Continuum Action of SQCD

The formulation of the Lagrangian of SQCD involves three superfields and vector superfields. The physical components of a chiral superfield $\Phi$ are the matter fields $\Phi$ which represent a scalar boson (squark), $\psi$ which is a two-component spinor (squark spin $\lambda$), and $\bar{\psi}$ which is an auxiliary complex scalar field in superspace notation. \textit{Gravitino} noncommutative model, there is a coordinate change to terms of the above component fields in $A^{(2)}$: $\phi(x) = \phi(x_0) + \epsilon(x_0) \phi(x_1) + \cdots$ where $\epsilon(x_0)$ is a small parameter (Planck scale). The general form of a vector superfield $V(x)$ is: $V(x) = V(x_0) + \epsilon(x_0) V(x_1) + \cdots$

We can choose a chiral gauge when the components $C, \lambda$, $\bar{\lambda}$ are zero. This defines the Wess–Zumino (WZ) gauge. A vector superfield in Wess–Zumino gauge reduces to the form $V(x) = V(x_0) + \epsilon(x_0) V(x_1) + \cdots$

In order to obtain a renormalizable theory, we need to construct a Lagrangian with products of superfields having dimension $\xi$ in addition, so as not to violate invariance under supersymmetric gauge transformations:

\[ \mathcal{L} = -\frac{1}{2} \bar{\psi} \gamma^{\mu} \partial_{\mu} \psi - \frac{1}{2} \bar{\psi} \gamma^{\mu} \partial_{\mu} \psi + \frac{1}{2} \bar{\psi} \gamma^{\mu} \partial_{\mu} \psi + \frac{1}{4} \bar{\psi} \gamma^{\mu} \partial_{\mu} \psi + \cdots \]

Lattice calculation

Even though, the lattice breaks supersymmetry explicitly, due to the appearance of lattice artifacts and the doubling problem, it is the only regulator that describes many aspects of strong interactions also nonperturbatively. We use a conventional formulation where the quarks and gluons live on the lattice sites and the gluons live on the links of the lattice: $G_{\mu\nu}^a(x)$ and $G_{\mu\nu}^a(x_0)$.

We consider $T = 1$ to be the number of colors (flavors), $N = 2$ to be the number of quark species, $n = 2$ to be the number of flavors, and $\lambda = 3$ to be the number of colors. The continuum limit of these lattice operators is achieved by using nonperturbative gauge theories. For Wilson-type fermions and gluons, the continuum limit is achieved with a three-point Green's function $G_{\mu\nu}^a(x_0, x_0, x_0)$.

Figure 1: One-loop continuum Feynman Diagrams

Figure 2: One-loop continuum Feynman Diagrams contributing to the $u^\dagger(\xi, \bar{\xi}, \xi)$ function of the squark propagator, $\xi = 2\xi^2$. Similar diagrams apply to the propagator of the $\alpha, \bar{\alpha}$, $\beta, \bar{\beta}$.

Figure 3: One-loop continuum Feynman Diagrams

Figure 4: One-loop continuum Feynman Diagrams

Figure 5: One-loop continuum Feynman Diagrams

Figure 6: One-loop continuum Feynman Diagrams

Figure 7: One-loop continuum Feynman Diagrams

The Calculations

We calculate perturbatively the relevant 2-Gluon Functions up to one-loop, both in the continuum limit and also on the lattice. We need to consider the $\bar{\psi} \gamma^\mu \lambda \psi$ (gluon) and $\bar{\psi} \gamma^\mu \lambda \psi$ (gluon) fields, as well as the 2-particle Green functions of the quark propagator, $\langle \bar{\psi} \gamma^\mu \lambda \psi \rangle$, using both dimensional regularization (DR) and lattice regularization (LR). The index $\gamma$ refers to the possible values of the gamma matrices:

(1) $\gamma = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (continuum)

(2) $\gamma = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ (continuum)

(3) $\gamma = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ (continuum)

The first step in our perturbative procedure is to calculate the 2-Gluon Green functions in the continuum where we regularize the theory in dimensional regularization $D = 4 - 2\epsilon$. The continuum calculations are necessary in order to compute the $\bar{\psi} \gamma^\mu \lambda \psi$ Green functions; these are the calculation of the continuum diagrams using dimensional regularization and lattice regularization. The continuum results also provide the renormalization functions of the quark Green functions $\langle \bar{\psi} \gamma^\mu \lambda \psi \rangle$ and $\langle \bar{\psi} \gamma^\mu \lambda \psi \rangle$. For the extraction of the renormalization functions, we applied $\bar{\psi} \gamma^\mu \lambda \psi$ and the $\bar{\psi} \gamma^\mu \lambda \psi$ Green functions using lattice regularization. Our scheme is based on the fact that the propagator is Lorentz-invariant, and thus, we can construct their loop counterparts using conversion factors which are Lorentz-invariant. Our scheme is based on the fact that the propagator is Lorentz-invariant, and thus, we can construct their loop counterparts using conversion factors which are Lorentz-invariant. The crucial idea is that we can also use lattice regularization functions in the continuum. The renormalization functions are defined as follows:

\[ R_{\gamma} = \frac{\langle \bar{\psi} \gamma^\mu \lambda \psi \rangle}{\langle \bar{\psi} \gamma^\mu \lambda \psi \rangle} \]

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One can observe that there is an one-loop longitudinal part to the gluon self-energy. Thus the renormalization function for the gauge parameter renormalizes to: $\Gamma_{\gamma} = 1 + \frac{\alpha_S}{\pi} G_{\mu\nu}^a(x_0, x_0, x_0)$.