Chiral symmetry breaking in continuum QCD

Mario Mitter

Ruprecht-Karls-Universität Heidelberg

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fQCD collaboration - QCD (phase diagram) with FRG:

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large part of this effort: vacuum YM-theory and QCD
QCD with the FRG

- use only perturbative QCD input
  - $\alpha_s(\Lambda = \mathcal{O}(10) \text{ GeV})$
  - $m_q(\Lambda = \mathcal{O}(10) \text{ GeV})$
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- Wetterich equation with initial condition $S[\Phi] = \Gamma_\Lambda[\Phi]$

\[
\partial_k \Gamma_k[A, \bar{c}, c, \bar{q}, q] = \frac{1}{2} \quad - \quad - \quad -
\]

$\Rightarrow$ effective action $\Gamma[\Phi] = \lim_{k \to 0} \Gamma_k[\Phi]$
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$$\partial_k \Gamma_k[A, \bar{c}, c, \bar{q}, q] = \frac{1}{2} \quad \Rightarrow \text{effective action } \Gamma[\Phi] = \lim_{k \to 0} \Gamma_k[\Phi]$$

- $\partial_k$: integration of momentum shells controlled by regulator
- full field-dependent equation with $(\Gamma^{(2)}[\Phi])^{-1}$ on rhs
- gauge-fixed approach (Landau gauge): ghosts appear
Vertex Expansion

- approximation necessary - vertex expansion

\[ \Gamma[\Phi] = \sum_n \int_{p_1, \ldots, p_{n-1}} \Gamma_{\Phi_1 \cdots \Phi_n}^{(n)}(p_1, \ldots, p_{n-1}) \Phi^1(p_1) \cdots \Phi^n(-p_1 - \cdots - p_{n-1}) \]
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- functional derivatives with respect to \( \Phi_i = A, \bar{c}, c, \bar{q}, q \):
  \( \Rightarrow \) equations for 1PI \( n \)-point functions, e.g. gluon propagator:

\[ \partial_t \begin{array}{c} 1 \\ \end{array} = \begin{array}{c} -2 \\ \end{array} + \frac{1}{2} \]
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- want “apparent convergence” of \( \Gamma[\Phi] = \lim_{k \to 0} \Gamma_k[\Phi] \)
Landau gauge QCD

- two crucial phenomena: $S\chi_{SB}$ and confinement
- similar scales - hard to disentangle
- quenched QCD: allows separate investigation:

see e.g. [Williams, Fischer, Heupel, 2015]
Landau gauge QCD

- two crucial phenomena: $S\chi_{SB}$ and confinement
- similar scales - hard to disentangle
- quenched QCD: allows separate investigation:
  - pure YM-theory (cf. talk Anton Cyrol)
  - quenched matter part
  - outlook: unquenching

See e.g. [Williams, Fischer, Heupel, 2015]

[Cyrol, Fister, MM, Pawlowski, Strodthoff, 2016]

[MM, Strodthoff, Pawlowski, 2014]

[Cyrol, MM, Strodthoff, Pawlowski, in preparation]
Chiral symmetry breaking

- $\chi_{SB} \Leftrightarrow$ resonance in 4-Fermi interaction $\lambda$ (pion pole):

\[
\frac{\partial}{\partial t} \lambda = a\lambda^2 + b\lambda\alpha + c\alpha^2, \\
b > 0, \\
a, c \leq 0
\]
Chiral symmetry breaking

- $\chi_{SB} \iff$ resonance in 4-Fermi interaction $\lambda$ (pion pole):
  - resonance $\Rightarrow$ singularity without momentum dependency

$$\partial_t \lambda = a \lambda^2 + b \lambda \alpha + c \alpha^2, \quad b > 0, \quad a, c \leq 0$$

[Braun, 2011]
(transverse) running couplings

agreement in perturbative regime required by gauge symmetry
non-degenerate in nonperturbative regime: reflects gluon mass gap
\( \alpha_{\bar{q}Aq} > \alpha_{cr} \): necessary for chiral symmetry breaking
area above \( \alpha_{cr} \) very sensitive to errors
4-Fermi vertex via dynamical hadronization

- change of variables: particular 4-Fermi channels $\rightarrow$ meson exchange
- efficient inclusion of momentum dependence $\Rightarrow$ no singularities
- calculation of model parameters from QCD

$$\partial_k \Gamma_k = \frac{1}{2}$$

[MM, Strodthoff, Pawlowski, 2014]

[Braun, Fister, Haas, Pawlowski, Rennecke, 2014]

[MM, Strodthoff, Pawlowski, 2014]
Vertex Expansion

[MM, Strodthoff, Pawlowski, 2014],

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\[ \Gamma_{AA}^{(2)}(p) \propto Z_A(p) \, p^2 \left( \delta^{\mu\nu} - p^\mu p^\nu / p^2 \right) \]

- band: family of decoupling solutions bounded by scaling solution
- more details ⇒ Talk Anton K. Cyrol, Thursday 6pm

Quark propagator

\[ \Gamma_{\bar{q}q}^{(2)}(p) \propto Z_q(p) (\not{p} + M(p)) \]

- FRG vs. lattice: bare mass, quenched, scale set via gluon propagator

Bowman et al., '05

1/Z_q

M_q
Quark propagator

- $\Gamma_{\bar{q}q}^{(2)}(p) \propto Z_q(p) (\not{p} + M(p))$

![Graph showing quark propagator dressings vs. momentum $p$ in GeV.](image)

- FRG vs. lattice: bare mass, quenched, scale set via gluon propagator
- Agreement not sufficient: need apparent convergence at $\mu \neq 0$

Outlook: unquenching
extended truncation:

\[
\Gamma_{AA}^{(2)}(p) \quad \Gamma_{\bar{c}c}^{(2)}(p) \quad \Gamma_{\bar{A}c}^{(3)}(p, q) \quad \Gamma_{A3}^{(3)}(p, q) \quad \Gamma_{A4}^{(4)}(p_{\text{sym}})
\]
classical tensor

\[
\Gamma_{\bar{q}q}^{(2)}(p) \quad \Gamma_{\bar{A}q}^{(3)}(p, q) \quad \Gamma_{A\bar{q}q}^{(4)}(p_{\text{sym}}) \quad \Gamma_{A\bar{q}q}^{(5)}(p_{\text{sym}}) \quad \Gamma_{\bar{q}qqq}^{(4)}(p, p, -p)
\]
\(\bar{q}D^n q\) complete, \(n \leq 3\)
mom.–ind. tensors

\[
\Gamma_{\phi \phi}^{(2)}(p) \quad \Gamma_{\bar{q}q\phi}^{(3)}(p, -p) \quad \Gamma_{qq\phi}^{(4)}(p, -p, 0) \quad \Gamma_{qq\phi\phi}^{(5)}(p, -p, 0, 0) \quad \Gamma_{\phi^n}^{(n)}(0)
\]
“classical” tensor

\(\phi \in \{\sigma, \vec{\pi}\}\)

systematics of improving the truncation?
Outlook: unquenching

extended truncation:

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⇒ BRST-invariant operators, e.g. $\bar{\psi} D^n \psi$
Outlook: running couplings

\[ \alpha_{cAc} = \frac{\left( \Gamma_{cAc}^{(3)}(p) \right)^2}{4\pi Z_A(p)Z_c(p)^2} \]

\[ \alpha_{AAA} = \frac{\left( \Gamma_{AAA}^{(3)}(p) \right)^2}{4\pi Z_A(p)^3} \]

\[ \alpha_{A^4} = \frac{\left( \Gamma_{A^4}^{(4)}(p) \right)}{4\pi Z_A(p)^2} \]

\[ \alpha_{qAq} = \frac{\left( \Gamma_{qAq}^{(3)}(p) \right)^2}{4\pi Z_A(p)Z_q(p)^2} \]

\[ \alpha_{cAc,prop} = \frac{1}{4\pi Z_A(p)Z_c(p)^2} \]
Outlook: gluon propagator

Sternbeck et al., '2012
FRG

Outlook: quark propagator

[Cyrol, MM, Pawlowski, Strodthoff, in preparation]

\begin{itemize}
  \item comparison FRG with lattice: bare mass and scale setting
\end{itemize}

**Summary and Outlook**

**QCD with functional RG**
- vertex expansion
- sole input $\alpha_S(\Lambda = \mathcal{O}(10)\text{ GeV})$ and $m_q(\Lambda = \mathcal{O}(10)\text{ GeV})$
- good agreement with lattice correlators

**Outlook**
- QCD phase diagram: order parameters, equation of state and fluct. of cons. charges
- bound-state properties (form factors, PDA, ...)
- more checks on convergence of vertex expansion

Poster: “fQCD: QCD with the Functional RG”

M. Mitter (U Heidelberg)
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