- Covariant generalization: $\boldsymbol{q} \to -q^2$

S.L *et al.* PRD **90**, 096003 (2014)



Initial state: either quark or antiquark on-shell

$$\langle V_L\phi\rangle(p) = \int \frac{d^3k}{(2\pi)^3}\frac{m}{E_k} V_L(p,\hat{k})\phi(\hat{k}) = -8\pi\sigma \int \frac{d^3k}{(2\pi)^3}\frac{m}{E_k}\frac{\phi(\hat{k})-\phi(\hat{p}_R)}{(p-\hat{k})^4}$$

$$\hat{k} = (E_k, \boldsymbol{k}) \qquad \hat{p}_R = (E_{p_R}, \boldsymbol{p_R})$$

value $\boldsymbol{k}$ at which kernel becomes singular

Lorentz structure of the kernel

- We use a kernel of the general form

$$\mathcal{V} = \left[(1-y)\left(1_1 \otimes 1_2 + \gamma_1^5 \otimes \gamma_2^5\right) - y\gamma_1^\mu \otimes \gamma_{\mu 2}\right]V_L - \gamma_1^\mu \otimes \gamma_{\mu 2}\left[V_{Coul.} + V_C\right]$$

where $V_L, V_{Coul.}, V_C$ are relativistic generalizations of a linear conning potential, a short-range Coulomb term and a global constant potential.

- The parameter y dials continuously between the two extreme cases $y = 1$ being pure vector coupling, and $y = 0$ pure scalar+pseudoscalar coupling.

- The reason for the presence of a pseudoscalar component is chiral symmetry. Although in general scalar interactions break chiral symmetry, it was shown that the CS equation with our relativistic linear confining kernel satisfies the axial-vector Ward-Takahashi identity when it is accompanied by an equal-weight pseudoscalar interaction.

PRD **90**, 096008 (2014).

- Finally, for any interaction kernel K, the 1CST-BS equation for the vertex function Γ, reads

$$\Gamma(p) = -\sum_K \int \frac{d^3k}{(2\pi)^3} \frac{m_1}{E_{1k}} V_K(p,k) \Theta_1^{K(\mu)} \frac{m_1 + \hat{\not{k}}_1}{2m_1} \Gamma(k) \frac{m_2 + \not{k}_2}{m_2^2 - k_2^2 - i\epsilon} \Theta_{2(\mu)}^K$$

Numerical solution of CST-BS

- Instead of solving the 1CST-BS directly for these structure functions, we prefer to first introduce relativistic "wave functions", defined as Dirac spinor matrix elements of the vertex function multiplied by the off-shell quark propagator and with definite orbital angular momentum (*why*? important when comparing to experimentally determined states).

- The 1CST-BS for the relativistic wave functions can be written as a generalized linear eigenvalue problem for the total bound-state mass μ.

- We solve this system by expanding the wave functions in a basis of B-splines.

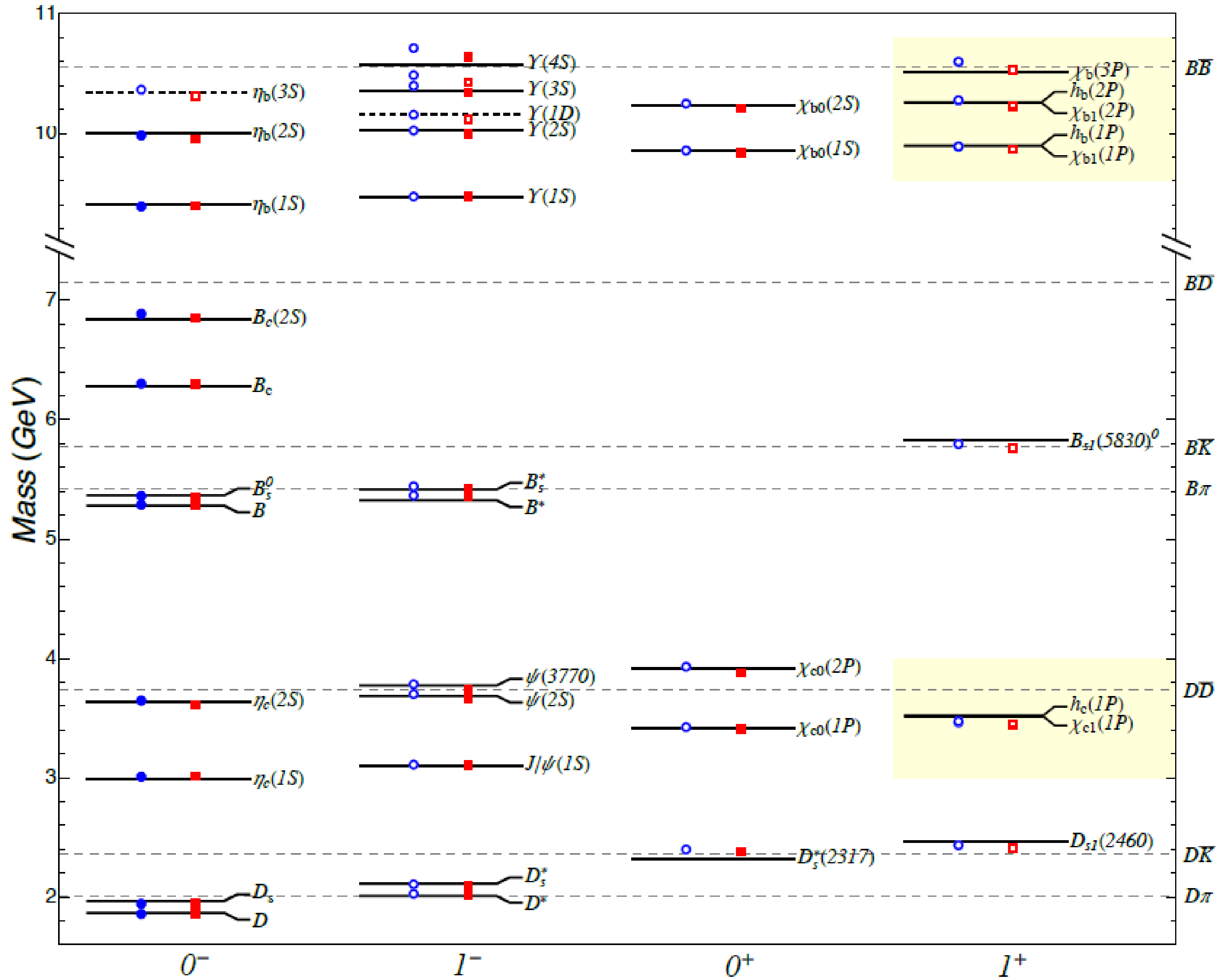
- Special attention is needed to treat the singularities in the kernel at $q^2 = \left(\hat{p}_1 - \hat{k}_1\right)^2 = 0$.

- Due to retardation effects, the loop integrals over the kernels do not converge. We use a standard Pauli-Villars regularization to cure this problem, at the expense of a momentum cut-off parameter Λ. It turns out that our results are very insensitive to this parameter ($\Lambda = 2\mathrm{m}_1$).

- We *set* the following masses

$$m_b = 4.892\, GeV, m_c = 1.600\, GeV, m_s = 0.448\, GeV \text{ and } m_u = m_d = 0.346\, GeV.$$

Results



- Early results clearly favored pure scalar+pseudoscalar confinement, so throughout this work we set $y = 0$.

Model P1 (fitted to pseudoscalar states *only*)

$$\sigma = 0.2493\ GeV^2, \alpha_s = 0.3643,$$
$$C = 0.3491\ GeV$$

- ● states fitted
- ○ predicted states

$$rms = 0.036\ GeV$$

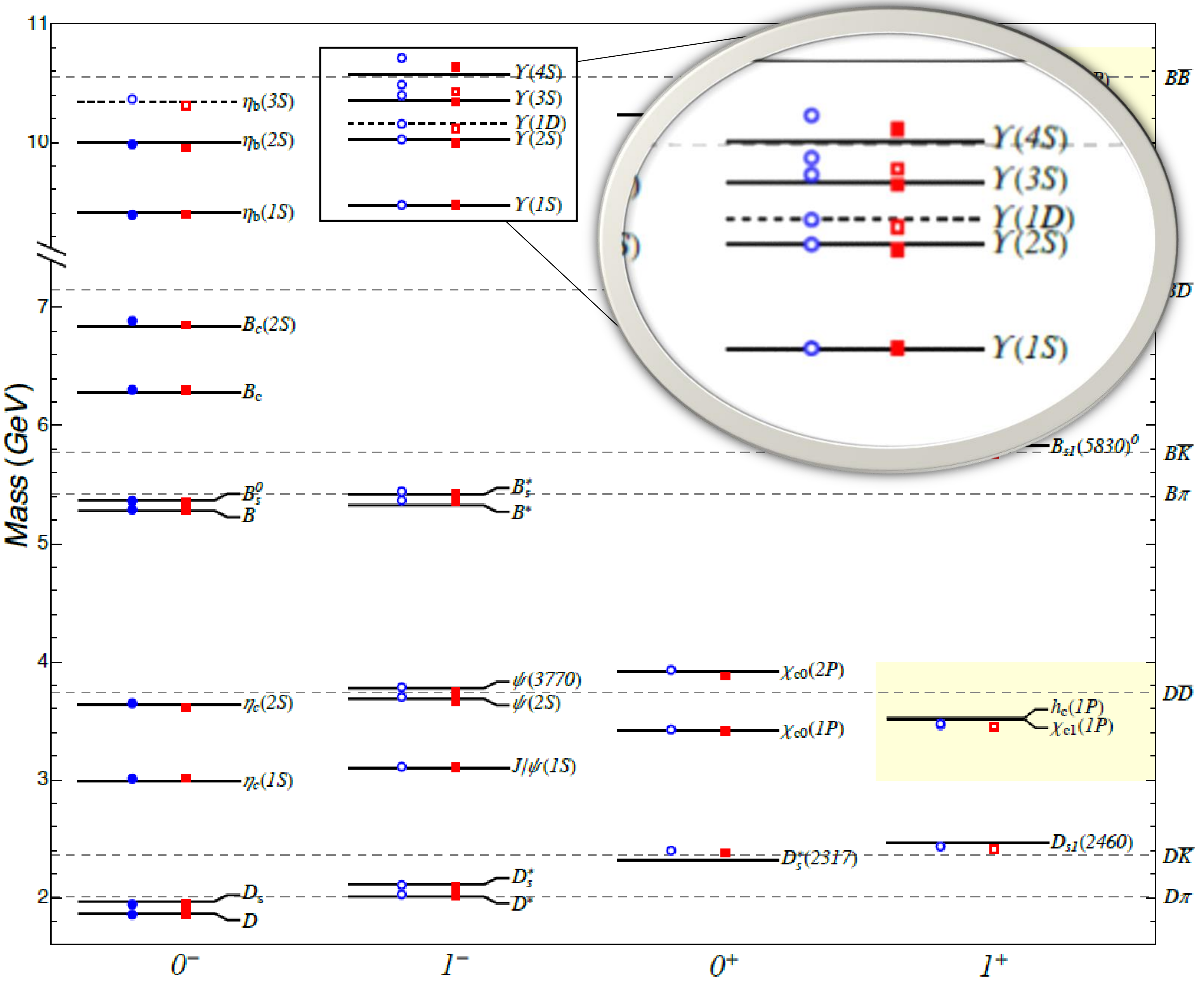
Model PSV1 (fitted to pseudoscalar, scalar and vector)

$$\sigma = 0.2247\ GeV^2, \alpha_s = 0.3614,$$
$$C = 0.3377\ GeV$$

- ■ states fitted
- □ predicted states

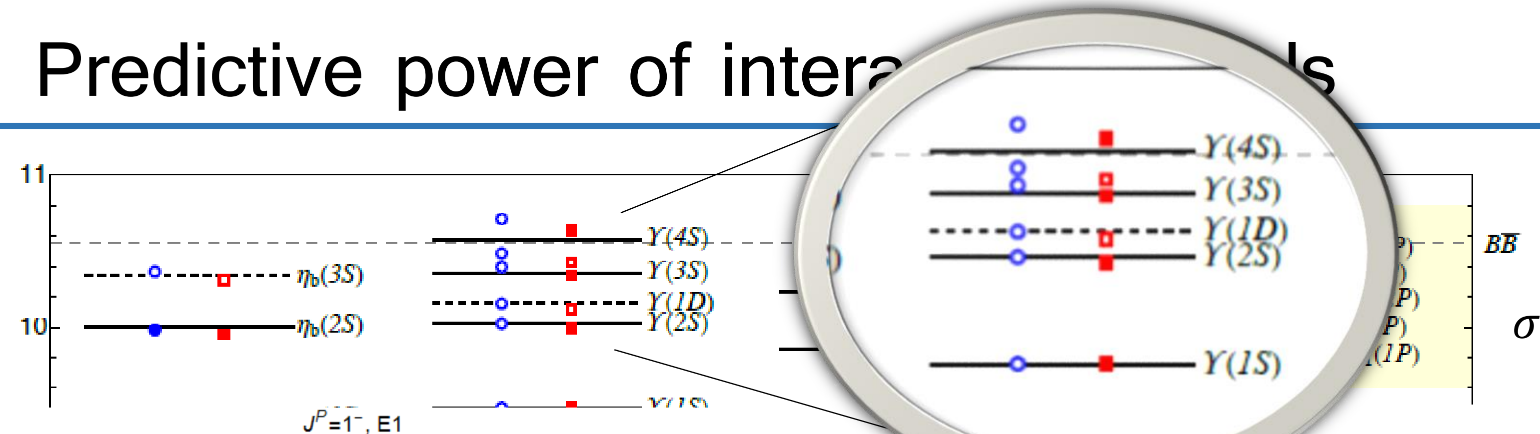
$$rms = 0.030\ GeV$$

Predictive power of interaction kernels



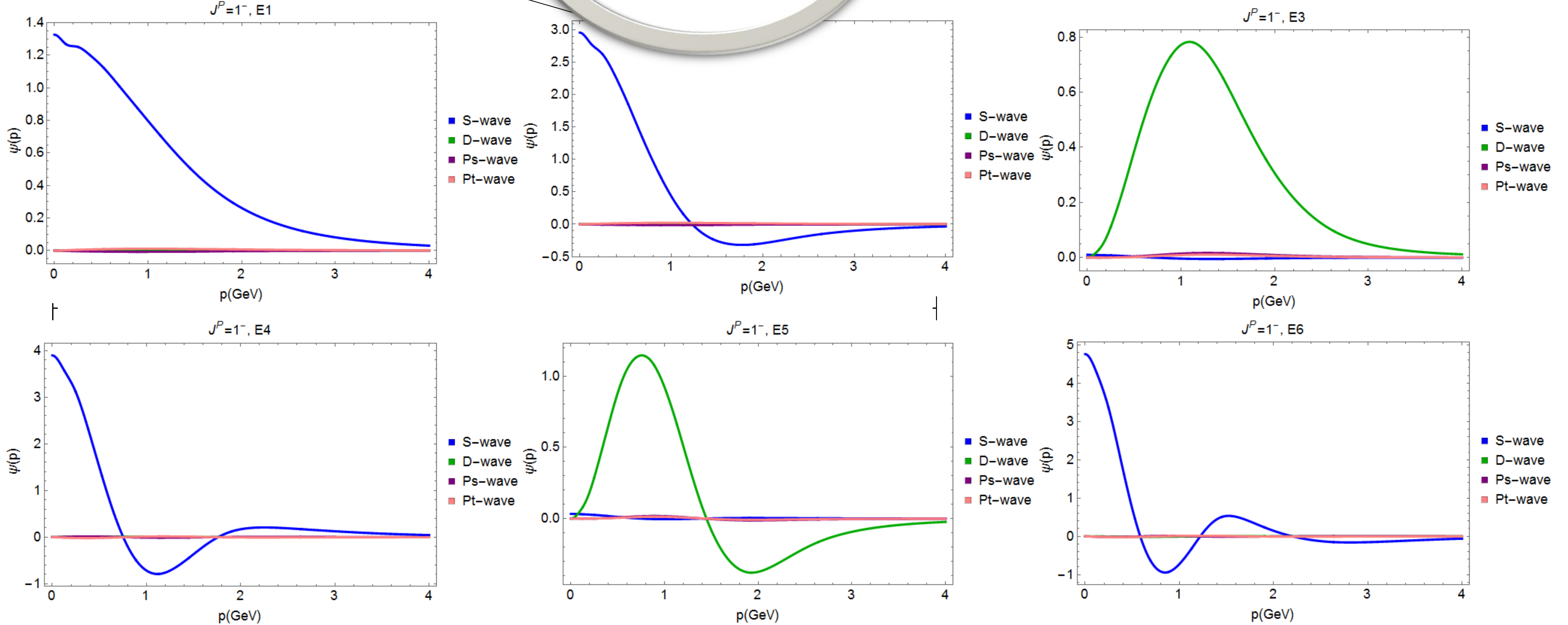
- Vector bottomonium: S or D states?

Predictive power of intera ls

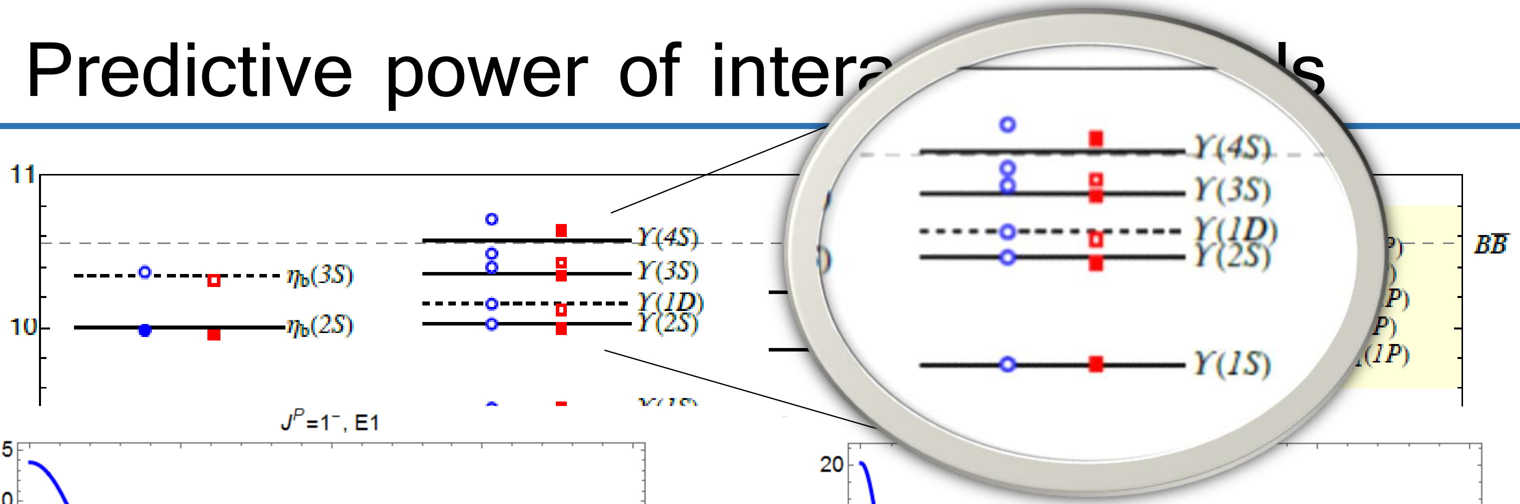


Model PSV1

$$\sigma = 0.2247\ GeV^2, \alpha_s = 0.3614, C = 0.3377\ GeV$$



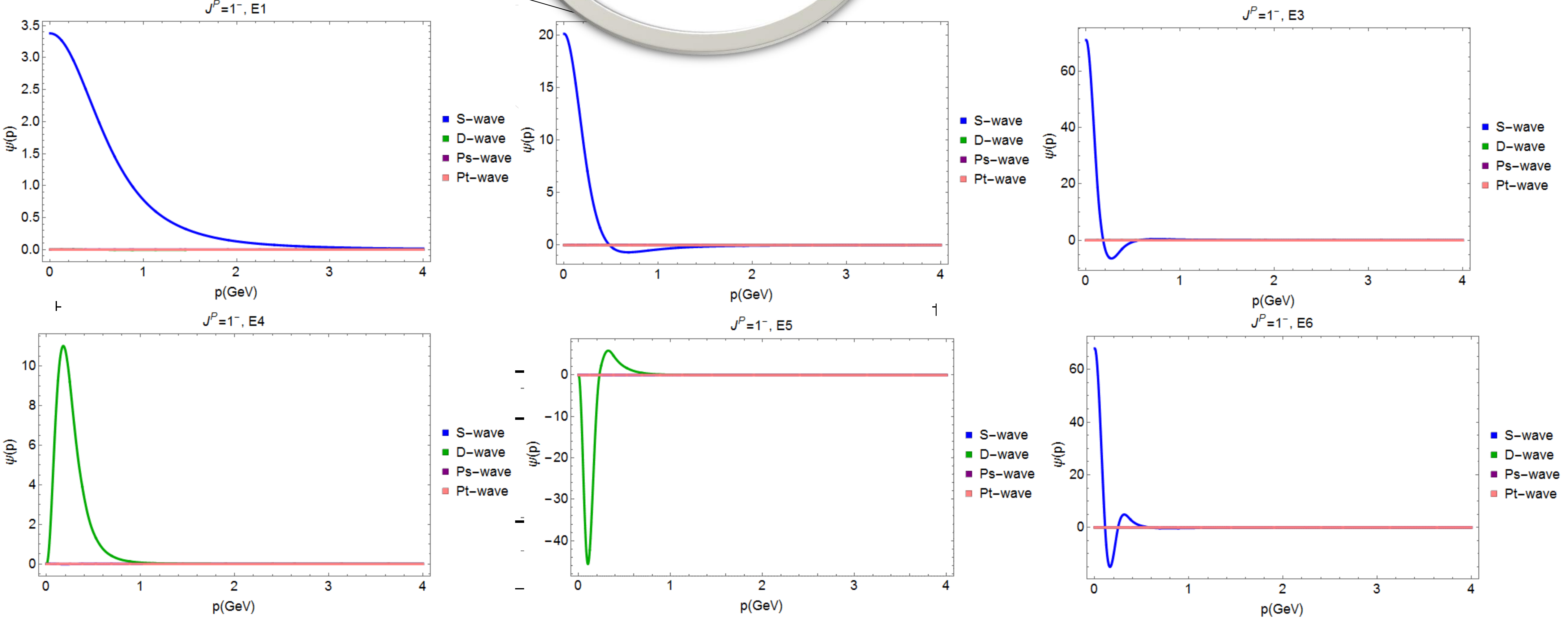
Predictive power of intera ls



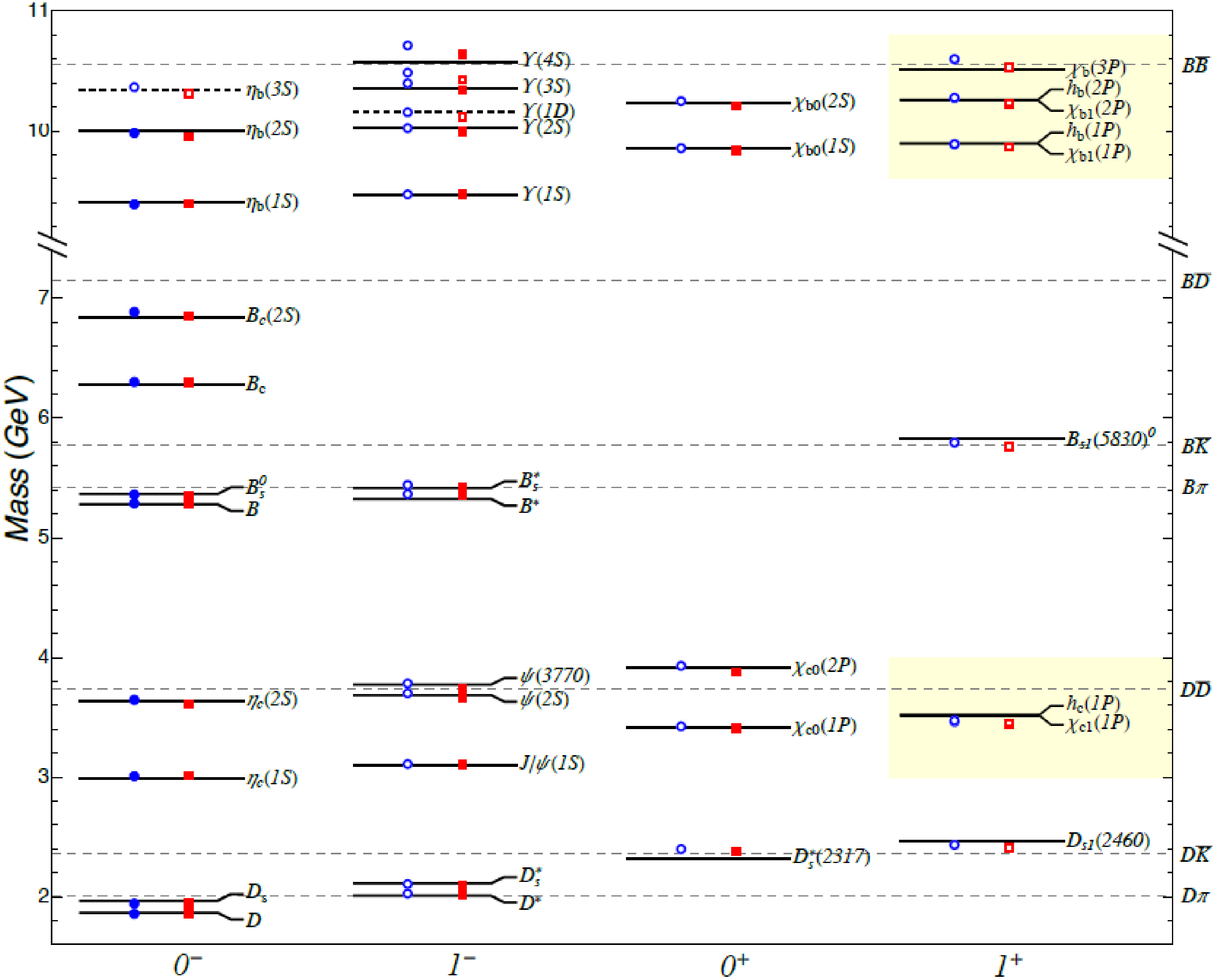
Just Coulomb

Switch off linear part

$\sigma = 0.0\ GeV^2, \alpha_s = 0.3614, C = 0.3377\ GeV$



Predictive power of *covariant* interaction kernels



- The observed meson spectrum is very well reproduced after setting a small number of model parameters (global fit).
- Remarkably, a fit to a few pseudoscalar meson states *only*, which are insensitive to spin-orbit and tensor forces and do not allow to separate the spin-spin from the central interaction, leads to essentially the same model parameters as a more general fit!
- Our covariant kernel correctly predicts the spin-dependent interactions solely based on their relation to the spin-independent interactions as dictated by covariance!

Summary and Outlook

- We reported on the recent developments of CST-BS formalism applied to heavy and heavy-light mesons.

- Very good mass-spectrum was obtained with just a few parameters—we are inclined to believe that a *global* description is possible.

- We have *tested* that *covariance indeed* leads to an accurate prediction of the spin-dependent quark-antiquark interactions.

In a near future we aim to

- include also tensor mesons (*in progress*),

- extend the formalism to the light sector consistently, i.e., by solving the CST-Dyson (mass gap) & CST-BS equations together,

- compute other observables besides mass spectra (decay rates, form factors, etc. …).

Thank you!

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