Strongly interacting matter under strong magnetic fields within non-local NJL-type models

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Abstract

We consider the effect of an external magnetic field on the behavior of strongly interacting matter in the context of nonlocal NJL-type models, that are extensions of the NJL model that represent a step towards a more realistic modeling of QCD. We find that at zero temperature the condensates display the well-known Magnetic Catalysis effect with predictions in good quantitative agreement with lattice QCD (LQCD). When extended to finite T we find that, contrary to what happens in the standard local NJL model, our results show that the Inverse Magnetic Catalysis effect is naturally incorporated. We also analyze the magnetic susceptibility of the QCD vacuum considering two different model parametrizations, and compare our numerical results to those obtained in other theoretical approaches and in LQCD calculations.

Formalism

We begin by stating the Euclidean action for the simplest nonlocal chiral quark model in the case of two light flavours

$$S_E = \int d^4x \left\{ \bar\psi(x)\left(-i\,\partial\!\!\!/ + m_c\right)\psi(x) - \frac{G}{2} j_a(x) j_a(x) \right\} , \qquad (1)$$

where m_c is the current quark mass, assumed to be equal for u and d quarks, and the nonlocal currents $j_a(x)$ are given by

$$j_a(x) = \int d^4z\, \mathcal{G}(z)\, \bar\psi(x+\frac{z}{2})\, \Gamma_a\, \psi(x-\frac{z}{2}) \, . \qquad (2)$$

Here $\Gamma_a = (1\!\!1, i\gamma_5\vec\tau)$, and $\mathcal{G}(z)$ is a nonlocal form factor that characterizes the effective interaction. In order to study the influence of a magnetic field, we introduce in Eq. (1) a coupling to an external electromagnetic (EM) gauge field $\mathcal{A}_\mu$. This can be done by performing the replacement $\partial_\mu \to \partial_\mu + i\,\hat Q \mathcal{A}_\mu(x)$, where $\hat Q = \mathrm{diag}(q_u, q_d)$, with $q_u/2 = -q_d = e/3$, is the EM quark charge operator, and also the appropriate change in the nonlocal currents appearing in the interaction terms,

$$\psi(x - z/2) \to W(x, x - z/2)\ \psi(x - z/2) \, , \qquad (3)$$

and a related change for $\bar\psi(x+z/2)$. Here $W(s,t) \; = \; \mathrm{P}\ \exp\left[i \int_s^t dr_\mu\, \hat Q \mathcal{A}_\mu(r)\right]$, where r runs over an arbitrary path connecting s with t that we take to be the straight line path.

At this point we perform a standard bosonization, introducing scalar and pseudoscalar fields $\sigma(x)$ and $\vec\pi(x)$. We consider the case of a constant and homogeneous magnetic field along the 3− axis, choosing the Landau gauge. In the mean field approximation (MFA) the pseudoscalar fields vanish and we assume $\bar\sigma$ is independent of x. Using the Ritus eigenfunction method [1] it is possible to write

$$\frac{S^{\rm MFA}_{\rm bos}}{V^{(4)}} = \frac{\bar\sigma^2}{2G} - N_c \sum_{f=u,d} \frac{|q_f B|}{2\pi} \int \frac{d^2\bar p}{(2\pi)^2} \left\{ \ln\left[\bar p^2 + M^{s_f,f\,2}_{\bar p,0}\right] + \sum_{k=1}^{\infty} \ln\left[\left(2k|q_fB| + \bar p^2 + M^{-s_f,f}_{\bar p,k} M^{s_f,f}_{\bar p,k}\right)^2 + \bar p^2\left(M^{s_f,f}_{\bar p,k} - M^{-s_f,f}_{\bar p,k}\right)^2\right] \right\}, \qquad (4)$$

$$M^{\lambda,f}_{\bar p,k} = (-1)^{k-\frac{1-s_f\lambda}{2}} \int_0^\infty dr\, r\ \exp(-r^2/2)\left[m_c + \bar\sigma\, g\left(\frac{|q_fB|}{2}r^2 + \bar p^2\right)\right] L_{k-\frac{1-s_f\lambda}{2}}(r^2) \qquad (5)$$

Here, $g(p^2)$ is the Fourier transform of $\mathcal{G}(z)$, $L_k(x)$ are the Laguerre polynomials, $s_f = \mathrm{sign}(q_fB)$, and k labels the Landau levels. We extend this result to finite T using the Matsubara formalism. In this way, the corresponding MFA thermodynamical potential $\Omega^{\rm MFA}$ and gap eq. can be obtained. By deriving $\Omega^{\rm MFA}$ with respect to the current quark masses we get the magnetic field dependent quark condensate for each flavor

$$\langle\bar\psi_f\psi_f\rangle_{B,T} = -\frac{N_c\,|q_fB|\,T}{\pi} \int \frac{dp_3}{(2\pi)} \sum_{k=0}^{\infty}\sum_{n=-\infty}^{\infty} \frac{M^{-s_f,f}_{\bar p_n,k}\left[\bar p_n^2 + 2k|q_fB| + M^{s_f,f\,2}_{\bar p_n,k}\right] + (+\leftrightarrow -)}{\left(2k|q_fB| + \bar p_n^2 + M^{-s_f,f}_{\bar p_n,k} M^{s_f,f}_{\bar p_n,k}\right)^2 + \bar p_n^2\left(M^{s_f,f}_{\bar p_n,k} - M^{-s_f,f}_{\bar p_n,k}\right)^2} \, . \qquad (6)$$

As customary in nonlocal models this quantity turns to be divergent [2]. We regularize it according to

$$<\bar\psi_f\psi_f>^{reg}_{B,T} = <\bar\psi_f\psi_f>_{B,T} - <\bar\psi_f\psi_f>^{free}_{B,T} + <\bar\psi_f\psi_f>^{free,reg}_{B,T} \, . \qquad (7)$$

In order to make contact with LQCD results of Ref.[3] we introduce the quantity

$$\Sigma^f_{B,T} = \frac{2m_c}{S^4}\left[<\bar\psi_f\psi_f>^{reg}_{B,T} - <\bar\psi_f\psi_f>^{reg}_{0,0}\right] + 1 \, , \quad \text{where} \quad S = (135\times 86)^{1/2}\text{MeV}, \qquad (8)$$

together with $\Delta\Sigma^f_{B,T} = \Sigma^f_{B,T} - \Sigma^f_{0,T}$ and $\Delta\bar\Sigma_{B,T} = (\Delta\Sigma^u_{B,T} + \Delta\Sigma^d_{B,T})/2$.

The results shown below were obtained using $g(p^2) = \exp(-p^2/\Lambda^2)$. In this way the model has three parameters: m_c, G, Λ, fixed to reproduce empirical values of f_π, m_π and a certain chosen value of the quark condensate at zero T and B, $\Phi_0 \equiv (-<\bar\psi_f\psi_f>^{reg}_{0,0})^{1/3}$. Details can be found in Ref.[4].

Results at $T = 0$

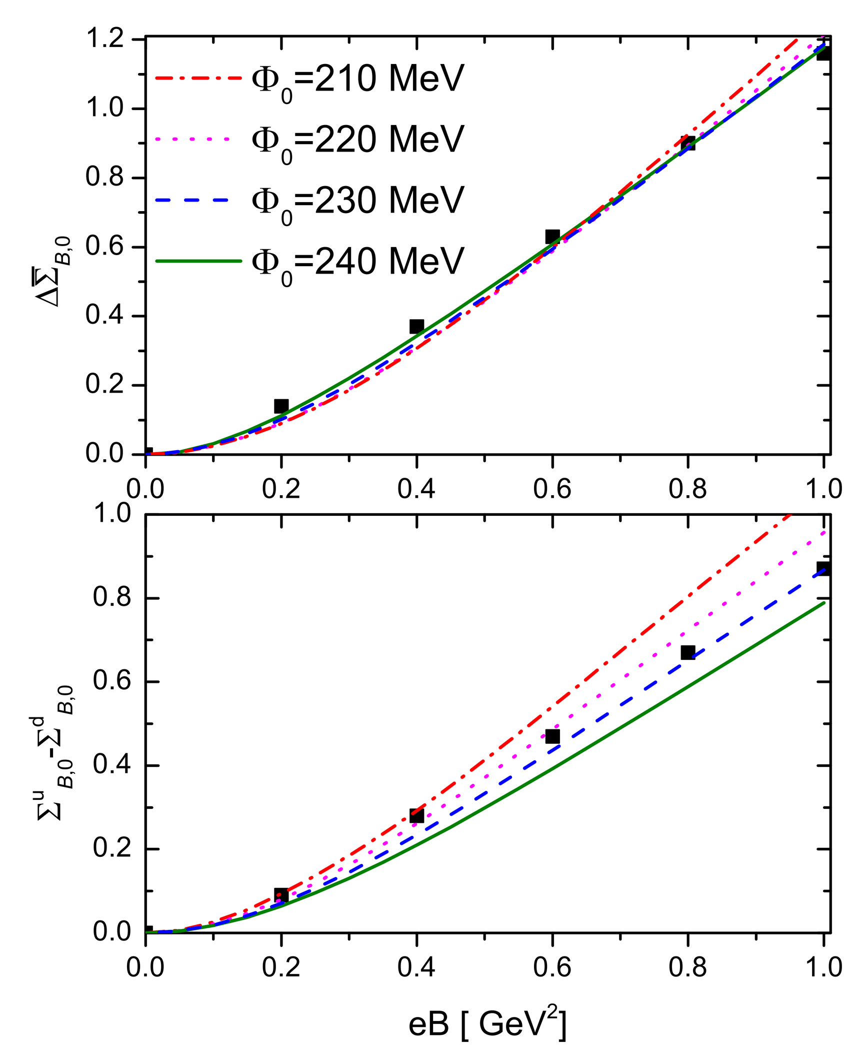

Fig.1: $\Delta\bar\Sigma_{B,0}$ as function of eB for various model parametrizations as compared with the LQCD of Ref.[3] are shown in the upper panel. Corresponding results for $\Sigma^u_{B,0} - \Sigma^d_{B,0}$ are shown in lower panel. The behavior of the condensates display the well-known magnetic catalysis effect, with predictions in good quantitative agreement with LQCD.

Results for finite temperature

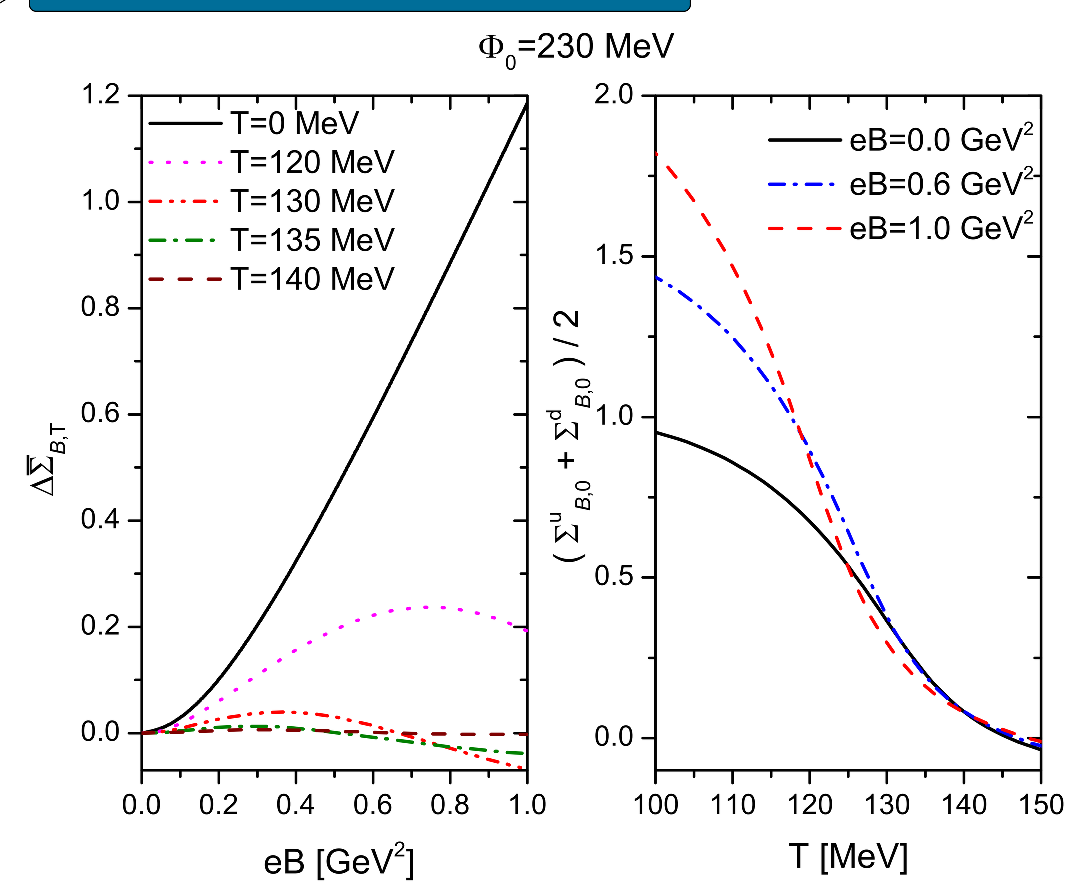

Fig.2: Left: Results for $\Delta\bar\Sigma_{B,T}$ as a function of eB for some representative values of T. Above a certain value of eB $\Delta\bar\Sigma_{B,T}$ decreases as eB increases implying that the present nonlocal model naturally exhibits the IMC effect found in the LQCD calculations. Right: $(\Sigma^u_{B,T} + \Delta\Sigma^d_{B,T})/2$ as a function of T for some selected values of eB. There is a crossover transition from the chiral symmetry broken phase to the (partially) restored one as the temperature increases that occurs at lower temperatures as the magnetic field increases.

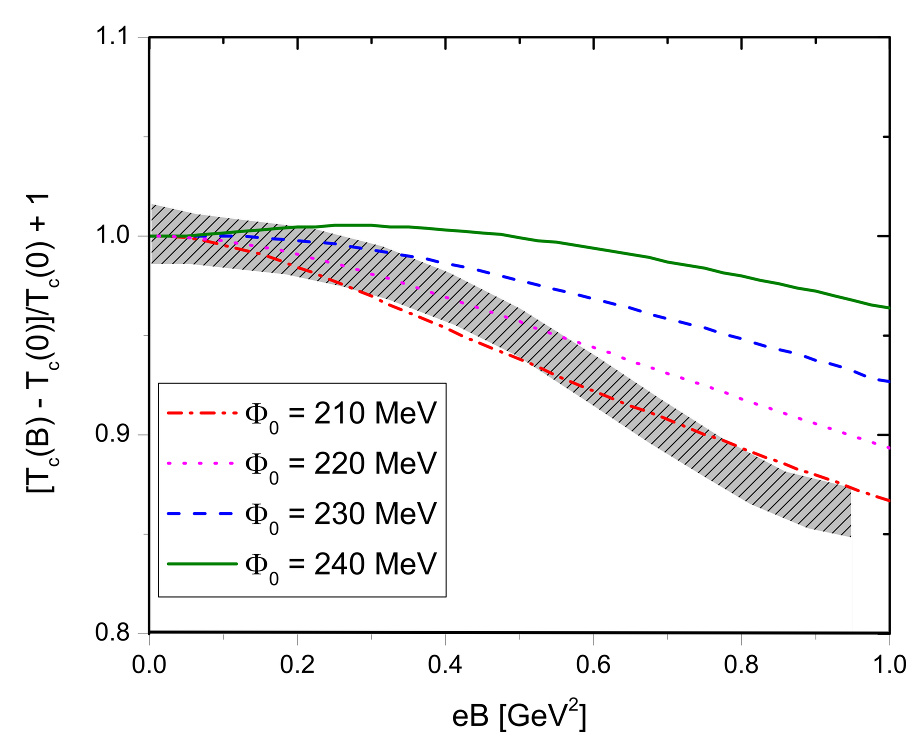

Fig.3: $(T_c(B) - T_c(0))/T_c(0) + 1$ as function of eB, where T_c is defined as the temperature at which the derivative of $(\Sigma^u_{B,T} + \Delta\Sigma^d_{B,T})/2$ with respect to T is maximum. LQCD results are also shown. For all the parametrization considered a decrease of the critical temperature as the magnetic field increases is found, i.e. the IMC effect, is observed. In fact, only for $\Phi_0 = 240$ MeV, a slight opposite behavior is found below that value. The best agreement with the LQCD result correspons to the case associated with the lowest value of Φ_0 considered.

Magnetic susceptibility of the QCD vacuum

A constant external EM field induces the existence of other nonvanishing condensates, which describe the response of the QCD vacuum to the source. We studied, in the context of a nlNJL model that also includes wave function renormalization (WFR) [5], the VEV of the tensor polarization operator $\langle\bar\psi\,\sigma_{\mu\nu}\,\psi\rangle$, where $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$ is the relativistic spin operator. To leading order in the external field one has

$$\langle\bar\psi_f\,\sigma_{\mu\nu}\,\psi_f\rangle_A \;=\; q_f F_{\mu\nu}\,\tau_f \, , \qquad (9)$$

where $F_{\mu\nu}$ is the field strength tensor and τ_f is the so-called tensor coefficient, that within our model reads

$$\tau_f \;=\; 4N_c \int \frac{d^4p}{(2\pi)^4}\, Z(p)\, \frac{M_f(p) - p^2 dM_f(p)/dp^2}{\left[p^2 + M_f(p)^2\right]^2} \, . \qquad (10)$$

Here $M_f(p)$ is the effective mass and $Z(p)$ the WFR. In the corresponding extension to finite T we also included a coupling to the Polyakov loop with different PL thermodynamical potentials found in the literature [6-9].

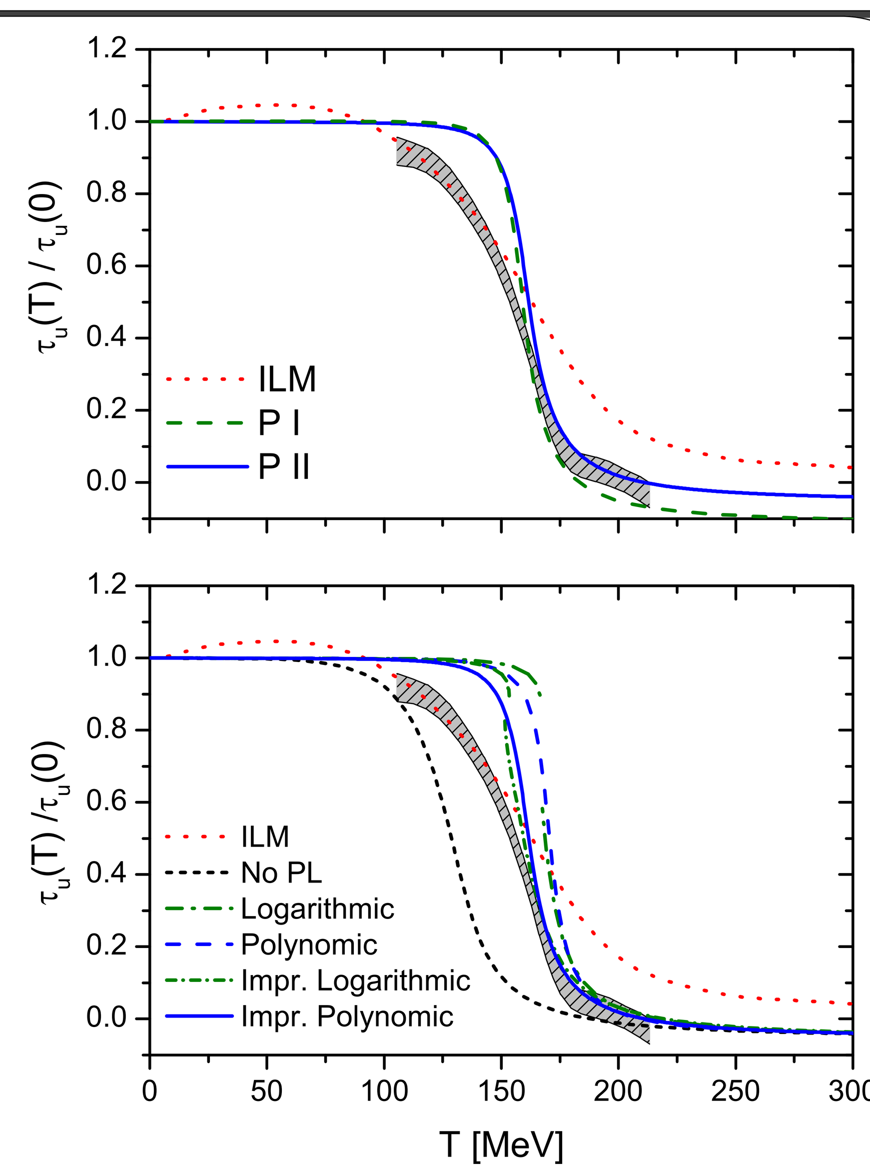

Fig.4: Normalized tensor coefficient vs. T. Upper panel: Results obtained considering an improved polynomial PL potential [6,9], and a parametrization with gaussian form factors (PI) and for a LQCD fitted parametrization (PII) [5]. Defining T_c as the temperature corresponding to the inflection point of τ_u we get $T_c = 158(160)$ MeV for PI (PII), while $T_c^{LQCD} \sim 162$ MeV [11]. We observe that when $T > T_c$ the shape of the curves obtained within our model are in reasonable agreement with LQCD calculations. The steep onset of the transition within nlPNJL models may be cured once the mesonic fluctuations are included.

Lower panel: results corresponding to parametrization PII, for various PL potentials [6-9]. Values obtained within the ILM [10] and results from LQCD [11] (dashed grey bands) are also shown. Whereas different PL potentials give rise to different shapes for $\tau_u(T)$ at T below the chiral transition, once the transition is surpassed the functions converge to a single curve that is in agreement with lattice estimates.

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