Nuclear Effects on Tetraquark Production by Double Parton Scattering

F. Carvalho and F. S. Navarra

XII Quark Confinement and Hadron Spectrum

Thessaloniki – Greece
1) To develop a model to estimate the production cross section of exotic mesons in pp collisions

2) To estimate the nuclear effects on these cross sections
In the last years, some exotic mesons have been measured and confirmed by different collaborations:

- **X(3872)** Belle in 2003 (Confirmed by CDF/D0, BaBar, CMS and LHCb);
- **Z(4430)** Belle in 2008 (Confirmed by BaBar, LHCb);

- Many others are being investigated (XYZ)

- The structure of these mesons is still not well defined: [1,2,3,4,5]: mesons molecules ➔ different conclusions among their results

- This work: Tetraquark ➔ was never applied before
Steps to produce a tetraquark

- First step: **To produce four quarks** in pp

Two main ways:

**Single Parton Scattering (SPS):** the standard QCD leading order gluon gluon scattering with **an extra pair** $q\bar{q}$ produced:

$$gg \rightarrow q\bar{q}q\bar{q}$$

**Double Parton Scattering (DPS):** two independent leading order gluon gluon scattering:

$$(2x) \quad gg \rightarrow q\bar{q}$$
In the present work, we are only interested in the DPS contribution for tetraquark production.

- Important at high energies:
  
  \[ \text{DPS } \propto g(x)^4 \]
  
  \[ \text{SPS } \propto g(x)^2 \]

  \( g(x) \) grows fast with the energy

DPS contribution for the cross section in the tetraquark approach has never been calculated before

We are leaving aside other possibilities, which also contribute to the cross section, because they have already been calculated by other groups.
Second step: To bind the four quarks

We used the same ideas of the color evaporation model (CEM).

In the charmonium production ($c\bar{c}$):

- invariant mass of the pair: larger than $2m_c$ and smaller than $2m_D$

- The $c\bar{c}$ pair will absorb or emit a soft gluon ($E \approx \Lambda_{QCD}$) in order to become color neutral (color evaporation).
Fig. 1. Two gluons collide and form a $q\bar{q}$ state with mass $M_{12}$, while other two gluons collide and form a second $q\bar{q}$ state with mass $M_{34}$. The two objects bind to each other forming a 4-quark state, with invariant mass $M$.

Note: these states are still not color singlets!
Kinematics of the figure 1:

Working with the usual CEM one-dimensional kinematics:

\[ M_{12} = \sqrt{x_1 x_2 s} \quad \text{– invariant mass of the pair 12} \]

\[ M_{34} = \sqrt{x_3 x_4 s} \quad \text{– invariant mass of the pair 34} \]

\[ y_{12} = \frac{1}{2} \ln \left( \frac{x_1}{x_2} \right) \quad \text{– rapidity of the pair 12} \]

\[ y_{34} = \frac{1}{2} \ln \left( \frac{x_3}{x_4} \right) \quad \text{– rapidity of the pair 34} \]

\[ M^2 = M_{12}^2 + M_{34}^2 + 2M_{12}M_{34} \cosh(y_{12} - y_{34}) \quad \text{– invariant mass of the 4-quark object} \]
We assumed: \( y_{12} = y_{34} \)

The “cosh” function grows very fastly with the argument:

any difference between \( y_1 \) and \( y_2 \) \( \Rightarrow \) big value of \( M \)

Thus the system formed by four quarks will have invariant mass

\[
M = M_{12} + M_{34}
\]

Finally, in order to produce the color singlet tetraquark the cluster with mass \( M \) emits or absorbs a soft gluon carrying an energy \( \Delta \) \( (\Delta \approx \Lambda_{\text{QCD}}) \):

\[
M_T = M \pm \Delta
\]
The cross section of the process shown in Fig. 1 can be calculated with the schematic DPS “pocket” formula:

\[ \sigma_{DPS} \propto \frac{\sigma_{SPS}^{12} \times \sigma_{SPS}^{34}}{\sigma_{eff}} \]

where \( \sigma_{eff} \) is a constant taken from experimental data.

\( \sigma_{SPS} \) is the standard convolution between the parton densities \( g(x) \) and the elementary cross sections \( \sigma_{gg\rightarrow q\bar{q}} \).

Expanding the above formula we have:
\[ \sigma_{DPS} = \frac{F_T}{\sigma_{\text{eff}}} \left[ \int_0^1 dx_1 \int_0^1 dx_2 \ g(x_1, \mu^2) g(x_2, \mu^2) \sigma_{g_{1g_2} \rightarrow q\bar{q}} \right] \]
\[ \times \left[ \int_0^1 dx_3 \int_0^1 dx_4 \ g(x_3, \mu^2) g(x_4, \mu^2) \sigma_{g_{3g_4} \rightarrow q\bar{q}} \right] \]
\[ \times \Theta(1 - x_1 - x_2) \Theta(1 - x_3 - x_4) \]
\[ \times \Theta(M_{12}^2 - 4m_q^2) \Theta(M_{34}^2 - 4m_q^2) \]
\[ \times \delta(y_{34} - y_{12}) \]

where:

- \( g(x, \mu^2) \) is the gluon distribution in the proton with the gluon fractional momentum \( x \) and at the factorization scale \( \mu^2 \)
- \( \sigma_{gg \rightarrow q\bar{q}} \) is the elementary cross section \( gg \rightarrow q\bar{q} \).
• The step functions $\Theta(1 - x_1 - x_2) \Theta(1 - x_3 - x_4)$ enforce momentum conservation in the projectile and in the target.

• The step functions $\Theta(M_{12}^2 - 4m_q^2) \Theta(M_{34}^2 - 4m_q^2)$ guarantee that we have enough mass to produce the quark pairs.

• The delta function $\delta(y_{34} - y_{12})$ implements our “binding condition”.

• $F_T$ is a constant to be taken from experimental data.
Ingredients for the calculations:

\[ X(3872) \rightarrow q\bar{q}c\bar{c} \quad \Rightarrow \quad \text{Some experimental information available} \]

\[ T_{4c} \rightarrow c\bar{c}c\bar{c} \quad \Rightarrow \quad \text{Easily calculable in pQCD} \]

1. Parton distribution functions \( g(x_1, \mu^2) \): CTEQ 6

[http://hep.pa.msu.edu/cteq/public/cteq6.html]

2. The elementary cross section \( gg \rightarrow q\bar{q}(c\bar{c}) \)

For \( gg \rightarrow c\bar{c} \) standard leading order QCD result:

\[
\sigma_{gg\rightarrow cc} = \frac{\pi \alpha_s^2(m^2)}{3m^2} \left\{ \left(1 + \frac{4m_c^2}{m^2} + \frac{m_c^4}{m^4}\right) \ln \left(\frac{1 + \beta}{1 - \beta}\right) - \frac{1}{4} \left(7 + \frac{31m_c^2}{m^2}\right) \beta \right\}
\]

Where \( m^2 = M_{12(34)}^2 \) for \( g_{12(34)} \) and \( \beta = \left[1 - \frac{4m_c^2}{m^2}\right]^{1/2} \)
- We used for the light quarks the same elementary cross section as in the heavy quark production with $m_q = 0.5 \text{ GeV}$

- With this mass we found $\alpha_s = 0.4$, which justifies the use of the perturbative formula

3. The constant $F_T$:


\[ \sigma_X = 30 \text{ nb} \quad \text{at} \quad \sqrt{s} = 7 \text{ TeV} \]
• \( F_{T4c} \):

We assume that the only difference is: \( q\bar{q} \rightarrow c\bar{c} \)

\[ \downarrow \]

gives rise to:  

more difficult to produce

Penalty factor:

\[
\frac{\sigma_{c\bar{c}c\bar{c}}}{\sigma_{c\bar{c}q\bar{q}}} \approx \frac{\sigma_{c\bar{c}} \sigma_{c\bar{c}}}{\sigma_{c\bar{c}} \sigma_{q\bar{q}}} \approx 0.12
\]

\[
F_{T4c} = 0.12 F_x
\]

4. Masses

\[ M_{X(3872)} = 3872 \text{ MeV} \]

\[ M_{T4c} = 5400 \text{ MeV} \]

\[ m_q = 500 \text{ MeV} \]

\[ m_c = 1200 - 1300 \text{ MeV} \]
The increase with energy

- $\sigma = 30 \text{nb} \text{ at } 7 \text{ TeV}$
- $\sigma = 45 \text{nb} \text{ at } 14 \text{ TeV}$

Fig. 2. $X(3872)$ production cross section as a function of the energy.
The increase with energy

- $\sigma = 3.6\,nb$ at 7 TeV
- $\sigma = 7\,nb$ at 14 TeV

Fig. 3. $T_{4c}$ production cross section as a function of the energy.
The nuclear effects on the cross section

Replacement: \( g_p(x) \rightarrow g_A(x) \)

\[
\sigma_{DPS} = \frac{F_T}{\sigma_{eff}} \left[ \int_0^1 dx_1 \int_0^1 dx_2 \, g(x_1, \mu^2) g_A(x_2, \mu^2) \sigma \rightarrow q\bar{q} \right] \\
\times \left[ \int_0^1 dx_3 \int_0^1 dx_4 \, g(x_3, \mu^2) g_A(x_4, \mu^2) \sigma \rightarrow q\bar{q} \right] \\
\times \Theta(1-x_1-x_2) \Theta(1-x_3-x_4) \\
\times \Theta(M_{12}^2 - 4m_q^2) \Theta(M_{34}^2 - 4m_q^2) \\
\times \delta(y_{34} - y_{12})
\]

g_A(x) predicted by the EKS (1999), DS (2004) and EPS09 2009) collab:

Checking the consistence:

The nuclear modification ratios: \( R_A = \frac{x g_A}{A x g_p} \) for \( A = 208 \) and \( Q^2 = 1.0 \text{ GeV}^2 \):
Very different predictions for the shadowing!

Another source of uncertainty in the results...
The $\sigma_{pPb}$ cross sections:

$X(3872)$

Fig. 5. $X(3872)$ production cross section as a function of the energy.

- The increase with energy
- Rapid growth with the atomic number ($\sigma_{pA} \approx 10^4 \sigma_{pp}$)
- Different results (EKS/EPS, DS)
The increase with energy

Rapid growth with the atomic number \((\sigma_{pA} \approx 10^4 \sigma_{pp})\)

Different results (EKS/ EPS, DS)

Fig.6. \(T_{4c}\) production cross section as a function of the energy.

\(\sigma_{pp}\) (\(\mu b\))

\(s^{1/2}\) (TeV)

\(\sigma_{pp}\) (\(\mu b\))

\(\sigma_{pA}\) = \(10^4 \sigma_{pp}\)

\(\sigma_{pA}\) more easily observable!!
We have developed a model for tetraquark production in pp and pA collisions which combines double parton scattering and the basic ideas of the color evaporation model.

We have made predictions for the $T_{4c}$ production cross sections and also for $X(3872)$ at higher energies, which may be confronted with the forthcoming LHC data taken at $\sqrt{s} = 14\,TeV$.

The dependence with the energy of our model is all contained in the gluon distribution function.

We have computed the production cross section in proton-nucleus collisions and found a rapid growth with the atomic number, despite the different predictions given by the nuclear gluon distribution functions.
The results presented contain some uncertainties:

(i) They do not include tetraquark production in SPS events.

(ii) The binding mechanism is probably too simple and insensitive to the quantum numbers of the involved particles.

(iii) In the case of $X(3872)$ production, the use of the elementary cross section for light quark production is questionable.

(iv) The differences among the nuclear parton distribution functions
15. W. Heupel, G. Eichmann, and C. S. Fischer (2012);
17. P. Bicudo et al (2015);