Light-front field theory in the description of hadrons

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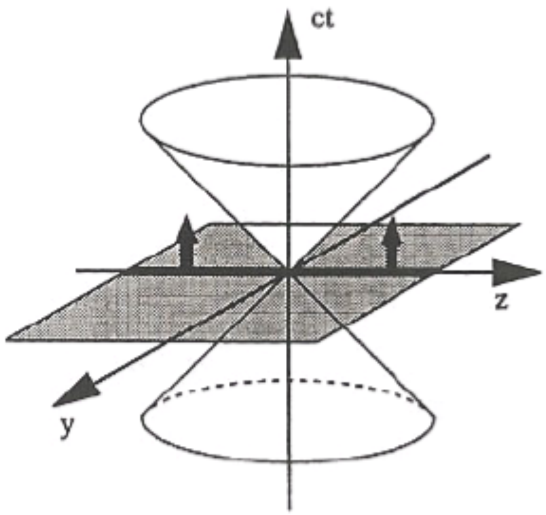
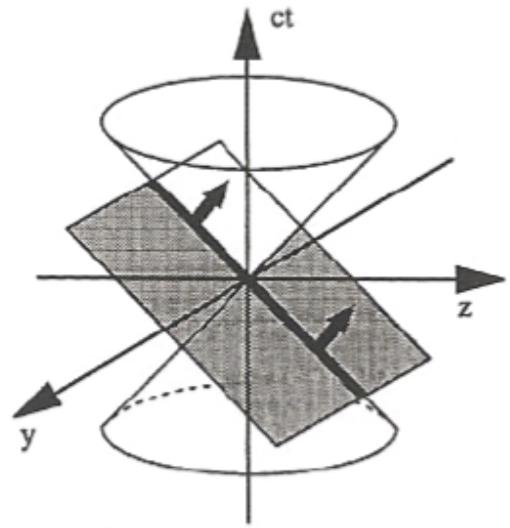
Thessaloniki, August 29, 2016

Dirac's Proposition



The instant form

1949



The front form

Traditional approach
evolved from NR dynamics

Close contact with
Euclidean space

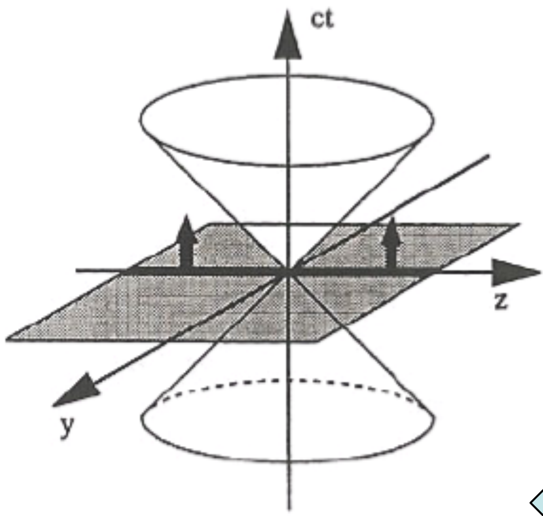
T-dept QFT, LQCD, etc.

Progressive approach
for relativistic dynamics

Strictly in Minkowski space

DIS, PDFs, DVCS, GPDs, etc.

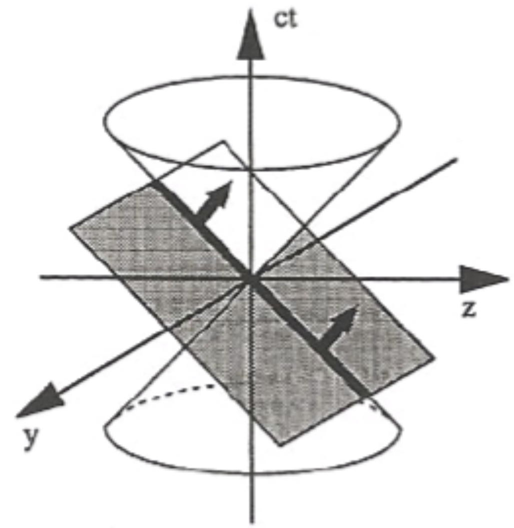
Dirac's Proposition



The instant form

1949

Can they be linked?



The front form

Traditional approach evolved from NR dynamics

Innovative approach for relativistic dynamics

Close contact with Euclidean space

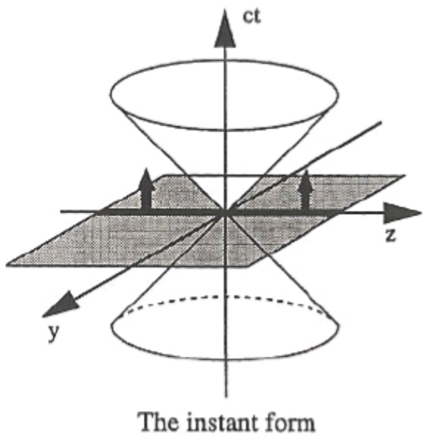
Strictly in Minkowski space

T-dept QFT, LQCD,etc.

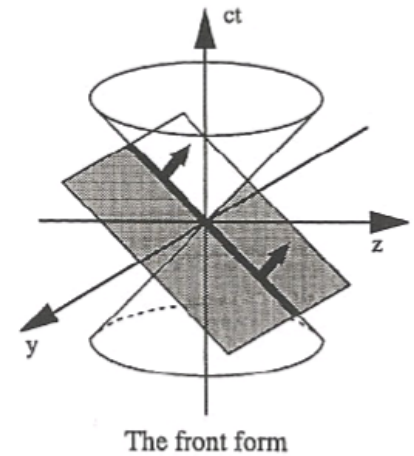
DIS, PDFs, DVCS, GPDs, etc.

Outline

- Interpolation between IFD and LFD
  - Whole landscape between the two
  - Clarification on IMF and LFD
  - LF Zero-mode (LFZM)
- Application to flavor asymmetry of proton sea
  - LNA as “GPS” in Phenomenology
  - Resolution of LNA Factor Difference with LFZM
  - SeaQuest preliminary result
- Conclusion and Outlook

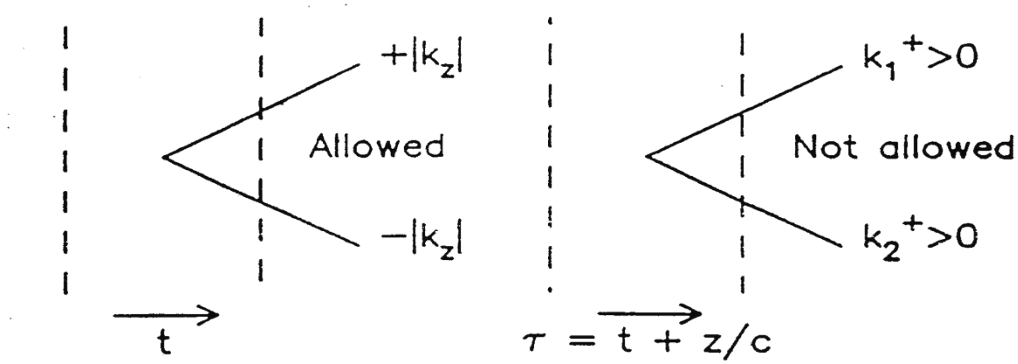


Equal t		*Equal* τ
p^0	$\leftrightarrow$	$p^- = p^0 - p^3$
(p^1, p^2)	$\leftrightarrow$	$\vec{p}_\perp$
p^3	$\leftrightarrow$	$p^+ = p^0 + p^3$



Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^{\,2} + m^2} \qquad\qquad p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$

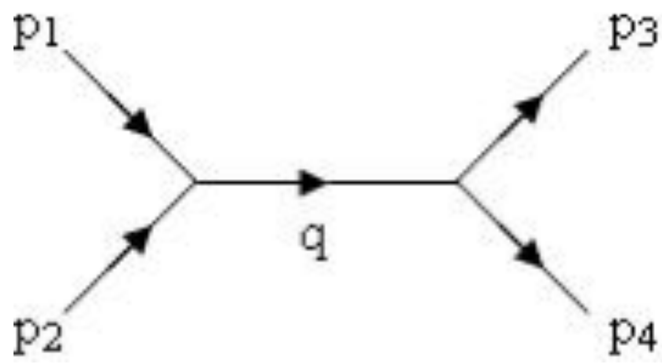


(a) (b)

Zero-modes
$k_1^+ = 0$, $k_2^+ = 0$

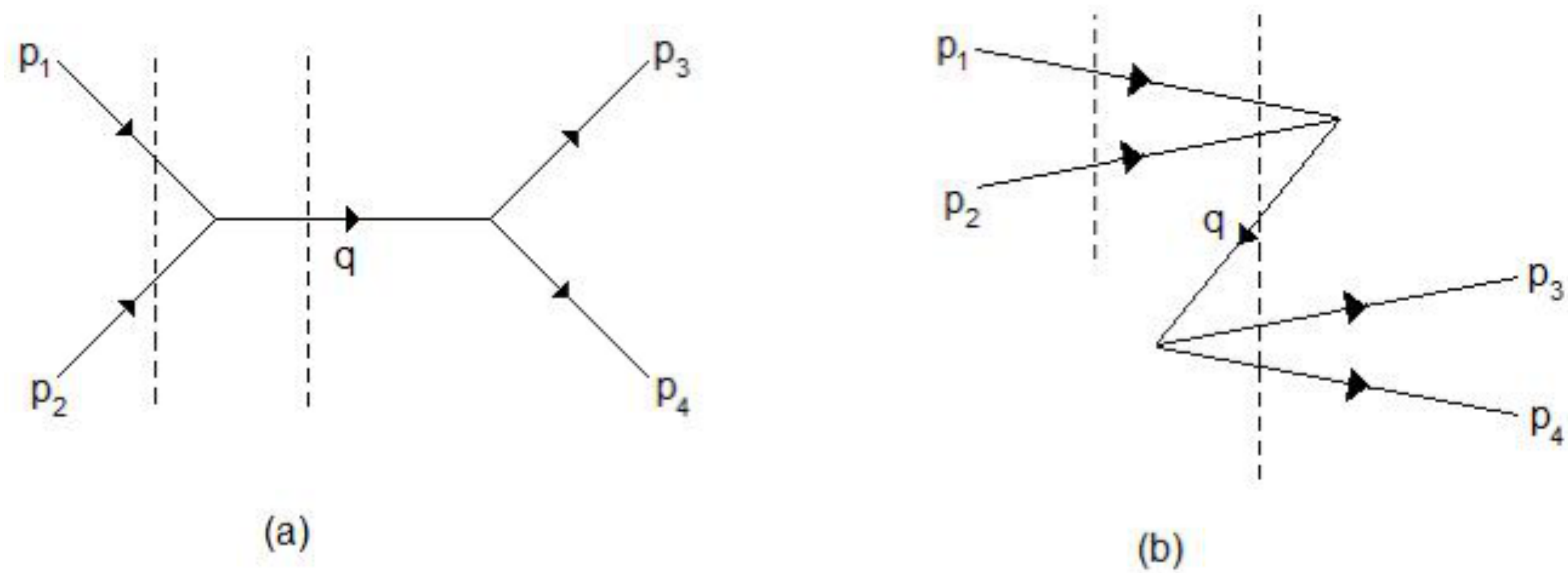
Stability Group

6 7 (maximum)



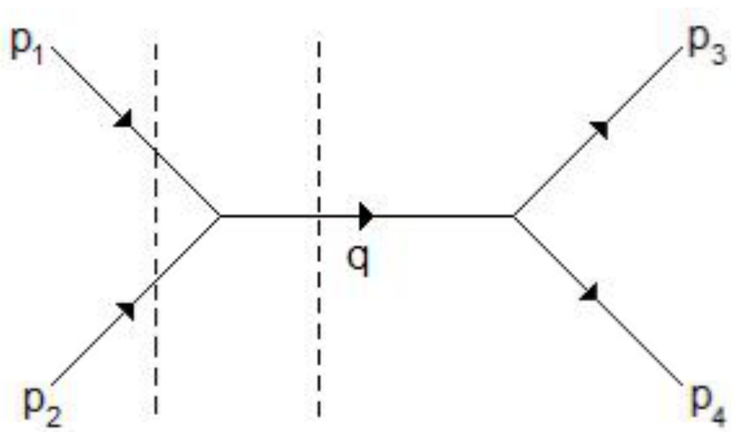
$$= \frac{1}{q^2 - m^2} = \frac{1}{s - m^2}$$

Feynman Diagram: Invariant under all Poincaré generators



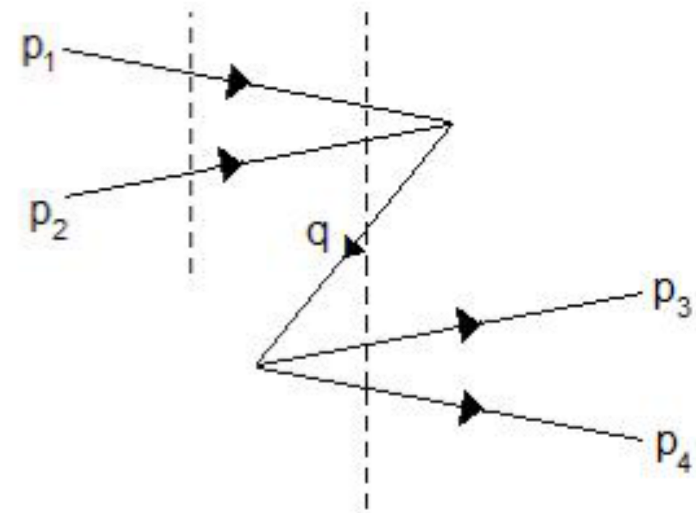
Individual Time-Ordered Diagrams: Invariant under stability group

Kinematic vs. Dynamic Generators



(a)

$$\frac{1}{E_1 + E_2 - Eq}$$



(b)

$$-\frac{1}{Eq + E_3 + E_4}$$

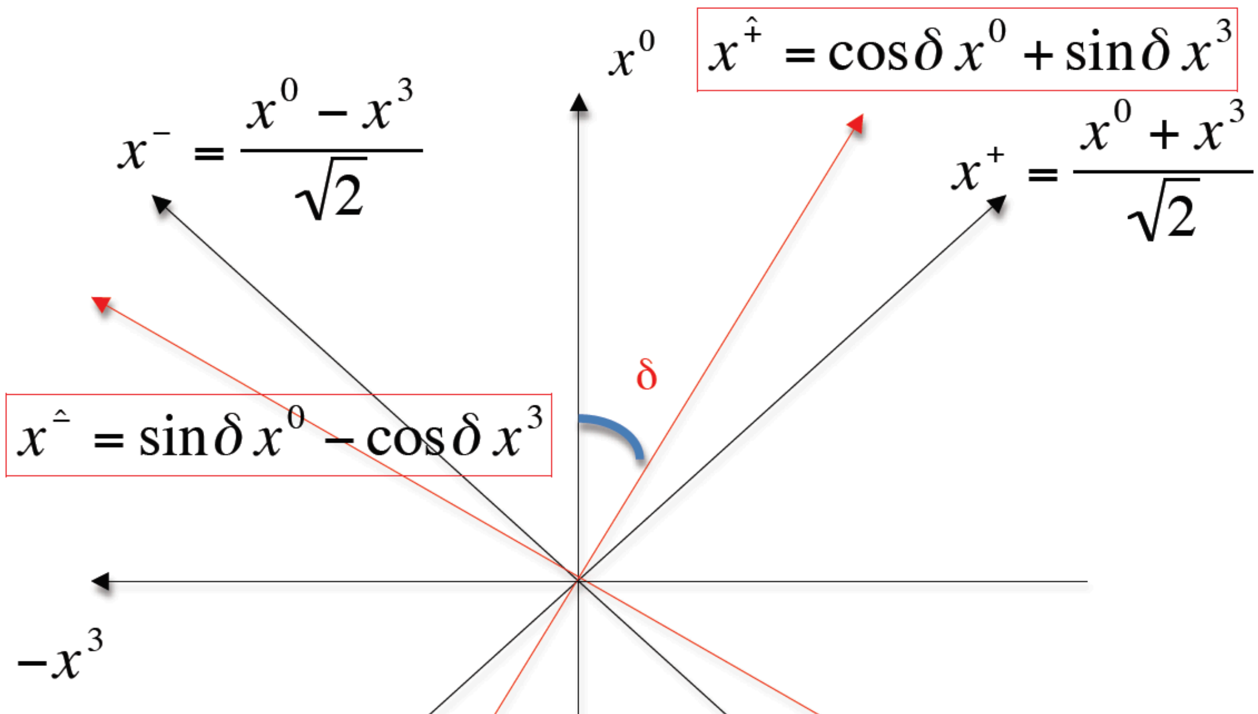
$$= -\frac{1}{Eq + E_1 + E_2}$$

$$\rightarrow \quad 0$$

S.Weinberg, PR158,1638(1967)
"Dynamics at Infinite Momentum"

Note however this is still in the instant form.

Interpolation between Instant and Front Forms



K. Hornbostel, PRD45, 3781 (1992) – RQFT

C.Ji and S.Rey, PRD53,5815(1996) – Chiral Anomaly

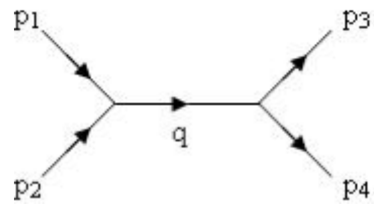
C.Ji and C. Mitchell, PRD64,085013 (2001) – Poincare Algebra

C.Ji and A. Suzuki, PRD87,065015 (2013) – Scattering Amps

C.Ji, Z. Li and A. Suzuki, PRD91, 065020 (2015) – EM Gauges

Z.Li, M. An and C.Ji, PRD92, 105014 (2015) – Spinors

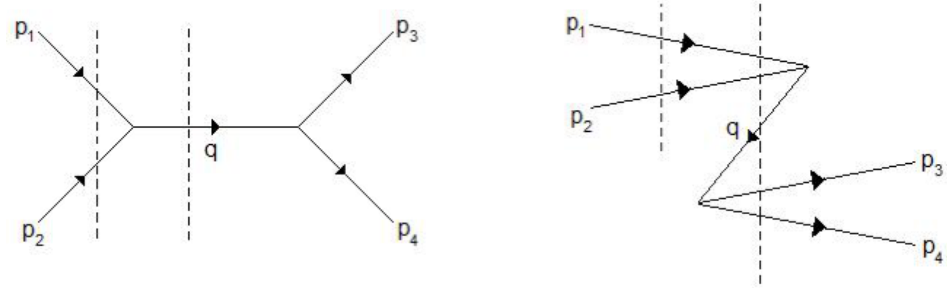
C.Ji, Z.Li, B.Ma and A.Suzuki, in prepartion – Fermion Prop.



$$\delta = 0$$
$$p_0 = p^0$$
$$-p_3 = p^3$$

$$0 < \delta < \pi/4$$
$$p_{\hat{+}} = p^0 \cos\delta - p^3 \sin\delta$$
$$p_{\hat{-}} = p^0 \sin\delta + p^3 \cos\delta$$

$$\delta = \pi/4$$
$$p_+ = p^-$$
$$p_- = p^+$$



(a) (b)

$$\frac{1}{2q^0}\left(\frac{1}{p_1^0 + p_2^0 - q^0} - \frac{1}{p_1^0 + p_2^0 + q^0}\right)$$

$$\frac{1}{2\omega_q}\left(\frac{1}{P_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} - \omega_q}{\mathbb{C}}} - \frac{1}{P_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} + \omega_q}{\mathbb{C}}}\right)$$

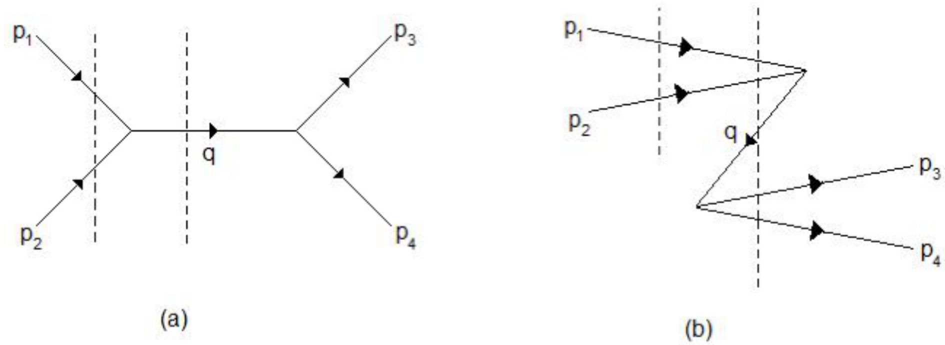
$$\omega_q = \sqrt{q_{\hat{-}}^2 + \mathbb{C}\left(\vec{\mathbf{q}}_{\hat{\perp}}^2 + m^2\right)}$$

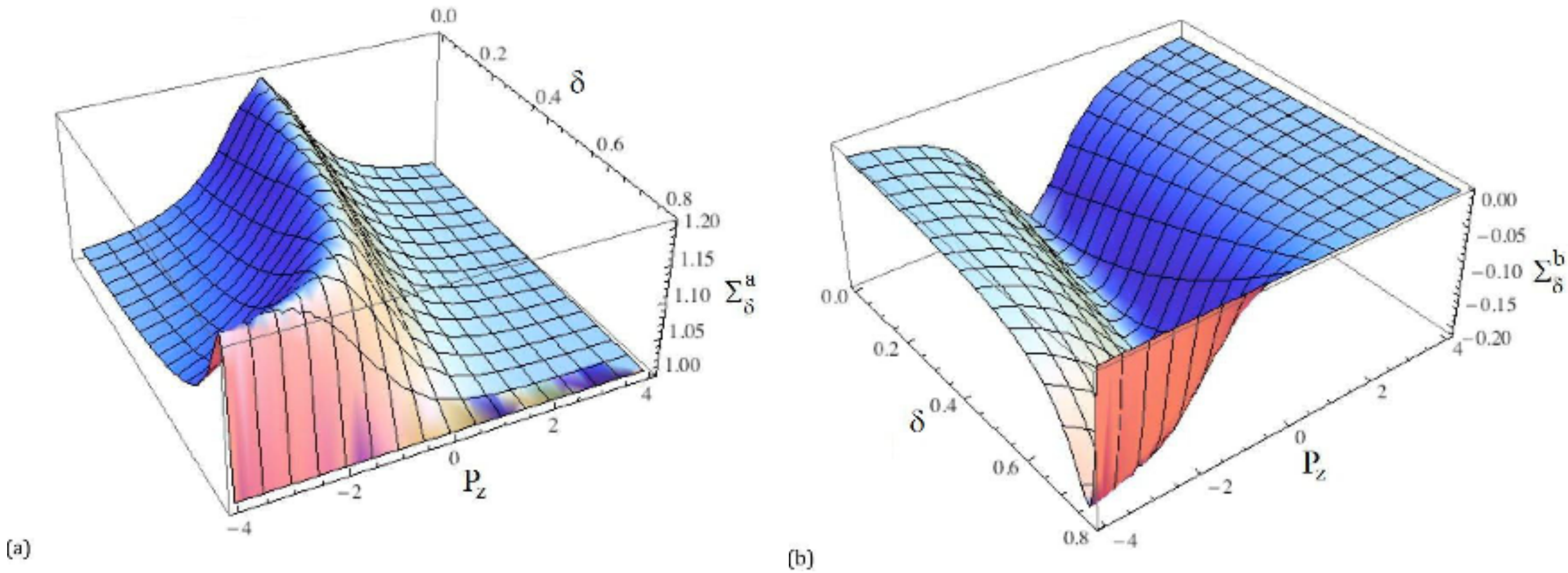
$$\mathbb{C} = \cos 2\delta$$

$$\mathbb{S} = \sin 2\delta$$

$$\frac{1}{P^+}\frac{1}{\left\{P^- - \frac{(\vec{\mathbf{P}}_\perp^2 + m^2)}{2P^+}\right\}}$$

$$\frac{\mathbb{S}q_{\hat{-}} + \omega_q}{\mathbb{C}} \to \frac{2}{\mathbb{C}} - \frac{\vec{\mathbf{q}}_{\hat{\perp}}^2 + m^2}{2q_{\hat{-}}} + \mathcal{O}(\mathbb{C})$$
$$\to \infty \quad \text{as } \mathbb{C} \to 0$$

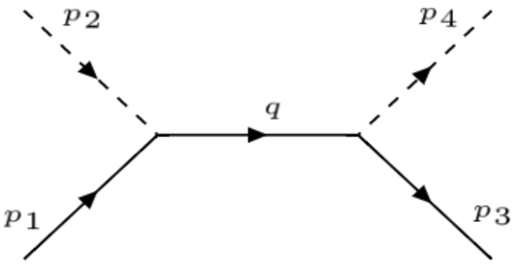




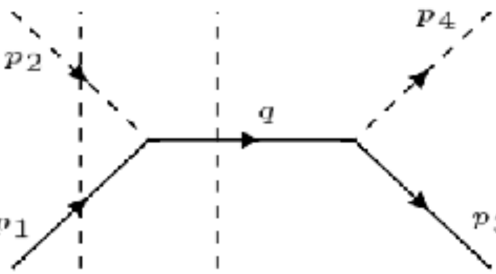
$\Sigma(a)+\Sigma(b)=1/(s-m^2)$; s=2 GeV^2, m=1GeV

J-shape peak & valley : $P_z = -\sqrt{\frac{s(1-C)}{2C}}$; $C = \cos(2\delta)$

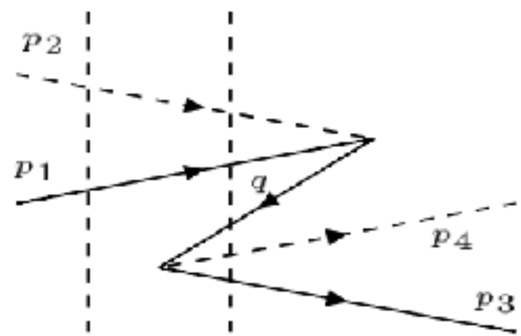
As C → 0, $P^+ = P^0 + P_z$ → 0 leads to LF Zero-modes.



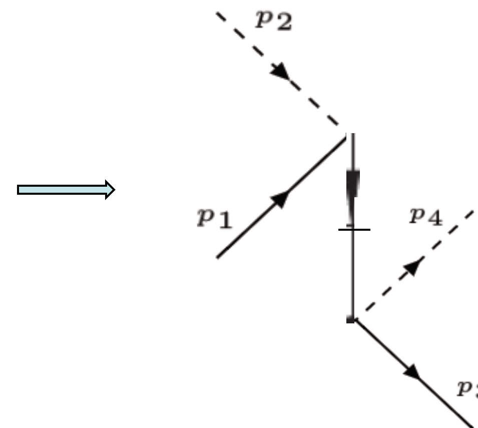
Fermion Propagator



(a)



(b)



$$\Sigma_a^{\mathsf{IFD}} + \Sigma_b^{\mathsf{IFD}} = \frac{1}{2q^0_{on}}\left(\frac{\not q + m}{q^0 - q^0_{on}} - \frac{\not q + m}{q^0 + q^0_{on}}\right)$$

$$= \frac{1}{2q^0_{on}} \frac{2q^0_{on}(\not q + m)}{(q^0)^2 - (q^0_{on})^2}$$

$$= \frac{\not q + m}{q^2 - m^2}$$

$$\Sigma_{a,\delta\to\frac{\pi}{4}} = \frac{\not q_{on} + m}{q^2 - m^2}$$

$$\Sigma_{b,\delta\to\frac{\pi}{4}} = \frac{\gamma^+}{2q^+}$$

$$\boxed{\frac{1}{\not q - m} = \frac{\sum_s u(q,s)\overline{u}(q,s)}{q^2 - m^2} + \frac{\gamma^+}{2q^+}}$$

S.-J.Chang and T.-M.Yan, PRD7,1147(1973)

LNA of $\bar{D} - \bar{U} \equiv \int_0^1 dx(\bar{d} - \bar{u})$

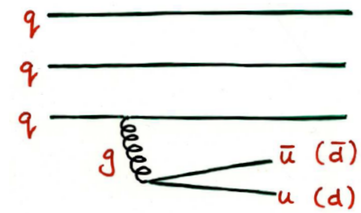
$$(\bar{D} - \bar{U})_{LNA} = \frac{3g_A^2 + 1}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

$$= \frac{4g_A^2 + (1 - g_A^2)}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

"on-shell" contribution

δ-function contribution

"Wally Melnitchouk's plenary talk this morning"



PDFs using ChPT

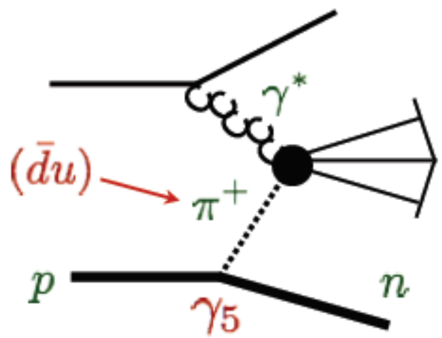
- *Motivation*: can one understand flavor asymmetries in the nucleon (*e.g.* $\bar{d} - \bar{u}$) from QCD?

$\longrightarrow$ origin of 5-quark Fock components $|qqq\,\bar{q}q\rangle$ of nucleon

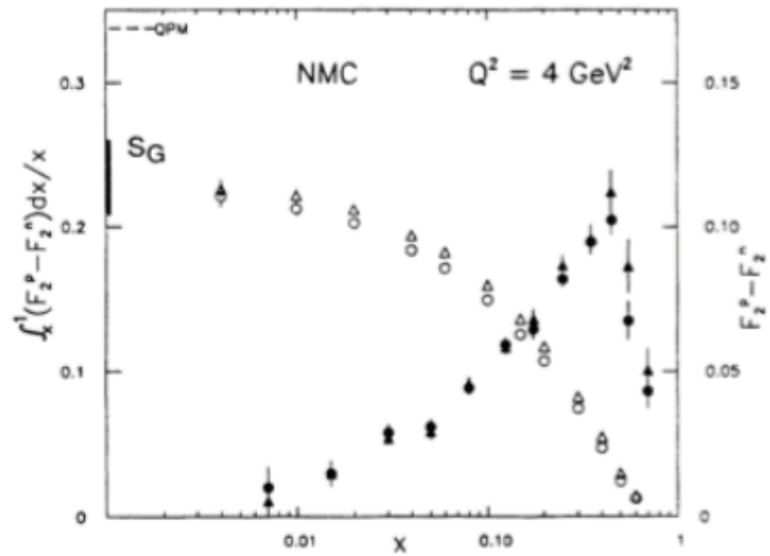
$$\int_0^1 dx\,(\bar{d}(x) - \bar{u}(x)) = 0.118 \pm 0.012$$ *E866 (Fermilab), PRD* ***64****, 052002 (2001)*

Sullivan process

$\boxed{\bar{d} > \bar{u}} \longleftrightarrow \boxed{\pi^+ n > \pi^- \Delta^{++}}$



Sullivan, PRD ***5****, 1732 (1972)*
Thomas, PLB ***126****, 97 (1983)*



$$\int_0^1 \frac{dx}{x}(F_2^p - F_2^n) = \frac{1}{3} - \frac{2}{3}\int_0^1 dx\,(\bar{d} - \bar{u}) = 0.235(26)$$

New Muon Collaboration, PRD ***50****, 1 (1994)*

Connection with QCD

- $(\bar{d}-\bar{u})(x) = \frac{2}{3}\int_x^1 \frac{dy}{y} f_\pi(y)\, \bar{q}^\pi(x/y)$ $\boxed{f_\pi(y) = \frac{3g^2_{\pi NN}}{16\pi^2}\, y \int dt \frac{-t\, \mathcal{F}^2_{\pi NN}(t)}{(t-m_\pi^2)^2}}$

$\longrightarrow$ *model-independent* leading nonanalytic (LNA) behavior *consistent with Chiral Symmetry of QCD.*

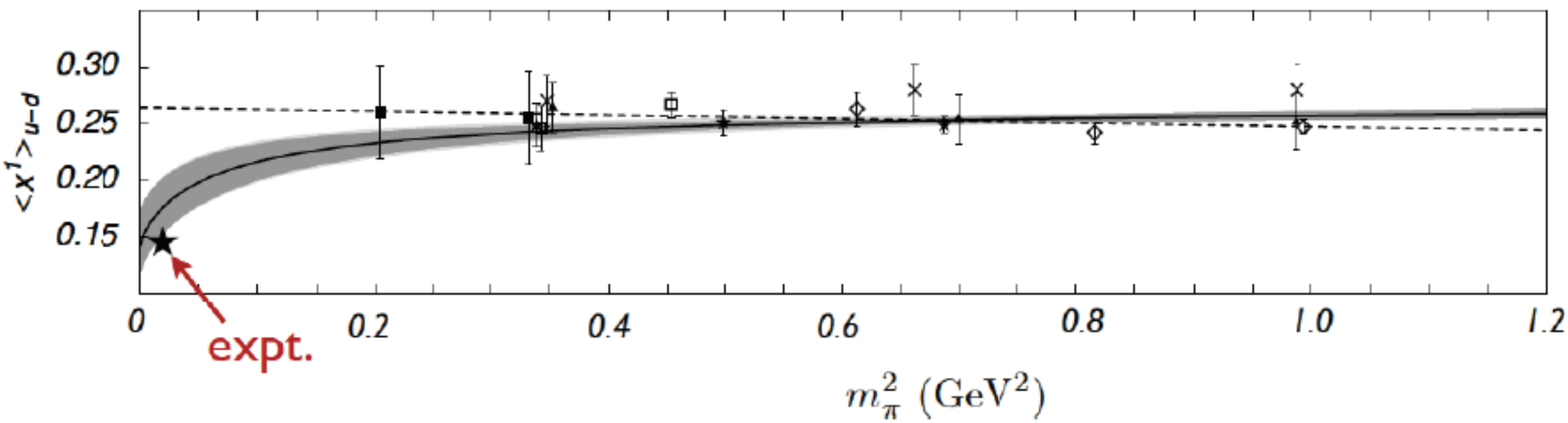
$$\langle x^0\rangle_{\bar{d}-\bar{u}} \equiv \int_0^1 dx(\bar{d}-\bar{u})$$

$$\boxed{m_\pi^2 f_\pi^2 = -2m_q < \bar{q}q >}$$

$$= \frac{2}{3}\int_0^1 dy\, f_\pi(y) = \frac{2g_A^2}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2/\mu^2) \ + \ \text{analytic terms}$$

- Nonanalytic behavior vital for chiral extrapolation of lattice data

Thomas, Melnitchouk, Steffens PRL **85**, 2892 (2000)



- Direct calculation of matrix elements of local twist-2 operators in ChPT disagrees with "Sullivan" result

$$\langle x^n \rangle_{u-d} = a_n \left(1 - \frac{(3g_A^2+1)}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2/\mu^2)\right) + \mathcal{O}(m_\pi^2)$$

Chen, X. Ji, PLB **523**, 107 (2001)
Arndt, Savage, NPA **692**, 429 (2002)

$\boxed{\text{cf. } 4g_A^2 \text{ in "Sullivan", via moments of } f_\pi(y)}$

Puzzle lingered more than a decade…

Thomas, Melnitchouk, Steffens, PRL **85**, 2892 (2000)

→ is there a problem with application of ChPT or "Sullivan process" to DIS?

→ is light-front treatment of pion loops problematic?

→ investigate relation between *covariant*, *instant-form*, and *light-front* formulations

C. Ji, W. Melnitchouk & A. Thomas, PRD80, 054018 (2009); PRL110, 179101 (2013); PRD88, 076005 (2013)

M. Burkardt, K. Hendricks, C. Ji, W. Melnitchouk & A. Thomas, PRD87, 056009 (2013)

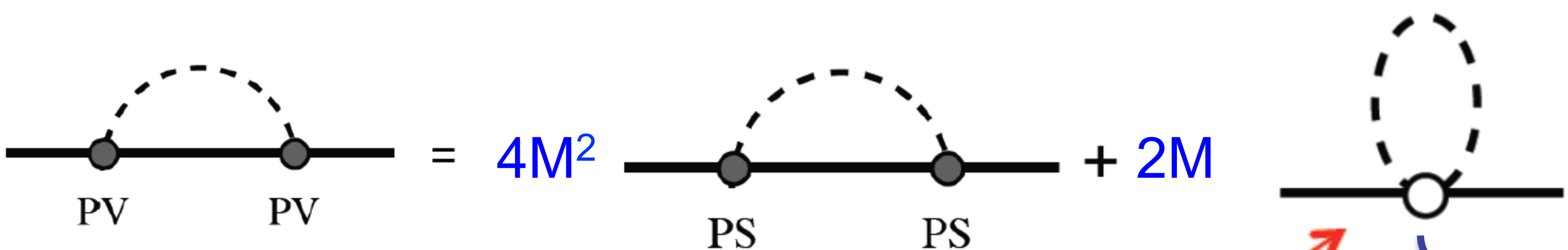
Relation between PV and PS Theories

Self-Energy

$$\Sigma^{PV} = \frac{1}{2}\sum_s \bar{u}(p,s)\hat{\Sigma}^{PV}\, u(p,s) \qquad \hat{\Sigma}^{PV} = -i\left(\frac{2g_A}{f_\pi}\right)^2 \vec{\tau}\cdot\vec{\tau}\int \frac{d^4k}{(2\pi)^4}\frac{\not{k}\gamma_5(\not{p}-\not{k}+M)\gamma_5\not{k}}{D_\pi D_N}$$

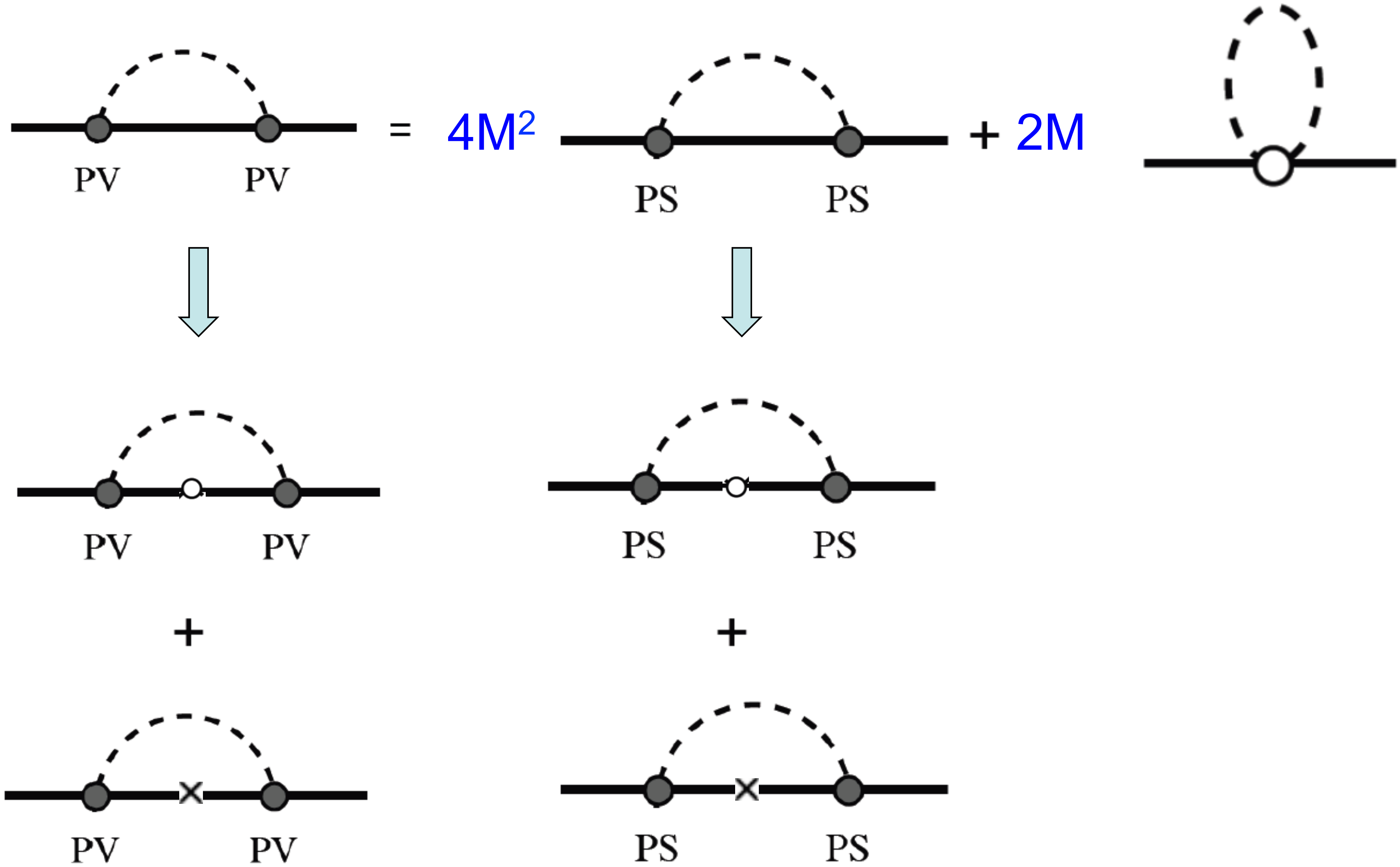
$$D_\pi = k^2 - m_\pi^2 + i\varepsilon \qquad D_N = (p-k)^2 - M^2 + i\varepsilon$$

$$\boxed{\begin{aligned} \bar{u}(p)\not{k}\gamma^5\frac{1}{\not{p}-\not{k}-M}\gamma^5\not{k}\,u(p) &= \bar{u}(p)[\not{k}-\not{p}+M]\gamma^5\frac{1}{\not{p}-\not{k}-M}\gamma^5[\not{k}-\not{p}+M]u(p) \\ &= 4M^2\,\bar{u}(p)\gamma^5\frac{1}{\not{p}-\not{k}-M}\gamma^5 u(p) + 2M\,\bar{u}(p)u(p) + \cancel{\bar{u}(p)\not{k}u(p)} \end{aligned}}$$

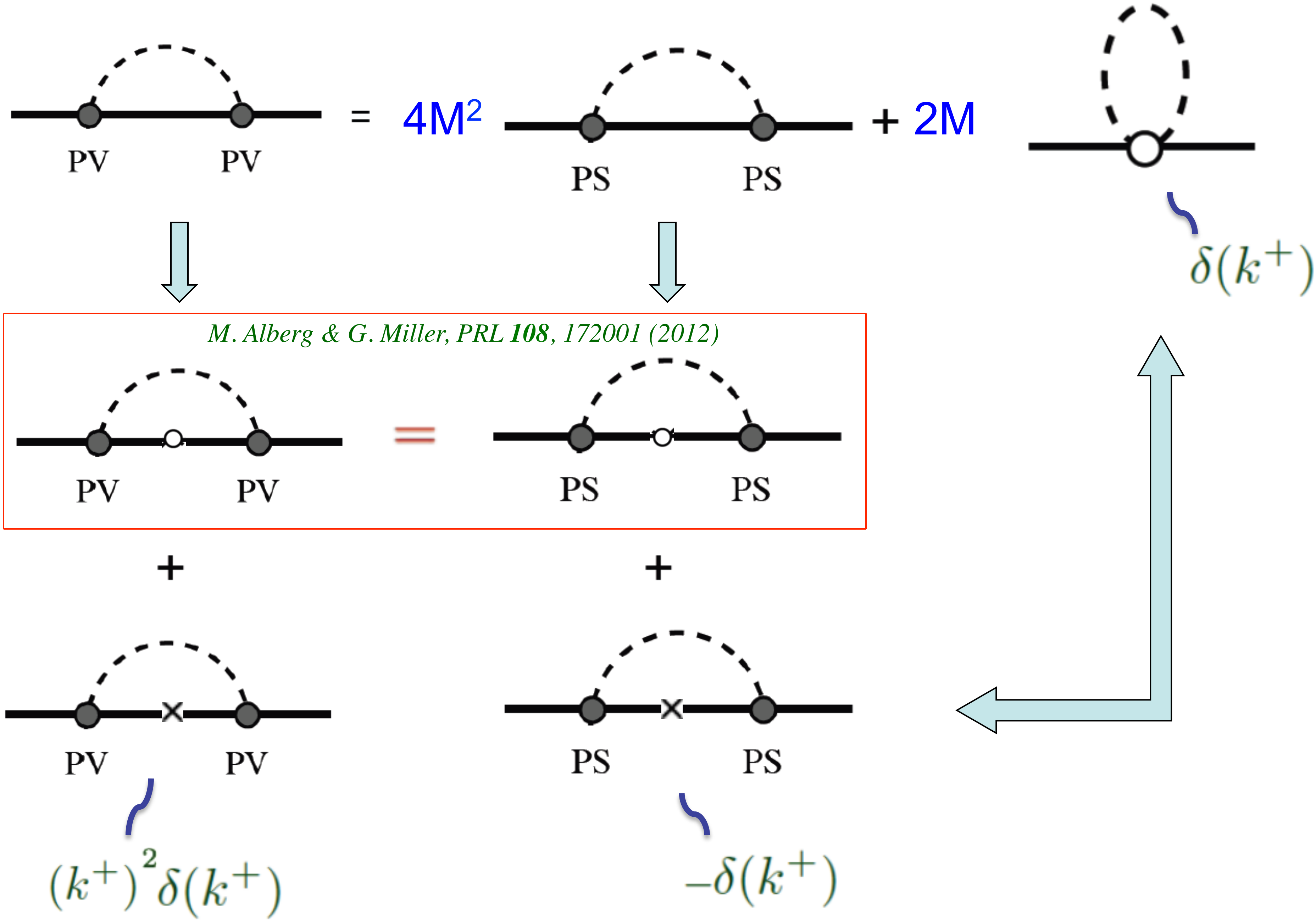


"treacherous" $k^+ = 0$ (end-point) term

$\delta(k^+)$



$$\boxed{\frac{1}{\not p - M} = \frac{\sum_s u(p,s)\bar{u}(p,s)}{p^2 - M^2} + \frac{\gamma^+}{2p^+}}$$

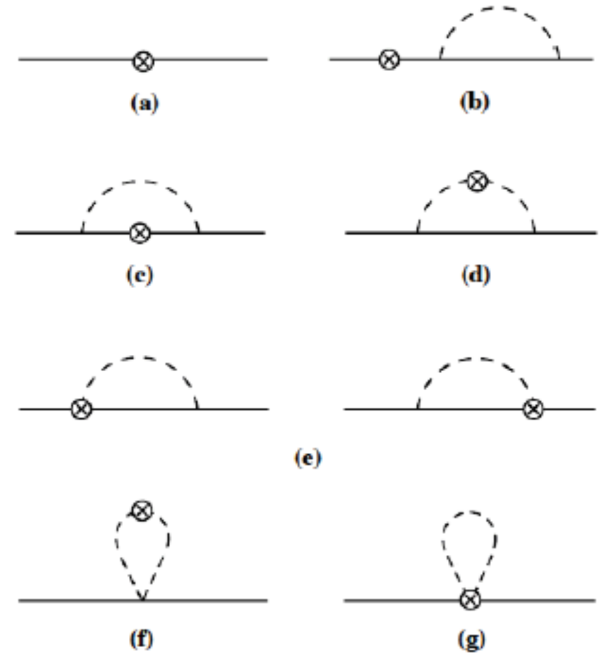


*C.Ji, W.Melnitchouk, A.W.Thomas, PRL **110**, 179191 (2013)*

Vertex corrections

- Pion cloud corrections to electromagnetic N coupling

  → N rainbow **(c)**, π rainbow **(d)**, Kroll-Ruderman **(e)**, π tadpole **(f)**, N tadpole **(g)**



- Vertex renormalization

$$(Z_1^{-1} - 1)\, \bar{u}(p)\, \gamma^\mu\, u(p) = \bar{u}(p)\, \Lambda^\mu\, u(p)$$

→ taking "+" components: $Z_1^{-1} - 1 \approx 1 - Z_1 = \dfrac{M}{p^+}\, \bar{u}(p)\, \Lambda^+\, u(p)$

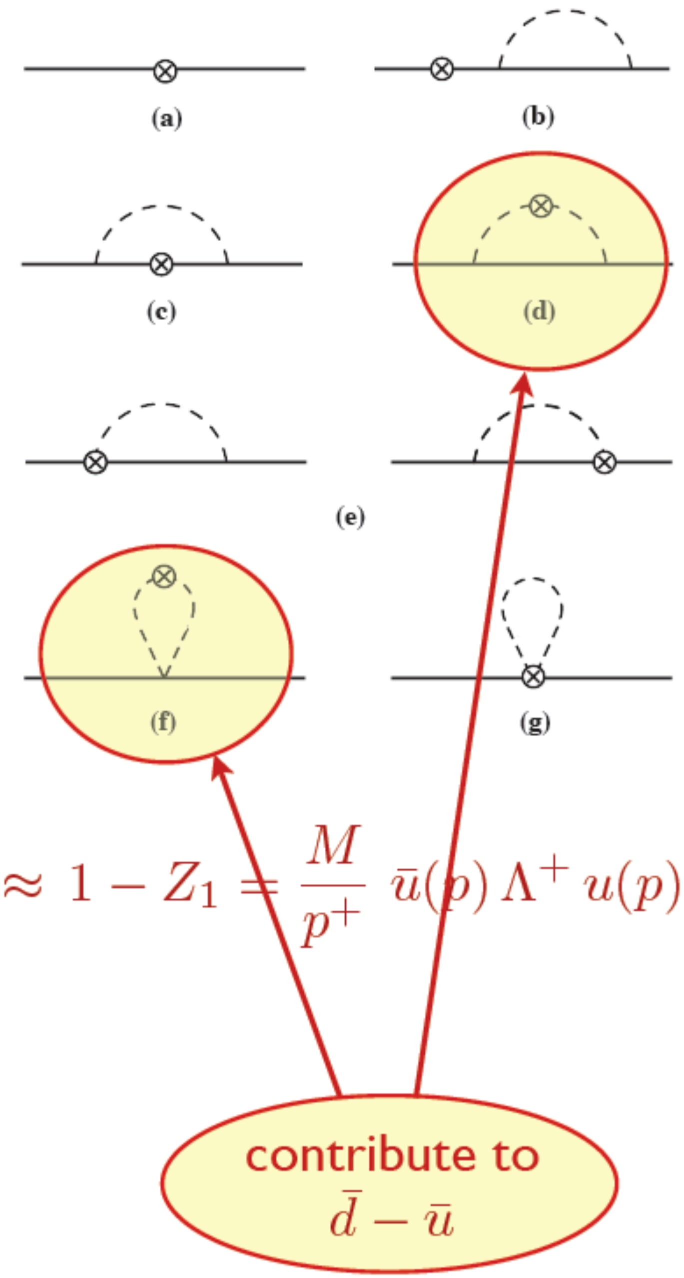
Ward-Takahashi identity $\Lambda^\mu = -\dfrac{\partial \hat{\Sigma}}{\partial p_\mu} \longrightarrow Z_1 = Z_2$

$$\frac{1}{\not{p} - M - \hat{\Sigma}} = \frac{Z_2}{\not{p} - (M + \delta M)} \quad ; \quad \hat{\Sigma} = -(Z_2^{-1} - 1)(\not{p} - M - \delta M) + \delta M$$

Flavor asymmetry

- Pion cloud corrections to electromagnetic N coupling

  → N rainbow **(c)**, π rainbow **(d)**, Kroll-Ruderman **(e)**, π bubble **(f)**, π tadpole **(g)**



- Vertex renormalization

$$(Z_1^{-1} - 1)\, \bar{u}(p)\, \gamma^\mu\, u(p) = \bar{u}(p)\, \Lambda^\mu\, u(p)$$

→ taking "+" components: $Z_1^{-1} - 1 \approx 1 - Z_1 = \dfrac{M}{p^+}\, \bar{u}(p)\, \Lambda^+\, u(p)$

→ *e.g.* for N rainbow contribution,

$$\Lambda_\mu^N = -\frac{\partial \hat{\Sigma}}{\partial p^\mu}$$

Nonanalytic behavior of vertex renormalization factors

	$1/D_\pi D_N^2$	$1/D_\pi^2 D_N$	$1/D_\pi D_N$	$1/D_\pi$ or $1/D_\pi^2$	sum (PV)	sum (PS)
$1 - Z_1^N$	g_A^2 *	0	$-\frac{1}{2}g_A^2$	$\frac{1}{4}g_A^2$	$\frac{3}{4}g_A^2$	g_A^2
$1 - Z_1^\pi$	0	g_A^2 *	0	$-\frac{1}{4}g_A^2$	$\frac{3}{4}g_A^2$	g_A^2
$1 - Z_1^{\mathrm{KR}}$	0	0	$-\frac{1}{2}g_A^2$	$\frac{1}{2}g_A^2$	0	0
$1 - Z_1^{N\,\mathrm{tad}}$	0	0	0	$-1/2$	$-1/2$	0
$1 - Z_1^{\pi\,\mathrm{tad}}$	0	0	0	$1/2$	$1/2$	0

* also in PS

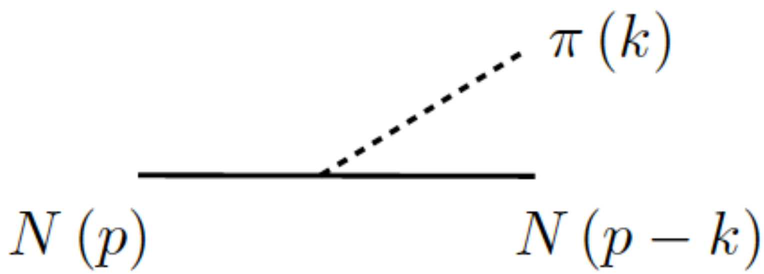
in units of $\frac{1}{(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$

→ origin of ChPT *vs*. Sullivan process difference clear!

$$\boxed{\left(1 - Z_1^{N\,(\mathrm{PV})}\right)_{\mathrm{LNA}} = \frac{3}{4}\left(1 - Z_1^{N\,(\mathrm{PS})}\right)_{\mathrm{LNA}}}$$

Pion splitting functions

- Each diagram can be represented by $N \to N\pi$ "splitting function" $f_i(y)$ (light-cone momentum distribution function)



$$y = \frac{k^+}{p^+}$$

- Vertex renormalization is k^+ moment of $f_i(y)$

$$1 - Z_1^i = \int dy\, f_i(y)$$

Pion splitting functions

- Summary of splitting functions:

$$1 - Z_1^i = \int dy\, f_i(y)$$

where
$$f_\pi(y) = f^{(\text{on})}(y) + f^{(\delta)}(y)$$
$$f_N(y) = f^{(\text{on})}(y) + f^{(\text{off})}(y) - f^{(\delta)}(y)$$
$$f_{\text{KR}}(y) = f^{(\text{off})}(y) - 2f^{(\delta)}(y)$$
$$f_{\text{tad}}(y) = -f_{\text{bub}}(y) \;=\; 2f^{(\text{tad})}(y)$$

with components
$$f^{(\text{on})}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2\, \frac{y(k_\perp^2 + y^2 M^2)}{[k_\perp^2 + y^2 M^2 + (1-y)m_\pi^2]^2}$$
$$f^{(\text{off})}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2\, \frac{y}{k_\perp^2 + y^2 M^2 + (1-y)m_\pi^2}$$
$$f^{(\delta)}(y) = \frac{g_A^2}{4(4\pi f_\pi)^2} \int dk_\perp^2\, \log\left(\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right) \delta(y)$$
$$f^{(\text{tad})}(y) = -\frac{4}{g_A^2}\, f^{(\delta)}(y)$$

tadpole & bubble
equal & opposite

$$(1 - Z_1^{\text{tad}}) = -(1 - Z_1^{\text{bub}})$$

$4g_A^2$ in "Sullivan", via moments of $f_\pi(y)$ *Chen, X. Ji, PLB **523**, 107 (2001)* *Arndt, Savage, NPA **692**, 429 (2002)*

$$\langle x^n \rangle_{u-d} = a_n \left(1 - \frac{(3g_A^2+1)}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2/\mu^2) \right) + \mathcal{O}(m_\pi^2)$$

C. Ji, W. Melnitchouk & A. Thomas, PRD88, 076005 (2013)

$$\mathcal{M}_N^{(p-n)} \xrightarrow{\text{LNA}} 1 - \frac{(4g_A^2 + [1-g_A^2])}{(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

$$\mathcal{M}_\pi^{(p-n)} \xrightarrow{\text{LNA}} \frac{(4g_A^2 + [1-g_A^2])}{(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$$

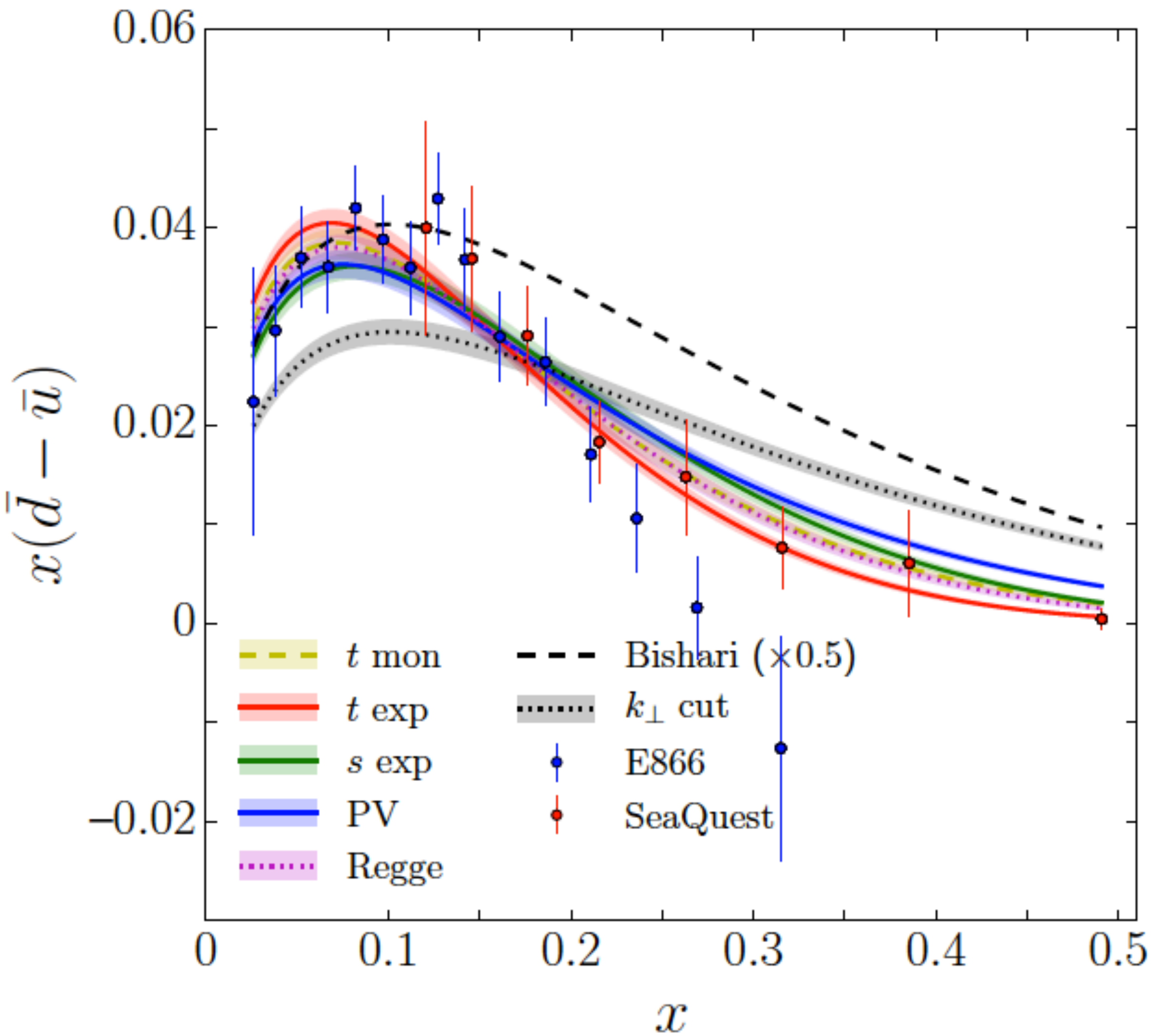
PS ("on-shell") contribution

δ-function contribution

J. McKenney, N. Sato, W. Melnitchouk, C.Ji, PRD93, 054011 (2016)

P. Barry, N. Sato, W. Melnitchouk, C.Ji, in progress

SeaQuest Preliminary Result: APS April 2016 Talk by B.Kerns



Conclusion and Outlook

- LFD is not just formal but consequential in the analysis of physical observables.
- Maximal stability group of LFD saves dynamic efforts.
- LFZM is treacherous, but useful in resolution of theoretical issues such as LNA factor difference in chiral effective theory.
- Quantitative analysis of LFZM contribution in hadron phenomenology is still open for progress.