Light-front field theory in the description of hadrons
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Dirac’s Proposition

Traditional approach evolved from NR dynamics

Progressive approach for relativistic dynamics

Close contact with Euclidean space

Strictly in Minkowski space

T-dept QFT, LQCD, etc.

DIS, PDFs, DVCS, GPDs, etc.

1949
Dirac’s Proposition

Traditional approach evolved from NR dynamics

Innovative approach for relativistic dynamics

Can they be linked?

Close contact with Euclidean space

Strictly in Minkowski space

T-dept QFT, LQCD, etc.

DIS, PDFs, DVCS, GPDs, etc.
Outline

• Interpolation between IFD and LFD
  - Whole landscape between the two
  - Clarification on IMF and LFD
  - LF Zero-mode (LFZM)
• Application to flavor asymmetry of proton sea
  - LNA as “GPS” in Phenomenology
  - Resolution of LNA Factor Difference with LFZM
  - SeaQuest preliminary result
• Conclusion and Outlook
Equal $t$

\[ p^0 \iff p^+ = p^0 + p^3 \]

\[ (p^1, p^2) \iff p_\perp \]

Equal $\tau$

\[ p^- = p^0 - p^3 \]

Energy-Momentum Dispersion Relations

\[ p^0 = \sqrt{p^2 + m^2} \]

\[ p^- = \frac{p_\perp^2 + m^2}{p^+} \]

Zero-modes

\[ k_1^+ = 0, \ k_2^+ = 0 \]

Stability Group

6

7 (maximum)
Feynman Diagram: Invariant under all Poincaré generators

\[ \frac{1}{q^2 - m^2} = \frac{1}{s - m^2} \]

Individual Time-Ordered Diagrams: Invariant under stability group

Kinematic vs. Dynamic Generators
\[
\frac{1}{E_1 + E_2 - Eq} - \frac{1}{Eq + E_3 + E_4} = -\frac{1}{Eq + E_1 + E_2} \rightarrow 0
\]

S. Weinberg, PR158, 1638 (1967)

“Dynamics at Infinite Momentum”

Note however this is still in the instant form.
Interpolation between Instant and Front Forms

\[ x^- = \frac{x^0 - x^3}{\sqrt{2}} \]

\[ x^\pm = \frac{x^0 \mp x^3}{\sqrt{2}} \]

\[ x^\pm = \sin \delta \, x^0 - \cos \delta \, x^3 \]

\[ x^+ = \frac{x^0 + x^3}{\sqrt{2}} \]

K. Hornbostel, PRD45, 3781 (1992) – RQFT
C. Ji and S. Rey, PRD53, 5815 (1996) – Chiral Anomaly
C. Ji and A. Suzuki, PRD87, 065015 (2013) – Scattering Amps
\[
\delta = 0 \\
p_0 = p^0 \\
-p_3 = p^3
\]

\[
\begin{align*}
0 < \delta < \pi / 4 & \quad p_+ = p^0 \cos \delta - p^3 \sin \delta \\
p_- = p^0 \sin \delta + p^3 \cos \delta
\end{align*}
\]

\[
\delta = \pi / 4 \\
p_+ = p^\pm
\]

\[
\begin{align*}
\frac{1}{2q^0} \left( \frac{1}{p_1^0 + p_2^0 - q^0} - \frac{1}{p_1^0 + p_2^0 + q^0} \right) & \quad \frac{1}{2\omega_q} \left( \frac{1}{P^+_q + \frac{S_{q^-} - \omega_q}{\mathcal{C}}} - \frac{1}{P^+_q + \frac{S_{q^-} + \omega_q}{\mathcal{C}}} \right) \\
\frac{1}{P^+} \left\{ p^- - \frac{(p_1^2 + m^2)}{2p^0} \right\} & \quad \frac{S_{q^-} + \omega_q}{\mathcal{C}} \to \frac{2}{\mathcal{C}} - \frac{q_\perp^2 + m^2}{2q_\perp} + \mathcal{O}(C) \quad \to \infty \quad \text{as } C \to 0
\end{align*}
\]

\[
\omega_q = \sqrt{q_\perp^2 + \mathcal{C} \left( q_\perp^2 + m^2 \right)} \\
\mathcal{C} = \cos 2\delta \\
S = \sin 2\delta
\]
\[ \Sigma(a) + \Sigma(b) = \frac{1}{(s-m^2)} ; \quad s=2 \text{ GeV}^2, \quad m=1\text{GeV} \]

**J-shape peak & valley:**

\[ P_z = -\sqrt{\frac{s(1-C)}{2C}} ; \quad C = \cos(2\delta) \]

As \( C \to 0 \), \( P^+ = P^0 + P_z \to 0 \) leads to LF Zero-modes.
Fermion Propagator

\[ \sum_{a}^{\text{IFD}} + \sum_{b}^{\text{IFD}} = \frac{1}{2q_{on}^{0}} \left( \frac{q^{0} + m}{q^{0} - q_{on}^{0}} - \frac{q^{0} + m}{q^{0} + q_{on}^{0}} \right) \]

\[ = \frac{1}{2q_{on}^{0}} \left( \frac{2q_{on}^{0}(q^{0} + m)}{(q^{0})^{2} - (q_{on}^{0})^{2}} \right) \]

\[ = \frac{q^{0} + m}{q^{2} - m^{2}} \]

S.-J. Chang and T.-M. Yan, PRD7, 1147(1973)
\[ \text{LNA of} \quad \bar{D} - \bar{U} \equiv \int_0^1 dx (\bar{d} - \bar{u}) \]

\[ (\bar{D} - \bar{U})_{LNA} = \frac{3g_A^2 + 1}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2 \]

\[ = \frac{4g_A^2 + (1 - g_A^2)}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2 \]

“on-shell” contribution

\(\delta\)-function contribution
Wally Melnitchouk’s plenary talk this morning

PDFs using ChPT

**Motivation:** can one understand flavor asymmetries in the nucleon (e.g. $\bar{d} - \bar{u}$) from QCD?

$\int_0^1 dx (\bar{d}(x) - \bar{u}(x)) = 0.118 \pm 0.012$  

_E866 (Fermilab), PRD 64, 052002 (2001)_

→ origin of 5-quark Fock components $|qqq \bar{q}q\rangle$ of nucleon

Sullivan process

$\bar{d} > \bar{u}$  

$\pi^+ n > \pi^- \Delta^{++}$

$\int_0^1 \frac{dx}{x} (F_2^p - F_2^n) = \frac{1}{3} - \frac{2}{3} \int_0^1 dx (\bar{d} - \bar{u}) = 0.235(26)$

_New Muon Collaboration, PRD 50, 1 (1994)_

_Sullivan, PRD 5, 1732 (1972)_

_Thomas, PLB 126, 97 (1983)_
Connection with QCD

\[(\bar{d} - \bar{u})(x) = \frac{2}{3} \int_x^1 \frac{dy}{y} f_\pi(y) \bar{q}(x/y)\]

\[f_\pi(y) = \frac{3g_{\pi NN}^2}{16\pi^2} y \int dt \frac{-t F_{\pi NN}^2(t)}{(t - m_{\pi}^2)^2}\]

→ model-independent leading nonanalytic (LNA) behavior consistent with Chiral Symmetry of QCD.

\[\langle x^0 \rangle_{\bar{d} - \bar{u}} \equiv \int_0^1 dx (\bar{d} - \bar{u}) = \frac{2}{3} \int_0^1 dy f_\pi(y) = \frac{2g_A^2}{(4\pi f_\pi)^2} m_{\pi}^2 \log(m_{\pi}^2/\mu^2) + \text{analytic terms}\]

Nonanalytic behavior vital for chiral extrapolation of lattice data

*Thomas, Meinichouk, Steffens PRL 85, 2892 (2000)*
Direct calculation of matrix elements of local twist-2 operators in ChPT disagrees with “Sullivan” result

\[ \langle x^n \rangle_{u-d} = a_n \left( 1 - \frac{(3g_A^2 + 1)}{(4\pi f_\pi)^2} m^2_\pi \log\left(m^2_\pi / \mu^2\right) \right) + \mathcal{O}(m^2_\pi) \]

*cf.* \(4g_A^2\) in “Sullivan”, via moments of \(f_\pi(y)\)

Chen, X. Ji, PLB 523, 107 (2001)
Arndt, Savage, NPA 692, 429 (2002)

Puzzle lingered more than a decade…

→ is there a problem with application of ChPT or “Sullivan process” to DIS?

→ is light-front treatment of pion loops problematic?

→ investigate relation between covariant, instant-form, and light-front formulations

Thomas, Melnitchouk, Steffens, PRL 85, 2892 (2000)

C. Ji, W. Melnitchouk & A. Thomas, PRD80, 054018 (2009); PRL110, 179101 (2013);
PRD88, 076005 (2013)
Relation between PV and PS Theories

Self-Energy

\[ \Sigma_{PV}^{PV} = \frac{1}{2} \sum_s \bar{u}(p,s) \Sigma_{PV}^{PV} u(p,s) \]

\[ \Sigma_{PV} = -i \left( \frac{2g_A}{f_\pi} \right)^2 \vec{\tau} \cdot \vec{\tau} \int \frac{d^4k}{(2\pi)^4} \frac{k \gamma_5 (p-k+M) \gamma_5 k}{D_{\pi} D_N} \]

\[ D_{\pi} = k^2 - m^2_{\pi} + i\epsilon \]

\[ D_N = (p-k)^2 - M^2 + i\epsilon \]

\[ \bar{u}(p) k \gamma^5 \frac{1}{p-k-M} \gamma^5 k u(p) = \bar{u}(p) [k-p+M] \gamma^5 \frac{1}{p-k-M} \gamma^5 [k-p+M] u(p) \]

\[ = 4M^2 \bar{u}(p) \gamma^5 \frac{1}{p-k-M} \gamma^5 u(p) + 2M \bar{u}(p)u(p) + \bar{u}(p)k u(p) \]

= 4M^2 \quad \text{PV} \quad \text{PV}

= 4M^2 \quad \text{PS} \quad \text{PS} + 2M

“treacherous” \( k^+ = 0 \) (end-point) term
\[
\frac{1}{p^2 - M^2} = \sum_s u(p, s) \bar{u}(p, s) \frac{\gamma^+}{p^2 - M^2} + \frac{\gamma^+}{2p^+}
\]
\[ PV \quad PV \quad = \quad 4M^2 \quad PS \quad PS \quad + \quad 2M \]

\[ PV \quad PV \quad = \quad \delta(k^+) \quad PS \quad PS \]

\[ (k^+)^2 \delta(k^+) \quad PS \quad PS \quad - \delta(k^+) \]


C. Ji, W. Melnitchouk, A. W. Thomas, PRL 110, 179191 (2013)
Vertex corrections

- Pion cloud corrections to electromagnetic $N$ coupling
  - $N$ rainbow (c), $\pi$ rainbow (d), Kroll-Ruderman (e), $\pi$ tadpole (f), $N$ tadpole (g)

- Vertex renormalization
  
  &lt;mathjax content="\begin{align*}(Z_1^{-1} - 1) \bar{u}(p) \gamma^\mu u(p) &= \bar{u}(p) \Lambda^\mu u(p)\
\text{taking "+" components: } Z_1^{-1} - 1 &\approx 1 - Z_1 = \frac{M}{p^+} \bar{u}(p) \Lambda^+ u(p)\
\text{Ward-Takahashi identity } &\quad \Lambda^\mu = -\frac{\partial \hat{\Sigma}}{\partial p_\mu} \rightarrow Z_1 = Z_2\
\frac{1}{p - M - \hat{\Sigma}} &= \frac{Z_2}{p - (M + \delta M)} \quad ; \quad \hat{\Sigma} = -(Z_2^{-1} - 1)(p - M - \delta M) + \delta M""/>
Flavor asymmetry

- Pion cloud corrections to electromagnetic $N$ coupling
  - $N$ rainbow (c), $\pi$ rainbow (d), Kroll-Ruderman (e), $\pi$ bubble (f), $\pi$ tadpole (g)

- Vertex renormalization

$$\left(Z_1^{-1} - 1\right) \bar{u}(p) \gamma^\mu u(p) = \bar{u}(p) \Lambda^\mu u(p)$$

- Taking “+” components:
  $$Z_1^{-1} - 1 \approx 1 - Z_1 = \frac{M}{p^+} \bar{u}(p) \Lambda^+ u(p)$$

- e.g. for $N$ rainbow contribution,

$$\Lambda^N_{\mu} = - \frac{\partial \hat{\Sigma}}{\partial p^\mu}$$
Nonanalytic behavior of vertex renormalization factors

<table>
<thead>
<tr>
<th>1/$D_\pi^2$</th>
<th>1/$D_\pi^2 D_N$</th>
<th>1/$D_\pi D_N$</th>
<th>1/$D_\pi$ or 1/$D_\pi^2$</th>
<th>sum (PV)</th>
<th>sum (PS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 - Z_1^{N}$</td>
<td>$g_A^2 *$</td>
<td>0</td>
<td>$-\frac{1}{2} g_A^2$</td>
<td>$\frac{1}{4} g_A^2$</td>
<td>$\frac{3}{4} g_A^2$</td>
</tr>
<tr>
<td>$1 - Z_1^\pi$</td>
<td>0</td>
<td>$g_A^2 *$</td>
<td>0</td>
<td>$-\frac{1}{4} g_A^2$</td>
<td>$\frac{3}{4} g_A^2$</td>
</tr>
<tr>
<td>$1 - Z_1^{KR}$</td>
<td>0</td>
<td>0</td>
<td>$-\frac{1}{2} g_A^2$</td>
<td>$\frac{1}{2} g_A^2$</td>
<td>0</td>
</tr>
<tr>
<td>$1 - Z_1^{N \text{ tad}}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$-1/2$</td>
<td>$-1/2$</td>
</tr>
<tr>
<td>$1 - Z_1^{\pi \text{ tad}}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
<td>1/2</td>
</tr>
</tbody>
</table>

* also in PS

in units of $\frac{1}{(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2$

→ origin of ChPT vs. Sullivan process difference clear!

\[
\left(1 - Z_1^{N \text{(PV)}}\right)_{\text{LNA}} = \frac{3}{4} \left(1 - Z_1^{N \text{(PS)}}\right)_{\text{LNA}}
\]
Pion splitting functions

- Each diagram can be represented by $N \rightarrow N \pi$
  “splitting function” $f_i(y)$ (light-cone momentum distribution function)

\[ y = \frac{k^+}{p^+} \]

- Vertex renormalization is $k^+$ moment of $f_i(y)$

\[ 1 - Z_1^i = \int dy f_i(y) \]
Pion splitting functions

Summary of splitting functions:

\[ 1 - Z_1^i = \int dy \, f_i(y) \]

where

\[
\begin{align*}
    f_\pi(y) &= f^{(\text{on})}(y) + f^{(\delta)}(y) \\
    f_N(y) &= f^{(\text{on})}(y) + f^{(\text{off})}(y) - f^{(\delta)}(y) \\
    f_{KR}(y) &= f^{(\text{off})}(y) - 2f^{(\delta)}(y) \\
    f_{\text{tad}}(y) &= -f_{\text{bub}}(y) = 2f^{(\text{tad})}(y)
\end{align*}
\]

with components

\[
\begin{align*}
    f^{(\text{on})}(y) &= \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \frac{y(k_\perp^2 + y^2 M^2)}{[k_\perp^2 + y^2 M^2 + (1 - y)m_\pi^2]^2} \\
    f^{(\text{off})}(y) &= \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \frac{y}{k_\perp^2 + y^2 M^2 + (1 - y)m_\pi^2} \\
    f^{(\delta)}(y) &= \frac{g_A^2}{4(4\pi f_\pi)^2} \int dk_\perp^2 \log \left( \frac{k_\perp^2 + m_\pi^2}{\mu^2} \right) \delta(y) \\
    f^{(\text{tad})}(y) &= -\frac{4}{g_A^2} f^{(\delta)}(y)
\end{align*}
\]

tadpole & bubble

equal & opposite

\[ (1 - Z_1^{\text{tad}}) = -(1 - Z_1^{\text{bub}}) \]
$4g_A^2$ in “Sullivan”, via moments of $f_\pi(y)$

\[
\langle x^n \rangle_{u-d} = a_n \left( 1 - \frac{(3g_A^2 + 1)}{(4\pi f_\pi)^2} m_\pi^2 \log\left(\frac{m_\pi^2}{\mu^2}\right) \right) + \mathcal{O}(m_\pi^2)
\]


\[
\mathcal{M}_{N}^{(p-n)} \xrightarrow{\text{LNA}} 1 - \frac{(4g_A^2 + [1 - g_A^2])}{(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2
\]

\[
\mathcal{M}_{\pi}^{(p-n)} \xrightarrow{\text{LNA}} \frac{(4g_A^2 + [1 - g_A^2])}{(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2
\]

PS (“on-shell”) contribution

δ-function contribution
SeaQuest Preliminary Result: APS April 2016 Talk by B. Kerns


P. Barry, N. Sato, W. Melnitchouk, C. Ji, in progress
Conclusion and Outlook

• LFD is not just formal but consequential in the analysis of physical observables.
• Maximal stability group of LFD saves dynamic efforts.
• LFZM is treacherous, but useful in resolution of theoretical issues such as LNA factor difference in chiral effective theory.
• Quantitative analysis of LFZM contribution in hadron phenomenology is still open for progress.