Quark Propagator with electroweak interactions in the Dyson-Schwinger approach

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Der Wissenschaftsfonds.

Motivation

- System of binary neutron stars mergers
- Source of gravitational waves
- Very high neutrino flux
- Very dense matter $\Rightarrow$ opaque for neutrinos
- Back coupling of neutrinos influences gravitational waves
- Measurement shows the inner structure of neutron star mergers
- Electroweak interactions play an important role
- Consider QCD + electroweak interactions non-perturbatively
- Additional symmetry breaking due to electroweak interactions (flavor, C, P etc.)

(Rosswog et al., Mon. Not. Roy. Astron. Soc. 342, 673 (2003), Y. Sekiguchi et al., PRL 107, 051102 (2011), J. A. Faber et al., Living Rev. Rel. 15, 8 (2012), D. Neilsen et al., PRD 89, 104029 (2014), C. Palenzuela et al., PRD 92, 044045 (2015), O. L. Caballero arXiv:1603.02755 [nucl-th], F. Foucart et al., PRD 93, 044019 (2016), . . .)

Quark Propagator with electroweak interactions

- **Effect of the parity violation on quark propagators**
- Non-vanishing propagator for the mixed flavor
- Parity violation ⇒ 4 instead of 2 Lorenz channels for the propagator
- Quark propagator from flavor A to flavor B

$$P_{AB}(p^2) = \tilde{A}_{AB}(p^2)\,\mathrm{i}\,\not{p} + \tilde{B}_{AB}(p^2)\mathbb{1} + \tilde{C}_{AB}(p^2)\,\mathrm{i}\,\not{p}\gamma^5 + \tilde{D}_{AB}(p^2)\gamma^5$$

- Very different energy scales + parity violation ⇒ Lattice calculation unfeasible
- Functional Methods (DSE, FRG, ...) prime candidate
  1. Continuous, covariant and non-perturbative formulation
  2. High and low energy scales accessible
  3. Infinite tower of coupled, nonlinear integral equations
  4. Needs truncation

Ansatz

- Approximation by breaking term

$$\begin{aligned}\mathcal{L} =& \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{Effective}} \\ =& \overline{\psi}_u \left[-\partial\!\!\!/ + m_u \right] \psi_u + \overline{\psi}_d \left[-\partial\!\!\!/ + m_d \right] \psi_d + g_s \overline{\psi}_u A\!\!\!/^i T^i \psi_u \\ &+ g_s \overline{\psi}_d A\!\!\!/^i T^i \psi_d - 2 g_{\text{w}} \left(\overline{\psi}^L_u \partial\!\!\!/ \psi^L_d + \overline{\psi}^L_d \partial\!\!\!/ \psi^L_u \right) + \mathcal{L}_{\text{Rest}} \\ \psi^L =& \frac{1}{2}(\mathbb{1} - \gamma^5)\psi\end{aligned}$$

- Violates charge conservation
- Couple with a reservoir of leptons

Ansatz

- Approximation by breaking term

$$
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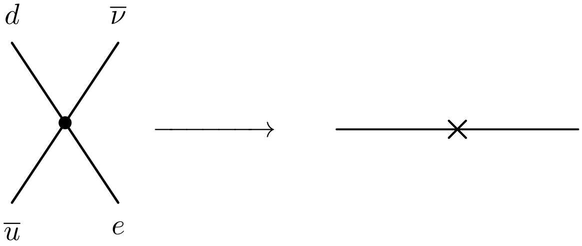
=& \overline{\psi}_u \left[-\not{\partial} + m_u \right] \psi_u + \overline{\psi}_d \left[-\not{\partial} + m_d \right] \psi_d + g_s \overline{\psi}_u \not{A}^i T^i \psi_u \\
& + g_s \overline{\psi}_d \not{A}^i T^i \psi_d - 2 g_{\text{w}} \left(\overline{\psi}^L_u \not{\partial} \psi^L_d + \overline{\psi}^L_d \not{\partial} \psi^L_u \right) + \mathcal{L}_{\text{Rest}} \\
\psi^L =& \frac{1}{2} (\mathbb{1} - \gamma^5) \psi
\end{aligned}
$$

- Violates charge conservation
- Couple with a reservoir of leptons



Tree-level Propagator

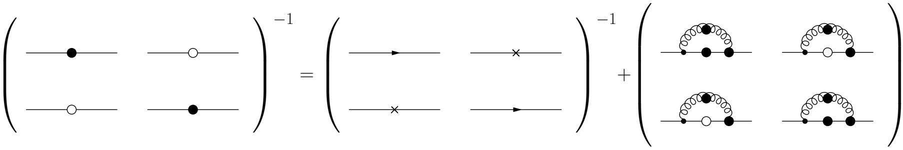
$$P_{0,uu}(p^2) = \frac{1}{N(p^2)} \left[(m_d^2 + (1 - 2g_\mathrm{w}^2)p^2)\, \mathrm{i}\, \not{p} + m_u(m_d^2 + p^2)\mathbb{1} + 2g_\mathrm{w}^2 p^2\, \mathrm{i}\, \not{p}\gamma^5 \right]$$

$$P_{0,ud}(p^2) = \frac{g_\mathrm{w}}{N(p^2)} \left[(m_u m_d - p^2)\, \mathrm{i}\, \not{p} - (m_u + m_d)p^2\mathbb{1} - (m_u m_d + p^2)\, \mathrm{i}\, \not{p}\gamma^5 \right] - \frac{g_\mathrm{w}(m_u - m_d)p^2}{N(p^2)}\gamma^5$$

$$N(p^2) = m_d^2 m_u^2 + (m_u^2 + m_d^2)p^2 + (1 - 4g_\mathrm{w}^2)p^4$$

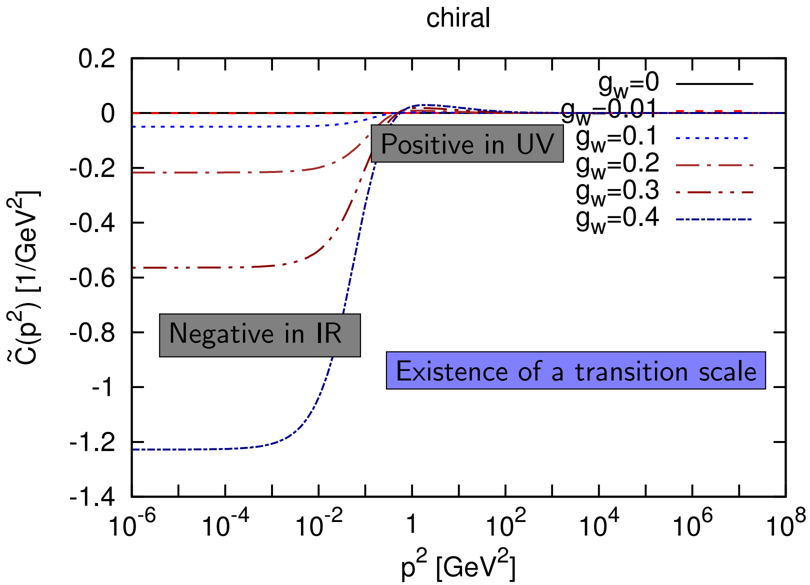
- Pseudo scalar channel of the mixed propagator (tree-level) is proportional to mass splitting
- $\Rightarrow$ All channels of full propagator have contributions from mass splitting

DSEs

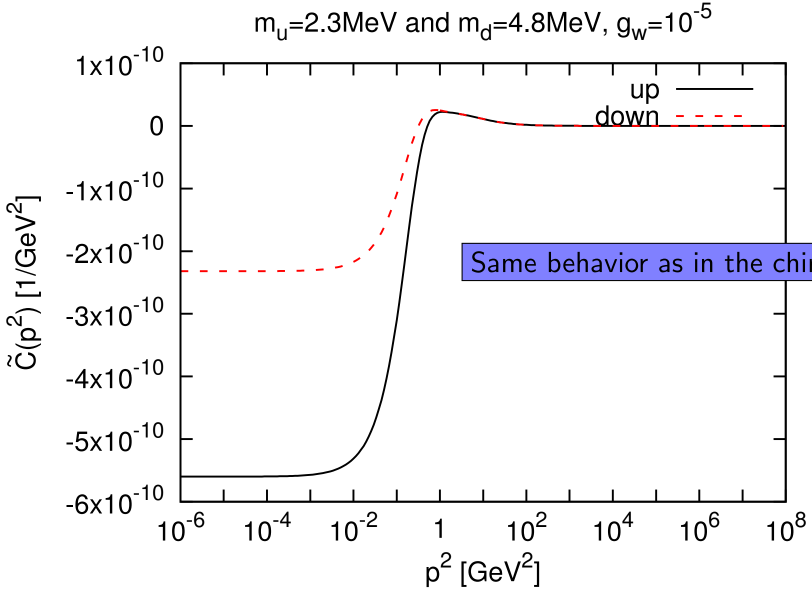


- Weak interaction: Non-vanishing off diagonal elements
- Rainbow-Truncation
- Effective coupling: Maris-Tandy (P. Maris and P. C. Tandy, PRC 60, 055214 (1999))
- Important features like dynamical mass generation and chiral symmetry particularly well implemented (R. Alkofer and L. von Smekal, Phys. Rept. 353, 281 (2001), C. S. Fischer J. Phys. G32, R253 (2006), C. D. Roberts, J. Phys. Conf. Ser. 706, 022003 (2016))

Axial channel

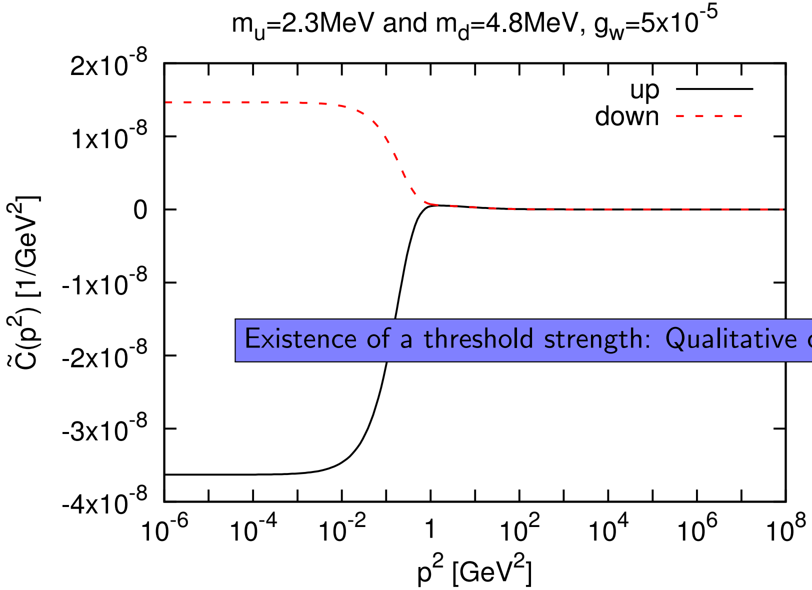


Axial channel



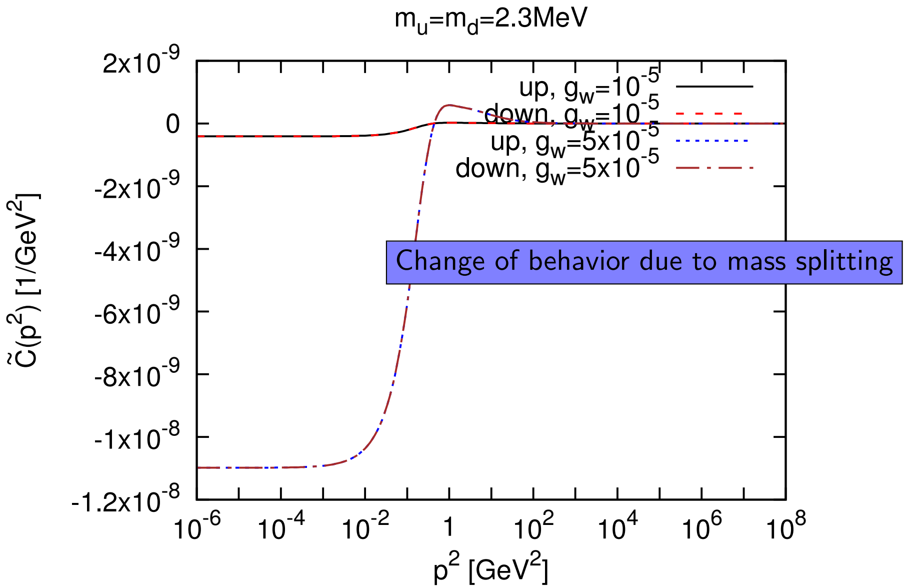
Same behavior as in the chiral limit

Axial channel

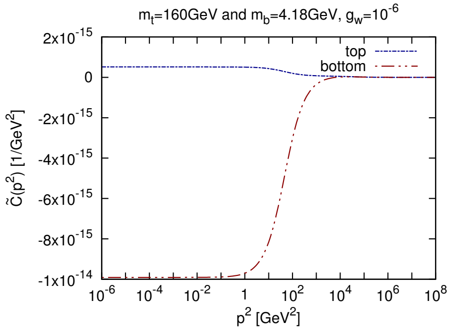


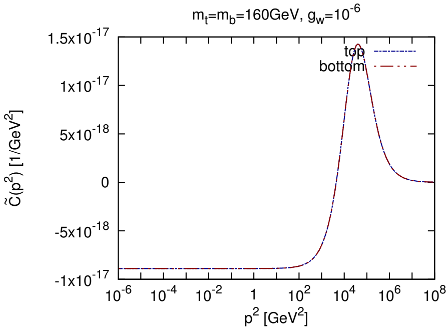
Existence of a threshold strength: Qualitative change

Effects of Mass Splitting on Axial channel



Effects of Mass Splitting on Axial channel





- Threshold value for g_w is increased for bigger mass splitting
- Transition scale is shifted to higher value

Right and Left Handed Projection

- Contributions from quark propagators to left $\tilde{L}$ and right handed $\tilde{R}$:

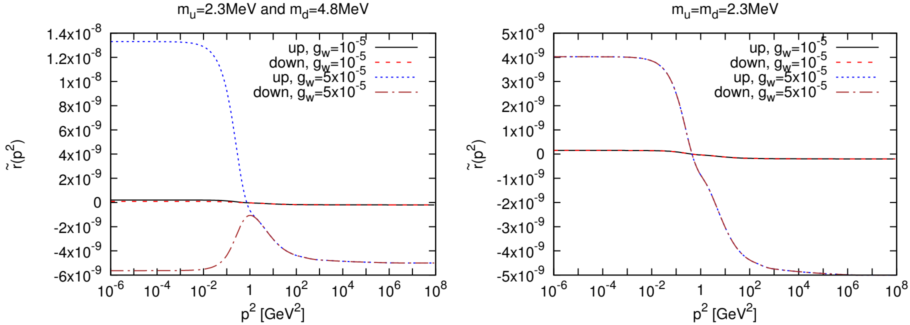
$$\begin{aligned}\tilde{L}_{AB} =& \tilde{A}_{AB} - \tilde{C}_{AB} \left(\propto \gamma^{\mu}(\mathbb{1} - \gamma^{5})\right)\\ \tilde{R}_{AB} =& \tilde{A}_{AB} + \tilde{C}_{AB} \left(\propto \gamma^{\mu}(\mathbb{1} + \gamma^{5})\right)\end{aligned}$$

- $\tilde{r}$: relative ratio for left handed to right handed contributions

$$\tilde{r}_{AB}(p^2) = \frac{\tilde{L}_{AB}(p^2) - \tilde{R}_{AB}(p^2)}{\tilde{L}_{AB}(p^2) + \tilde{R}_{AB}(p^2)} = -\frac{\tilde{C}_{AB}(p^2)}{\tilde{A}_{AB}(p^2)}$$

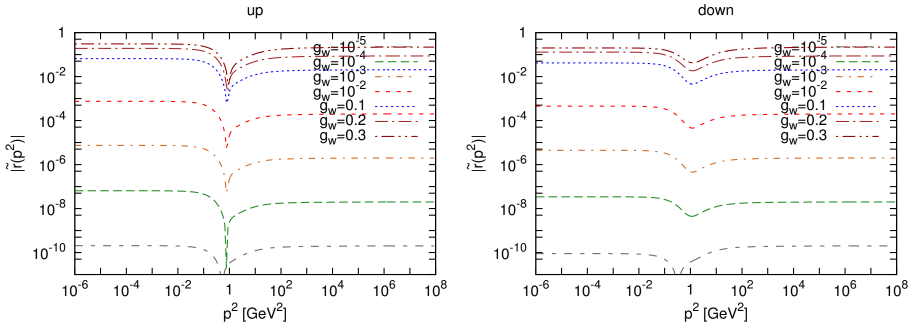
- More left handed or right handed contributions related to sign of $\tilde{C}$ for pure flavor quark propagators ($\tilde{A}$ always positive)

Parity Violation



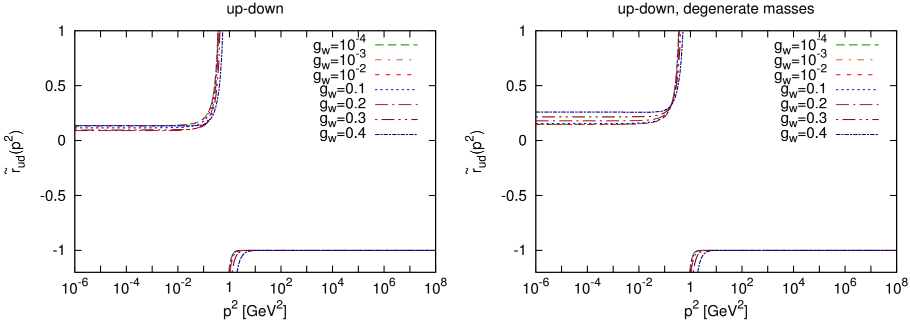
- Below the threshold strength
  1. Dominantly right handed in the UV
  2. Dominantly left handed in the IR
- Above the threshold strength
  1. No qualitative change for up quark
  2. Change for down quark: Dominantly right handed in UV and IR
- Absolute value for the ratio is increased due to mass splitting

Parity Violation



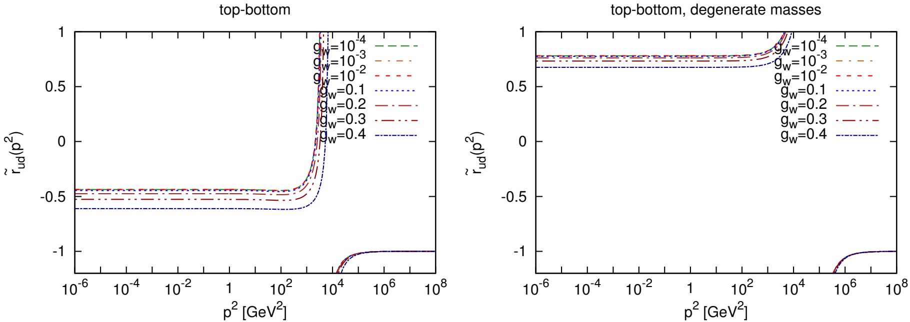
- Absolute value is increased by two order of magnitude, when g_w is increased by one order of magnitude

Parity Violation



- Propagator from up to down
  1. UV: Dominantly right handed
  2. IR: Dominantly left handed
- No qualitative change due to mass splitting
- Quantitative shifting towards right handed for mass splitting

Parity Violation



- UV and IR dominant contribution from right handed quarks
- Qualitative change due to mass splitting (from left handed to right handed in the IR)

Conclusion

- Influence of broken flavor and C and P symmetry on the quark propagator
- Existence of a scale p^2: Change from dominantly right handed to left handed contribution for quark propagator from UV to IR
- For pure flavor propagators
  - Existence of a threshold electroweak strength for mass splitting: Above the threshold strength dominantly right handed contribution in UV **and** IR for heavier quark
  - **Parity violation in different direction for heavier and lighter quarks above the threshold strength**
  - **Qualitative change of Parity violation even for small electroweak strength due to explicit isospin breaking**
- For mixed flavor propagators
  - Presence of mass splitting: Change of qualitative behavior depending on the absolute value of the mass splitting
- **Caution with perturbative extrapolation**