$Z_c(3900)$: experiment, theory, lattice


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Outline

1. Experiment
2. Theory
3. Lattice
4. Conclusions
Experimental information on $Z_c(3885)/Z_c(3900)$

- **$Z_c(3900)$** first seen by BESIII and Belle Collabs. in $J/\psi\pi^\pm$ invariant mass spectrum in $e^+e^- \rightarrow Y(4260) \rightarrow J/\psi\pi^+\pi^-$
  
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- Later on, CLEO-c data confirmed $Z_c(3900)$ in $e^+e^- \rightarrow \psi(4160) \rightarrow J/\psi\pi^+\pi^-$
  
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![Graph 1](image1.png)

![Graph 2](image2.png)
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“One of the most interesting resonances”: couples strongly to charmonium ($\sim \bar{c}c$) and yet it has charge ($\sim \bar{u}d$). Minimal quark constituent is four $[\bar{c}c\bar{u}d]$.

Many different interpretations:
- Tetraquark
- $\bar{D}^*D$ molecular state
- Simply a kinematical effect (ruled out)
- Hadrocharmonium
- Searched for in lattice QCD

Recent reviews (2015-2016):
- [Olsen, Front. Phys. 10, 121('15)]
- [Chen et al.,Phys. Rept. 639, 1('16)]
- [Hosaka et al.,PTEP 2016, 062C01('16)]

What is still missing?

A joint study of both reactions in which the $Z_c$ structure has been seen
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Coupling $\bar{D}^* D$ and $J/\psi \pi$ channels

**Coupling $\bar{D}^* D$ and $J/\psi \pi$ channels**

**Coupled channel** formalism is needed, because $Z_c(3900)$:
- is expected to be dynamically generated in $\bar{D}^* D$ channel (#2),
- but it is also seen in $J/\psi \pi$ channel (#1).

$$T = (\mathbb{I} - V \cdot G)^{-1} \cdot V,$$

$$V_{ij} = 4 \sqrt{m_{i1} m_{i2}} \sqrt{m_{j1} m_{j2}} \, e^{-q_i^2 / \Lambda_i^2} \, e^{-q_j^2 / \Lambda_j^2} \, C_{ij},$$

- $G(E)$ are loop functions (Regularized with standard gaussian regulator)
- $J/\psi \pi \rightarrow J/\psi \pi$: known to be tiny, $C_{11} = 0$.
- $\bar{D}^* D \rightarrow J/\psi \pi$: we make the simplest possible assumption, $C_{12} \equiv \tilde{C}$ (constant)
- $\bar{D}^* D \rightarrow \bar{D}^* D$: In a momentum expansion (HQSS), simply a constant, $C_{22} \equiv C_{1Z}$.

**Problem**: no resonance in the complex plane above threshold with only constant potentials (even with coupled channels).

We introduce some energy dependence,

$$C_{22}(E) = C_{1Z} + b \,(E - m_D - m_{D^*})$$
**Amplitudes:** $Y(4260) \rightarrow (J/\psi \pi^-) \pi^+, (D^*-D^0) \pi^+$

\[
\left| \mathcal{M}_2(s, t) \right|^2 = \left| \frac{1}{t - m_{D_1}^2} + l_3(s) T_{22}(s) \right|^2 q_\pi^4(s) + |\beta (1 + T_{22}(s) G_{22}(s))|^2
\]

- $s$ (Mandelstam) $\bar{D}^* D$ invariant mass squared
- $l_3(s)$: three meson loop propagator
- $\bar{D}^* D$ rescattering enters through $T_{22}(s)$
- $q_\pi^2(s) = \lambda(M_Y^2, s, m_\pi^2)/(4M_Y^2)$
Amplitudes: $Y(4260) \rightarrow (J/\psi \pi^-)\pi^+, (D^* - D^0)\pi^+$

- The decay proceeds mainly through $[T_{12}(s)]$
  $Y \rightarrow (\bar{D}^* D)\pi \rightarrow (J/\psi)\pi$
- Some direct production included through $\alpha$
- $s, t$ (Mandelstam) $J/\psi \pi^-, J/\psi \pi^+$
  invariant mass squared

$$\left| \mathcal{M}_1(s, t) \right|^2 = |\tau(s)|^2 q_\pi^4(s) + |\tau(t)|^2 q_\pi^4(t) + \frac{3\cos^2 \theta - 1}{4} \left( \tau(s)\tau(t)^* + \tau(s)^* \tau(t) \right) q_\pi^2(s)q_\pi^2(t)$$

$$\tau(s) = \sqrt{2}I_3(s)T_{12}(s) + \alpha$$
Events distributions and Experimental data

- Events distributions $\mathcal{N}_i$:
  \[ \mathcal{N}_i(s) = K_i \left( A_i(s) + B_i(s) \right) \]
  \[ A_i(s) = \int_{t_i, -}^{t_i, +} dt \left| \mathcal{M}_i(s, t) \right|^2 \]

- $K_i$ (unknown) global normalization constants

- $B_i$ are background functions (parametrized as in the experimental analyses) ($B_2 = 0$)

- “Branching ratio”:
  \[ R_{\text{exp}} = \frac{\Gamma \left( Z_c \to D\bar{D}^* \right)}{\Gamma \left( Z_c \to J/\psi\pi \right)} = 6.2 \pm 2.9 \]

- Theoretically estimated as the (physical) ratio of areas around $Z_c(3900)$ mass
  \[ R_{\text{th}} = \frac{\int ds A_2(s)}{\int ds A_1(s)} \]
Results: comparison with experiment(s)

- Four different fits: $b = \{\text{free}, 0\}$, $\Lambda_2 = \{0.5, 1.0\}$ GeV
- Only the $T$-matrix parameters are shown (not shown: normalization, ...)
- All fits have $\chi^2 \simeq 1$ ($\simeq 1.4$ for $b = 0$), and are within the error band of the best one
- Reproduction of the data is excellent
Results: comparison with experiment(s)

<table>
<thead>
<tr>
<th>$\Lambda_2$ (GeV)</th>
<th>$C_1Z$ (fm$^2$)</th>
<th>$b$ (fm$^3$)</th>
<th>$\bar{C}$ (fm$^2$)</th>
<th>$\chi^2$/dof</th>
<th>$R_{th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>$-0.19 \pm 0.08 \pm 0.01$</td>
<td>$-2.0 \pm 0.7 \pm 0.4$</td>
<td>$0.39 \pm 0.10 \pm 0.02$</td>
<td>1.02</td>
<td>$6.0 \pm 3.5 \pm 0.5$</td>
</tr>
<tr>
<td>0.5</td>
<td>$+0.01 \pm 0.21 \pm 0.03$</td>
<td>$-7.0 \pm 0.4 \pm 1.4$</td>
<td>$0.64 \pm 0.16 \pm 0.02$</td>
<td>1.09</td>
<td>$6.5 \pm 3.6 \pm 0.2$</td>
</tr>
<tr>
<td>1.0</td>
<td>$-0.27 \pm 0.08 \pm 0.07$</td>
<td>0 (fixed)</td>
<td>$0.34 \pm 0.14 \pm 0.01$</td>
<td>1.31</td>
<td>$10.3 \pm 9.0 \pm 1.1$</td>
</tr>
<tr>
<td>0.5</td>
<td>$-0.27 \pm 0.16 \pm 0.13$</td>
<td>0 (fixed)</td>
<td>$0.54 \pm 0.16 \pm 0.02$</td>
<td>1.36</td>
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- Only the $T$-matrix parameters are shown (not shown: normalization, ...)
- All fits have $\bar{\chi}^2 \sim 1$ ($\sim 1.4$ for $b = 0$), and are within the error band of the best one
- Reproduction of the data is **excellent**
Two different scenarios:

1. \( b \neq 0 \): \( Z_c \) is a \( \bar{D}^*D \) resonance very close to threshold

   (Differences with experiments are related to Breit-Wigner parametrizations)

2. \( b = 0 \): \( Z_c \) is a virtual state

In both scenarios,

- Data are very well reproduced
- A single structure (not two) \( Z_c(3885)/Z_c(3900) \) is needed
**Z_c(3900) on the lattice**

- LQCD simulations on Z_c(3900) still scarce:
  - [Prelovsek et al., PR,D91,014504(’15)] \( (m_\pi = 266\text{ MeV}) \) “no additional candidate”
  - [Y. Ikeda et al. [HAL QCD], arXiv:1602.03465] \( (m_\pi \geq 410\text{ MeV}) \)
    Virtual poles with very low masses and deep in the complex plane.
  - [Y. Chen et al., PR,D89,094506(’14)]
  - [L. Liu et al., PoS LATTICE 2014, 117(’14)]
  - [S. H. Lee et al., arXiv:1411.1389]

- Results are not conclusive (large pion masses, etc...)

- We can predict energy levels in a finite box. **Cooperation** between (unitary) EFTs and LQCD simulations is useful to understand the hadron spectrum.
  - [M. Doring, U. G. Meissner, E. Oset and A. Rusetsky, EPJ,A47,139(’11)]

**Periodic boundary conditions: discrete momenta**

<table>
<thead>
<tr>
<th>infinite volume</th>
<th>finite volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{q}$ continuous</td>
<td>$\vec{q} = \frac{2\pi}{L} \vec{n}$, $\vec{n} \in \mathbb{Z}^3$</td>
</tr>
<tr>
<td>$\int_{\mathbb{R}^3} \frac{d^3 q}{(2\pi)^3} e^{-\frac{2(q^2 - k^2)}{\Lambda^2}} \frac{E - \omega_{D\bar{D}^*}(q)}{L^3} \sum_{\vec{n} \in \mathbb{Z}^3} e^{-\frac{2(q^2 - k^2)}{\Lambda^2}}$</td>
<td>$\frac{1}{L^3} \sum_{\vec{n} \in \mathbb{Z}^3} E - \omega_{D\bar{D}^*}(q)$</td>
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<td>$T^{-1}(E) = V^{-1}(E) - G(E)$</td>
<td>$\tilde{T}^{-1}(E, L) = V^{-1}(E) - \tilde{G}(E, L)$</td>
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- $\omega_{D\bar{D}^*}^{\text{the}}(q) = m_D + m_{D^*} + \frac{m_D + m_{D^*}}{2m_D m_{D^*}} q^2$ (non relativistic)
- Finite volume $\rightarrow$ box of edge $L$: it is an infinite square well potential (like QM)
- Energy levels: bound states in the box. Given by $\tilde{T}^{-1}(E_m(L), L) = 0$
- Goal: to compare with LQCD energy levels reported in [Prelovsek et al., PR,D91,014504(15)].
- We use the same masses and energy-momentum dispersion relation ($\omega_{D\bar{D}^*}^{\text{lat}}(q)$) used in the LQCD simulation.
Our aim is to compare with an actual LQCD simulation

[Prelovsek et al., PR,D91,014504('15) [arXiv:1405.7623]]

Calculations done at $L = 1.98$ fm, 
$m_\pi = 266$ MeV.

Three separate regions, all theoretical predictions in good agreement with LQCD

Except for this point?

$E_{th} = 4000^{+24}_{-13}$ MeV

$E_{lat} = 4070 \pm 30$ MeV

$\Delta E = 70 \pm 40$ MeV ($< 2\sigma$ dev.)

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- $Z_c(3900)$ is a most-interesting, exotic, structure. A candidate for “tetraquark”, or a $D^*\bar{D}$ molecule...


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- Data are well reproduced in all fits ($\chi^2 \simeq 1$)

- Two different scenarios are found:
  1. ($b \neq 0$) $Z_c(3900)$ is a $\bar{D}^*D$ resonance
  2. ($b = 0$) $Z_c(3900)$ is a virtual state

- In any case, a single structure for $Z_c(3885)/Z_c(3900)$ is needed.

- Improved data on $J/\psi\pi$ invariant mass spectrum are necessary


- We have used our $T$-matrix to compute energy levels in a finite volume

- Good agreement is found for both scenarios (resonance and virtual state) with the energy levels reported in a LQCD simulation [Prelovsek et al., PR,D91,014504(15)]

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Z_c(3900): experiment, theory, lattice


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Thanks for your attention
Reflection of threshold and $Z_c(3900)$ in $J/\psi\pi^+\pi^-$ spectrum

When $M_{J/\psi\pi^-} \equiv \sqrt{s} \in (3.40, 3.55)$ GeV

$\Downarrow$

$M_{J/\psi\pi^+} \equiv \sqrt{t}$ can be at $\sqrt{t} = 3.9$ GeV

($D\bar{D}^*$ threshold, $Z_c(3900)$ mass)

This explains the enhancement (reflection)
Bound state, resonance, virtual ...

Well known example: $NN$ scattering and the deuteron

Triplet ($^3S_1 - ^3D_1$):
- $a_t \simeq 5$ fm.
- In this wave there is a bound state. The deuteron is a well known, really physical particle.

Singlet ($^1S_0$):
- $a_s \simeq -24$ fm.
- In this wave there is a virtual state.

- **A virtual state** does not correspond to a real particle. (Wavefunction not localized.)
- It produces effects at the threshold similar to those of a bound state or a nearby resonance.

![Graph showing events per 20 MeV/c² vs. $M_{J/\psi \pi^-}$ in MeV]

Data $b \neq 0$ (resonance)
$b = 0$ (virtual)
Complex plane & poles: First scenario (resonance)

- Pole located at $3894 - i30$ MeV
- Plot: unphysical Riemann sheet connected to the physical one above $D^* \bar{D}$
- Shift of the pole towards higher energies (interference!)
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Results

Results for the discrete energy levels as a function of box size ($L$)

- $J/\psi\pi$ channel not essential:
  - Always a level close to a free $J/\psi\pi$ one.
  - Coupled channels case levels follow single channel case levels (except near the free $J/\psi\pi$ levels).
- Level below threshold (attractive interaction) goes to threshold for $L \to \infty$: no bound state
- **Relevant** energy level: the one above threshold. Shift w.r.t. free levels is larger for the resonance case.
- No “additional(extra” energy level.
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- Both scenarios (resonance and virtual) agree with both cutoffs ($\Lambda^2 = 0.5$ GeV and 1 GeV). What to do?
- One possibility is to study volume dependence (several volumes)
- We compare here two predictions:
  - Resonance scenario with $\Lambda^2 = 0.5$ GeV (blue bands)
  - Virtual scenario with $\Lambda^2 = 1.0$ GeV (orange bands)

Both are indistinguishable around $L \simeq 2$ fm (say $1.9$ fm $< L < 2.2$ fm)

But they are clearly different at $L \simeq 2.4$ fm (say $2.3$ fm $< L < 2.5$ fm)
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  - Resonance scenario with $\Lambda_2 = 0.5 \text{ GeV}$ (blue bands)
  - Virtual scenario with $\Lambda_2 = 1.0 \text{ GeV}$ (orange bands)

- Both are indistinguishable around $L \approx 2 \text{ fm}$ (say $1.9 \text{ fm} < L < 2.2 \text{ fm}$)
- But they are clearly different at $L \approx 2.4 \text{ fm}$ (say $2.3 \text{ fm} < L < 2.5 \text{ fm}$)