

# The non-perturbative unquenched quark model

*XIIth Quark Confinement and the Hadron Spectrum*



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# Introduction

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- The naive quark model has been very successful classifying the particle spectra
- However higher Fock components should be included (already considered in the Cornell model)
- Unquenching the quark model includes these effects in a similar way as lattice unquenched calculations using dynamical quarks
- The higher Fock components became more relevant in 2003 with the  $X(3872)$  and other XYZ states
- The  $X(3872) \rightarrow J/\Psi\pi\pi$  decay is a clear indication that it can not be a pure  $c\bar{c}$  state and can be naturally explained if a  $DD^*$  component is present.
- Excited states can be close to a hadron-hadron threshold and the multiquark component can be enhanced
- One expects ground states (usually far from thresholds) to have small multiquark components

# Introduction

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- Different approaches
  - Mass shift and mixing (as Phys. Rev. D17, 3090)
  - Systems of one or more  $q\bar{q}$  coupled with close two meson channels (as Phys. Rev. D21, 772)
  - Unitarized quark models including two meson loops (as Phys. Rev. D29, 110)
- Models usually study the influence of two-meson loops on the  $c\bar{c}$  states
  - E. Eichten et al., Phys. Rev. D21 203
  - E. van Beveren and G. Rupp, hep-ph/0304105
  - J. Ferretti et al., Phys. Rev. D90 054010
- A slightly different point of view P.G. Ortega et al., PRD 81, 054023
- The **influence on the dynamics of two meson channels of  $q\bar{q}$  states is studied**
- This allows to study the shifts in mass and width of naive quark model states but also generates new states.
- The  $q\bar{q}$  components are always expanded as an expansion on the Eigenstates of the  $q\bar{q}$  Hamiltonian.
- **Naive quark model states have to be chosen a priori**
- In this contribution we propose a new method to include **all states** leaving the  $q\bar{q}$  wave function as an unknown, **in order to check the convergence of the expansion on Eigenstates**.

# The Cornell Model

E. Eichten et al., Phys. Rev. D 17, 3090

The Hamiltonian is

$$H_I = \frac{1}{2} \int d^3x d^3y : \rho_a(\vec{x}) \frac{3}{4} V(\vec{x} - \vec{y}) \rho_a(\vec{y}) :$$

$$\rho_a(\vec{x}) = \sum_{\text{flavors}} \psi^\dagger(\vec{x}) \frac{1}{2} \lambda_a \psi(\vec{x})$$

$$\Omega_{nm}(z) = \sum_{\tau} \int d^3p_1 d^3p_2$$

$$\times \frac{\langle n | U | \tau \vec{p}_1 \vec{p}_2 \rangle \langle m | U | \tau \vec{p}_1 \vec{p}_2 \rangle^*}{z - E_1 - E_2}. \quad (3.23)$$

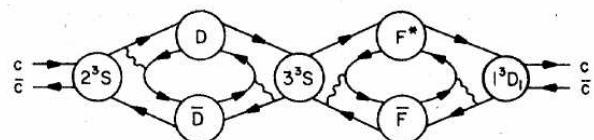
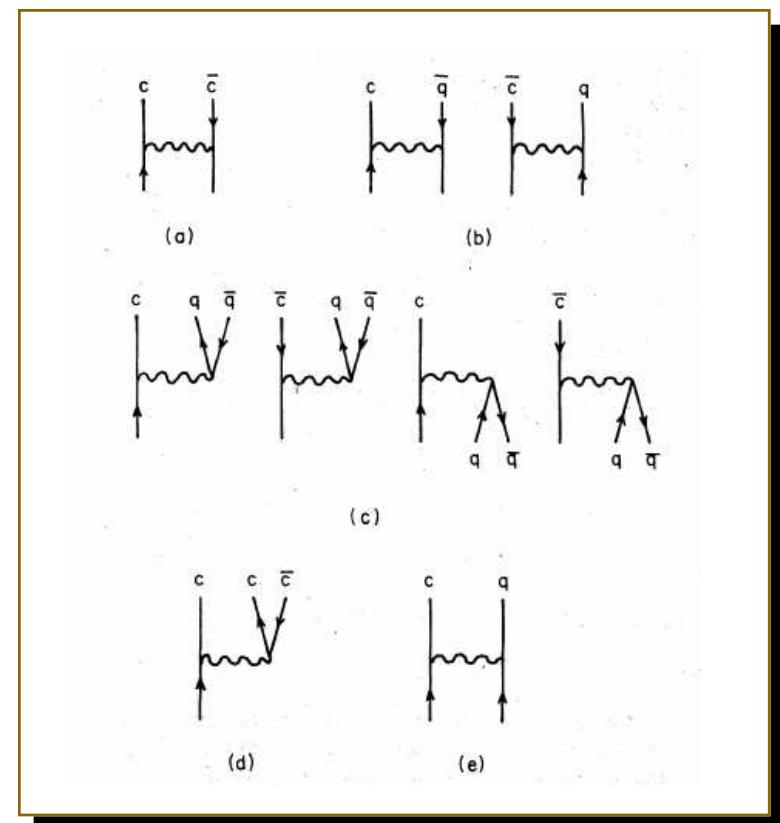


FIG. 8. The propagation of a  $c\bar{c}$  pair in the presence of open and closed decay channels as described in the Green's function  $\mathcal{G}$ .



$$\text{Det} | (z - \epsilon_n) \delta_{nm} - \Omega_{nm}(z) | = 0. \quad (3.28)$$

# Framework

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The needed pieces are

- The interaction between quarks: **the Chiral Quark Model**
- The interaction between mesons: **obtained from the quark interactions using the RGM**
- Coupling between one meson and two meson states: **the  $^3P_0$  model**
- **Solve the coupled system:** Different approaches
  - Perturbatively: Mass shifts, widths and mixing
  - Study the two meson dynamics including coupling coupling with the one meson sector
  - Solving the coupled equations allows to obtain states above threshold

# The Chiral Quark Model

J. Vijande *et al.*, J. Phys. G 31

- Spontaneous Chiral Symmetry Breaking →  
→ Goldstone bosons

$$\mathcal{M} = \bar{\Psi}(i\gamma^\mu \partial_\mu - MU^{\gamma_5})\Psi$$

$$U^{\gamma_5} = e^{i\pi^a \lambda^a \gamma_5 / f_\pi} \sim 1 + \frac{1}{f_\pi} \gamma_5 \lambda^a \pi^a - \frac{1}{2f_\pi^2} \pi^a \pi^a$$

- Goldstone bosons exchange  
→ Scalar boson exchanges

- Gluon coupling

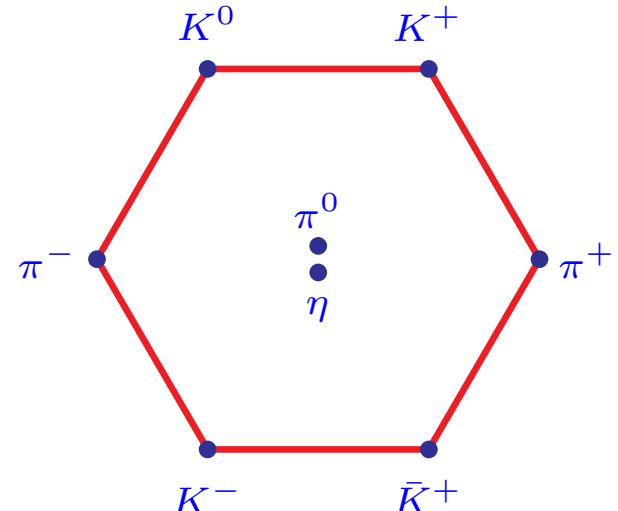
$$\mathcal{L}_{gqq} = i\sqrt{4\pi\alpha_s}\bar{\Psi}\gamma_\mu G_c^\mu \lambda^c \Psi$$

- One gluon exchange

- Confinement

- Interactions:

$$V_{q_i q_j} = \begin{cases} q_i q_j = nn \Rightarrow V_{CON} + V_{OGE} + V_{GBE} + V_{SBE} \\ q_i q_j = nQ \Rightarrow V_{CON} + V_{OGE} \\ q_i q_j = QQ \Rightarrow V_{CON} + V_{OGE} \end{cases}$$



# The RGM

The hadron wave function

$$\psi_H = \phi_H(\vec{p}_{\xi_H}) \chi_{SF} \xi_c$$

The two hadron wave function

$$\begin{aligned}\psi_{H_1 H_2} &= \mathcal{A} \left[ \chi(\vec{P}) \psi_{H_1 H_2}^{SF} \right] \\ &= \mathcal{A} \left[ \phi_{H_1}(\vec{p}_{\xi_{H_1}}) \phi_{H_2}(\vec{p}_{\xi_{H_2}}) \chi(\vec{P}) \chi_{H_1 H_2}^{SF} \xi_c \right]\end{aligned}$$

Rayleigh-Ritz variational principle (Resonating Group Method)

$$(\mathcal{H} - E_T) |\psi\rangle = 0 \Rightarrow \langle \delta\psi | (\mathcal{H} - E_T) |\psi\rangle = 0$$

$$\left( \frac{\vec{P}'^2}{2\mu} - E \right) \chi(\vec{P}') + \int \left( {}^{\text{RGM}} V_D(\vec{P}', \vec{P}_i) + {}^{\text{RGM}} K(\vec{P}', \vec{P}_i) \right) \chi(\vec{P}_i) d\vec{P}_i = 0$$

$$T_\alpha^{\alpha'}(z; p', p) = V_\alpha^{\alpha'}(p', p) + \sum_{\alpha''} \int dp'' p''^2 V_{\alpha''}^{\alpha'}(p', p'') \frac{1}{z - E_{\alpha''}(p'')} T_\alpha^{\alpha''}(z; p'', p)$$

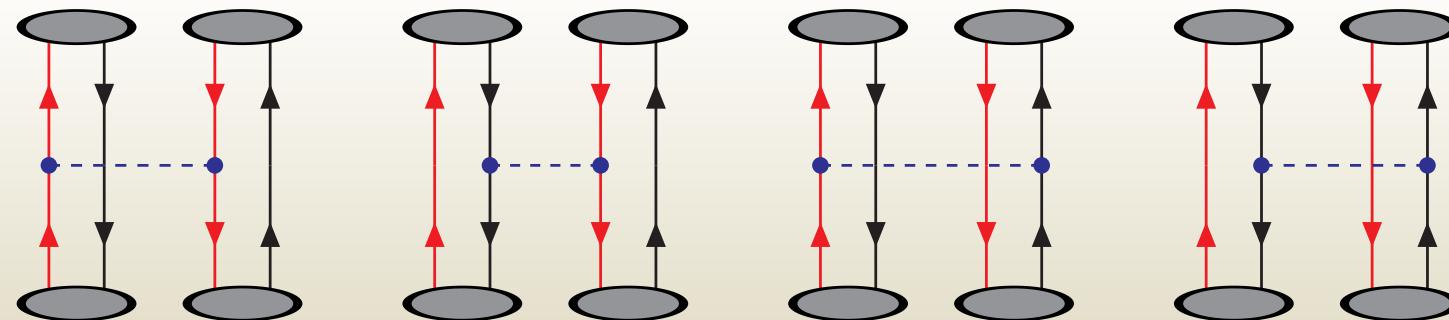
Lippmann-Schwinger Equation

# The $M_1 M_2$ system

- Quark interactions → Cluster interaction.
- For the  $DD^*$  system only direct RGM Potential:

$${}^{RGM}V_D(\vec{P}', \vec{P}_i) = \sum_{i \in A, j \in B} \int d\vec{p}_{\xi'_A} d\vec{p}_{\xi'_B} d\vec{p}_{\xi_A} d\vec{p}_{\xi_B} \phi_A^*(\vec{p}_{\xi'_A}) \phi_B^*(\vec{p}_{\xi'_B}) V_{ij}(\vec{P}', \vec{P}_i) \phi_A(\vec{p}_{\xi_A}) \phi_B(\vec{p}_{\xi_B})$$

- $\phi_C(\vec{p}_C)$  is the wave function for cluster  $C$  solution of Schrödinger's equation using Gaussian Expansion Method.



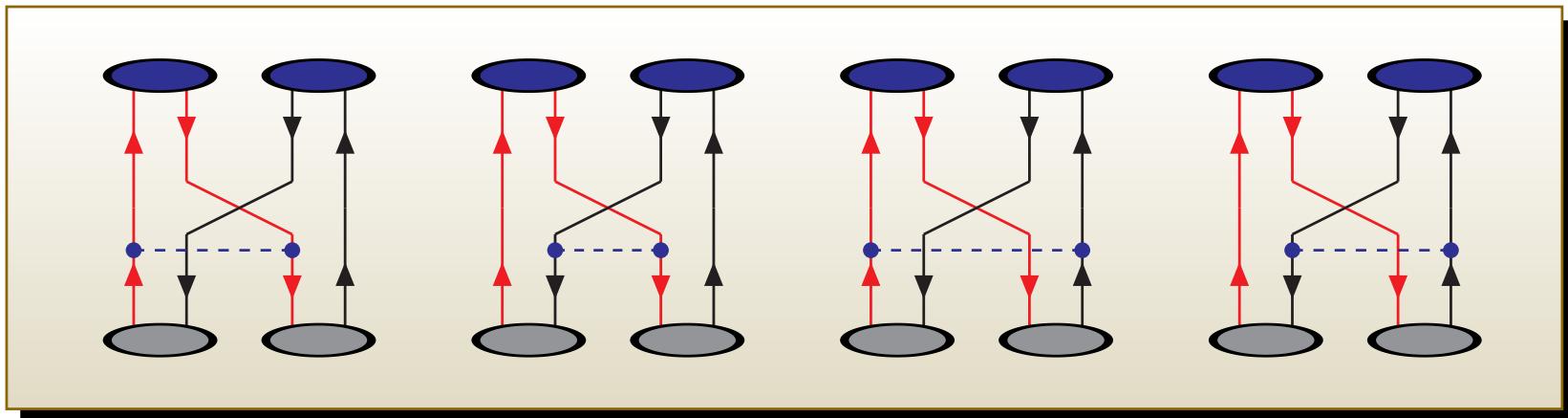
# The $M_1 M_2$ system

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- $\phi_C(\vec{p}_C)$  is the wave function for cluster  $C$  solution of Schrödinger's equation using Gaussian Expansion Method.

Rearrangement processes (like  $DD^* \rightarrow J/\psi\omega$ )



# $^3P_0$ model

■ Pair creation Hamiltonian:

$$\mathcal{H} = g \int d^3x \bar{\psi}(x) \psi(x)$$

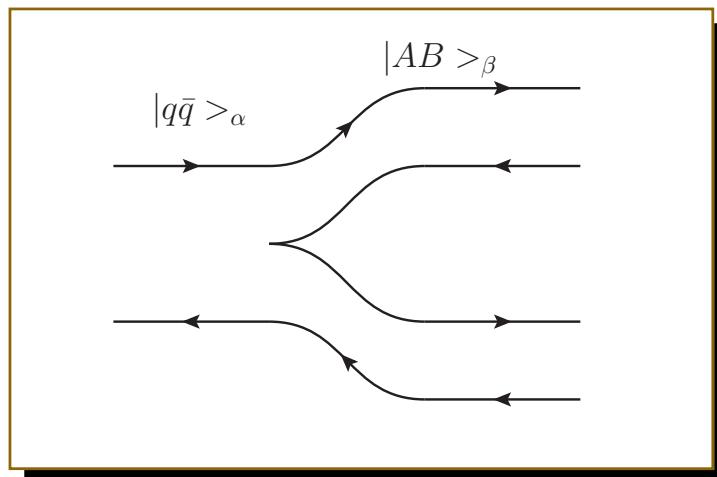
■ Non relativistic reduction:

$$T = -3\sqrt{2}\gamma' \sum_{\mu} \int d^3p d^3p' \delta^{(3)}(p + p') \left[ \mathcal{Y}_1 \left( \frac{p - p'}{2} \right) b_{\mu}^{\dagger}(p) d_{\nu}^{\dagger}(p') \right]^{C=1, I=0, S=1, J=0}$$

with  $\gamma' = 2^{5/2}\pi^{1/2}\gamma$ ,  $\gamma = \frac{g}{2m}$  (in the light quark sector)

■ Transition potential:

$$\langle \phi_{M_1} \phi_{M_2} \beta | T | \psi_{\alpha} \rangle = P h_{\beta\alpha}(P) \delta^{(3)}(\vec{P}_{cm})$$



# $^3P_0$ model

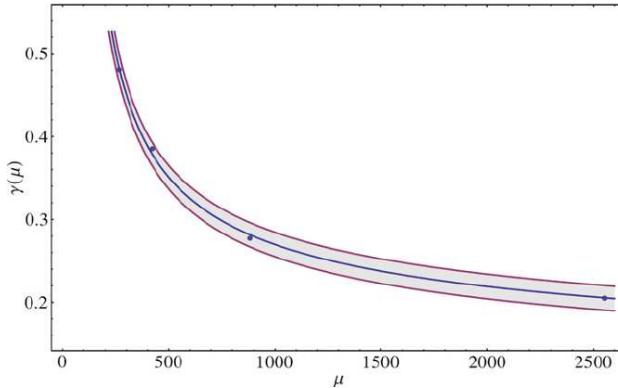
J. Segovia, DRE, F. Fernández, Phys. Lett. B 715, 322 (2012)

## Running coupling

$$\gamma(\mu) = \frac{\gamma_0}{\log(\frac{\mu}{\mu_0})}$$

$$\gamma_0 = 0.81 \pm 0.02$$

$$\mu_0 = (49.84 \pm 2.58) \text{ MeV}$$



**Table 1**

Meson decay widths which have been taken into account in the fit of the scale-dependent strength,  $\gamma$ . Some properties of these mesons are also shown.

Meson	$I$	$J$	$P$	$C$	Mass (MeV)	$\Gamma_{\text{Exp.}}$ (MeV)	
$D_1(2420)^{\pm}$	1/2	1	+1	-	$2423.4 \pm 3.1$	$25 \pm 6$	[21]
$D_2^*(2460)^{\pm}$	1/2	2	+1	-	$2464.4 \pm 1.9$	$37 \pm 6$	[21]
$D_{s1}(2536)^{\pm}$	0	1	+1	-	$2535.12 \pm 0.25$	$1.03 \pm 0.13$	[22]
$D_{s2}^*(2575)^{\pm}$	0	2	+1	-	$2571.9 \pm 0.8$	$17 \pm 4$	[21]
$\psi(3770)$	0	1	-1	-1	$3778.1 \pm 1.2$	$27.5 \pm 0.9$	[21]
$\Upsilon(4S)$	0	1	-1	-1	$10579.4 \pm 1.2$	$20.5 \pm 2.5$	[21]

# $^3P_0$ model

**Table 3**

Strong total decay widths calculated through the  $^3P_0$  model of the mesons which belong to charmed, charmed-strange, hidden charm and hidden bottom sectors. The value of the parameter  $\gamma$  in every sector is given by Eq. (10).

Meson	<i>I</i>	<i>J</i>	<i>P</i>	<i>C</i>	<i>n</i>	Mass (MeV)	$\Gamma_{\text{Exp.}}$ (MeV)	[21]	$\Gamma_{\text{The.}}$ (MeV)
$D^+(2010)^{\pm}$	0.5	1	-1	-	1	$2010.28 \pm 0.13$	$0.096 \pm 0.004 \pm 0.022$		0.036
$D_0^+(2400)^{\pm}$	0.5	0	+1	-	1	$2403 \pm 14 \pm 35$	$283 \pm 24 \pm 34$		212.01
$D_1(2420)^{\pm}$	0.5	1	+1	-	1	$2423.4 \pm 3.1$	$25 \pm 6$		25.27
$D_1(2430)^0$	0.5	1	+1	-	2	$2427 \pm 26 \pm 25$	$384^{+107}_{-75} \pm 74$		229.12
$D_2^+(2460)^{\pm}$	0.5	2	+1	-	1	$2464.4 \pm 1.9$	$37 \pm 6$		64.07
$D(2550)^0$	0.5	0	-1	-	2	$2539.4 \pm 4.5 \pm 6.8$	$130 \pm 12 \pm 13$	[29]	132.07
$D^+(2600)^0$	0.5	1	-1	-	2	$2608.7 \pm 2.4 \pm 2.5$	$93 \pm 6 \pm 13$	[29]	96.91
$D_J(2750)^0$	0.5	$\begin{bmatrix} 2 \\ 3 \end{bmatrix}$	-1	-	1	$2752.4 \pm 1.7 \pm 2.7$	$71 \pm 6 \pm 11$	[29]	$\begin{bmatrix} 229.86 \\ 107.64 \end{bmatrix}$
$D_J^+(2760)^0$	0.5	1	-1	-	3	$2763.3 \pm 2.3 \pm 2.3$	$60.9 \pm 5.1 \pm 3.6$	[29]	338.63
$D_{s1}(2536)^{\pm}$	0	1	+1	-	1	$2535.12 \pm 0.25$	$1.03 \pm 0.13$	[22]	0.99
$D_{s2}^+(2575)^{\pm}$	0	2	+1	-	1	$2571.9 \pm 0.8$	$17 \pm 4$		18.67
$D_{s1}^+(2710)^{\pm}$	0	1	-1	-	2	$2710 \pm 2^{+12}_{-7}$	$149 \pm 7^{+39}_{-52}$	[30]	170.76
$D_{sJ}^+(2860)^{\pm}$	0	$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$	-1	-	$\begin{bmatrix} 3 \\ 1 \end{bmatrix}$	$2862 \pm 2^{+5}_{-2}$	$48 \pm 3 \pm 6$	[30]	$\begin{bmatrix} 153.19 \\ 85.12 \\ 301.52 \end{bmatrix}$
$D_{sJ}(3040)^{\pm}$	0	1	+1	-	$\begin{bmatrix} 3 \\ 4 \end{bmatrix}$	$3044 \pm 8^{+30}_{-5}$	$239 \pm 35^{+46}_{-42}$	[30]	432.54
$\psi(3770)$	0	1	-1	-1	3	$3778.1 \pm 1.2$	$27.5 \pm 0.9$		26.47
$\psi(4040)$	0	1	-1	-1	4	$4039 \pm 1$	$80 \pm 10$		111.27
$\psi(4160)$	0	1	-1	-1	5	$4153 \pm 3$	$103 \pm 8$		115.95
$X(4360)$	0	1	-1	-1	6	$4361 \pm 9 \pm 9$	$74 \pm 15 \pm 10$	[31]	113.92
$\psi(4415)$	0	1	-1	-1	7	$4421 \pm 4$	$62 \pm 20$		159.02
$X(4640)$	0	1	-1	-1	8	$4634^{+8+5}_{-7-8}$	$92^{+40+10}_{-24-21}$	[32]	206.37
$X(4660)$	0	1	-1	-1	9	$4664 \pm 11 \pm 5$	$48 \pm 15 \pm 3$	[31]	135.06
$\Upsilon(4S)$	0	1	-1	-1	6	$10579.4 \pm 1.2$	$20.5 \pm 2.5$		20.59
$\Upsilon(10860)$	0	1	-1	-1	8	$10876 \pm 11$	$55 \pm 28$		27.89
$\Upsilon(11020)$	0	1	-1	-1	10	$11019 \pm 8$	$79 \pm 16$		79.16

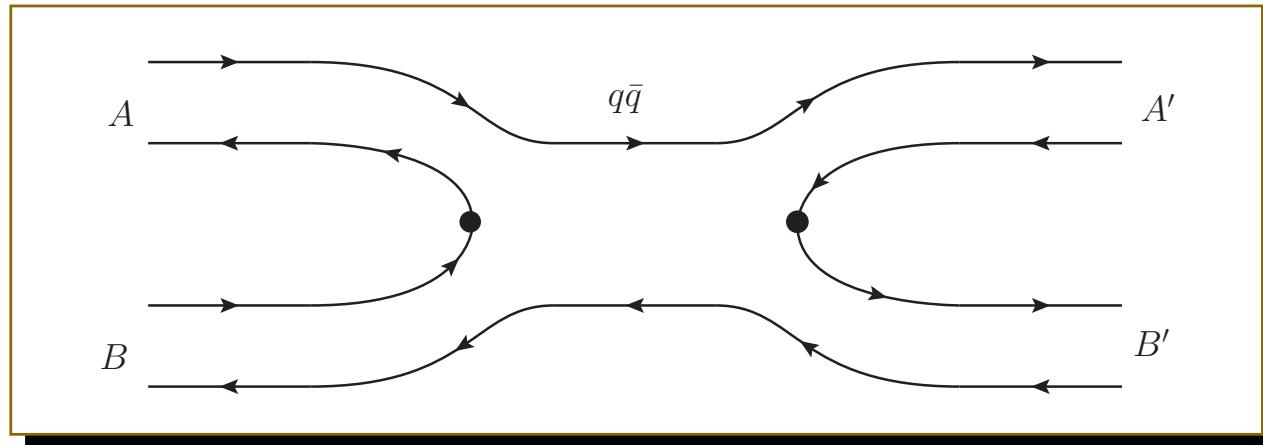
# Coupling $q\bar{q}$ and $q\bar{q}\bar{q}q$ sectors

- Hadronic state:  $|\Psi\rangle = \sum_{\alpha} c_{\alpha} |\psi\rangle + \sum_{\beta} \chi_{\beta}(P) |\phi_{M1}\phi_{M2}\beta\rangle$
- Solving the coupling with  $c\bar{c}$  states → Schrödinger type equation:

$$\sum_{\beta} \int \left( H_{\beta'\beta}^{M_1 M_2}(P', P) + V_{\beta'\beta}^{eff}(P', P) \right) \chi_{\beta}(P) P^2 dP = E \chi_{\beta'}(P')$$

with

$$V_{\beta'\beta}^{eff}(P', P) = \sum_{\alpha} \frac{h_{\beta'\alpha}(P') h_{\alpha\beta}(P)}{E - M_{\alpha}}$$



- The  $c\bar{c}$  amplitudes are given by,

$$c_{\alpha} = \frac{1}{E - M_{\alpha}} \sum_{\beta} \int h_{\alpha\beta}(P) \chi_{\beta}(P) P^2 dP$$

# Resonance states

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Lippmann-Schwinger equation

$$T^{\beta' \beta}(E; P', P) = V_T^{\beta' \beta}(P', P) + \sum_{\beta''} \int dP'' P''^2 V_T^{\beta' \beta''}(P', P'') \frac{1}{E - E_{\beta''}(P'')} T^{\beta'' \beta}(E; P'', P)$$

with  $V_T^{\beta' \beta}(P', P) = V^{\beta' \beta}(P', P) + V_{eff}^{\beta' \beta}(P', P)$ ,  $V_{\beta' \beta}^{eff}(P', P) = \sum_{\alpha} \frac{h_{\beta' \alpha}(P') h_{\alpha \beta}(P)}{E - M_{\alpha}}$

# Resonance states

## Lippmann-Schwinger equation

$$T^{\beta'\beta}(E; P', P) = V_T^{\beta'\beta}(P', P) + \sum_{\beta''} \int dP'' P''^2 V_T^{\beta'\beta''}(P', P'') \frac{1}{E - E_{\beta''}(P'')} T^{\beta''\beta}(E; P'', P)$$

with  $V_T^{\beta'\beta}(P', P) = V^{\beta'\beta}(P', P) + V_{eff}^{\beta'\beta}(P', P)$ ,  $V_{\beta'\beta}^{eff}(P', P) = \sum_{\alpha} \frac{h_{\beta'\alpha}(P') h_{\alpha\beta}(P)}{E - M_{\alpha}}$

Solution (Baru et al. Eur. Phys. Jour. A 44, 93 (2010))

$$T^{\beta'\beta}(E; P', P) = T_V^{\beta'\beta}(E; P', P) + \sum_{\alpha, \alpha'} \phi^{\beta'\alpha'}(E; P') \Delta_{\alpha'\alpha}^{-1}(E) \bar{\phi}^{\alpha\beta}(E; P)$$

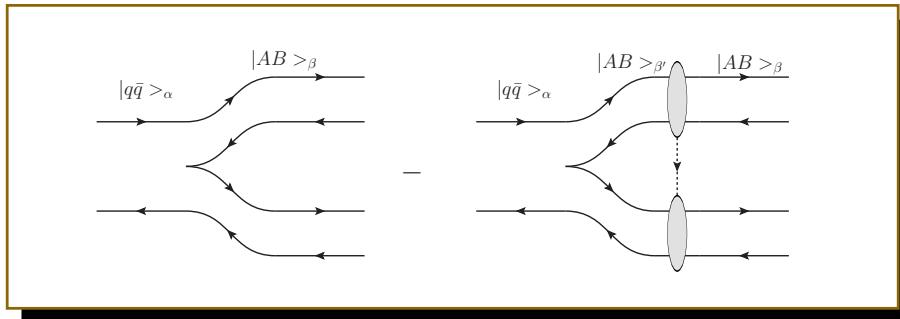
■ Non resonant contribution

■ Resonant contribution

with

$$T_V^{\beta'\beta}(E; P', P) = V^{\beta'\beta}(P', P) + \sum_{\beta''} \int dP'' P''^2 V^{\beta'\beta''}(P', P'') \frac{1}{z - E_{\beta''}(P'')} T_V^{\beta''\beta}(E; P'', P)$$

# Resonance states



**Solution (Baru et al. Eur. Phys. Jour. A 44, 93 (2010))**

$$T^{\beta'\beta}(E; P', P) = T_V^{\beta'\beta}(E; P', P) + \sum_{\alpha, \alpha'} \phi^{\beta'\alpha'}(E; P') \Delta_{\alpha'\alpha}^{-1}(E) \bar{\phi}^{\alpha\beta}(E; P)$$

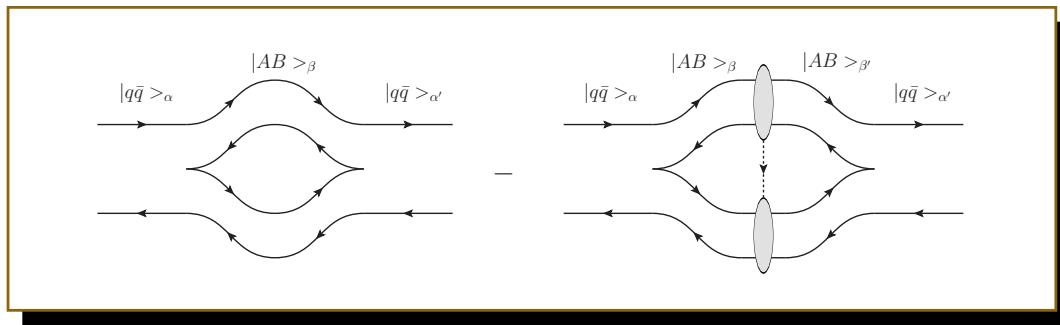
■ Non resonant contribution

■ Resonant contribution

with

$$\begin{aligned} \phi^{\alpha\beta'}(E; P) &= h_{\alpha\beta'}(P) - \sum_{\beta} \int \frac{T_V^{\beta'\beta}(E; P, q) h_{\alpha\beta}(q)}{q^2/2\mu - E} q^2 dq, \\ \bar{\phi}^{\alpha\beta}(E; P) &= h_{\alpha\beta}(P) - \sum_{\beta'} \int \frac{h_{\alpha\beta'}(q) T_V^{\beta'\beta}(E; q, P)}{q^2/2\mu - E} q^2 dq \end{aligned}$$

# Resonance states



**Solution (Baru et al. Eur. Phys. Jour. A 44, 93 (2010))**

$$T^{\beta' \beta}(E; P', P) = T_V^{\beta' \beta}(E; P', P) + \sum_{\alpha, \alpha'} \phi^{\beta' \alpha'}(E; P') \Delta_{\alpha' \alpha}^{-1}(E) \bar{\phi}^{\alpha \beta}(E; P)$$

■ Non resonant contribution

■ Resonant contribution

with

$$\Delta^{\alpha' \alpha}(E) = \left\{ (E - M_\alpha) \delta^{\alpha' \alpha} + \mathcal{G}^{\alpha' \alpha}(E) \right\}$$

$$\mathcal{G}^{\alpha' \alpha}(E) = \sum_{\beta} \int dq q^2 \frac{\phi^{\alpha \beta}(q, E) h_{\beta \alpha'}(q)}{q^2/2\mu - E}$$

# Resonance states

## ■ Resonance mass (pole position)

$$\left| \Delta^{\alpha' \alpha}(\bar{E}) \right| = \left| (\bar{E} - M_\alpha) \delta^{\alpha' \alpha} + \mathcal{G}^{\alpha' \alpha}(\bar{E}) \right| = 0$$

## ■ Bare $c\bar{c}$ probabilities

$$\left\{ M_\alpha \delta^{\alpha \alpha'} - \mathcal{G}^{\alpha' \alpha}(\bar{E}) \right\} c_{\alpha'}(\bar{E}) = \bar{E} c_\alpha(\bar{E})$$

## ■ Molecular wave function

$$\chi_{\beta'}(P') = -2\mu_{\beta'} \sum_\alpha \frac{\phi_{\beta' \alpha}(E; P') c_\alpha}{P'^2 - k_{\beta'}^2}$$

## ■ Normalization

$$\sum_\alpha |c_\alpha|^2 + \sum_\beta \langle \chi_\beta | \chi_\beta \rangle = 1$$

# Non-perturbative version

- Hadronic state:  $|\Psi\rangle = \sum_{\alpha} \mathcal{N}_{\alpha} |\psi_{\alpha}\rangle + \sum_{\beta} \chi_{\beta}(P) |\phi_{M1}\phi_{M2}\beta\rangle$
- Here  $\alpha$  doesn't mean a bare  $c\bar{c}$  state but a  $c\bar{c}$  partial wave (for example  ${}^3P_1$  or  ${}^3P_2$  and  ${}^3F_2$  for a coupled case)
- We don't make an expansion of the  $c\bar{c}$  component in Eigenstates of the  $c\bar{c}$  Hamiltonian but in a basis (the GEM basis):

$$|\psi_{\alpha}\rangle = \sum_{n=1}^{n_{\max}} c_n^{\alpha} \phi_{nl}^G(r) |ljm\rangle |\xi_c\rangle$$

- We follow the same procedure but now the vertex function is

$$h_{\beta\alpha}(P) = \sum_{n=1}^{n_{\max}} c_n^{\alpha} h_{\beta\alpha}^n(P)$$

- And the Equations become

$$\begin{aligned} \sum_{\alpha, n} \left[ \mathcal{H}_{n'n}^{\alpha'\alpha} - \mathcal{G}_{n'n}^{0\alpha'\alpha}(E) \right] c_n^{\alpha} &= EN_{n'n}^{\alpha'} c_n^{\alpha'} \\ \sum_{\beta} \int H_{\beta'\beta}(P', P) \chi_{\beta}(P) P^2 dP + \sum_{\alpha} h_{\beta'\alpha}(P') &= E \chi_{\beta'}(P') \end{aligned}$$

with

$$\mathcal{G}_{n'n}^{0\alpha'\alpha}(E) = -\delta^{\alpha'\alpha} \delta_{n'n} \sum_{\beta} \int h_{\alpha'\beta}^{n'}(P) \chi_{\beta}(P) P^2 dP$$

# Non-perturbative version

■ Again we use the solution  $T_V$

$$T_V^{\beta'\beta}(E; P', P) = V^{\beta'\beta}(P', P) + \sum_{\beta''} \int dP'' P''^2 V^{\beta'\beta''}(P', P'') \frac{1}{z - E_{\beta''}(P'')} T_V^{\beta''\beta}(E; P'', P)$$

■ Now

$$\phi_{\alpha\beta'}(E; P) = h_{\alpha\beta'}(P) - \sum_{\beta} \int \frac{T_V^{\beta'\beta}(E; P, q) h_{\alpha\beta}(q)}{q^2/2\mu - E} q^2 dq$$

and so

$$\phi_{\alpha\beta'}^n(E; P) = h_{\alpha\beta'}^n(P) - \sum_{\beta} \int \frac{T_V^{\beta'\beta}(E; P, q) h_{\alpha\beta}^n(q)}{q^2/2\mu - E} q^2 dq$$

■ We end up with an Schrödinger like equation

$$\sum_{\alpha, n} \left[ \mathcal{H}_{n'n}^{\alpha'\alpha} - \mathcal{G}_{n'n}^{\alpha'\alpha}(E) \right] c_n^\alpha = EN_{n'n}^{\alpha'\alpha} c_n^{\alpha'}$$

with the energy-dependent complete mass-shift matrix

$$\mathcal{G}_{n'n}^{\alpha'\alpha}(E) = \sum_{\beta} \int dq q^2 \frac{\phi_{\alpha'\beta}^{n'}(q, E) h_{\beta\alpha}^n(q)}{q^2/2\mu - E - i0^+}$$

# Non-perturbative version

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## ■ Molecular wave function

$$\chi_{\beta'}(P') = -2\mu_{\beta'} \sum_{\alpha,n=1}^{n_{\max}} \frac{\phi_{\beta'\alpha}(E; P') c_n^\alpha}{P'^2 - k_{\beta'}^2 - i0^+}$$

## ■ Normalization

$$\sum_{\alpha',\alpha} \sum_{n',n=1}^{n_{\max}} c_{n'}^{\alpha'*} N_{n'n}^{\alpha'\alpha} c_n^\alpha + \sum_{\beta} \langle \chi_\beta | \chi_\beta \rangle = 1$$

# The $1^{++}$ sector

Charge symmetry breaking is included with the right threshold positions of charged states

$$|D^\pm D^* \mp\rangle = \frac{1}{\sqrt{2}} (|DD^* I=0\rangle - |DD^* I=1\rangle)$$

$$|D^0 D^{*0}\rangle = \frac{1}{\sqrt{2}} (|DD^* I=0\rangle + |DD^* I=1\rangle)$$

Lower states in the  $1^{++}$  channel

$\gamma^3 P_0$	$M(MeV)$	$c\bar{c}$	$D^0 D^{*0}$	$D^\pm D^{*\pm}$	I=0	I=1
0.260	3949	56,71 %	22,47 %	20,82 %	43,10 %	0,25 %
	3867	30,22 %	51,37 %	18,40 %	64,72 %	5,06 %
	3468	95,70 %	2,18 %	2,12 %	4,30 %	0,0 %
0.218	3944	56,82 %	22,10 %	21,07 %	42,89 %	0,51 %
	3871	3,94 %	93,46 %	2,61 %	55,79 %	38,90 %
	3481	97,10 %	1,47 %	1,43 %	2,9 %	0,0 %

In order to obtain the correct binding energy of the  $X(3872)$  we fine tune  $\gamma$

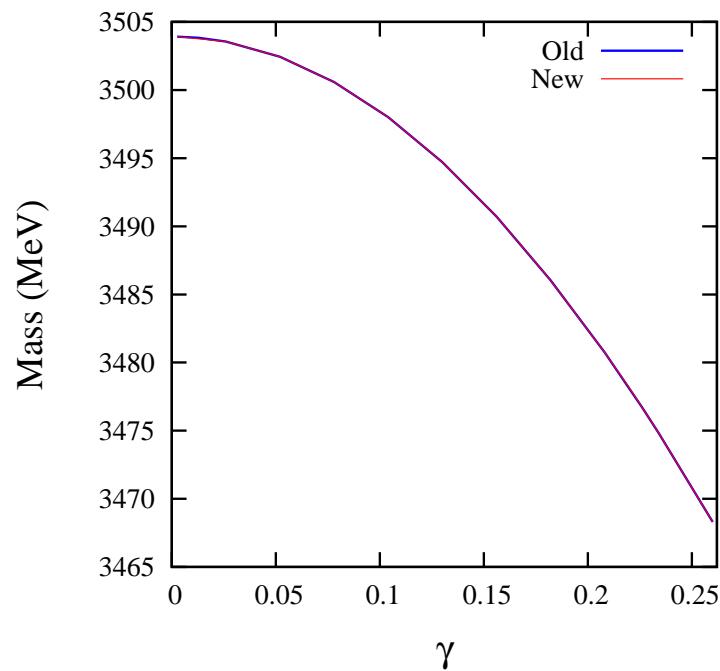
# The $1^{++}$ sector

We can project on the naive quark model basis

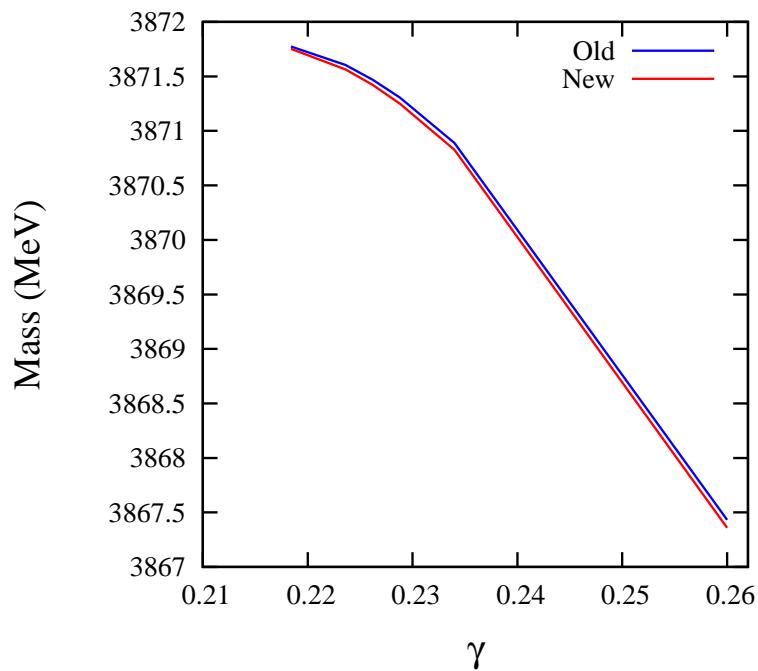
$\gamma^3 P_0$	$M(MeV)$	$c\bar{c}$	$1^3 P_1$	$2^3 P_1$	$3^3 P_1$	$4^3 P_1$
<b>0.260</b>	3949	56,71 %	1,61 %	96,33 %	1,28 %	0,78 %
	3867	30,22 %	1,80 %	98,14 %	0,06 %	0,0 %
	3468	95,70 %	99,99 %	0,01 %	0,0 %	0,0 %
<b>0.218</b>	3944	56,82 %	0,61 %	99,01 %	0,38 %	0,0 %
	3871	3,94 %	2,11 %	97,75 %	0,14 %	0,0 %
	3481	97,10 %	100,0 %	0,0 %	0,0 %	0,0 %

# The $1^{++}$ sector

$\chi_{c1}(1P)$



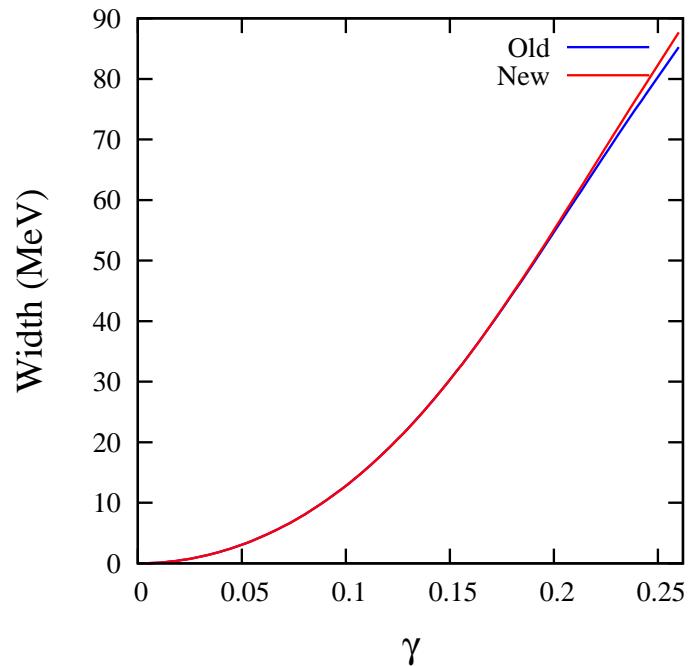
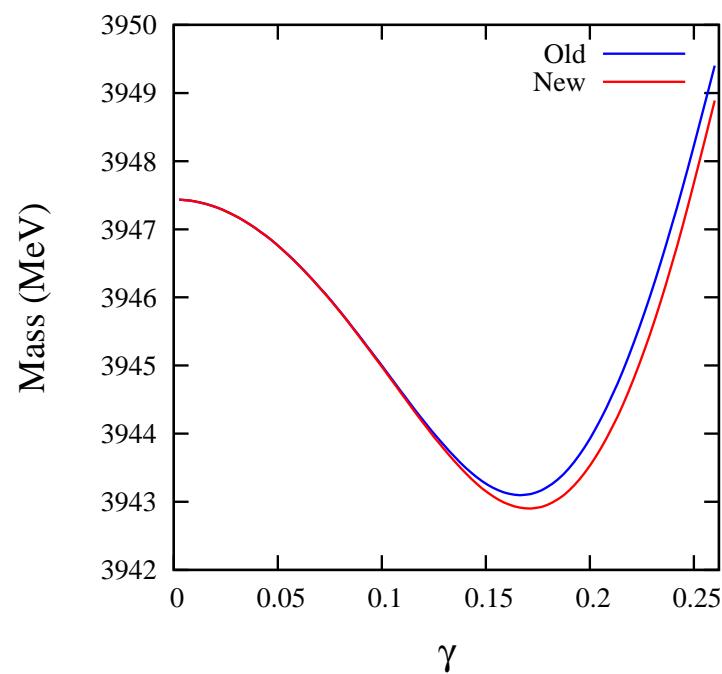
$X(3872)$



Coupling only with  $1P$  and  $2P$  bare states in the old approach  
Coupling with all states in the new approach

# The $1^{++}$ sector

$X(3940)$



Coupling only with  $1P$  and  $2P$  bare states in the old approach

Coupling with all states in the new approach

# Summary

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- The unquenched quark model is a useful tool to study the  $X Y Z$  states
- In the unquenched quark model we include
  - The  $q\bar{q}$  interaction to obtain naive quark model states.
  - The interaction between meson derived from the same quark interaction through the RGM.
  - The coupling between one meson and two meson states using the  $^3P_0$  model.
- We have developed the framework to include all states of the bare Hamiltonian on the  $q\bar{q}$  content of physical states
- On the small coupling limit the two approaches agree
- In the charmonium sector the coupling is small and only small variations on previous results are found.
- We will study lighter sectors where the coupling is bigger