Short-distance current correlators on the lattice

based on the work by the JLQCD collaboration:
M. Tomii et al, arXiv:1604.08702 [hep-lat]
K. Nakayama, B. Fahy, SH, arXiv:1606.01002 [hep-lat]

Shoji Hashimoto (KEK, Sokendai)
@ CONF12,
Thessaloniki, Aug 29, 2016

計算基礎科学連携拠点
JICFuS Joint Institute for Computational Fundamental Science
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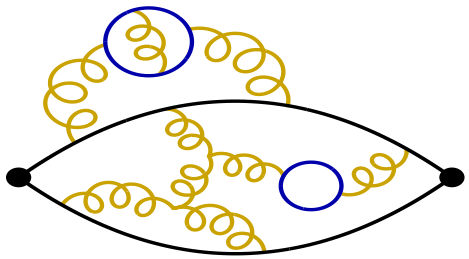
JLQCD collaboration

- Members
  - KEK: Y. Aoki, B. Fahy, S. Hashimoto, T. Kaneko, H. Matsufuru, K. Nakayama, M. Tomii
  - Osaka: H. Fukaya, T. Onogi
  - Kyoto: S. Aoki
  - Tsukuba: Y. Taniguchi
  - RIKEN: N. Yamanaka
  - Columbia: X. Feng
  - Wuhan: A. Tomiya
  - Edinburgh: G. Cossu
- Machines @ KEK
  - Hitachi SR16000 M1
  - IBM Blue Gene /Q

Current correlator

$$\Pi(x) \equiv \langle 0 | \mathrm{T}\, J(x) J(0) | 0 \rangle$$



Long distances
- Hadron spectrum from exp(-mt)
- non-perturbative, main use of LQCD

Short distances
- perturbation theory + power corrections (= OPE)
- α_s, quark mass, …

LQCD can calculate the both.

HPQCD (2008~) achieved precision m_c determination.

Lattice QCD

- Ab initio calculation of QCD
  → Use as an "experimental" facility of QCD

- Limitation due to finite lattice spacing
  - In order to match perturbation theory,

$$a \ll x \ll \Lambda_{\mathrm{QCD}}^{-1}$$

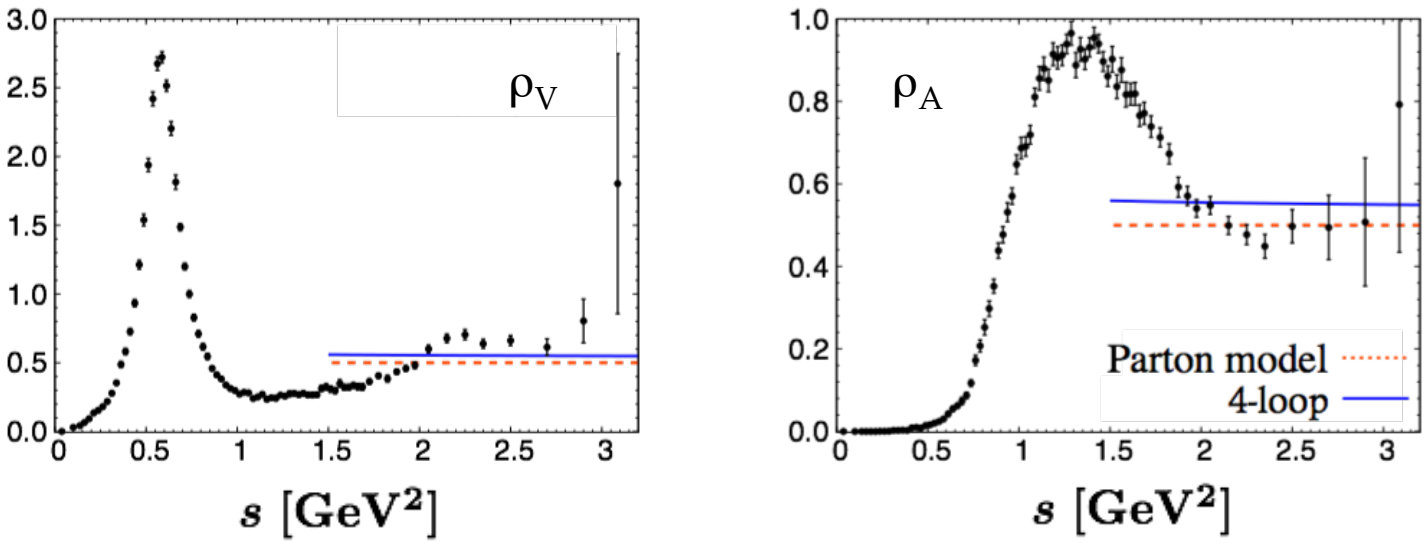
To avoid large discretization effects

To avoid too large non-perturbative effects

Need sufficiently small *a*. How small? See the data.

Observables at short distances

- Spectral function $\rho(s) \propto \mathrm{Im}\,\widetilde{\Pi}(s)$
  - available, e.g. from hadronic τ decays (ALEPH, 2013/14)

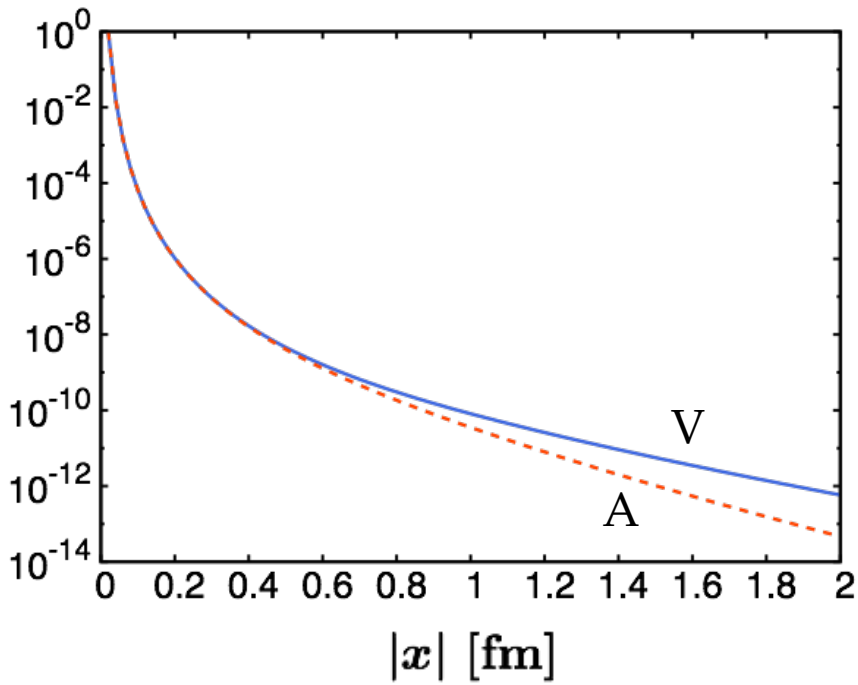


  - Resonances at low energy; perturbative at high energy
  - Testing QCD for the whole region? Yes, should be done.

On the coordinate space…

$$\Pi_{V/A}(x) = \frac{3}{8\pi^4}\int_0^\infty ds\; s^{3/2}\rho_{V/A}(s)\frac{K_1(\sqrt{s}|x|)}{|x|}$$

Schafer, Shuryak, 2001



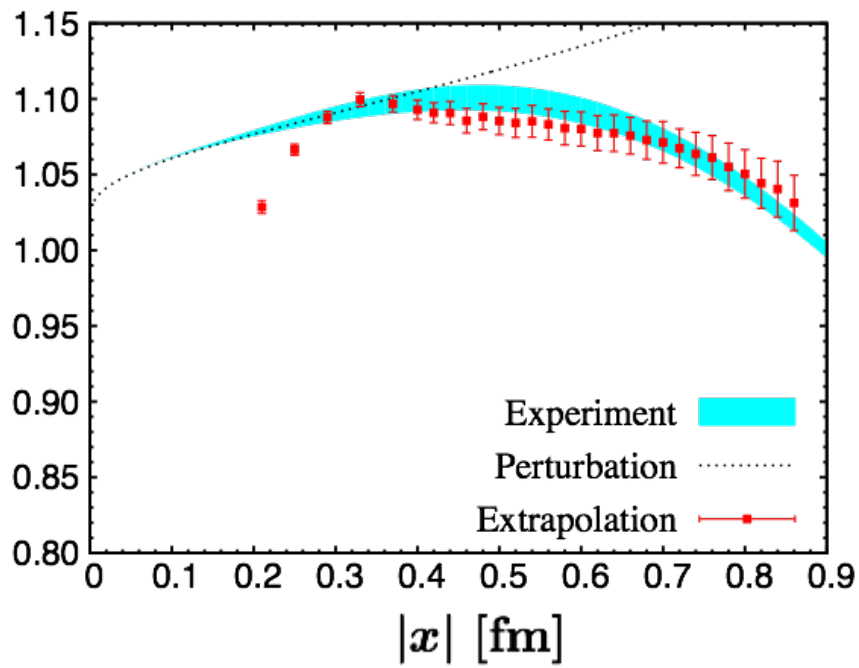
Lattice calculation

Tomii @ Lattice 2016

- on 2+1-flavor lattices with Mobius domain-wall fermion
- after taking the continuum limit with lattices at a = 0.08, 0.055, 0.044 fm.

V+A channel:

$$R_{V+A}(x) = \frac{\Pi_V(x) + \Pi_A(x)}{2\Pi_V^{\text{free}}(x)|_{m_q=0}}$$



Good agreement at |x| > 0.3 fm.

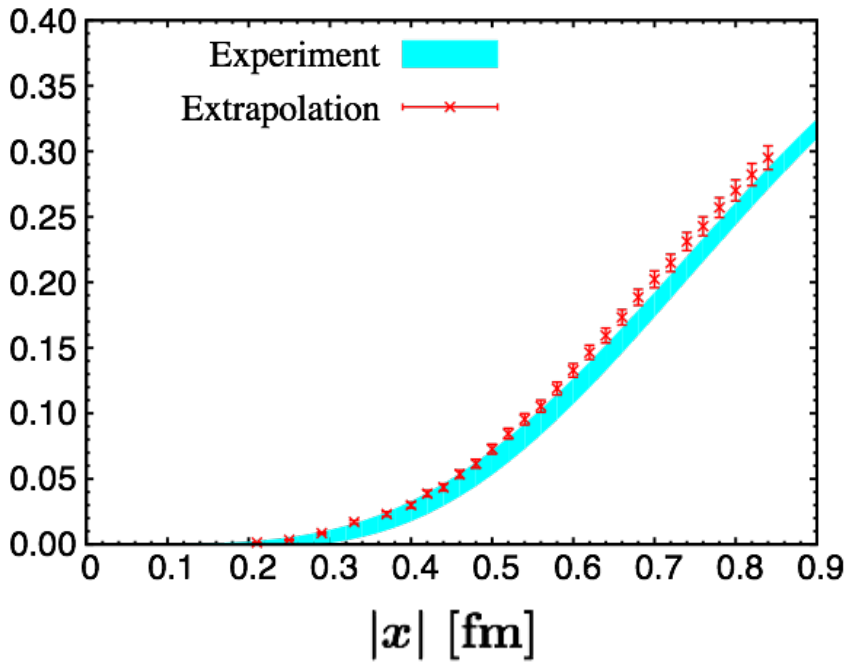
Lattice calculation

Tomii @ Lattice 2016

- on 2+1-flavor lattices with Mobius domain-wall fermion
- after taking the continuum limit with lattices at a = 0.08, 0.055, 0.044 fm.

V−A channel:

- vanishes in the perturbative regime.



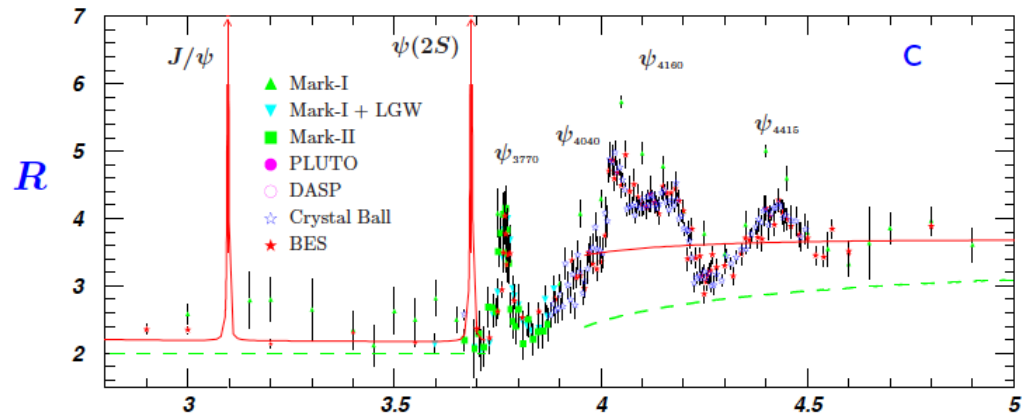
Good agreement at short distances.

Exp, Pert, Lattice

- Different sources of info:
  - **Exp**erimental data:
    - e^+e^- scattering, τ decays, …
  - **Pert**urbation theory:
    - useful at short distances
    - available up to $O(\alpha_s^3)$ or even $O(\alpha_s^4)$
  - **Lattice** calculation:
    - useful for any distances, in principle.
    - continuum limit is now a common practice
- Consistency
  - provides a test of QCD/Lattice
  - determination of parameters of QCD

Charmonium correlator

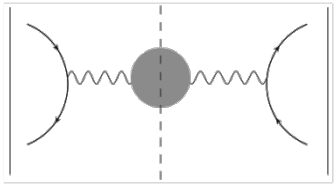
- Available from e^+e^-

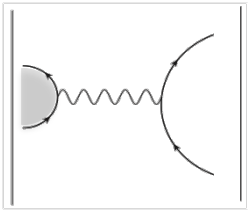


- Optical theorem:

$$(q_\mu q_\nu - q^2 g_{\mu\nu})\Pi_V^{(1)}(q^2) = i\int d^4x\, e^{ipx}\langle 0|T[j_\mu(x)j_\nu(0)]|0\rangle$$

$$\frac{1}{n!}\left(\frac{\partial}{\partial q^2}\right)^n (\Pi(q^2))_{q^2=0} = \frac{1}{12\pi Q_f^2}\int \mathrm{d}s\,\frac{1}{s^{n+1}}R(s)_{e^+e^-\to\ \mathrm{hadron}}$$





Temporal moments on the lattice

- Coordinate space

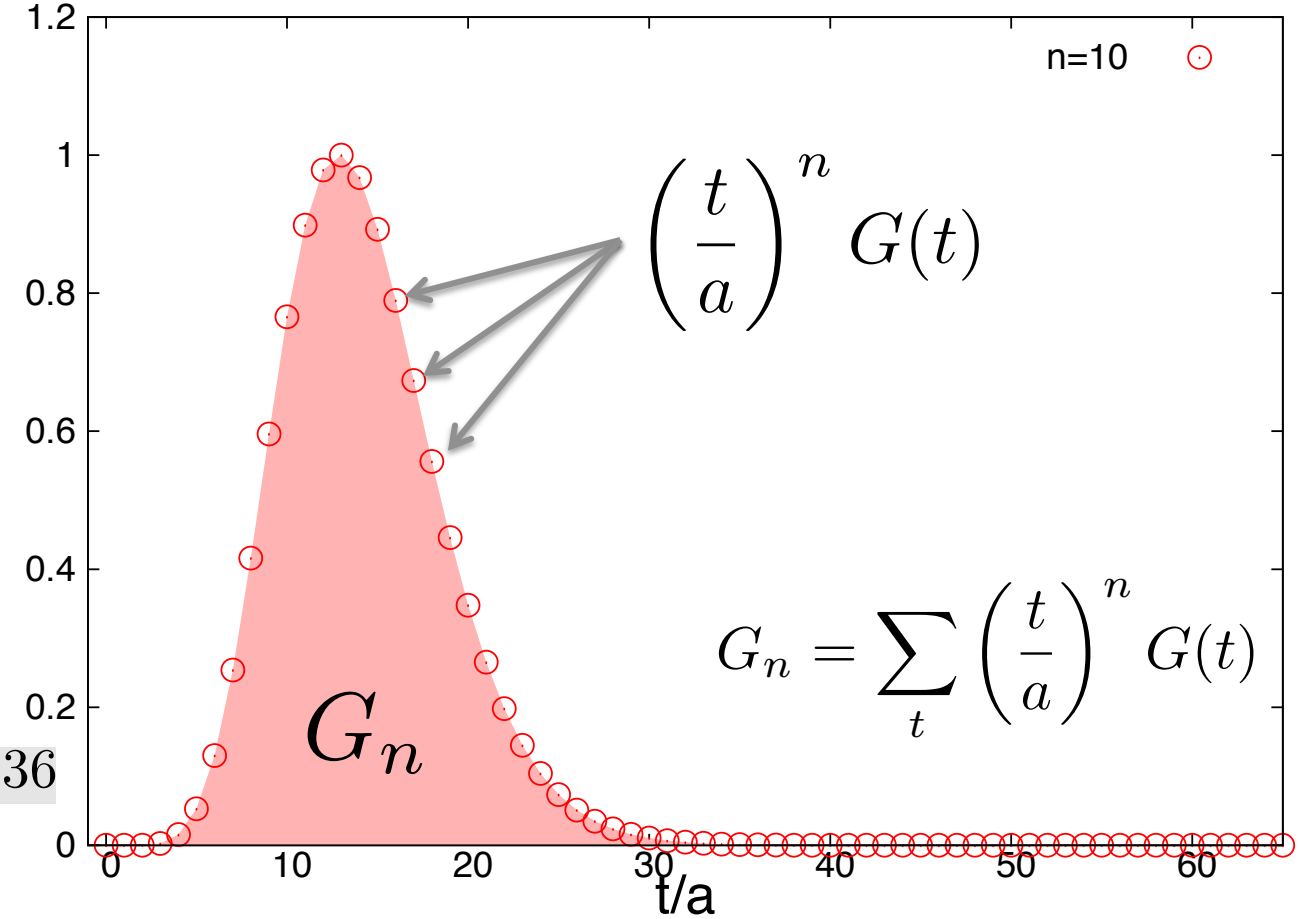
$$i\int \mathrm{d}x \frac{1}{n!}\left(\frac{\partial}{\partial q^2}\right)^n \mathrm{e}^{iqt} \longrightarrow a^4 \sum_x t^{2n} \qquad \text{(temporal moments)}$$

- On the Euclidean lattice,

$$G_V(t) = a^6 \sum_x \langle 0 | j_k(x,t) j_k(0,0) | 0 \rangle, \quad G_{V,n} = \sum_t (t/a)^n G_V(t)$$

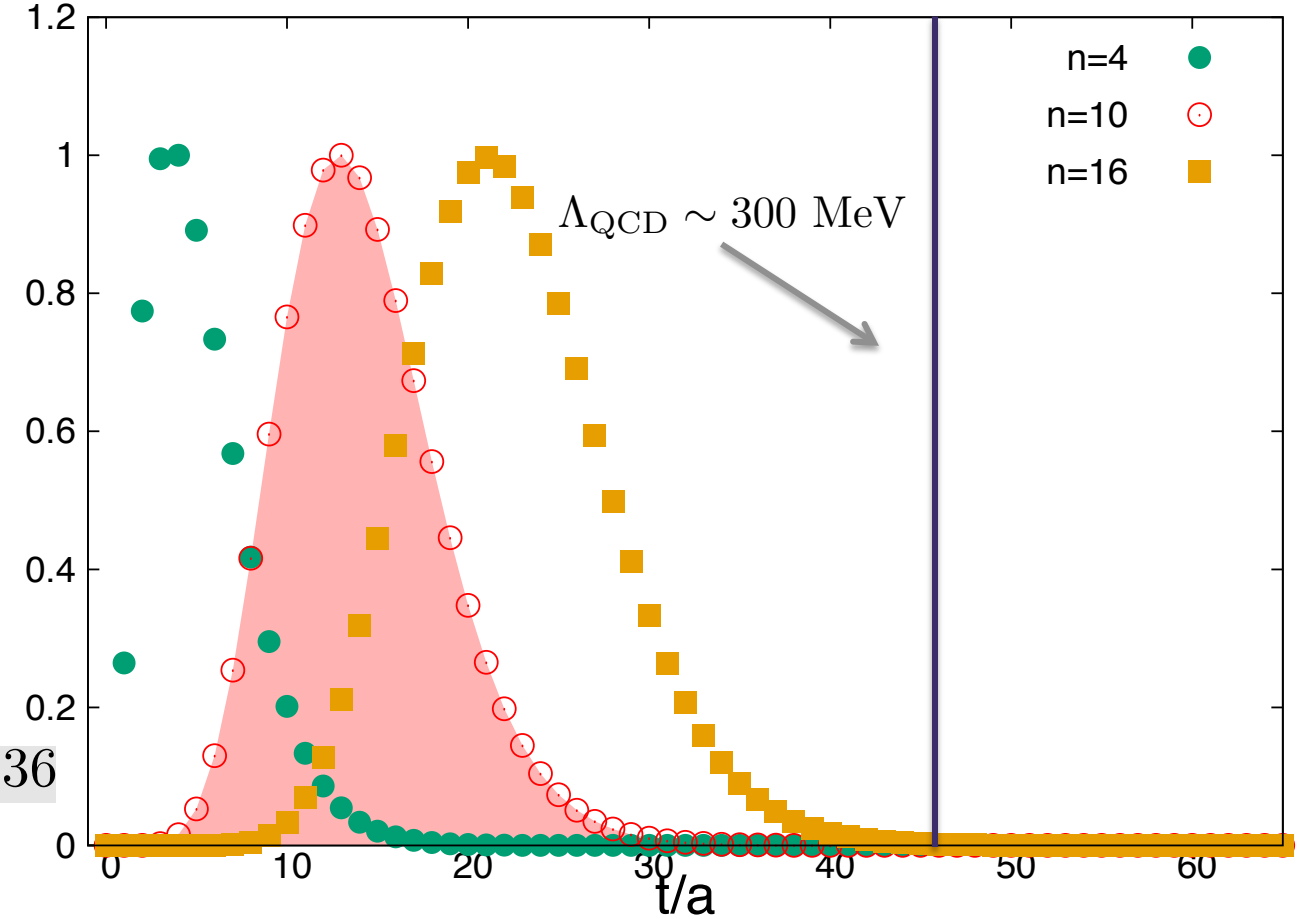
  - $G_V(t)$ represents a J/ψ correlator, ~exp(-$m_{J/\psi}$t), plus its excited states, continuum, etc.

Sum to define the moment



$am_{\eta_c} \sim 0.6636$

Sum to define the moment



$am_{\eta_c} \sim 0.6636$

Lattices with domain-wall

New set of lattice ensembles by JLQCD:

- 2+1 flavors of dynamical quarks described by domain-wall fermions
  - Good chiral symmetry on the lattice, m_{res} ~ 1 MeV or less.
- Fine lattices: $1/a$ ~ 2.4, 3.6, 4.6 GeV
  - Enough resolution to treat charm
- Sea quark mass coverage
  - m_π = 230 ~ 500 MeV
  - sandwich m_s

Lattices with domain-wall

$\beta = 4.17$, $1/a \sim 2.4$ GeV, $32^3 \times 64$ (x12)

m_{ud}	m_π [MeV]	MD time
$m_s = 0.030$		
0.007	310	10,000
0.012	410	10,000
0.019	510	10,000
$m_s = 0.040$		
0.0035	230	10,000
0.0035 ($48^3 \times 96$)	230	10,000
0.007	320	10,000
0.012	410	10,000
0.019	510	10,000

$\beta = 4.35$, $1/a \sim 3.6$ GeV, $48^3 \times 96$ (x8)

m_{ud}	m_π [MeV]	MD time
$m_s = 0.018$		
0.0042	300	10,000
0.0080	410	10,000
0.0120	500	10,000
$m_s = 0.025$		
0.0042	300	10,000
0.080	410	10,000
0.0120	510	10,000

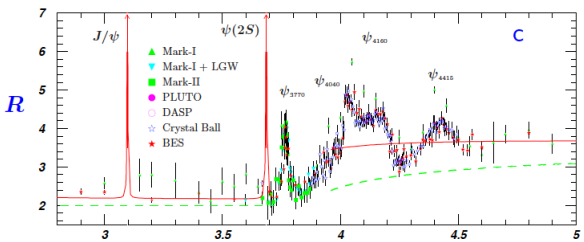
$\beta = 4.47$, $1/a \sim 4.6$ GeV, $64^3 \times 128$ (x8)

0.0030	~ 300	10,000

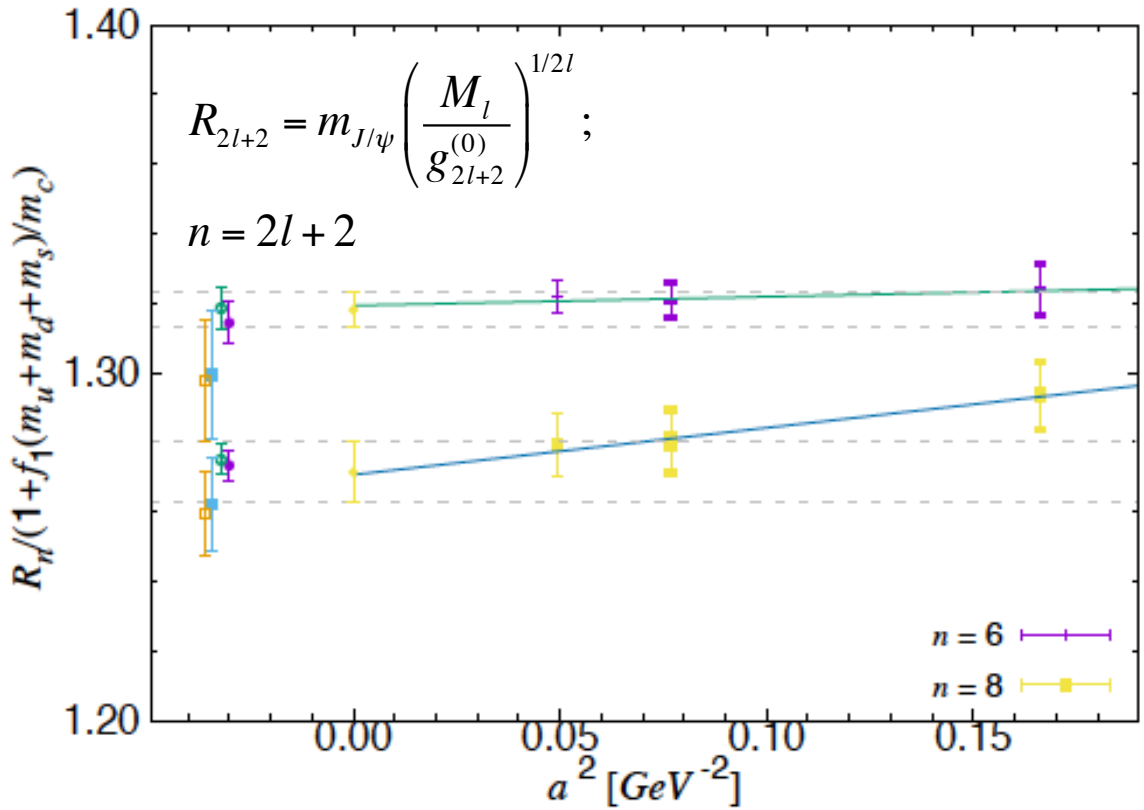
Lattice versus "exp+pheno"

Vector channel:

"exp" data from
- Dehnadi et al. (2015)
- Kuhn et al. (2007)



Lattice data after (perturbative) corrections of charm quark loop.



Determination of m_c and α_s

- Following the method of the pioneering work by the HPQCD + Karlsruhe group (2008~).
  - Allison et al., PRD78, 054513 (2008); McNeile et al., PRD82, 034512 (2010); Chakraborty et al., PRD91, 054508 (2015).

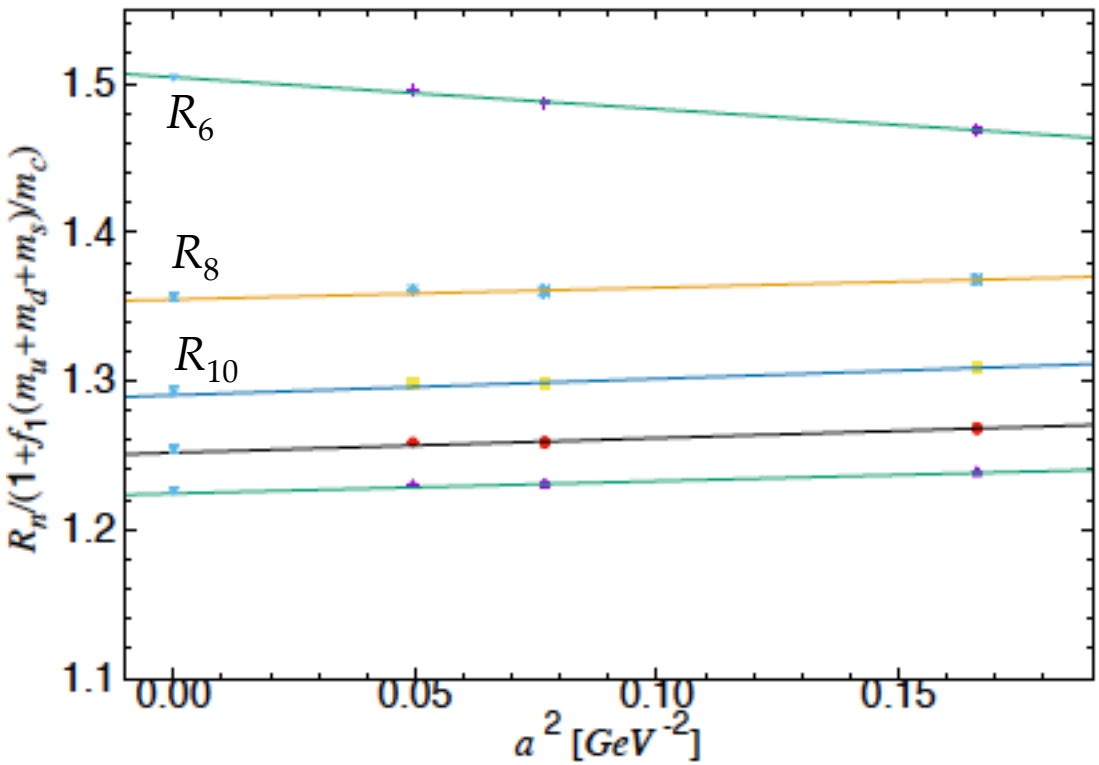
Lattice data
after continuum extrap

Continuum perturbation theory
known to α_s^3.

$$R_n = \frac{am_{\eta_c}^{(\mathrm{exp})}}{2a\bar{m}_c(\mu)} r_n(\mu; m_c(\mu), \alpha_s(\mu))$$

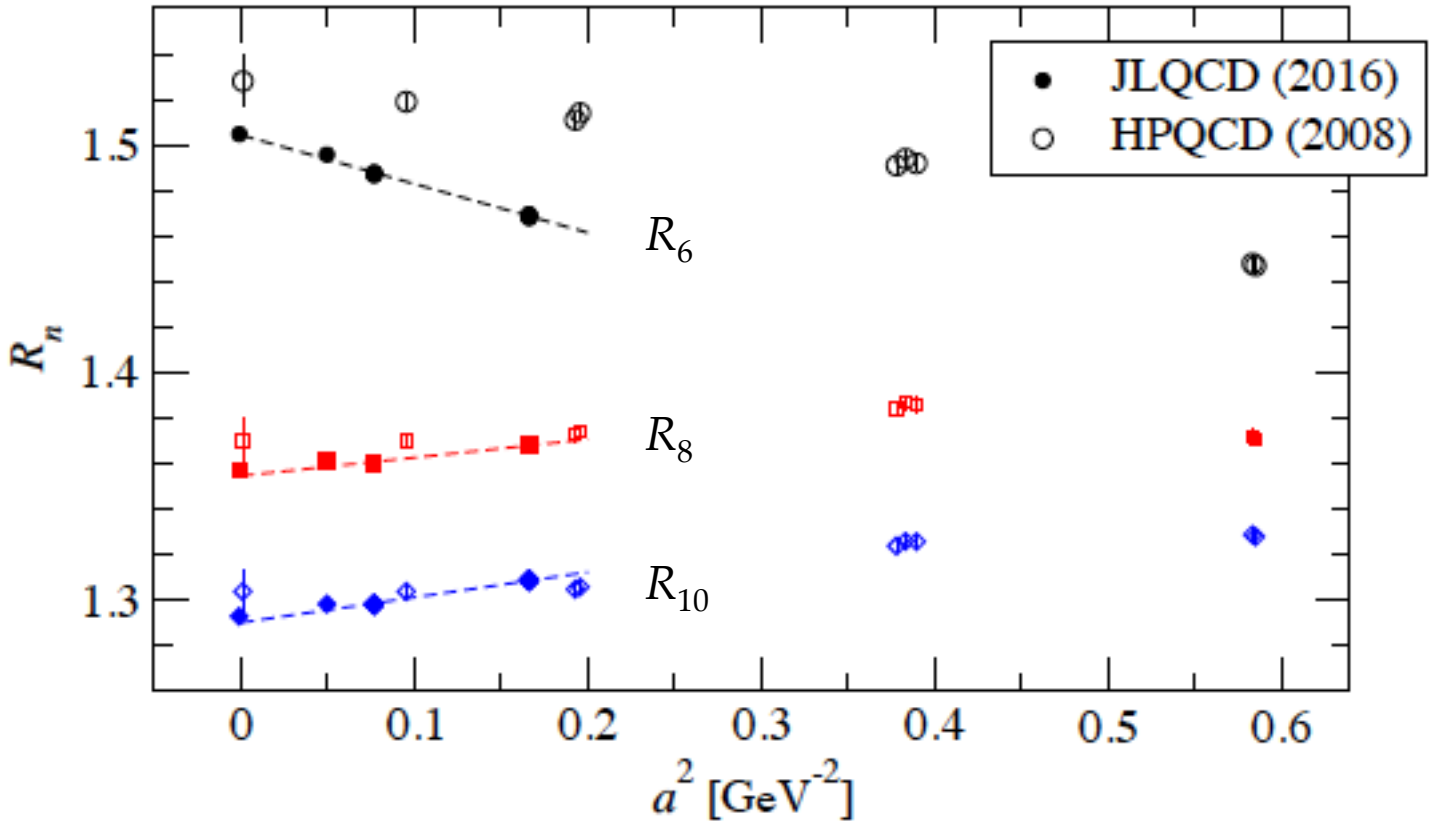
- Solve equations with different n's (n = 6, 8, 10, in this work).
- Use the pseudo-scalar density correlator for both lattice and pert. Perturbative coefficients known to α_s^3.

LHS: Continuum extrapolation of the lattice data



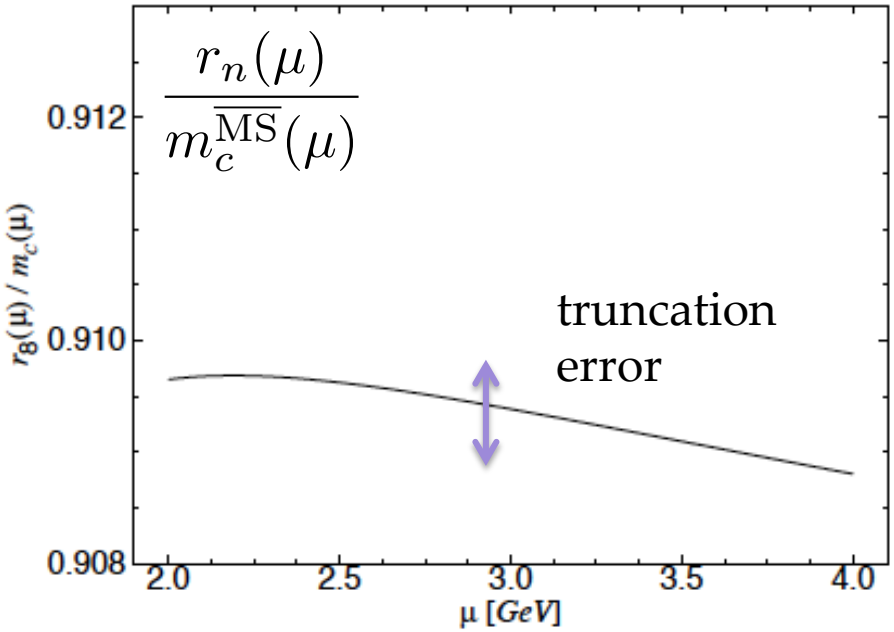
Sea quark mass dependence is invisible in this scale.
Fit with a linear function in $m_u+m_d+m_s$.

LHS: Continuum extrapolation of the lattice data



Larger slope than those of HPQCD with Highly Improved Staggered Quark (HISQ). Fine lattices are crucial.

RHS: Estimate of the perturbative truncation error.

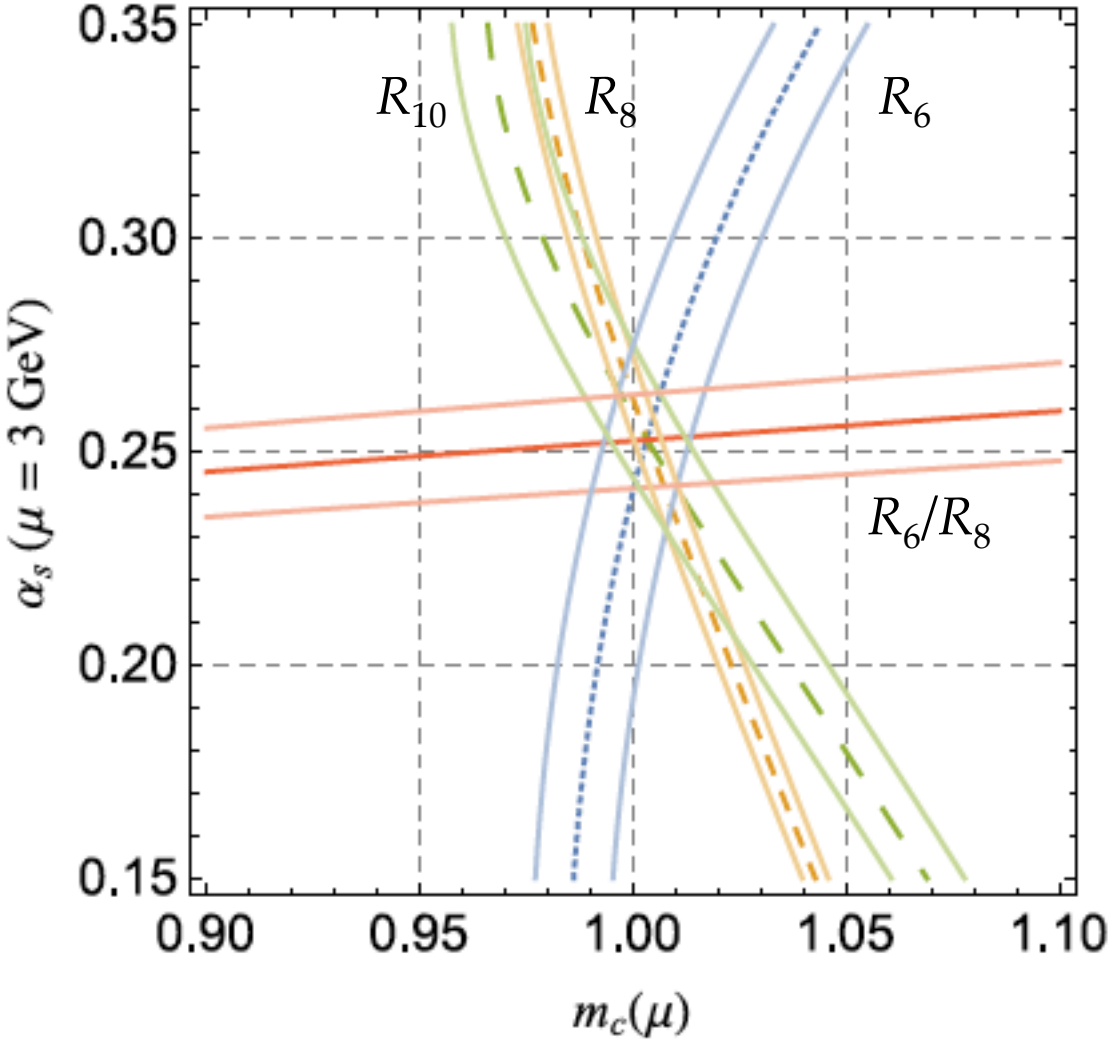


~ R_n: should be independent of μ.

Included up to $O(\alpha_s^3)$.
Also included the variation with $\mu_m \neq \mu_\alpha$.

$$
\begin{aligned}
(r_6)^2 = 1 & \\
& + \left(3.9 + 2.0\,\log\frac{m_c(\mu)^2}{\mu^2}\right)\frac{\alpha_s}{\pi} \\
& + \left(13.6 + 3.0\,\log\frac{m_c(\mu)^2}{\mu^2} - 0.08\left(\log\frac{m_c(\mu)^2}{\mu^2}\right)^2\right)\left(\frac{\alpha_s}{\pi}\right)^2 \\
& + \left(13.2 + 14.2\,\log\frac{m_c(\mu)^2}{\mu^2} + 1.03\left(\log\frac{m_c(\mu)^2}{\mu^2}\right)^2 + 0.06\left(\log\frac{m_c(\mu)^2}{\mu^2}\right)^3\right)\left(\frac{\alpha_s}{\pi}\right)^3
\end{aligned}
$$

$$R_n = \frac{m_{\eta_c}^{\rm exp}}{2m_c^{\overline{\rm MS}}} r_n(m_c^{\overline{\rm MS}}, \alpha_{\overline{\rm MS}})$$



Determine
$m_c(\mu)$, $\alpha_s(\mu)$, $\langle G_{\mu\nu}^2 \rangle$
with 3 moments.

$\alpha_s(\mu)$ is consistent, but its error is not competitive.

$m_c(3\ \mathrm{GeV}) = 1.003(10)\ \mathrm{GeV}$

Error budgets

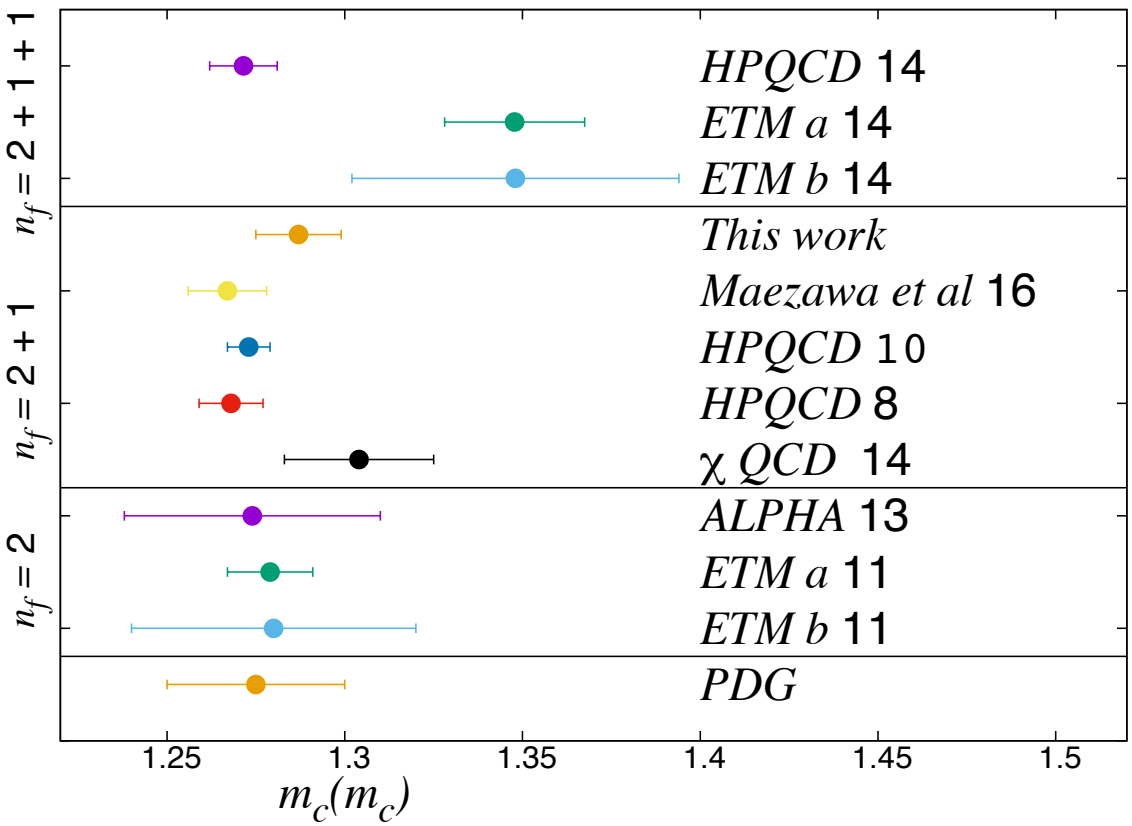
		pert	$t_0^{1/2}$	stat	$O(a^4)$	vol	$m_{\eta_c}^{\rm exp}$	disc	EM
$m_c(3\mathrm{GeV})$ [GeV]	1.0033(96)	(77)	(49)	(4)	(30)	(4)	(3)	(4)	(6)
$\alpha_s(3\mathrm{GeV})$	0.2528(127)	(120)	(32)	(2)	(26)	(1)	(0)	(0)	(1)

Dominant = truncation of perturbative expansion

lattice scale
Δa ~ 1%

discretization effect

Comparison


$n_f = 2+1+1$
$n_f = 2+1$
$n_f = 2$
HPQCD 14
ETM a 14
ETM b 14
This work
Maezawa et al 16
HPQCD 10
HPQCD 8
χ QCD 14
ALPHA 13
ETM a 11
ETM b 11
PDG
1.25
1.3
1.35
1.4
1.45
1.5
$m_c(m_c)$

Summary

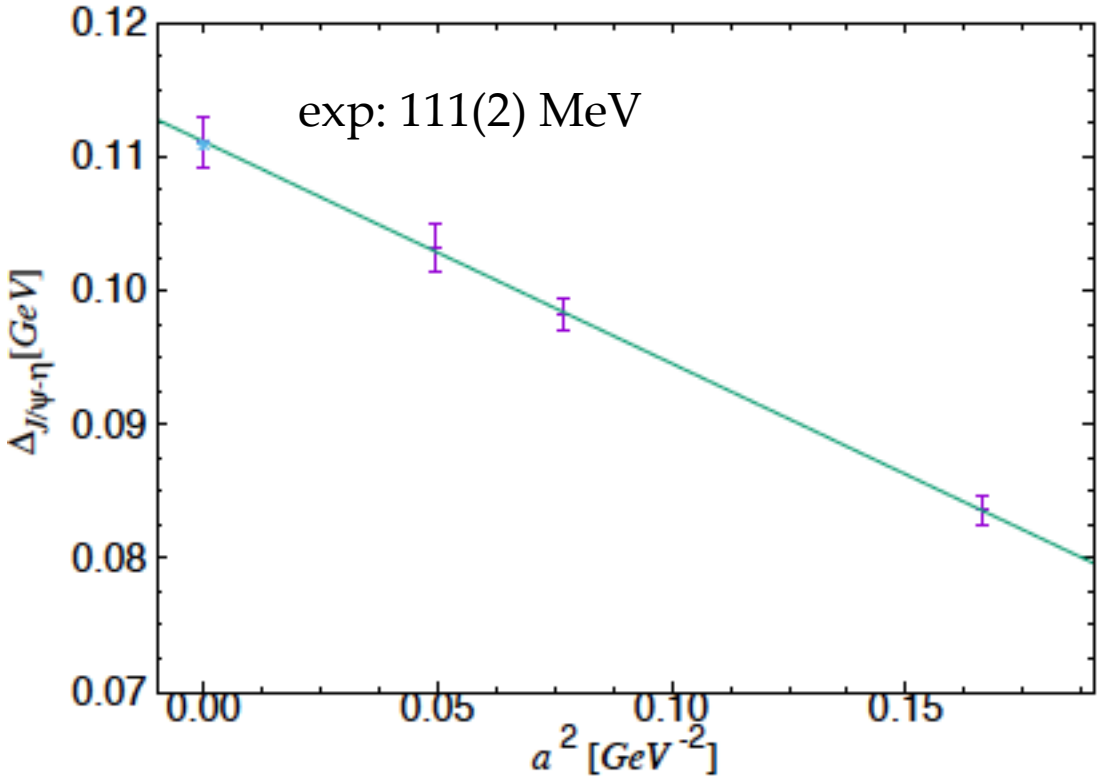
- Test of LQCD at short distances
  - For light-hadron and charmonium correlators.
  - Fine lattices ($1/a$ ~ 3-4 GeV) are necessary to keep the discretization errors under control.
  - Lattice results are consistent with the experimental data, also at short distances.

- Determination of $m_c(\mu)$
  - LQCD may provide an extra input (pseudo-scalar channel) to feed in the perturbative analysis.
  - m_c and α_s obtained with similar size of error as HPQCD.

backup

LHS: Test of the discretization effect

Hyperfine splitting $J/\psi - \eta_c$ is sensitive to the discretization effect.



consistent after extrapolation

RHS: Non-perturbative correction

$$r_n^{n-4} = \frac{1}{C_{n/2-1}^{(0)}} \left(C_{n/2-1} + \frac{16\pi^2}{3} \frac{\langle (\alpha_s/\pi) G_{\mu\nu}^2 \rangle}{(2m_{\rm OS})^4} \left(a_{n/2} + \frac{\alpha_s}{\pi} c_{n/2} \right) \right)$$

perturbative expansion

gluon condensate from OPE
No precise determination available.

$\langle (\alpha_s/\pi) G_{\mu\nu}{}^2 \rangle \sim 0.00\text{-}0.02\ \mathrm{GeV}^4$

Keep it as a free parameter and determine with the charmonium moments. Result:

$\langle (\alpha_s/\pi) G_{\mu\nu}{}^2 \rangle \sim 0.00(1)\ \mathrm{GeV}^4$

From the vector channel: