We consider the effect of an external magnetic field on the behavior of strongly interacting matter in the context of nonlocal NJL-type models, that are extensions of the NJL model that represent a step towards a more realistic modeling of QCD. We find that at zero temperature the condensates display the well-known Magnetic Catalysis effect with predictions in good quantitative agreement with lattice QCD (LQCD). When extended to finite T we find that, contrary to what happens in the standard local NJL model, our results show that the Inverse Magnetic Catalysis effect is naturally incorporated. We also analyze the magnetic susceptibility of the QCD vacuum considering two different model parametrizations, and compare our numerical results to those obtained in other theoretical approaches and in LQCD calculations.

Magnetic susceptibility of the QCD vacuum

A constant external EM field induces the existence of other nonvanishing condensates, which describe the response of the QCD vacuum to the source. We studied, in the context of a nNJL model that also includes wave function renormalization (WFR) [4], the VEV of the tensor polarization operator \( \langle \bar{\psi} \sigma_{\mu\nu} \psi \rangle \), where \( \sigma_{\mu\nu} = i [\gamma_{\mu}, \gamma_{\nu}] / 2 \) is the relativistic spin operator. To leading order in the external field one has

\[
\langle \bar{\psi} \sigma_{\mu\nu} \psi \rangle / \Lambda^4 = qF_B r_T,
\]

where \( F_B \) is the field strength tensor and \( r_T \) is the so-called tensor coefficient, that within our model reads

\[
r_T = 4N_c \int \frac{d^2 p}{(2\pi)^2} Z(p) M_3(p) \mu^2 M(p)^2 / \mu^2 M_3(p)^2.
\]

Here \( M_3(p) \) is the effective mass and \( Z(p) \) the WFR. In the corresponding extension to finite T we also included a coupling to the Polyakov loop with different PL thermodynamical potentials found in the literature [6-9].

Formalism

We begin by stating the Euclidean action for the simplest nonlocal chiral quark model in the case of two light flavours

\[
S_E = \int d^4 x \left\{ \phi(x)(-i\hat{\gamma}_\mu + m_0)\phi(x) - \frac{G}{2} \phi(x)\phi(x) \right\},
\]

where \( m_0 \) is the current quark mass, assumed to be equal for \( u \) and \( d \) quarks, and the nonlocal currents \( j_\mu(x) \) are given by

\[
j_\mu(x) = \int d^4 y \, \phi^\dagger(y) \frac{\hat{\gamma}_\mu}{2} \phi(y) \Gamma \phi(x - x/y).
\]

Here \( \Gamma \) is the Fourier transform of \( \phi(z) \), \( L_\mu(x) \) is the Laguerre polynomials, \( s_y = \log(i y) \), and \( k \) labels the Landau levels. We extend this result to finite T using the Matsubara formalism. In this way, the corresponding MAR thermodynamical potential \( \Pi(T) \) and gap eq can be obtained. By deriving \( \Pi(T) \) with respect to the current quark masses we get the magnetic field dependent quark condensate for each flavor

\[
\langle \bar{\psi} \gamma_\mu \psi \rangle / T = \frac{N_c}{2\pi} \int_0^\infty d\rho \frac{\rho^3}{\cosh(\rho T)} \int_0^\infty \frac{d\rho_\perp}{\rho_\perp} \rho_\perp^2 \left[ M_2^2(\mu) - M_2^2(\mu_0) \right],
\]

where \( M_2(\mu) \) is the Fourier transform of \( q(x) \), \( L_\mu(x) \) are the Laguerre polynomials, \( s_y = \log(i y) \), and \( k \) labels the Landau levels. We extend this result to finite T using the Matsubara formalism. In this way, the corresponding MAR thermodynamical potential \( \Pi(T) \) and gap eq can be obtained. By deriving \( \Pi(T) \) with respect to the current quark masses we get the magnetic field dependent quark condensate for each flavor

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\]

As customary in nonlocal models this quantity tends to diverge [2]. We regularize it according to

\[
\langle \bar{\psi} \gamma_\mu \psi \rangle / T = \frac{N_c}{2\pi} \int_0^\infty d\rho \frac{\rho^3}{\cosh(\rho T)} \int_0^\infty \frac{d\rho_\perp}{\rho_\perp} \rho_\perp^2 \left[ M_2^2(\mu) - M_2^2(\mu_0) \right],
\]

for both light and strange quarks. For all the parametrization considered a decrease of the gap \( \Delta(\mu) \) is observed as \( T \) increases. As the temperature increases the Higgs condensate becomes more important as a source of mass, and the magnetic field induces a new condensate, that is the magnetic catalysis effect, that is, a decrease of the gap \( \Delta(\mu) \) as \( T \) increases, with predictions in good quantitative agreement with lattice QCD (LQCD). When extended to finite T we find that, contrary to what happens in the standard local NJL model, our results show that the Inverse Magnetic Catalysis effect is naturally incorporated. We also analyze the magnetic susceptibility of the QCD vacuum considering two different model parametrizations, and compare our numerical results to those obtained in other theoretical approaches and in LQCD calculations.

References