One-loop calculations in Supersymmetric Lattice QCD

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Abstract

In recent years the prospects of extracting nonperturbative information for Supersymmetric Theories through lattice simulations are being studied extensively, from a number of viewpoints. There are a number of important physical questions regarding SUSY to be ultimately addressed on the lattice, such as the nature of SUSY breaking, and the phase diagram of SUSY models. Such questions have become increasingly relevant in recent years, in the context of investigations of BSM Physics. Many notorious problems arise when formulating SUSY models on the lattice, such as the emergence of a plethora of counterterms in the action and the need for fine-tuning of masses and coupling constants.The present investigates these problems using, as a representative nontrivial model, supersymmetric $\mathcal{N}=1$ Quantum Chromodynamics (SQCD).
We study the self energies of all particles which appear in a lattice regularization of supersymmetric QCD ($\mathcal{N}=1$). We compute, perturbatively to one-loop, the relevant two-point Green's functions using both the dimensional and the lattice regularizations. Our lattice formulation involves a variety of discretizations for the gluino and quark fields, including Wilson, clover and overlap fermions. For gluons we employ the Wilson action, as well as Symanzik improved variants. For scalar fields (squarks) we use naive discretization. The gauge group that we consider is $SU(N_c)$ while the number of colors, N_c and the number of flavors, N_f, are kept as generic parameters. We have also searched for relations among the propagators which are computed from our one-loop results. We have obtained analytic expressions for the renormalization functions of the quark field (Z_ψ), gluon field (Z_u), gluino field (Z_λ) and squark field ($Z_{A_\pm}$). In this study we also describe the perturbative calculation of the renormalization of quark bilinear operators which, unlike the non-supersymmetric case, exhibit a rich pattern of operator mixing at the quantum level.

Continuum Action of SQCD

The construction of the Lagrangian of SQCD involves chiral superfields and vector superfields. The physical components of a chiral superfield Φ, are the matter fields: A which represents a scalar boson (squark), ψ which is a two-component spinor (quark - spin $\frac{1}{2}$) and F which is an auxiliary complex scalar field. In superspace notation (x: spacetime coordinates, $\theta/\bar\theta$: anticommuting coordinates) the chiral superfield Φ in terms of the above component fields is:

$$\begin{aligned}\Phi(x,\theta,\bar\theta) &= A(x)+\sqrt{2}\,\theta\psi(x)+\theta\theta\,F(x)+i\theta\,\sigma^\mu\,\bar\theta\,\partial_\mu A(x)\\ &+\frac{i}{\sqrt{2}}\theta\theta\,\bar\theta\,\bar\sigma^\mu\,\partial_\mu\psi(x)+\frac{1}{4}\,\theta\theta\,\bar\theta\bar\theta\,\partial_\mu\partial^\mu A(x)\end{aligned} \tag{1}$$

The general form of a vector superfield $V(x,\theta,\bar\theta)$ is:

$$\begin{aligned}V(x,\theta,\bar\theta) &= C(x)+i\theta\chi(x)-i\bar\theta\bar\chi(x)+\frac{i}{2}\theta\theta\left[M(x)+iN(x)\right]-\frac{i}{2}\bar\theta\bar\theta\left[M(x)-iN(x)\right]\\ &-\theta\,\sigma^\mu\,\bar\theta\,u_\mu(x)+i\theta\theta\,\bar\theta\left[\bar\lambda(x)+\frac{i}{2}\bar\sigma^\mu\partial_\mu\chi(x)\right]-i\bar\theta\bar\theta\,\theta\left[\lambda(x)+\frac{i}{2}\sigma^\mu\partial_\mu\bar\chi(x)\right]\\ &+\frac{1}{2}\theta\theta\,\bar\theta\bar\theta\left[D(x)+\frac{1}{2}\partial_\mu\partial^\mu C(x)\right].\end{aligned} \tag{2}$$

We can choose a special gauge where the components C,χ,M,N are zero. This defines the Wess $-$ Zumino (WZ) gauge. A vector superfield in Wess Zumino gauge reduces to the form:

$$V(x,\theta,\bar\theta)=-\theta\,\sigma^\mu\,\bar\theta\,u_\mu(x)+i\theta\theta\,\bar\theta\bar\lambda(x)-i\bar\theta\bar\theta\,\theta\lambda(x)+\frac{1}{2}\theta\theta\,\bar\theta\bar\theta\,D(x)\,. \tag{3}$$

where u_μ^α is the gluon field, λ^α is the gluino field which transform in the adjoint representation of the gauge group and D^α is an auxiliary real scalar field.

In order to obtain a renormalizable theory, we need to construct a Lagrangian with products of superfields having dimensionality ≤ 4; in addition, we require Lorentz invariance as well as invariance under supersymmetric gauge transformations:

$$\begin{aligned}\Phi'_+ &= e^{-i\Lambda}\Phi_+\\ \Phi'_- &= \Phi_- e^{i\Lambda}\\ e^{V'} &= e^{-i\Lambda^\dagger}e^V e^{i\Lambda}\end{aligned} \tag{4}$$

$$\mathcal{L}=\frac{1}{16kg}\mathrm{Tr}\left(W^\alpha\,W_\alpha|_{\theta\theta}+\bar W_{\dot\alpha}\,\bar W^{\dot\alpha}|_{\bar\theta\bar\theta}\right)+\left(\Phi_+^\dagger\,e^V\,\Phi_++\Phi_-\,e^{-V}\,\Phi_-^\dagger\right)|_{\theta\theta\bar\theta\bar\theta}+m\left(\Phi_-\Phi_+|_{\theta\theta}+\Phi_+^\dagger\Phi_-^\dagger|_{\bar\theta\bar\theta}\right) \tag{5}$$

where $\mathrm{Tr}(T^\alpha T^\beta)=k\,\delta^{\alpha\beta}$, $W_\alpha=-\frac{1}{4}\bar{\mathcal{D}}\bar{\mathcal{D}}\,e^{-V}\,\mathcal{D}_\alpha\,e^V$ is the supersymmetric field strength and the supersymmetric covariant derivative is defined:
$\mathcal{D}_\alpha=\frac{\partial}{\partial\theta^\alpha}+i\sigma^\mu_{\alpha\dot\alpha}\,\bar\theta^{\dot\alpha}\,\partial_\mu$, $\bar{\mathcal{D}}_{\dot\alpha}=-\frac{\partial}{\partial\bar\theta^{\dot\alpha}}-i\theta^\alpha\,(\sigma^\mu)_{\alpha\dot\alpha}\,\partial_\mu$. Combining the components of Φ_+ with Φ_- we can construct a 4 component Dirac Spinor (ψ_D).
Upon functionally integrating over the auxiliary fields; restoring the coupling g by rescaling $V\longrightarrow 2\,g\,V$ and after a Wick rotation, the form of the Euclidean action in 4 dimensions in Dirac notation $\mathcal{S}^E_{\mathrm{SQCD}}$ is:

$$\begin{aligned}\mathcal{S}^E_{\mathrm{SQCD}} &= \int d^4x\Big[\frac{1}{4}u^{(\alpha)}_{\mu\nu}u^{(\alpha)}_{\mu\nu}+\frac{1}{2}\bar\lambda^{(\alpha)}_M\gamma^E_\mu\mathcal{D}_\mu\lambda^{(\alpha)}_M\\ &+\mathcal{D}_\mu A_+^\dagger\mathcal{D}_\mu A_++\mathcal{D}_\mu A_-\mathcal{D}_\mu A_-^\dagger+\bar\psi_D\gamma^E_\mu\mathcal{D}_\mu\psi_D\\ &+i\sqrt{2}g\big(A_+^\dagger\bar\lambda^{(\alpha)}_M T^{(\alpha)}P^E_+\psi_D-\bar\psi_D P^E_-\lambda^{(\alpha)}_M T^{(\alpha)}A_++A_-\bar\lambda^{(\alpha)}_M T^{(\alpha)}P^E_-\psi_D-\bar\psi_D P^E_+\lambda^{(\alpha)}_M T^{(\alpha)}A_-^\dagger\big)\\ &+\frac{1}{2}g^2(A_+^\dagger T^{(\alpha)}A_+-A_-T^{(\alpha)}A_-^\dagger)^2-m(\bar\psi_D\psi_D-mA_+^\dagger A_+-mA_-A_-^\dagger)\Big].\end{aligned} \tag{6}$$

where $\lambda_M=\begin{pmatrix}\lambda_a\\ \bar\lambda^{\dot a}\end{pmatrix}$ and $\psi_D^T=\begin{pmatrix}\psi_{+a}\\ \bar\psi_-^{\dot a}\end{pmatrix}$, $P^E_\pm=\frac{1\pm\gamma^E_5}{2}$, $\gamma^E_5=\gamma^E_0\gamma^E_1\gamma^E_2\gamma^E_3$. $\mathcal{S}^E_{\mathrm{SQCD}}$ is invariant under supersymmetric transformations (ξ_M: Majorana spinor parameter):

$$\begin{aligned}\delta_\xi A_+ &= -\sqrt{2}\bar\xi_M P^E_+\psi_D\,,\\ \delta_\xi A_- &= -\sqrt{2}\bar\psi_D P^E_+\xi_M\,,\\ \delta_\xi(P^E_+\psi_D) &= \sqrt{2}(\mathcal{D}_\mu A_+)P^E_+\gamma^E_\mu\xi_M-\sqrt{2}mP^E_+\xi_M A_-^\dagger\,,\\ \delta_\xi(P^E_-\psi_D) &= \sqrt{2}(\mathcal{D}_\mu A_-)^\dagger P^E_-\gamma^E_\mu\xi_M-\sqrt{2}mA_+P^E_-\xi_M\,,\\ \delta_\xi u^{(\alpha)}_\mu &= -\bar\xi_M\gamma^E_\mu\lambda^{(\alpha)}_M,\\ \delta_\xi\lambda^{(\alpha)}_M &= \frac{1}{4}u^{(\alpha)}_{\mu\nu}[\gamma^E_\mu,\gamma^E_\nu]\xi_M-2ig\gamma^E_5\xi_M(A_+^\dagger T^{(\alpha)}A_+-A_-T^{(\alpha)}A_-^\dagger)\,.\end{aligned} \tag{7}$$

Vertices arising from this action are shown in Fig. 1.

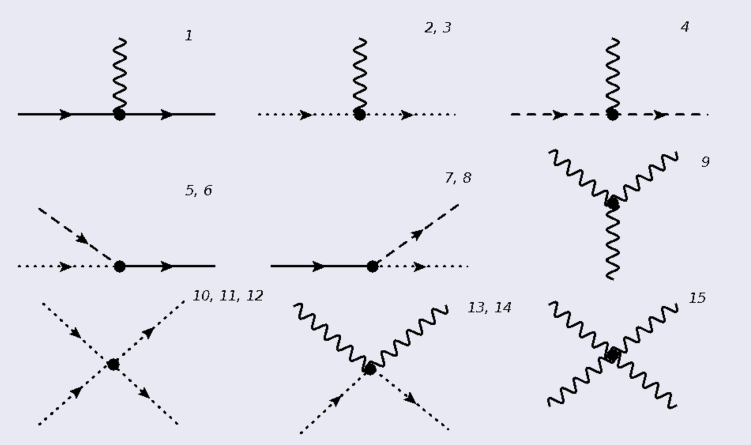

Figure 1: Interaction Vertices of $\mathcal{S}^E_{\mathrm{SQCD}}$. A wavy (solid) line represents gluons (quarks). A dotted (dashed) line corresponds to squarks (gluinos). Note also that there exist similara vertices which involve A_+ and A_-. In the next diagrams we write $\pm$ in order to distinguish them.

The Calculation

We calculate perturbatively the relevant 2-pt Green's functions up to one-loop, both in the continuum and on the lattice. The quantities that we study are the self energies of the quark (ψ), gluon (u_μ), squark (A) and gluino (λ) fields, as well as the 2-pt Green's functions of the quantities $\mathcal{O}_i(x)=\bar\psi(x)\Gamma_i\psi(x)$, using both dimensional regularization (DR) and lattice regularization (L). The index "i" refers to the possibilities of the gamma matrices:

(scalar) $\Gamma_S=1$,
(pseudoscalar) $\Gamma_P=\gamma_5$,
(vector) $\Gamma_V=\gamma_\mu$,
(axial) $\Gamma_A=\gamma_5\gamma_\mu$,
(tensor) $\Gamma_T=\gamma_\mu\gamma_\nu$.

The first step in our perturbative procedure is to calculate the 2-pt Green's functions in the continuum where we regularize the theory in D-dimensions ($D=4-2\,\epsilon$). The continuum calculations are necessary in order to compute the $\overline{\mathrm{MS}}$-renormalized Green's functions; these enter the calculation of the corresponding Green's functions using lattice regularization and $\overline{\mathrm{MS}}$ renormalization. The continuum results also provide the renormalization functions of the quark field (Z_ψ), squark field ($Z_{A_\pm}$), gluon field (Z_u) and gluino field (Z_λ) and for a complete set of quark bilinear operators $\mathcal{O}_i$ (Z_i). For the extraction of the renormalization functions, we applied the $\overline{\mathrm{MS}}$ scheme at a scale $\bar\mu$. Once we have computed the renormalization functions in the $\overline{\mathrm{MS}}$ scheme we can construct their RI$'$ counterparts using conversion factors which are immediately extracted from our computation to the required perturbative order.Being regularization independent, these same conversion factors can then be also used for lattice renormalization functions.
The aforementioned renormalization functions are defined as follows:

$$\begin{aligned}\psi^R &= \sqrt{Z_\psi}\,\psi^B, &(8)\\ A^R_\pm &= \sqrt{Z_{A_\pm}}\,A^B_\pm, &(9)\\ u^R_\mu &= \sqrt{Z_u}\,u^B_\mu, &(10)\\ \lambda^R &= \sqrt{Z_\lambda}\,\lambda^B, &(11)\\ \mathcal{O}^R_i &= Z_i\mathcal{O}^B_i. &(12)\end{aligned}$$

One loop continuum Feynman Diagrams

The one-loop Feynman diagrams (one-particle irreducible (1PI)) contributing to $\langle\psi(x)\bar\psi(y)\rangle$ are shown in Fig. 2, those contributing to $\langle A(x)A^\dagger(y)\rangle$ in Fig. 3. One-loop Feynman diagrams contributing to the Green's function $\langle u^{(\alpha)}_\mu(x)u^{(\beta)}_\nu(y)\rangle$ and $\langle\lambda^{(\alpha)}(x)\bar\lambda^{(\beta)}(y)\rangle$ are shown in Fig. 4 and Fig. 5, respectively. The Feynman diagrams that enter the calculation of the Green's functions $\langle\psi(x)\mathcal{O}_i(z)\bar\psi(y)\rangle$ up to one-loop are shown in Fig. 6.

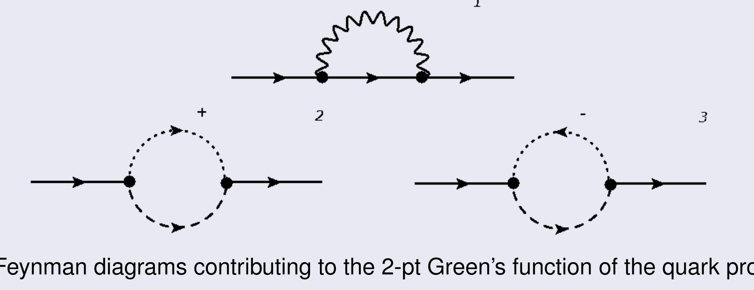

Figure 2: One-loop Feynman diagrams contributing to the 2-pt Green's function of the quark propagator, $\langle\psi(x)\bar\psi(y)\rangle$.

One loop continuum Feynman Diagrams

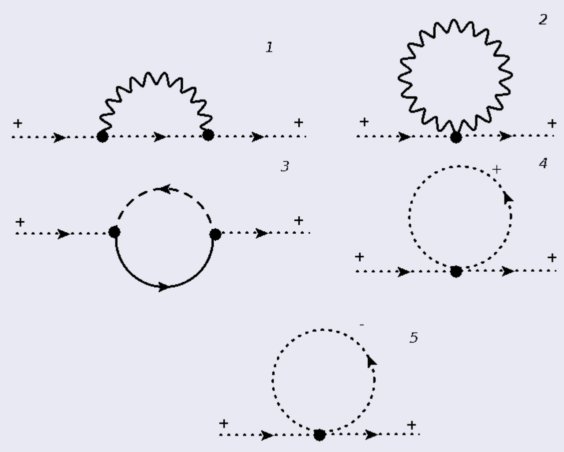

Figure 3: One-loop Feynman diagrams contributing to the 2-pt Green's function of the squark propagator, $\langle A_+(x)A_+^\dagger(y)\rangle$. Similar diagrams apply to the propagator of the A_- field.

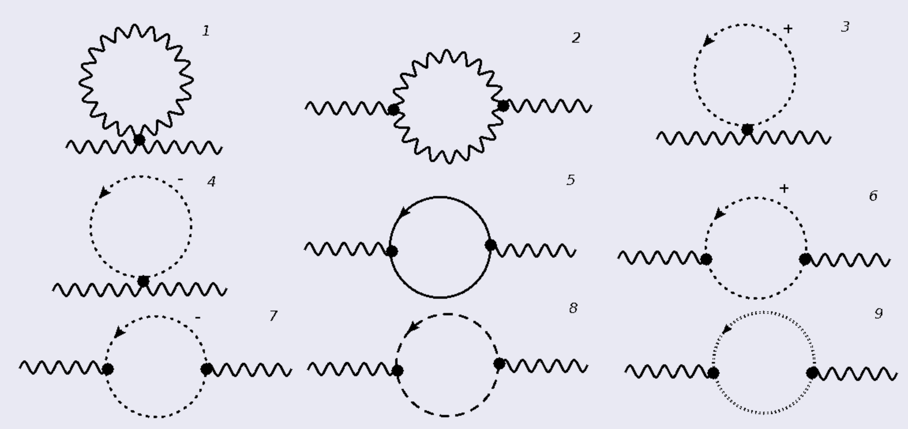

Figure 4: One-loop Feynman diagrams contributing to the 2-pt Green's function of the gluon propagator, $\langle u^{(\alpha)}_\mu(x)u^{(\beta)}_\nu(y)\rangle$. The last Feynman diagram is the one with a closed ghost loop coming from the ghost part of the action.

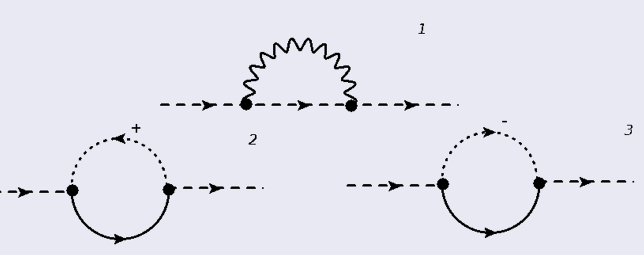

Figure 5: One-loop Feynman diagrams contributing to the 2-pt Green's function of the gluino propagator, $\langle\lambda^\alpha(x)\bar\lambda^\beta(y)\rangle$.

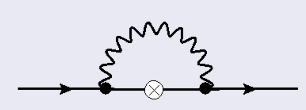

Figure 6: One-loop Feynman diagram contributing to the 2-pt Green's function of $\langle\psi(x)\mathcal{O}_i(z)\bar\psi(y)\rangle$, where $\mathcal{O}_i$ are all local quark bilinear operators. The circle cross denotes the quark bilinear operator insertion.

Continuum Results

Here we can collect all our results for the 2-pt Green's functions:

$$\langle\psi(x)\bar\psi(y)\rangle^{DR}_{\mathrm{amp}}\Big|_{m=0}=i\,\not{q}\left[1-\frac{g^2\,C_F}{16\,\pi^2}\left(\frac{2+\alpha}{\epsilon}+4+\alpha+(2+\alpha)\log\left(\frac{\bar\mu^2}{q^2}\right)\right)\right] \tag{13}$$

where N_c (N_f) is the number of colors (flavors), $C_F=(N_c^2-1)/(2\,N_c)$ is the quadratic Casimir operator in the fundamental representation, α is the gauge parameter ($\alpha=1(0)$ corresponds to the Feynman (Landau) gauge). A Kronecker delta for color indices is understood in Eqs. (13) and (14).

$$\langle A_\pm(x)A_\pm^\dagger(y)\rangle^{DR}_{\mathrm{amp}}\Big|_{m=0}=q^2\left[1-\frac{g^2\,C_F}{16\,\pi^2}\left(\frac{1+\alpha}{\epsilon}+\frac{16}{3}+(1+\alpha)\log\left(\frac{\bar\mu^2}{q^2}\right)\right)\right] \tag{14}$$

$$\begin{aligned}\langle u^{(\alpha)}_\mu(x)u^{(\beta)}_\nu(y)\rangle^{DR}_{\mathrm{amp}}\Big|_{m=0} &= \frac{1}{\alpha}\delta^{(\alpha)\,(\beta)}q_\mu q_\nu+\delta^{(\alpha)\,(\beta)}\left(q^2\delta_{\mu\nu}-q_\mu q_\nu\right)\times\\ &\Big[1-\frac{g^2\,N_f}{16\,\pi^2}\left(\frac{1}{\epsilon}+2+\log\left(\frac{\bar\mu^2}{q^2}\right)\right)\\ &-\frac{g^2\,N_c}{16\,\pi^2}\frac{1}{2}\left(\frac{1+\alpha}{\epsilon}+\frac{7}{2}-\alpha-\frac{\alpha^2}{2}+(1+\alpha)\log\left(\frac{\bar\mu^2}{q^2}\right)\right)\Big]\end{aligned} \tag{15}$$

$$\begin{aligned}\langle\lambda^{(\alpha)}(x)\bar\lambda^{(\beta)}(y)\rangle^{DR}_{\mathrm{amp}}\Big|_{m=0} &= \frac{i}{2}\delta^{(\alpha)\,(\beta)}\;\not{q}\Big[1-\frac{g^2\,N_f}{16\,\pi^2}\left(2+\frac{1}{\epsilon}+\log\left(\frac{\bar\mu^2}{q^2}\right)\right)\\ &-\frac{g^2\,N_c}{16\,\pi^2}\left(4+\frac{4\,\alpha}{\epsilon}+4\,\alpha\log\left(\frac{\bar\mu^2}{q^2}\right)\right)\Big]\end{aligned} \tag{16}$$

$$\langle\psi(x)\mathcal{O}_S(z)\bar\psi(y)\rangle^{DR}_{\mathrm{amp}}\Big|_{m=0}=\mathbb{1}\left[1+\frac{g^2\,C_F}{16\,\pi^2}\left(\frac{3+\alpha}{\epsilon}+4+2\alpha+(3+\alpha)\log\left(\frac{\bar\mu^2}{q^2}\right)\right)\right] \tag{17}$$

$$\langle\psi(x)\mathcal{O}_P(z)\bar\psi(y)\rangle^{DR}_{\mathrm{amp}}\Big|_{m=0}=\gamma_5\left[1+\frac{g^2\,C_F}{16\,\pi^2}\left(\frac{3+\alpha}{\epsilon}+12+2\alpha+(3+\alpha)\log\left(\frac{\bar\mu^2}{q^2}\right)\right)\right] \tag{18}$$

$$\langle\psi(x)\mathcal{O}_V(z)\bar\psi(y)\rangle^{DR}_{\mathrm{amp}}\Big|_{m=0}=\gamma_\mu\left[1+\frac{g^2\,C_F}{16\,\pi^2}\alpha\left(\frac{1}{\epsilon}+1+\log\left(\frac{\bar\mu^2}{q^2}\right)\right)\right]-2\alpha\frac{q_\mu\,\not{q}}{q^2}\frac{g^2\,C_F}{16\,\pi^2} \tag{19}$$

$$\begin{aligned}\langle\psi(x)\mathcal{O}_A(z)\bar\psi(y)\rangle^{DR}_{\mathrm{amp}}\Big|_{m=0} &= \gamma_5\gamma_\mu\left[1+\frac{g^2\,C_F}{16\,\pi^2}\left(\frac{\alpha}{\epsilon}+4+\alpha+\alpha\log\left(\frac{\bar\mu^2}{q^2}\right)\right)\right]\\ &-2\alpha\gamma_5\frac{q_\mu\,\not{q}}{q^2}\frac{g^2\,C_F}{16\,\pi^2}\end{aligned} \tag{20}$$

$$\langle\psi(x)\mathcal{O}_T(z)\bar\psi(y)\rangle^{DR}_{\mathrm{amp}}\Big|_{m=0}=\gamma_\mu\gamma_\nu\left[1+\frac{g^2\,C_F}{16\,\pi^2}(\alpha-1)\left(\frac{1}{\epsilon}+\log\left(\frac{\bar\mu^2}{q^2}\right)\right)\right]. \tag{21}$$

One can observe that there is no one-loop longitudinal part for the gluon self-energy. Thus the renormalization function for the gauge parameter receives no one-loop contribution. From the above results we can extract the renormalization functions:

$$Z^{DR,\overline{\mathrm{MS}}}_\psi = 1+\frac{g^2\,C_F}{16\,\pi^2}\frac{1}{\epsilon}(2+\alpha) \tag{22}$$

$$Z^{DR,\overline{\mathrm{MS}}}_{A_\pm} = 1+\frac{g^2\,C_F}{16\,\pi^2}\frac{1}{\epsilon}(1+\alpha) \tag{23}$$

$$Z^{DR,\overline{\mathrm{MS}}}_u = 1+\frac{g^2}{16\,\pi^2}\frac{1}{\epsilon}\left(\frac{1+\alpha}{2}N_c+N_f\right) \tag{24}$$

$$Z^{DR,\overline{\mathrm{MS}}}_\lambda = 1+\frac{g^2}{16\,\pi^2}\frac{1}{\epsilon}(4\,\alpha\,N_c+N_f) \tag{25}$$

$$Z^{DR,\overline{\mathrm{MS}}}_S = 1-\frac{g^2\,C_F}{16\,\pi^2}\frac{1}{\epsilon} \tag{26}$$

$$Z^{DR,\overline{\mathrm{MS}}}_P = 1-\frac{g^2\,C_F}{16\,\pi^2}\frac{1}{\epsilon} \tag{27}$$

$$Z^{DR,\overline{\mathrm{MS}}}_V = 1+\frac{g^2\,C_F}{16\,\pi^2}\frac{2}{\epsilon} \tag{28}$$

$$Z^{DR,\overline{\mathrm{MS}}}_A = 1+\frac{g^2\,C_F}{16\,\pi^2}\frac{2}{\epsilon} \tag{29}$$

$$Z^{DR,\overline{\mathrm{MS}}}_T = 1+\frac{g^2\,C_F}{16\,\pi^2}\frac{3}{\epsilon}. \tag{30}$$

Lattice calculation

Even though, the lattice breaks supersymmetry explicitly, due to the appearance of lattice artifacts and the doubling problem, it is the only regulator that describes many aspects of strong interactions also nonperturbatively. We use a standard discretization where the quarks, squarks and gluinos live on the lattice sites and the gluons live on the links of the lattice: $U_\mu(x)=e^{igaT^a u^a_\mu(x+a\hat\mu/2)}$. Our procedure is based on Wilson's formulation of non-supersymmetric gauge theories. For Wilson-type fermions and gluons, the Euclidean action $\mathcal{S}^{E,L}_{\mathrm{SQCD}}$ on the lattice becomes:

$$\begin{aligned}\mathcal{S}^{E,L}_{\mathrm{SQCD}} &= a^4\sum_x\Big[\frac{2N_c}{g^2}\sum_{\mu\,\nu}\left(1-\frac{1}{N_c}Re\,\mathrm{Tr}U_{\mu\,\nu}\right)+\mathrm{Tr}\left(\bar\lambda_M\gamma^E_\mu\mathcal{D}_\mu\lambda_M\right)\\ &+\mathcal{D}_\mu A_+^\dagger\mathcal{D}_\mu A_++\mathcal{D}_\mu A_-\mathcal{D}_\mu A_-^\dagger+\bar\psi_D\gamma^E_\mu\mathcal{D}_\mu\psi_D\\ &+i\sqrt{2}g\big(A_+^\dagger\bar\lambda^{(\alpha)}_M T^{(\alpha)}P^E_+\psi_D-\bar\psi_D P^E_-\lambda^{(\alpha)}_M T^{(\alpha)}A_++A_-\bar\lambda^{(\alpha)}_M T^{(\alpha)}P^E_-\psi_D-\bar\psi_D P^E_+\lambda^{(\alpha)}_M T^{(\alpha)}A_-^\dagger\big)\\ &+\frac{1}{2}g^2(A_+^\dagger T^{(\alpha)}A_+-A_-T^{(\alpha)}A_-^\dagger)^2-m(\bar\psi_D\psi_D-mA_+^\dagger A_+-mA_-A_-^\dagger)\Big].\end{aligned} \tag{31}$$

where: $U_{\mu\nu}(x)=U_\mu(x)U_\nu(x+a\hat\mu)U^\dagger_\mu(x+a\hat\nu)U^\dagger_\nu(x)$,

$$\begin{aligned}\mathcal{D}_\mu\lambda_M &= \frac{1}{2a}\Big[U_\mu(x)\lambda_M(x+a\hat\mu)U^\dagger_\mu(x)-U^\dagger_\mu(x-a\hat\mu)\lambda_M(x-a\hat\mu)U^\dagger_\mu(x-a\hat\mu)\Big]\\ &-\frac{r}{2a}\Big[U_\mu(x)\lambda_M(x+a\hat\mu)U^\dagger_\mu(x)-2\lambda_M(x)+U^\dagger_\mu(x-a\hat\mu)\lambda_M(x-a\hat\mu)U^\dagger_\mu(x-a\hat\mu)\Big]\end{aligned} \tag{32}$$

$$\begin{aligned}\mathcal{D}_\mu\psi_D(x) &= \frac{1}{2a}\Big[U_\mu(x)\psi_D(x+a\hat\mu)-U^\dagger_\mu(x-a\hat\mu)\psi_D(x-a\hat\mu)\Big]\\ &-\frac{r}{2a}\Big[U_\mu(x)\psi_D(x+a\hat\mu)-2\psi_D(x)+U^\dagger_\mu(x-a\hat\mu)\psi_D(x-a\hat\mu)\Big]\end{aligned} \tag{33}$$

$$\mathcal{D}_\mu A_+ = \frac{1}{2a}\Big[U_\mu(x)A_+(x+a\hat\mu)-U^\dagger_\mu(x-a\hat\mu)A_+(x-a\hat\mu)\Big] \tag{34}$$

$$\mathcal{D}_\mu A_+^\dagger = \frac{1}{2a}\Big[A_+^\dagger(x+a\hat\mu)U^\dagger_\mu(x)-A_+^\dagger(x-a\hat\mu)U_\mu(x-a\hat\mu)\Big] \tag{35}$$

$$\mathcal{D}_\mu A_- = \frac{1}{2a}\Big[A_-(x+a\hat\mu)U^\dagger_\mu(x)-A_-(x-a\hat\mu)U_\mu(x-a\hat\mu)\Big] \tag{36}$$

$$\mathcal{D}_\mu A_-^\dagger = \frac{1}{2a}\Big[U_\mu(x)A_-^\dagger(x+a\hat\mu)-U^\dagger_\mu(x-a\hat\mu)A_-^\dagger(x-a\hat\mu)\Big] \tag{37}$$

Calculating the same Green's functions as before on the lattice, and combining them with our results from the continuum, we will be able to extract Z^L_ψ, Z^L_u, Z^L_λ, $Z^L_{A_\pm}$ and Z^L_Γ in the $\overline{\mathrm{MS}}$ scheme and on the lattice. On the lattice we have to calculate all the diagrams which were presented here as well as further tadpole diagrams containing closed gluon loops. For the algebraic operations involved in evaluating Feynman diagrams, we make use of our symbolic package in Mathematica.

Mixing of quark bilinear operators with gluino bilinear operators

In general, the renormalization of quark bilinears using other 2-pt Green's functions, e.g. $\langle\lambda(x)\mathcal{O}_i(z)\bar\lambda(y)\rangle$ is nontrivial. A serious complication in this case is that operators of equal and lower dimensionality, with the same quantum numbers, potentially including also non gauge invariant quantities, can mix with $\mathcal{O}_i$ at the quantum level. We identified all operators which can possibly mix with $\mathcal{O}_i$ and all Green's functions which must be calculated in order to compute those elements of the mixing matrix which are relevant for the renormalization of $\mathcal{O}_i$. We list these operators in Table 1 for the flavor non-singlet case ($\bar\psi\Gamma_i\psi\equiv\bar\psi_f\Gamma_i\psi_{f'}$) and in Table 2 for the flavor singlet case ($\bar\psi\Gamma_i\psi\equiv\sum_f\bar\psi_f\Gamma_i\psi_f$).

$\bar\psi\psi$	$A_-A_-^\dagger$	$A_+^\dagger A_+$	A_-A_+	$A_+^\dagger A_-^\dagger$
$\bar\psi\gamma_5\psi$	no mixing			
$\bar\psi\gamma_\mu\psi$	$A_-\partial_\mu A_-^\dagger$ $A_-D_\mu A_-^\dagger$	$A_+^\dagger\partial_\mu A_+$ $A_+^\dagger D_\mu A_+$	$A_-\partial_\mu A_+$ $A_-D_\mu A_+$	$A_+^\dagger\partial_\mu A_-^\dagger$ $A_+^\dagger D_\mu A_-^\dagger$
$\bar\psi\gamma_5\gamma_\mu\psi$	no mixing			
$\bar\psi\gamma_\mu\gamma_\nu\psi$	no mixing			

Table 1: Mixing patterns in the flavor non-singlet case. Flavor non-singlet operators in the leftmost column can mix at the quantum level with the remaining operators in the same row.

$\bar\psi\psi$	$\bar\lambda\lambda$	$A_-A_-^\dagger$	$A_+^\dagger A_+$	A_-A_+	$A_+^\dagger A_-^\dagger$
$\bar\psi\gamma_5\psi$	$\bar\lambda\gamma_5\lambda$				
$\bar\psi\gamma_\mu\psi$	$\bar\lambda\gamma_\mu\lambda$	$A_-\partial_\mu A_-^\dagger$ $A_-D_\mu A_-^\dagger$	$A_+^\dagger\partial_\mu A_+$ $A_+^\dagger D_\mu A_+$	$A_-\partial_\mu A_+$ $A_-D_\mu A_+$	$A_+^\dagger\partial_\mu A_-^\dagger$ $A_+^\dagger D_\mu A_-^\dagger$
$\bar\psi\gamma_5\gamma_\mu\psi$	$\bar\lambda\gamma_5\gamma_\mu\lambda$				
$\bar\psi\gamma_\mu\gamma_\nu\psi$	$\bar\lambda\gamma_\mu\gamma_\nu\lambda$				

Table 2: Mixing patterns in the flavor singlet case. Flavor singlet operators in the leftmost column can mix at the quantum level with the remaining operators in the same row.

Additional 3-squark operators may mix with some of the $\mathcal{O}_i$ operators for $N_f\geq 3$. In order to calculate the mixing coefficients we must evaluate Feynman diagrams shown in Fig. 7. Diagrams in the same row of Fig. 7 lead to the values of the corresponding row of the mixing matrix.

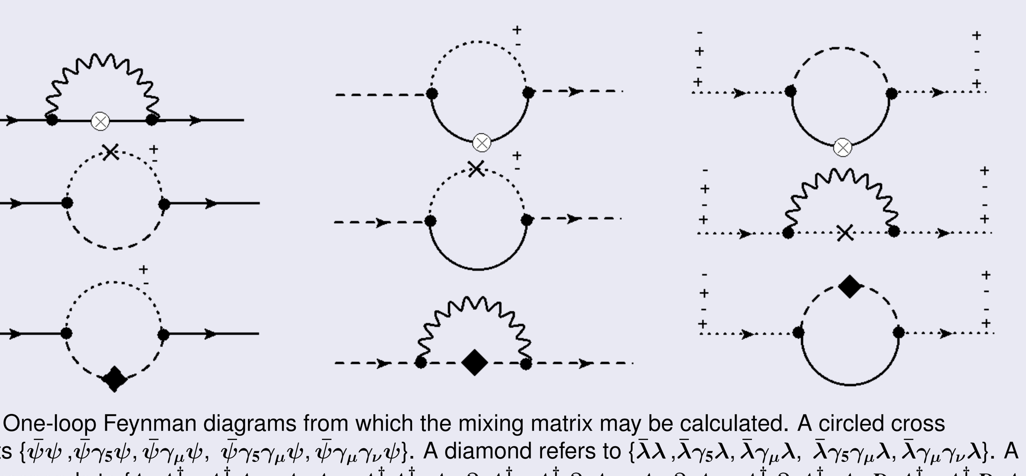

Figure 7: One-loop Feynman diagrams from which the mixing matrix may be calculated. A circled cross represents $\{\bar\psi\psi\,,\bar\psi\gamma_5\psi,\bar\psi\gamma_\mu\psi,\ \bar\psi\gamma_5\gamma_\mu\psi,\bar\psi\gamma_\mu\gamma_\nu\psi\}$. A diamond refers to $\{\bar\lambda\lambda\,,\bar\lambda\gamma_5\lambda,\bar\lambda\gamma_\mu\lambda,\ \bar\lambda\gamma_5\gamma_\mu\lambda,\bar\lambda\gamma_\mu\gamma_\nu\lambda\}$. A cross corresponds to $\{A_-A_-^\dagger,\ A_+^\dagger A_+,\ A_-A_+,\ A_+^\dagger A_-^\dagger, A_-\partial_\mu A_-^\dagger,\ A_+^\dagger\partial_\mu A_+,\ A_-\partial_\mu A_+,\ A_+^\dagger\partial_\mu A_-^\dagger, A_-D_\mu A_-^\dagger,\ A_+^\dagger D_\mu A_+,\ A_-D_\mu A_+,\ A_+^\dagger D_\mu A_-^\dagger\ \}$.

Future Plans - Conclusion

- Determination of the Renormalization Functions of all fields and parameters which appear in $\mathcal{S}^E_{\mathrm{SQCD}}$ and of a complete set of quark bilinear operators on the lattice, which are necessary ingredients in the prediction of physical probability amplitudes from lattice matrix elements.
- Investigation of relationships between different Green's functions involved in SUSY Ward identities.
- Study of the mixing among operators.
- Innovation of this computation: This will be the *first* calculation of the renormalization functions for SQCD on the lattice, providing a thorough set of results for all counterterms, mixing coefficients and parameter fine-tuning.

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