

Chiral symmetry breaking in continuum QCD

Mario Mitter

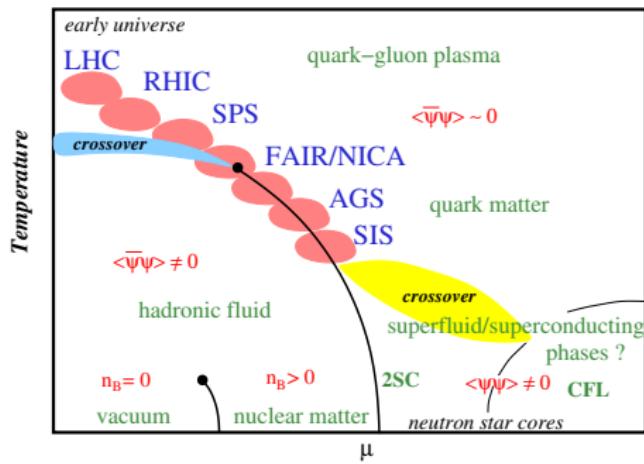
Ruprecht-Karls-Universität Heidelberg

Thessaloniki, August 2016



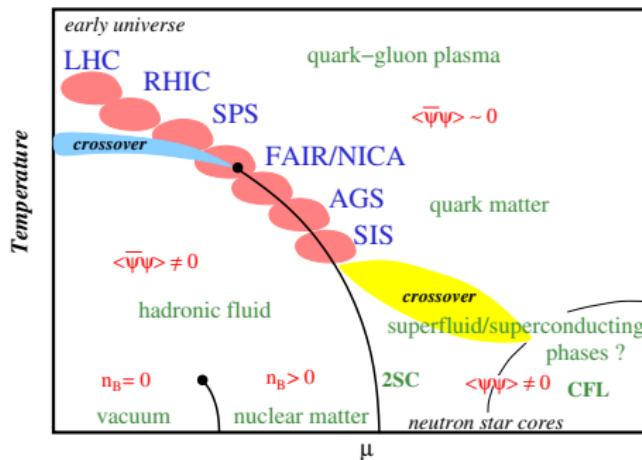
fQCD collaboration - QCD (phase diagram) with FRG:

J. Braun, L. Corell, A. K. Cyrol, L. Fister, W. J. Fu, M. Leonhardt, MM,
J. M. Pawłowski, M. Pospiech, F. Rennecke, N. Strodthoff, N. Wink ...



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large part of this effort: vacuum YM-theory and QCD

QCD with the FRG

- use only perturbative QCD input
 - ▶ $\alpha_S(\Lambda = \mathcal{O}(10) \text{ GeV})$
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- Wetterich equation with initial condition $S[\Phi] = \Gamma_\Lambda[\Phi]$

$$\partial_k \Gamma_k[A, \bar{c}, c, \bar{q}, q] = \frac{1}{2} \quad - \quad \begin{array}{c} \text{---} \\ \text{---} \end{array} \quad - \quad \begin{array}{c} \text{---} \\ \text{---} \end{array}$$

$$\Rightarrow \text{effective action } \Gamma[\Phi] = \lim_{k \rightarrow 0} \Gamma_k[\Phi]$$

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- ∂_k : integration of momentum shells controlled by regulator
- full field-dependent equation with $(\Gamma^{(2)}[\Phi])^{-1}$ on rhs
- gauge-fixed approach (Landau gauge): ghosts appear

Vertex Expansion

- approximation necessary - vertex expansion

$$\Gamma[\Phi] = \sum_n \int_{p_1, \dots, p_{n-1}} \Gamma_{\Phi_1 \dots \Phi_n}^{(n)}(p_1, \dots, p_{n-1}) \Phi^1(p_1) \dots \Phi^n(-p_1 - \dots - p_{n-1})$$

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- functional derivatives with respect to $\Phi_i = A, \bar{c}, c, \bar{q}, q$:
⇒ equations for 1PI n -point functions, e.g. gluon propagator:

$$\partial_t \text{ (gluon loop)}^{-1} = \text{ (gluon loop with insertion)} - 2 \text{ (gluon loop with insertion)} + \frac{1}{2} \text{ (gluon loop with insertion)}$$

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- want “apparent convergence” of $\Gamma[\Phi] = \lim_{k \rightarrow 0} \Gamma_k[\Phi]$

Landau gauge QCD

- two crucial phenomena: $S\chi$ SB and confinement
- similar scales - hard to disentangle see e.g. [Williams, Fischer, Heupel, 2015]
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- quenched QCD: allows separate investigation:
- pure YM-theory (cf. talk Anton Cyrol) [Cyrol, Fister, MM, Pawłowski, Strodthoff, 2016]
- quenched matter part [MM, Strodthoff, Pawłowski, 2014]
- outlook: unquenching [Cyrol, MM, Strodthoff, Pawłowski, in preparation]

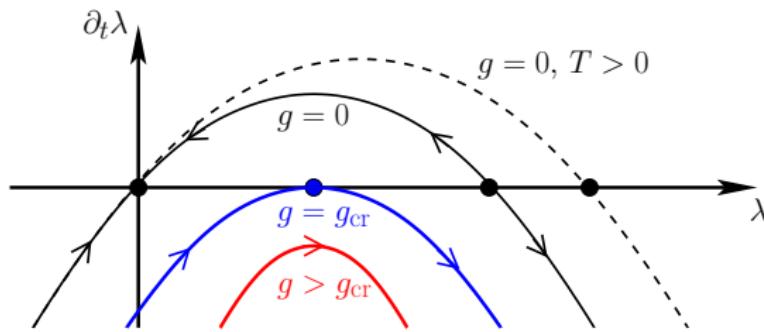
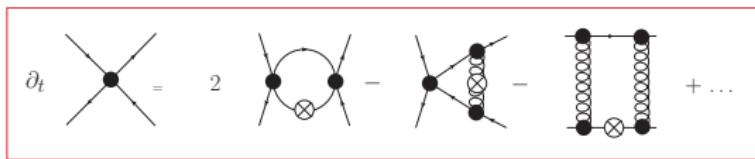
Chiral symmetry breaking

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- resonance \Rightarrow singularity without momentum dependency

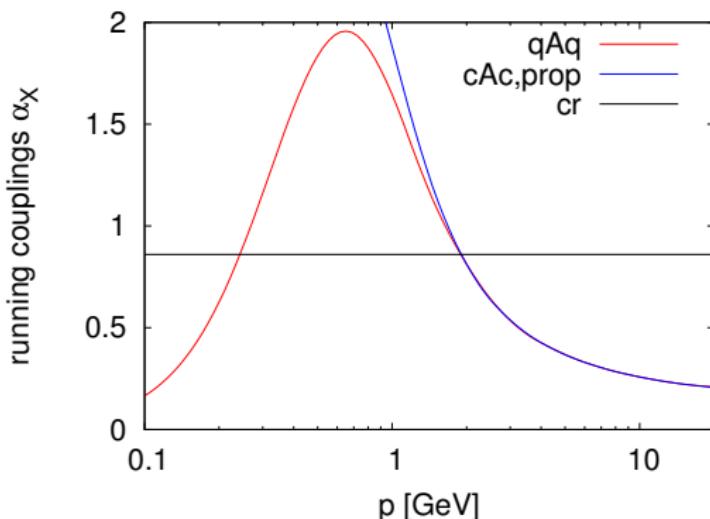
$$\partial_t \lambda = a \lambda^2 + b \lambda \alpha + c \alpha^2, \quad b > 0, \quad a, c \leq 0$$



[Braun, 2011]

(transverse) running couplings

[Cyrol, MM, Pawłowski, Strodthoff, in preparation]



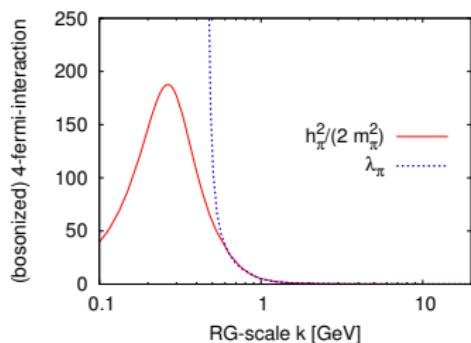
- agreement in perturbative regime required by gauge symmetry
- non-degenerate in nonperturbative regime: reflects gluon mass gap
- $\alpha_{\bar{q}Aq} > \alpha_{cr}$: necessary for chiral symmetry breaking
- area above α_{cr} very sensitive to errors

4-Fermi vertex via dynamical hadronization

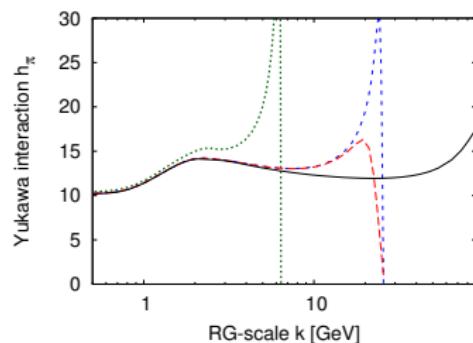
[Gies, Wetterich, 2002]

- change of variables: particular 4-Fermi channels \rightarrow meson exchange
- efficient inclusion of momentum dependence \Rightarrow no singularities
- calculation of model parameters from QCD

$$\partial_k \Gamma_k = \frac{1}{2} \left(\text{Diagram 1} - \text{Diagram 2} \right) + \frac{1}{2} \left(\text{Diagram 3} - \text{Diagram 4} \right)$$



[MM, Strodthoff, Pawlowski, 2014]



[Braun, Fister, Haas, Pawlowski, Rennecke, 2014]

[MM, Strodthoff, Pawlowski, 2014]

Vertex Expansion

[MM, Strodthoff, Pawlowski, 2014],

[Cyrol, Fister, MM, Strodthoff, Pawlowski, 2016]

$$\partial_t \text{---}^{-1} =$$
$$+ \frac{1}{2}$$
$$-$$
$$+$$
$$-$$
$$-$$

$$\partial_t \text{---} =$$
$$-\frac{1}{2}$$
$$+$$
$$2$$
$$-$$
$$+$$
$$\text{perm.}$$

$$\partial_t \text{---} - 2$$
$$-$$
$$-$$
$$-$$
$$-$$
$$-$$
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$$\partial_t \text{---}^{-1} = \begin{array}{c} \text{Diagram 1} \\ + \end{array} \begin{array}{c} \text{Diagram 2} \\ + \frac{1}{2} \end{array} \begin{array}{c} \text{Diagram 3} \\ - \end{array} \begin{array}{c} \text{Diagram 4} \\ + \end{array} \begin{array}{c} \text{Diagram 5} \\ - \end{array} \begin{array}{c} \text{Diagram 6} \\ - \end{array}$$

$$\partial_t \text{---}^{-1} = \begin{array}{c} \text{Diagram 1} \\ + \end{array}$$

$$\partial_t \text{---}^{-1} = \begin{array}{c} \text{Diagram 1} \\ - 2 \end{array} \begin{array}{c} \text{Diagram 2} \\ + \frac{1}{2} \end{array}$$

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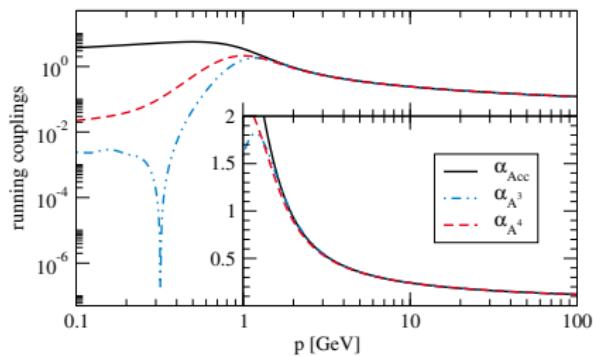
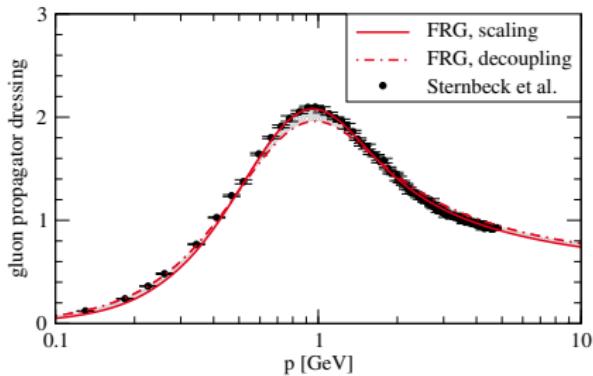
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YM theory

[Cyrol, Fister, MM, Pawłowski, Strodthoff, 2016]

- $\Gamma_{AA}^{(2)}(p) \propto Z_A(p) p^2 (\delta^{\mu\nu} - p^\mu p^\nu / p^2)$

- running couplings



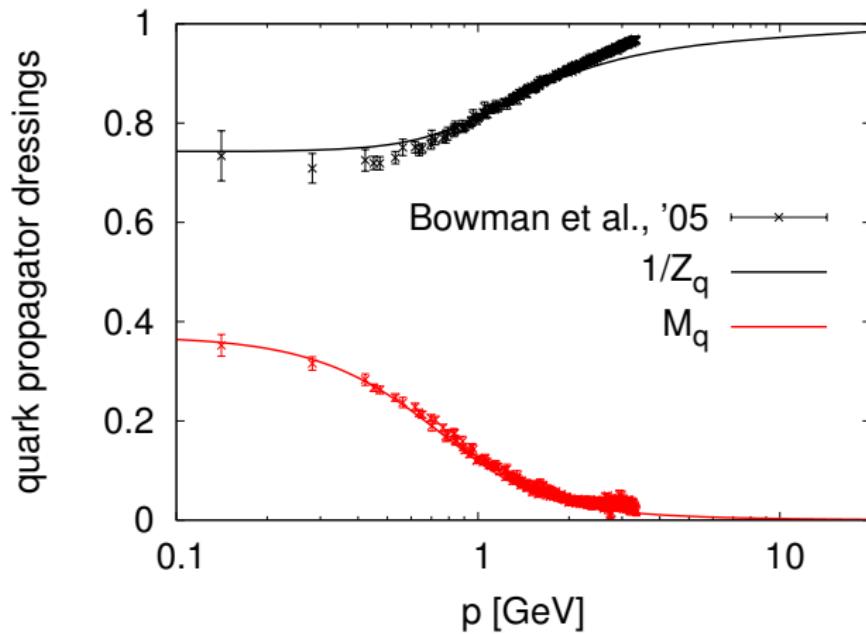
- band: family of decoupling solutions bounded by scaling solution
- more details \Rightarrow Talk Anton K. Cyrol, Thursday 6pm

lattice data: A. Sternbeck, E. M. Ilgenfritz, M. Müller-Preussker, A. Schiller, and I. L. Bogolubsky, PoS LAT2006, 076.

Quark propagator

[MM, Pawłowski, Strodthoff, 2014]

- $\Gamma_{\bar{q}q}^{(2)}(p) \propto Z_q(p) (\not{p} + M(p))$

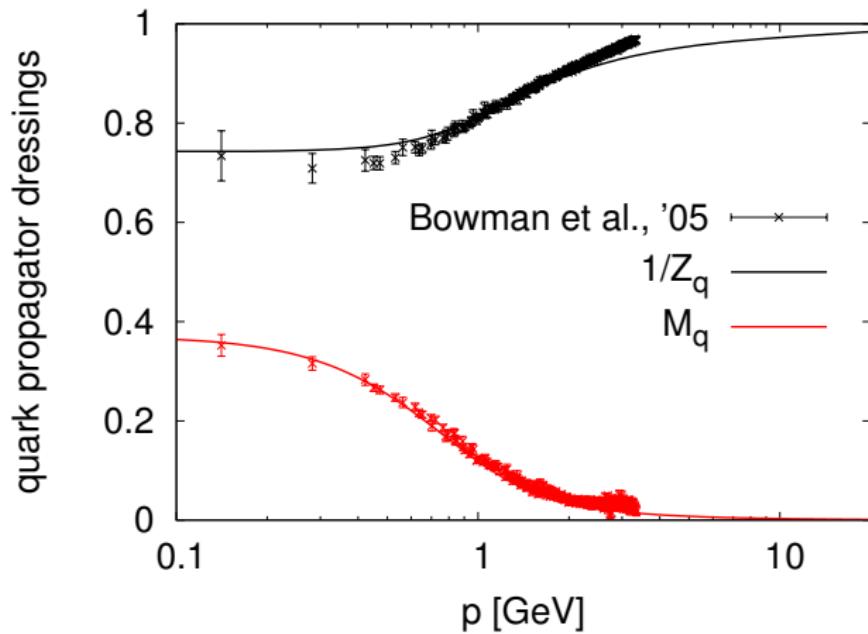


- FRG vs. lattice: bare mass, quenched, scale set via gluon propagator

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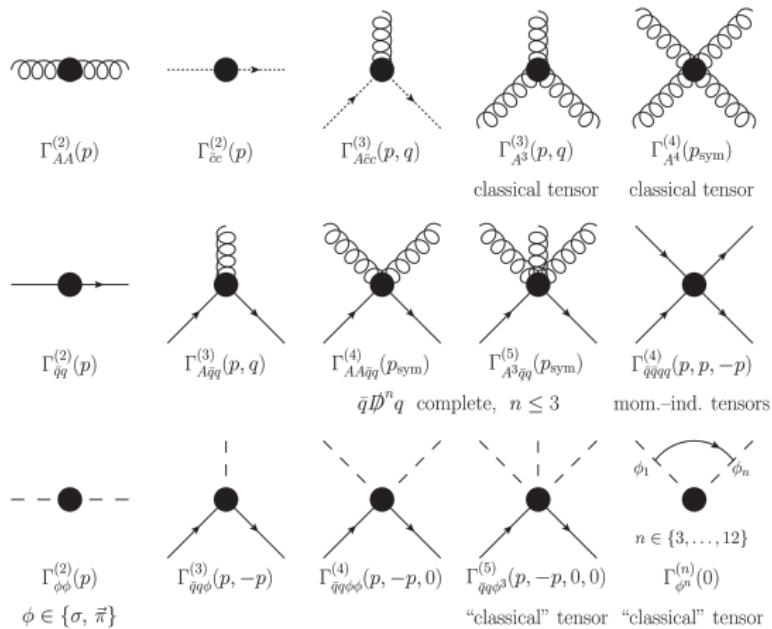
- FRG vs. lattice: bare mass, quenched, scale set via gluon propagator
- agreement not sufficient: need apparent convergence at $\mu \neq 0$

lattice data: Bowman, Heller, Leinweber, Parappilly, Williams, Zhang , Phys. Rev. D71, 054507 (2005).

Outlook: unquenching

[Cyrol, MM, Pawłowski, Strodthoff, in prep.]

extended truncation:

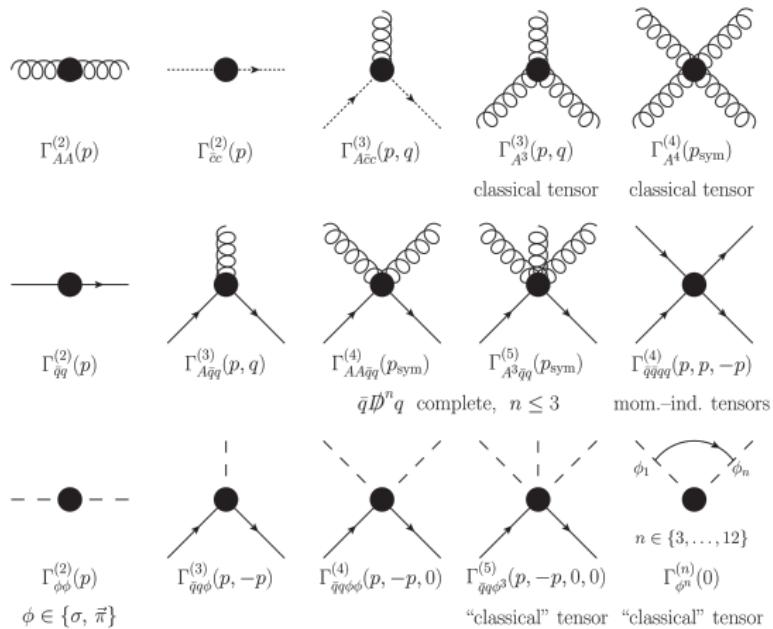


systematics of improving the truncation?

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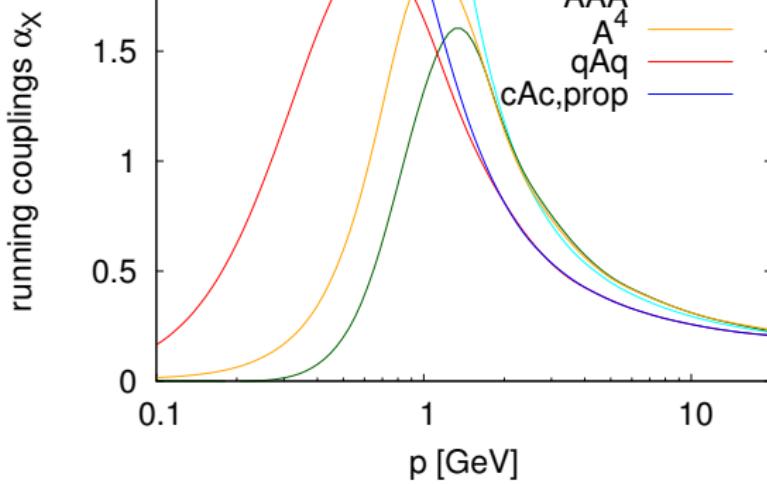
systematics of improving the truncation?

⇒ BRST-invariant operators, e.g. $\bar{\psi} \not{D}^n \psi$

Outlook: running couplings

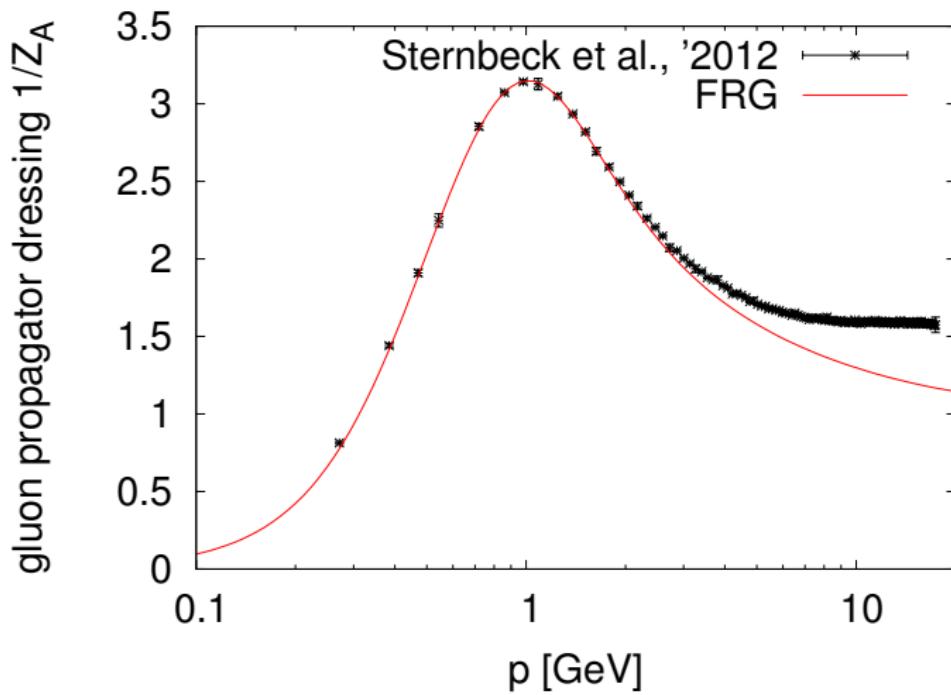
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- $\alpha_{cAc} = \frac{\left(\Gamma_{cAc}^{(3)}(p)\right)^2}{4\pi Z_A(p)Z_c(p)^2}$
- $\alpha_{AAA} = \frac{\left(\Gamma_{AAA}^{(3)}(p)\right)^2}{4\pi Z_A(p)^3}$
- $\alpha_{A^4} = \frac{\left(\Gamma_{A^4}^{(4)}(p)\right)}{4\pi Z_A(p)^2}$
- $\alpha_{qAq} = \frac{\left(\Gamma_{qAq}^{(3)}(p)\right)^2}{4\pi Z_A(p)Z_q(p)^2}$
- $\alpha_{cAc,\text{prop}} = \frac{1}{4\pi Z_A(p)Z_c(p)^2}$



Outlook: gluon propagator

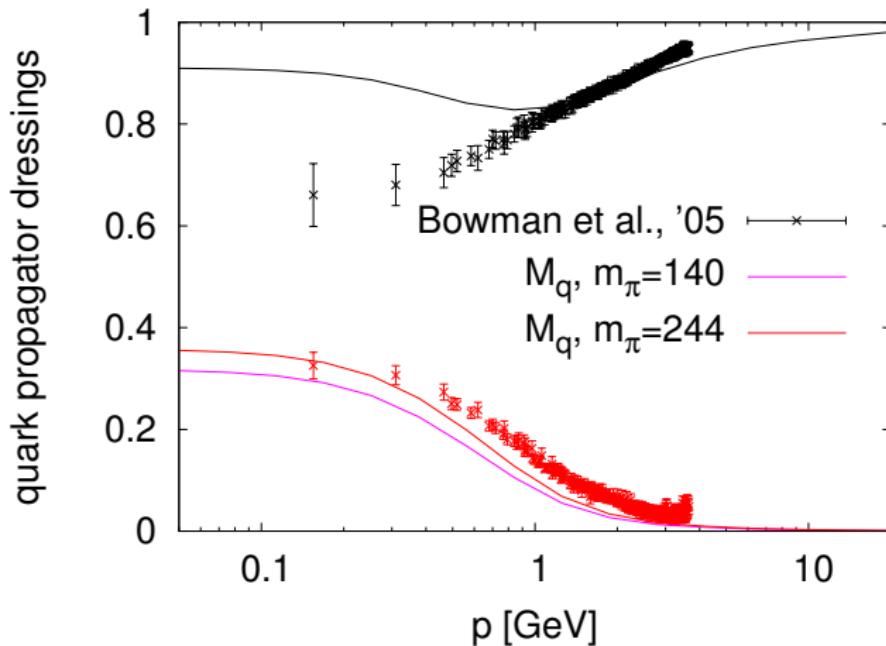
[Cyrol, MM, Pawłowski, Strodthoff, in preparation]



lattice data: A. Sternbeck, K. Maltman, M. Müller-Preussker, L. von Smekal, PoS LATTICE2012 (2012) 243.

Outlook: quark propagator

[Cyrol, MM, Pawłowski, Strodthoff, in preparation]



- comparison FRG with lattice: bare mass and scale setting

lattice data: Bowman, Heller, Leinweber, Parappilly, Williams, Zhang , Phys. Rev. D71, 054507 (2005).

Summary and Outlook

QCD with functional RG

- vertex expansion
- sole input $\alpha_S(\Lambda = \mathcal{O}(10) \text{ GeV})$ and $m_q(\Lambda = \mathcal{O}(10) \text{ GeV})$
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- QCD phase diagram:
order parameters, equation of state and fluct. of cons. charges
- bound-state properties (form factors, PDA...)
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Poster: “fQCD: QCD with the Functional RG”