Nucleon matrix elements and charges in lattice QCD

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European Twisted Mass Collaboration

HPC-LEAP
EUROPEAN JOINT DOCTORATES

Outline

1. Wilson twisted mass lattice QCD
2. Nucleon matrix elements
3. Nucleon charges: g_A, g_s, g_T
4. Nucleon σ-terms
5. Conclusions

Wilson twisted mass lattice QCD

Simulations by the European Twisted Mass Collaboration (ETMC)

We report on the analysis of an $N_f = 2$ ensemble of twisted mass plus a clover term simulated at a physical value of the pion mass, referred as the *Physical ensemble*, (ETMC) A. Abdel-Rehim *et al.* :1507.04936, 1507.05068, 1411.6842, 1311.4522

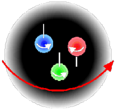
Parameters: lattice size $48^3 \times 96$, $a = 0.093(1)$ fm, $m_\pi = 0.1312(13)$ GeV

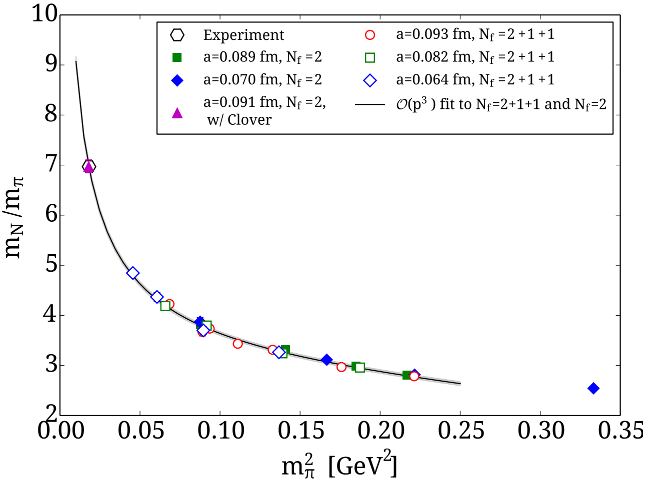
Wilson tmQCD at maximal twist, R. Frezzotti, G. C. Rossi, JHEP 0408 (2004) 007

- Automatic $O(a)$ improvement
- No operator improvement needed, renormalization simplified $\rightarrow$ important for hadron structure

- Use exact deflation to speed-up the inversions and do multiple sources on each gauge configuration
- Use domain decomposition multi-grid (DD-αAMG) adapted for twisted mass fermions $\rightarrow$ cost of inversions the same as at $m_\pi = 300$ MeV, S. Bacchio, J. Finkenrath, A. Frommer

The nucleon



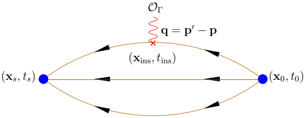


- Cut-off effects small for these lattice spacings
- LO fit with $m_\pi < 375$ MeV does not include the physical point
- Determine lattice spacing using the $\mathcal{O}(p^3)$ result

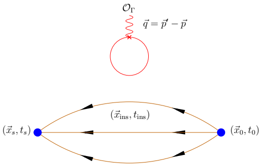
Evaluation of matrix elements

Three-point functions:

$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\rm ins}) = \sum_{\vec{x}_s, \vec{x}_{\rm ins}} e^{i\vec{x}_{\rm ins}\cdot\vec{q}}\, \Gamma_{\beta\alpha}\, \langle J_\alpha(\vec{x}_s, t_s)\mathcal{O}_\Gamma^{\mu\nu}(\vec{x}_{\rm ins}, t_{\rm ins})\bar{J}_\beta(\vec{x}_0, t_0)\rangle$$

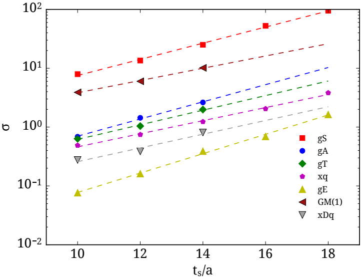


connected contribution



disconnected contribution

$$R(t_s, t_{\rm ins}, t_0) \xrightarrow[(t_s - t_{\rm ins})\Delta \gg 1]{(t_{\rm ins} - t_0)\Delta \gg 1} \mathcal{M}[1 + \ldots e^{-\Delta(\mathbf{p})(t_{\rm ins} - t_0)} + \ldots e^{-\Delta(\mathbf{p}')(t_s - t_{\rm ins})}]$$



- $\mathcal{M}$ the desired matrix element
- t_s, $t_{\rm ins}$, t_0 the sink, insertion and source time-slices
- $\Delta(\mathbf{p})$ the energy gap with the first excited state

To ensure ground state dominance need multiple sink-source time separations ranging from 0.9 fm to 1.5 fm

Extracting nucleon matrix elements

- Plateau method:

$$R(t_s, t_{\rm ins}, t_0) \xrightarrow[(t_s - t_{\rm ins})\Delta \gg 1]{(t_{\rm ins} - t_0)\Delta \gg 1} \mathcal{M}[1 + \ldots e^{-\Delta(\mathbf{p})(t_{\rm ins}-t_0)} + \ldots e^{-\Delta(\mathbf{p}')(t_s - t_{\rm ins})}]$$

  - $\mathcal{M}$ the desired matrix element
  - $t_s, t_{\rm ins}, t_0$ the sink, insertion and source time-slices
  - $\Delta(\mathbf{p})$ the energy gap with the first excited state

- Excited states contributions are different for different operators and pion mass $\rightarrow$ need to carefully check
- Need to include disconnected contributions unless shown to be negligible

- Summation method: Summing over $t_{\rm ins}$:

$$\sum_{t_{\rm ins}=t_0}^{t_s} R(t_s, t_{\rm ins}, t_0) = \text{Const.} + \mathcal{M}[(t_s - t_0) + \mathcal{O}(e^{-\Delta(\mathbf{p})(t_s - t_0)}) + \mathcal{O}(e^{-\Delta(\mathbf{p}')(t_s - t_0)})].$$

  Excited state contributions are suppressed by exponentials decaying with $t_s - t_0$, rather than $t_s - t_{\rm ins}$ and/or $t_{\rm ins} - t_0$
  However, one needs to fit the slope rather than to a constant or take differences and then fit to a constant
  L. Maiani, G. Martinelli, M. L. Paciello, and B. Taglienti, Nucl. Phys. B293, 420 (1987); S. Capitani *et al.*, arXiv:1205.0180

- Fit keeping the first excited state, T. Bhattacharya *et al.*, arXiv:1306.5435

All should yield the same answer in the end of the day!

Nucleon charges: g_A, g_s, g_T

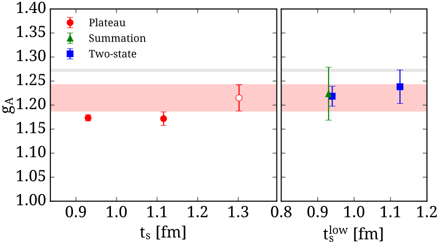
- scalar operator: $\mathcal{O}_S^a = \bar{\psi}(x)\frac{\tau^a}{2}\psi(x)$
- axial-vector operator: $\mathcal{O}_A^a = \bar{\psi}(x)\gamma^\mu\gamma_5\frac{\tau^a}{2}\psi(x)$
- tensor operator: $\mathcal{O}_T^a = \bar{\psi}(x)\sigma^{\mu\nu}\frac{\tau^a}{2}\psi(x)$

$\Longrightarrow$ extract from ratio: $\langle N(\vec{p}')\mathcal{O}_X N(\vec{p})\rangle|_{q^2=0}$ to obtain g_s, g_A, g_T

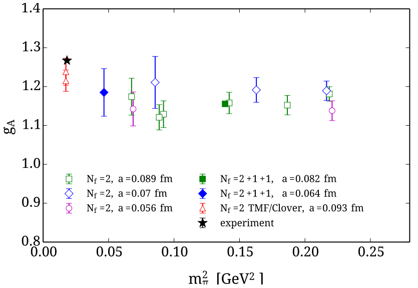
- isovector combination has no disconnected contributions;
- g_A well-known experimentally;
- Predict transverse spin g_T, to be measured at JLab;
- Predict g_S relevant for BSM β-decay

Nucleon charges: g_A

- $N_f = 2$ twisted mass plus clover, $48^3 \times 96$, $a = 0.093(1)$ fm, $m_\pi = 131$ MeV
- 9264 statistics
- 3 sink-source time separations ranging from 0.9 fm to 1.5 fm



Isovector axial charge (t_s is the sink-source time separation and t_s^{low} is the lowest value of t_s used in the fits)



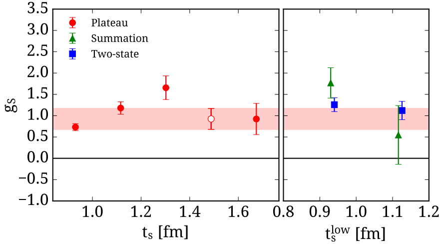
Results using twisted mass fermions

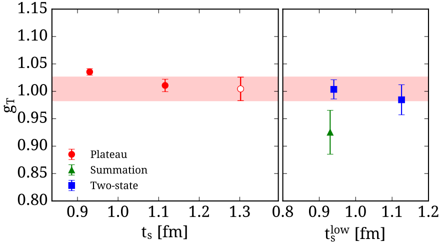
At the physical point we find from the plateau method: $g_A = 1.22(3)(2)$, where the first error is statistical and the second systematic determined by the difference between the values from the plateau and two-state fits.

A. Abdel-Rehim *et al.* (ETMC):1507.04936, 1507.05068, 1411.6842, 1311.4522

Nucleon charges: g_S, g_T

Updated results using $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1)$ fm with ~ 9260 statistics for $t_s/a = 10, 12, 14$, ~ 48000 for $t_s/a = 16$ and ~ 70000 for $t_s/a = 18$.





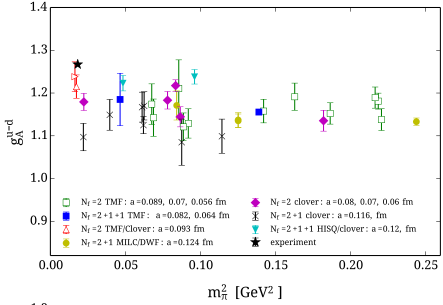
At the physical point we find from the plateau method:

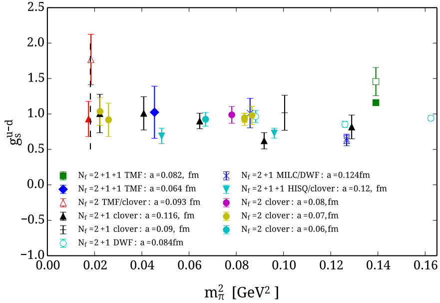
- $g_S^{\text{isov}} = 0.93(25)(33)$
- $g_T^{\text{isov}} = 1.00(2)(1)$

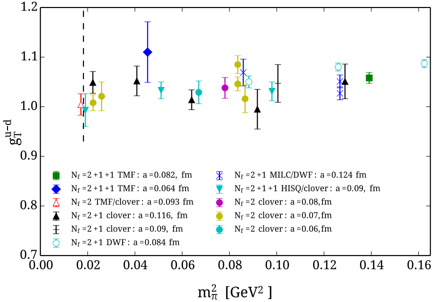
where the first error is statistical and the second systematic determined by the difference between the values from the plateau and two-state fits.

Summary of results on nucleon charges: g_A, g_S, g_T

Isovector



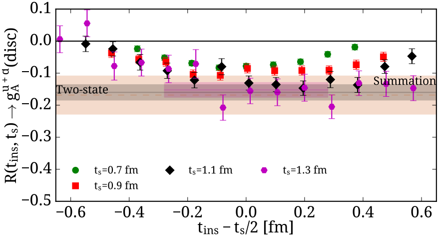




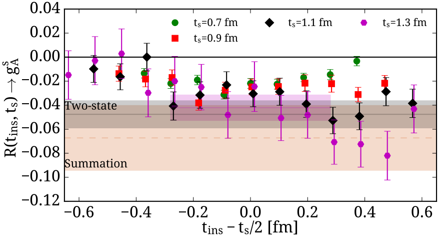
- g_A at the physical point requires further study for larger t_s. Important to keep constant error → we need large statistics
- New analysis of COMPASS and Belle data: $g_T^{u-d} = 0.81(44)$, M. R. A. Courtoy, A. Bacchettad, M. Guagnellia, arXiv: 1503.03495
- For g_S increasing the sink-source time separation to ~ 1.5 fm is crucial but more statistics are needed to settle its value.

Disconnected contributions to g_A^q

Updated results using $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1)$ fm



Disconnected isoscalar axial charge



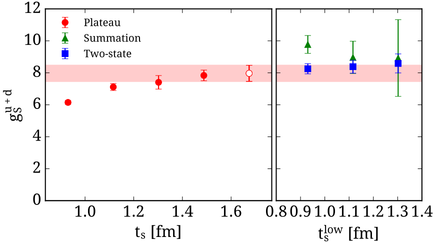
Strange axial charge

We find from the plateau method:

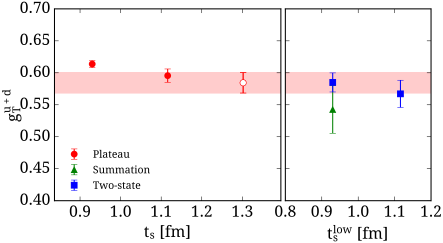
- $g_A^{u+d} = -0.15(2)$ with 854,400 statistics
- Combining with the isovector we find: $g_A^u = 0.828(21)$, $g_A^d = -0.387(21)$
- $g_A^s = -0.042(10)$ with 861,200 statistics

Scalar g_S and Tensor g_T charges

Updated results using $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1)$ fm with ~ 9260 statistics for $t_s/a = 10, 12, 14$, ~ 48000 for $t_s/a = 16$ and ~ 70000 for $t_s/a = 18$.



Connected isoscalar scalar charge, $\overline{\text{MS}}$ at 2 GeV



Connected isoscalar tensor charge, $\overline{\text{MS}}$ at 2 GeV

At the physical point we find from the plateau method:

- g_S^{u+d}= 8.25(51)(13) (conn);
- g_T^{u+d}=0.584(16)(17) (conn);

where the first error is statistical and the second error on the connected is the systematic determined by the difference between the values from the plateau and two-state fits.

A. Abdel-Rehim *et al.* (ETMC):1507.04936, 1507.05068, 1411.6842, 1311.4522

Scalar g_S and Tensor g_T charges

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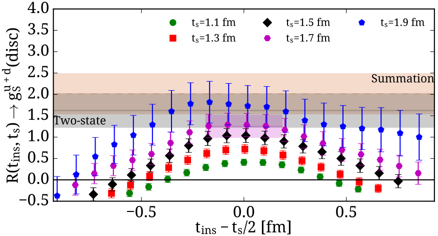

Disconnected isoscalar scalar charge, $\overline{\text{MS}}$ at 2 GeV

At the physical point we find from the plateau method:

- g_S^{u+d}= 8.25(51)(13) (conn); 1.25(26) (disconn) $\rightarrow$ $g_S^u = 5.21(31)$, $g_S^d = 4.28(31)$
- g_T^{u+d}=0.584(16)(17) (conn); 0.0007(11) (disconn) $\rightarrow$ $g_T^u = 0.795(13)$, $g_T^d = -0.210(13)$

where the first error is statistical and the second error on the connected is the systematic determined by the difference between the values from the plateau and two-state fits.

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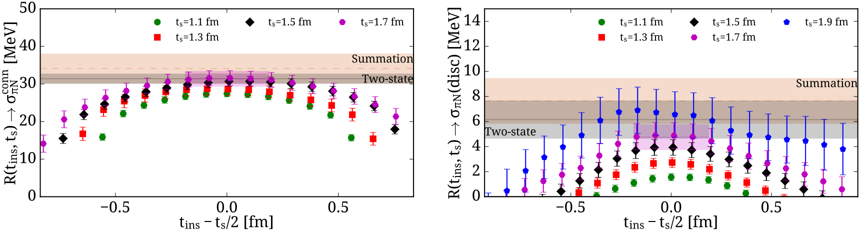
The quark content of the nucleon

- $\sigma_f \equiv m_f \langle N|\bar{q}_f q_f|N\rangle$: measures the explicit breaking of chiral symmetry
  Largest uncertainty in interpreting experiments for dark matter searches - Higgs-nucleon coupling depends on σ, J. Ellis, K. Olive, C. Savage, arXiv:0801.3656
- In lattice QCD:
  - Feynman-Hellmann theorem: $\sigma_l = m_l \frac{\partial m_N}{\partial m_l}$
    Similarly $\sigma_s = m_s \frac{\partial m_N}{\partial m_s}$
  - Direct computation of the scalar matrix element, A. Abdel-Rehim *et al.* arXiv:1601.3656, PRL116 (2016) 252001



With our increased statistics we find $\sigma_{\pi N} = 36(2)$ MeV, $\sigma_s = 37(8)$ MeV, $\sigma_c = 83(17)$ MeV

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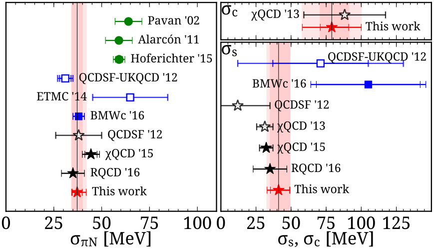

With our increased statistics we find $\sigma_{\pi N} = 36(2)$ MeV, $\sigma_s = 37(8)$ MeV, $\sigma_c = 83(17)$ MeV

Conclusions

- Results at the physical point are now directly accessible
  High statistics and careful cross-checks are needed → noise reduction techniques are crucial e.g. AMA, TSM, smearing etc
- Evaluation of quark loop diagrams has become feasible even at the physical point!
- Confirmation of experimentally known quantities such as g_A will enable reliable predictions of others → provide insight into the structure of hadrons and input that is crucial for new physics such as the nucleon σ-terms, g_s and g_T

Thank you for your attention