Pion exchange for $P_c(4450)^+$ and related states

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[T.B. & E. Swanson (ongoing)]
$P_c(4380)$ and $P_c(4450)$

$J/\psi p$ states in $\Lambda_b \to J/\psi pK^-$ and $\Lambda_b \to J/\psi p\pi^-$. 

$uudc\bar{c} = \text{exotic flavour}$  

[LHCb(2015,2016); S. Neubert talk]
$P_c(4380)$ and $P_c(4450)$

<table>
<thead>
<tr>
<th></th>
<th>$P_c(4380)^+$</th>
<th>$P_c(4450)^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>$4380 \pm 8 \pm 29$</td>
<td>$4449.8 \pm 1.7 \pm 2.5$</td>
</tr>
<tr>
<td>Width</td>
<td>$205 \pm 18 \pm 86$</td>
<td>$35 \pm 5 \pm 19$</td>
</tr>
<tr>
<td>Assignment 1</td>
<td>$3/2^-$</td>
<td>$5/2^+$</td>
</tr>
<tr>
<td>Assignment 2</td>
<td>$3/2^+$</td>
<td>$5/2^-$</td>
</tr>
<tr>
<td>Assignment 3</td>
<td>$5/2^+$</td>
<td>$3/2^-$</td>
</tr>
<tr>
<td>Assignment 4</td>
<td>$5/2^-$</td>
<td>$3/2^+$</td>
</tr>
</tbody>
</table>

|     | $\Sigma^+_c \bar{D}^0$ | $(udc)(u\bar{c})$ | $4382.3 \pm 2.4$ |
|     | $\Sigma^+_c \bar{D}^{*0}$ | $(udc)(u\bar{c})$ | $4459.9 \pm 0.5$ |
|     | $\Lambda^+_c(1P) \bar{D}^0$ | $(udc)(u\bar{c})$ | $4457.09 \pm 0.35$ |
|     | $\chi_{c1}\rho$ | $(udu)(c\bar{c})$ | $4448.93 \pm 0.07$ |
Molecules

Molecular approaches:

- Wu, Molina, Oset, Zou, Xiao, Nieves, Uchino, Liang, Roca, Magas, Feijoo, Ramos, ... (2010-2016)
- Yang, Sun, He, Liu, Zhu (2011)
- He (2015)
- Shimizu, Suenaga, Harada (2016)
- Yamaguchi, Santopinto (2016)
- Yang, Ping (2015)
- Ortega, Entem, Fernandez (2016)
- ...
Pion-exchange
Pion exchange: basics

Potential between hadrons due to quark-pion vertices, with parameters fit to the deuteron:

\[ V(\vec{r}) = \sum_{ij} \left[ C(r) \vec{\sigma}_i \cdot \vec{\sigma}_j + T(r) S_{ij}(\hat{r}) \right] \vec{\tau}_i \cdot \vec{\tau}_j \]

For \( uudc\bar{c} \), each constituent must have:

▶ light quarks: \((u dc)(u \bar{c})\) but not \((u ud)(c \bar{c})\)
▶ non-zero isospin: \(\Sigma_c, \Sigma^*_c\), but not \(\Lambda_c\).
▶ non-zero spin: \(\bar{D}^*\) but not \(\bar{D}\).

This leaves

▶ \(\Sigma_c \bar{D}^*\) [\(P_c(4450)\)]
▶ \(\Sigma^*_c \bar{D}^*\)
Pion exchange: basics

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- non-zero isospin: \(\Sigma_c, \Sigma^*_c\), but not \(\Lambda_c\).
- non-zero spin: \(\tilde{D}^*\) but not \(\tilde{D}\).

This leaves

- \(\Sigma_c \tilde{D}^* \) [\(P_c(4450)\)]
- \(\Sigma^*_c \tilde{D}^*\)
Pion exchange: basics

Potential between hadrons due to quark-pion vertices, with parameters fit to the deuteron:

\[ V(\vec{r}) = \sum_{ij} \left[ C(r) \vec{\sigma}_i \cdot \vec{\sigma}_j + T(r) S_{ij}(\hat{r}) \right] \vec{\tau}_i \cdot \vec{\tau}_j \]

For \( uuud\bar{c}\), each constituent must have:

- light quarks: \((udc)(u\bar{c})\) but not \((uud)(c\bar{c})\)
- non-zero isospin: \(\Sigma_c, \Sigma^*_{c}\), but not \(\Lambda_c\).
- non-zero spin: \(\bar{D}^*\) but not \(\bar{D}\).

This leaves

- \(\Sigma_c \bar{D}^*\) \([P_c(4450)]\)
- \(\Sigma^*_{c} \bar{D}^*\)
Pion exchange: central potential

\[ V(\vec{r}) = \sum_{ij} \left[ C(r) \vec{\sigma}_i \cdot \vec{\sigma}_j + T(r) S_{ij}(\hat{r}) \right] \vec{\tau}_i \cdot \vec{\tau}_j \]

The potential is “attractive” if \( \langle \sum_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\tau}_i \cdot \vec{\tau}_j \rangle < 0 \), and for NN this is guaranteed by Pauli stats.

- \(-C(r)\) without delta term
- \(-C(r)\) with delta term

[See also Close & Thomas PRD78,034007(2008)]
Pion exchange: central potential

The coefficients \( \langle \sum_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\tau}_i \cdot \vec{\tau}_j \rangle \) of \( C(r) \) are:

<table>
<thead>
<tr>
<th>( \Sigma_c \vec{D}^* )</th>
<th>( \Sigma^* \vec{D}^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2(1/2(^-)) +16/3</td>
<td>1/2(1/2(^-)) +20/3</td>
</tr>
<tr>
<td>1/2(3/2(^-)) -8/3</td>
<td>1/2(3/2(^-)) +8/3</td>
</tr>
<tr>
<td></td>
<td>1/2(5/2(^-)) -4</td>
</tr>
<tr>
<td>3/2(1/2(^-)) -8/3</td>
<td>3/2(1/2(^-)) -10/3</td>
</tr>
<tr>
<td>3/2(3/2(^-)) +4/3</td>
<td>3/2(3/2(^-)) -4/3</td>
</tr>
<tr>
<td></td>
<td>3/2(5/2(^-)) +2</td>
</tr>
</tbody>
</table>

Two attractive channels in \( I = 1/2 \):
- \( \Sigma c \vec{D}^* \) matches \( P_c(4450) \)
- \( \Sigma^* \vec{D}^* \) \( \rightarrow J/\psi_p \) in D-wave with spin violation

Several attractive \( I = 3/2 \) states.

But... the central potential alone is too weak to bind!
Pion exchange: central potential

The coefficients $\langle \sum_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\tau}_i \cdot \vec{\tau}_j \rangle$ of $C(r)$ are:

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</tr>
<tr>
<td>$3/2(3/2^-)$</td>
<td>$+4/3$</td>
</tr>
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<td></td>
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</tr>
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Two attractive channels in $I = 1/2$:

- $\Sigma c \bar{D}^* \frac{1}{2}(1/2^-)$ matches $Pc(4450)$
- $\Sigma^* c \bar{D}^* \frac{1}{2}(3/2^-)$:

Several attractive $I = 3/2$ states. But... the central potential alone is too weak to bind!
Pion exchange: central potential

The coefficients \( \langle \sum_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\tau}_i \cdot \vec{\tau}_j \rangle \) of \( C(r) \) are:

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</tr>
<tr>
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</tr>
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Two attractive channels in \( I = 1/2 \):

- \( \Sigma_c \bar{D}^* 1/2(3/2^-) \): matches \( P_c(4450) \)
- \( \Sigma_c \bar{D}^* 1/2(5/2^-) \): \( \rightarrow J/\psi p \) in D-wave with spin violation
Pion exchange: central potential

The coefficients $\langle \sum_{ij} {\vec{\sigma}_i \cdot {\vec{\sigma}_j}} {\vec{\tau}_i \cdot {\vec{\tau}_j}} \rangle$ of $C(r)$ are:

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<td>$1/2(3/2^-)$  $+8/3$</td>
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<tr>
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</tr>
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Two attractive channels in $I = 1/2$:

- $\Sigma_c {\bar{D}}^*$  $1/2(3/2^-)$: matches $P_c(4450)$
- $\Sigma^* {\bar{D}}^*$  $1/2(5/2^-)$: $\rightarrow J/\psi p$ in D-wave with spin violation

Several attractive $I = 3/2$ states.
Pion exchange: central potential

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<td>$-4/3$</td>
</tr>
<tr>
<td>$1/2(5/2^-)$</td>
<td>$-4$</td>
</tr>
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Two attractive channels in $I = 1/2$:

- $\Sigma_c \bar{D}^*$ $1/2(3/2^-)$: matches $P_c(4450)$
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Several attractive $I = 3/2$ states.
But... the central potential alone is too weak to bind!
Pion exchange: central and tensor

The full potential

\[ V(\vec{r}) = \sum_{ij} \left( C(r) \vec{\sigma}_i \cdot \vec{\sigma}_j + T(r) S_{ij}(\hat{r}) \right) \vec{\tau}_i \cdot \vec{\tau}_j \]

is a matrix problem, with tensor mixing $S$- and $D$-waves.

E.g. for the the $P_c(4450)$ candidate state $\Sigma_c \bar{D}^* 1/2(3/2^-)$:

\[
\begin{array}{ccc}
\langle ^4S_{3/2} \rangle & | ^4 S_{3/2} \rangle & | ^2 D_{3/2} \rangle & | ^4 D_{3/2} \rangle \\
\langle ^2 D_{3/2} \rangle & -\frac{8}{3} C & -\frac{8}{3} T & -\frac{16}{3} T \\
\langle ^4 D_{3/2} \rangle & -\frac{16}{3} T & +\frac{16}{3} C & +\frac{8}{3} T \\
\end{array}
\]

As with the deuteron, including the tensor facilitates binding, and binding energies depend (strongly) on the form factor cutoff.
Pion exchange: central and tensor

Dipole cut-offs $\Lambda$ (GeV) required to achieve binding:

<table>
<thead>
<tr>
<th></th>
<th>$NN$</th>
<th>$\Sigma_c^<em>\bar{D}^</em>$</th>
<th>$\Sigma_c\bar{D}^*$</th>
<th>other</th>
<th>other</th>
</tr>
</thead>
<tbody>
<tr>
<td>No delta</td>
<td>0.8</td>
<td>0.9</td>
<td>1.1</td>
<td>$\geq 1.4$</td>
<td>$\geq 1.7$</td>
</tr>
<tr>
<td>Delta</td>
<td>1.0</td>
<td>1.2</td>
<td>1.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For both extremes (with or without the $\delta$ term):

- $\Sigma_c^*(\bar{D})^*$ binding with modest increase in $\Lambda$ cf. deuteron
- $I = 1/2$ binding c.f. central potential, but not $I = 3/2$
- over large range of $\Lambda$ there are (only) two bound states: the $P_c(4450)$ candidate, plus (more deeply bound) $\Sigma_c^*\bar{D}^* \frac{1}{2}(\frac{5}{2}^-)$.
- $\Lambda$ is constrained: if additional states bind, $P_c(4450)$ is too bound, and the additional states cannot be "explained away".

N.B.: an attractive delta function core spoils the pattern.
$\Xi_c \bar{D}^*$ molecules
The \( \Xi_c \) family also couple to pions.

\[
\begin{align*}
\Lambda_c &= ((ud)_0 c)_{1/2} \quad \Rightarrow \quad \Xi_c &= ((us)_0 c)_{1/2} \\
\Sigma_c &= ((ud)_1 c)_{1/2} \quad \Rightarrow \quad \Xi'_c &= ((us)_1 c)_{1/2} \\
\Sigma_c^* &= ((ud)_1 c)_{3/2} \quad \Rightarrow \quad \Xi^*_c &= ((us)_1 c)_{3/2}
\end{align*}
\]

and so potentially form molecules

\[
\begin{align*}
\Sigma_c \bar{D}^* &\quad \Rightarrow \quad \Xi'_c \bar{D}^* \\
\Sigma_c^* \bar{D}^* &\quad \Rightarrow \quad \Xi^*_c \bar{D}^*
\end{align*}
\]
The potential matrices (central + tensor) are directly related:

\[
\begin{array}{|c|c|c|c|}
\hline
\Sigma_c^{(*)} \bar{D}^* & \Xi_c^{(r,*)} \bar{D}^* & \Sigma_c^{(*)} \bar{D}^* & \Xi_c^{(r,*)} \bar{D}^* \\
I = 1/2 & I = 0 & I = 3/2 & I = 1 \\
+4 & +3 & -2 & -1 \\
\hline
\end{array}
\]

Thus anticipate some \( I = 0 \) but no \( I = 1 \) states:

\[
\begin{align*}
\Sigma_c^* \bar{D}^* & \ 1/2(5/2^-) \quad \Rightarrow \quad \Xi_c^* \bar{D}^* & \ 0(5/2^-) \\
\Sigma_c \bar{D}^* & \ 1/2(3/2^-) \quad \Rightarrow \quad \Xi_c' \bar{D}^* & \ 0(3/2^-)
\end{align*}
\]

<table>
<thead>
<tr>
<th>( )</th>
<th>( NN )</th>
<th>( \Sigma_c^* \bar{D}^* )</th>
<th>( \Sigma_c \bar{D}^* )</th>
<th>( \Xi_c^* \bar{D}^* )</th>
<th>( \Xi_c' \bar{D}^* )</th>
<th>other</th>
<th>other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delta</td>
<td>0(1^-)</td>
<td>1/2(5/2^-)</td>
<td>1/2(3/2^-)</td>
<td>0(5/2^-)</td>
<td>0(3/2^-)</td>
<td>1/2(J^-)</td>
<td>3/2(J^-)</td>
</tr>
<tr>
<td>No delta</td>
<td>0.8</td>
<td>0.9</td>
<td>1.1</td>
<td>1.2</td>
<td>1.4</td>
<td>1.4</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>1.2</td>
<td>1.4</td>
<td>1.5</td>
<td>1.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\(\Xi_c\bar{D}^*\) molecules

Pion-exchange: \(\Xi_c\bar{D}^* \ 0(5/2^-)\)
Local-hidden gauge: \(\Xi_c\bar{D}^* \ 0(3/2^-), \Xi_c'\bar{D}^* \ 0(3/2^-)\)


[see talk V. Magas]

The \(\Xi_c\bar{D}^* \ 0(5/2^-)\) state is

- weakly bound, with mass \(\approx 4652\ \text{MeV}\)
- narrow, decaying into \(J/\psi \Lambda\)
- produced in \(\Lambda_0^0 \rightarrow J/\psi \Lambda \eta, \Lambda_0^0 \rightarrow J/\psi \Lambda \phi\)
- produced via similar diagrams to \(P_c(4450)\)
Isospin mixing
Isospin mixing: $P_c(4380)$ and $P_c(4450)$

The $uudc\bar{c}$ combination is

$$
\begin{aligned}
(u dc)(u\bar{c}) &= \Sigma_c^+ \bar{D}^0 \\
(u uc)(d\bar{c}) &= \Sigma_c^{++} D^-
\end{aligned}
$$

Mass gap is significant on the scale of the binding energies,

$$
\begin{align*}
P_c(4380) &= 4380 \pm 8 \pm 29 \\
\Sigma_c^{*+} \bar{D}^0 &= 4382.3 \pm 2.4 \\
\Sigma_c^{*++} D^- &= 4387.5 \pm 0.7
\end{align*}
$$

so the $P_c$ states have mixed isospin,

$$
|P_c\rangle = \cos \phi |\frac{1}{2}, \frac{1}{2}\rangle + \sin \phi |\frac{3}{2}, \frac{1}{2}\rangle
$$

and can decay into $J/\psi \Delta^+$ and $\eta_c \Delta^+$, with weights:

$$
\begin{align*}
J/\psi p : J/\psi \Delta^+ : \eta_c \Delta^+ &= 2 \cos^2 \phi : 5 \sin^2 \phi : 3 \sin^2 \phi \\
J/\psi p : J/\psi \Delta^+ : \eta_c \Delta^+ &= \cos^2 \phi : 10 \sin^2 \phi : 6 \sin^2 \phi
\end{align*}
$$
Isospin mixing: other states

\[ \Sigma_c^* \bar{D}^* \, \frac{1}{2}(5/2^-) \]
\[ \Xi_c^* \bar{D}^* \, 0(5/2^-) \]

\[ \Sigma_c^{*+} \bar{D}^{*0} = 4524.4 \pm 2.4 \]
\[ \Sigma_c^{*++} D^*^- = 4528.2 \pm 0.7 \]

Eigenstate of mixed isospin:
\[ |P\rangle = \cos \phi |\frac{1}{2}, \frac{1}{2}\rangle + \sin \phi |\frac{3}{2}, \frac{1}{2}\rangle \]

\[ \Xi_c^{*0} \bar{D}^{*0} = 4652.9 \pm 0.6 \]
\[ \Xi_c^{*+} D^*^- = 4656.2 \pm 0.7 \]

Eigenstate of mixed isospin:
\[ |P\rangle = \cos \phi |0, 0\rangle + \sin \phi |1, 0\rangle \]

\( J/\psi p: \) D-wave, spin flip
\( J/\psi \Delta: \) S-wave, spin cons.
\[ \implies I = 3/2 \text{ decay enhanced.} \]

\( J/\psi \Lambda: \) D-wave, spin flip
\( J/\psi \Sigma^*: \) S-wave, spin cons.
\[ \implies I = 1 \text{ decay enhanced.} \]
Conclusions

- Pion exchange (normalised to the deuteron) binds a $\Sigma^*_c \bar{D}^*$ molecule, consistent with $P_c(4450)$.

- Within a significant (and constrained) parameter range, and independently of the poorly-known short-distance potential, only one $\Sigma^*_c \bar{D}^*$ partner is expected.

- A corresponding $\Xi^* \bar{D}^*$ molecule is also bound, and is produced via similar diagrams to the $P_c$ states.

- Small isospin admixtures in all states could be observed due to enhanced decays.
Backup slides
Pion exchange: central potential

For channels with $\langle \sum_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j \cdot \vec{\tau}_i \cdot \vec{\tau}_j > 0 \rangle$, the central potential with delta term has a deeply attractive core.

$$\Sigma_c \vec{D}^* \ (I=1/2, \ J=3/2)$$

[Chen, Liu, Li & Zhu, PRL115, 132002(2015)]

But should it be trusted?
Cusps and triangle singularities

Models:
- Guo, Meißner, Wang, Yang [PRD92, 07152(2015)]
- Mikhasenko [1507.06552]
- Liu, Wang, Zhao [PLB757, 231(2015)]
Cusps and triangle singularities

\[ \Lambda_b \rightarrow \Lambda_c/\Sigma_c \]

\[ D_s \rightarrow \bar{D} \]

\[ J/\psi, \chi_c, \ldots \]

\[ \Lambda \rightarrow p, K^- \]

\[ \bar{D} \rightarrow J/\psi, p, K^- \]
$P_c(4380)$ and $P_c(4450)$: partner states

$\chi_{c1}p$ scenario:
- neutral $\chi_{c1}n$ partner heavier by $\approx 1.29$ MeV
- $1/2^−, 3/2^−$ and $5/2^−$ partners ($P$-wave is required)

$\Lambda_c^{+*}\bar{D}^0$ scenario:
- neutral $\Lambda_c^{+*}D^-$ partner heavier by $\approx 4.77$ MeV
- other $J^P$ partners

$\Sigma_c^{(*)}\bar{D}^{(*)}$ scenario:
- neutral $I = 1/2$ partner
- possible $I = 3/2$ partners including doubly-charged, decaying into $J/\psi\Delta$
- possible $J^P$ partners

Compact pentaquark scenario:
- many partners with different flavours and $J^P$
$P_c(4450)$: parallels with $X(3872)$

$X(3872)$

$\bar{D}^* - \bar{D} = 142.1$

Nearby $J/\psi \rho$ & $J/\psi \omega$

Isospin violation

$P_c(4450)$

$\Lambda_c^* - \Sigma_c = 139.4$

Nearby $\chi_{c1}\rho$

Isospin violation?

Enhanced binding ($S$-wave vertex)?
Cusps and triangle diagrams
Cusps and triangle diagrams

\[ \Lambda_c \]

\[ D_{s} \]

\[ \bar{D} \]

\[ K^- \]

\[ J/\psi \]

\[ p \]

\[ \text{mass, but} \]

\[ S\text{-wave} = \frac{1}{2} + \cdot \text{P-wave} = \frac{1}{2} - \frac{3}{2} - \frac{5}{2} \]

\[ \text{mass, and} \]

\[ \text{doubly suppressed} \]

\[ \text{what restricts} \]

\[ J^P? \]

\[ \text{why not} \]

\[ \Sigma_{\bar{c}D} \approx P_{\Lambda_c}(4450) \]

\[ \Sigma_{\bar{c}D}^{\ast} \approx P_{\Lambda_c}(4450) \]
Cusps and triangle diagrams

\[ \Lambda^c \rightarrow J/\psi, \bar{D}, p \]

\[ D_s \]

\[ \Lambda^b \]

\[ u, d, c, \bar{s}, \bar{c} \]

\[ b, \bar{b}, \bar{c} \]

\[ u, d, s \]
Cusps and triangle diagrams

\[ \Lambda_{b} \rightarrow J/\psi, \chi_{c}, \ldots \]

\[ \Lambda \rightarrow p, K^{-} \]

\[ \Lambda_{c} \rightarrow J/\psi \]

\[ D_{s} \rightarrow p, K^{-} \]

\[ \bar{D} \rightarrow p, K^{-} \]
Cusps and triangle diagrams

\[ \begin{align*}
\Lambda_b & \quad \Lambda_c \\
D_s & \quad \bar{D} \\
J/\psi & \quad p \\
K^- & \quad K^- \\
J/\psi, \chi_c, \ldots & \quad J/\psi, p \\
\Lambda & \quad p \\
\end{align*} \]
Cusps and triangle diagrams

\[
\begin{align*}
\text{bs} & \rightarrow \Lambda_b \rightarrow J/\psi, \chi_c, \ldots \\
\text{bs} & \rightarrow \Lambda_b \rightarrow J/\psi, \chi_c, \ldots \\
\text{sc} & \rightarrow \Xi_c \rightarrow \Lambda_c/\Sigma_c \\
\text{sc} & \rightarrow \Xi_c \rightarrow \Lambda_c/\Sigma_c
\end{align*}
\]
Cusps and triangle diagrams

Enhancements expected at
\[ \Lambda_c \bar{D} = 1/2^- \]
\[ \Lambda_c \bar{D}^* = 1/2^-, 3/2^- \]
not seen at LHCb
Cusps and triangle diagrams

\[ \Lambda_c^* \bar{D} \approx P_c(4450) \text{ mass, but} \]
- S-wave = 1/2^+
- P-wave = 1/2^-, 3/2^-
- why no \( \Lambda_c^* \bar{D}^* \) states?

- \( \Lambda_b \)
- \( \Lambda_c \)
- \( D_s \)
- \( \bar{D} \)
- \( J/\psi \)
- \( p \)
- \( K^- \)

**Diagram 1:**
- \( \Lambda_b \)
- \( \Lambda_c \)
- \( D_s \)
- \( \bar{D} \)
- \( J/\psi \)
- \( p \)
- \( K^- \)

**Diagram 2:**
- \( \Lambda_b \)
- \( \Lambda \)
- \( p \)
- \( J/\psi \)
- \( K^- \)
- \( \chi_c, \ldots \)

**Diagram 3:**
- \( \Lambda_b \)
- \( \bar{D} \)
- \( J/\psi \)
- \( p \)
- \( K^- \)

**Diagram 4:**
- \( \Lambda_b \)
- \( \Xi_c \)
- \( \Lambda_c/\Sigma_c \)
- \( p \)
- \( K^- \)
Cusps and triangle diagrams

\[ \Lambda_c^* \bar{D} \approx P_c(4450) \] mass, but
\[ \cdot \text{S-wave} = 1/2^+ \]
\[ \cdot \text{P-wave} = 1/2^-, 3/2^- \]
\[ \cdot \text{why no } \Lambda_c^* \bar{D}^* \text{ states?} \]

\[ \chi_{c1} p = P_c(4450) \] mass, but
\[ \cdot \text{doubly suppressed} \]
\[ \cdot \text{S-wave} = 1/2^+, 3/2^+ \]
\[ \cdot \text{P-wave} = 1/2^-, 3/2^-, 5/2^- \]
Cusps and triangle diagrams

$\Lambda_c^* \bar{D} \approx P_c (4450) \text{ mass, but}$
- S-wave $= 1/2^+$
- P-wave $= 1/2^-, 3/2^-$
- why no $\Lambda_c^* \bar{D}^*$ states?

$\chi_{c1} p = P_c (4450) \text{ mass, but}$
- doubly suppressed
- S-wave $= 1/2^+, 3/2^+$
- P-wave $= 1/2^-, 3/2^-, 5/2^-$

$\Sigma_c^* \bar{D} \approx P_c (4380) \text{ mass, and}$
$\Sigma_c \bar{D}^* \approx P_c (4450) \text{ mass, but}$
- doubly suppressed
- what restricts $J^P$?
- why not $\Sigma_c \bar{D}, \Sigma_c^* \bar{D}^*$?
$P_c$ states in the Cabibbo-suppressed mode

\[ \Lambda_b \to J/\psi p K^- \quad \Lambda_b \to J/\psi p \pi^- \]

Before $P_c$ discovery LHCb had previously observed $\Lambda_b \to J/\psi p \pi^-$, and reported no sign of a $J/\psi p$ structure.

[LHCb, JHEP07(2014)103]
$P_c$ states in the Cabibbo-suppressed mode
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$P_c(4380)$ and $P_c(4450)$

(a) LHCb

Events/(15 MeV)

$m_{Kp} [\text{GeV}]$
Amplitudes for $P_c$ states
Cusps and triangle singularities

These effects are also connected to thresholds.

Belle study of decays

\[ \Upsilon(5S) \to \Upsilon(nS)\pi^+\pi^- \]
\[ \Upsilon(5S) \to h_b(nS)\pi^+\pi^- \]

discovers charged \( Z_b \) states in \( \Upsilon(nS)\pi^\pm \) and \( h_b(nS)\pi^\pm \), just above \( B^*\bar{B} \) and \( B^*\bar{B}^* \) thresholds.

BESIII study of decays

\[ \Upsilon(4260) \to J/\psi\pi^+\pi^- \]

discovers charged \( Z_c \) states in \( J/\psi\pi^\pm \), just above \( D^*\bar{D} \) and \( D^*\bar{D}^* \) thresholds.
Cusps and triangle singularities

An example for the $Z_b$ states:

[Swanson PRD91,034009(2015)]
$P_c(4380)$ and $P_c(4450)$

LHCb amplitude analysis of the three-body decay $\Lambda_b \rightarrow J/\psi pK^-$, studying the $pK^-$ and exotic $J/\psi p$ mass spectra.

[LHCb, PRL115, 072001, 2015]