

Strong Couplings of Three Mesons with Charm(ing) Involvement

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Couplings, form factors & decay constants

By use of a relativistic dispersion approach relying on the constituent-quark model, we study the **strong couplings** $g_{PP'V}$ and $g_{PV'V}$ for vector mesons V , with polarization vectors ε_μ , and pseudoscalar mesons P , in the amplitudes

$$\langle P'(p_2) V(q) | P(p_1) \rangle = -\frac{g_{PP'V}}{2} (p_1 + p_2)^\mu \varepsilon_\mu^*(q) ,$$

$$\langle V'(p_2) V(q) | P(p_1) \rangle = -g_{PV'V} \epsilon_{\varepsilon^*(q) \varepsilon^*(p_2) p_1 p_2} , \quad q \equiv p_1 - p_2 .$$

Of particular interest to us will be the case when there is, at least, one of the charmonia J/ψ or η_c among this meson triple. These strong couplings enter into the residues of poles in corresponding **transition form factors** for $q^2 > 0$ arising from intermediate meson states. The form factors relevant for us are

$$\langle P'(p_2) | \bar{q}_2 \gamma_\mu q_1 | P(p_1) \rangle = F_+^{P \rightarrow P'}(q^2) (p_1 + p_2)_\mu + \dots ,$$

$$\langle V(p_2) | \bar{q}_2 \gamma_\mu q_1 | P(p_1) \rangle = \frac{2 V^{P \rightarrow V}(q^2)}{M_P + M_V} \epsilon_{\mu \varepsilon^*(p_2) p_1 p_2} ,$$

$$\langle V(p_2) | \bar{q}_2 \gamma_\mu \gamma_5 q_1 | P(p_1) \rangle = i q_\mu (\varepsilon^*(p_2) p_1) \frac{2 M_V}{q^2} A_0^{P \rightarrow V}(q^2) + \dots .$$

With the [decay constants](#) of the vector and pseudoscalar mesons f_V and f_P , defined in terms of the meson-to-vacuum transition amplitude of the vector quark current $\bar{q}_2 \gamma_\mu q_1$ or axial-vector quark current $\bar{q}_2 \gamma_\mu \gamma_5 q_1$ according to

$$\langle 0 | \bar{q}_1 \gamma_\mu q_2 | V(q) \rangle = f_V M_V \varepsilon_\mu(q) , \quad \langle 0 | \bar{q}_1 \gamma_\mu \gamma_5 q_2 | P(q) \rangle = i f_P q_\mu ,$$

the [poles](#), at pseudoscalar and vector resonances P_R and V_R , are of the form

$$\begin{aligned} F_+^{P \rightarrow P'}(q^2) &= \frac{g_{PP'V_R} f_{V_R}}{2 M_{V_R}} \frac{1}{1 - \frac{q^2}{M_{V_R}^2}} + \dots , \\ V^{P \rightarrow V}(q^2) &= \frac{(M_V + M_P) g_{PVV_R} f_{V_R}}{2 M_{V_R}} \frac{1}{1 - \frac{q^2}{M_{V_R}^2}} + \dots , \\ A_0^{P \rightarrow V}(q^2) &= \frac{g_{PP_R V} f_{P_R}}{2 M_V} \frac{1}{1 - \frac{q^2}{M_{P_R}^2}} + \dots . \end{aligned}$$

Quark-model-backed dispersion approach

We compute the form factors $F_+^{P \rightarrow P'}(q^2)$, $V^{P \rightarrow V}(q^2)$, and $A_0^{P \rightarrow V}(q^2)$ within the framework of a relativistic [constituent-quark](#) picture [1]. To this end, we must relate the currents defining the form factors to their constituent-quark (Q) counterparts: this task is easily accomplished for heavy-quark currents,

$$\bar{q}_1 \gamma_\mu q_2 = g_V \bar{Q}_1 \gamma_\mu Q_2 + \dots , \quad \bar{q}_1 \gamma_\mu \gamma_5 q_2 = g_A \bar{Q}_1 \gamma_\mu \gamma_5 Q_2 + \dots ,$$

by introducing form factors g_V and g_A , but not so simple for light quarks [2]. Numerically, we employ [3], for the constituent-quark masses and couplings, $m_d = m_u = 0.23$ GeV, $m_s = 0.35$ GeV, $m_c = 1.45$ GeV and $g_V = g_A = 1$. The relativistic dispersion formalism allows us to represent the quantities of interest by integrals, over invariant masses of intermediate quark–antiquark states, of [spectral densities](#) derived from (one-loop) Feynman diagrams and [wave functions](#) of the involved pseudoscalar or vector mesons [4], of the form

$$\begin{aligned} \phi_{P,V}(s) &= \frac{\pi}{\sqrt{2}} \frac{\sqrt{s^2 - (m_1^2 - m^2)^2}}{\sqrt{s - (m_1 - m)^2}} \frac{w_{P,V}(k^2)}{s^{3/4}} , \\ k^2 &= \frac{(s - m_1^2 - m^2)^2 - 4 m_1^2 m^2}{4 s} , \quad \int dk k^2 w_{P,V}^2(k^2) = 1 . \end{aligned}$$

For the radial meson wave functions $w_{P,V}(k^2)$, we assume Gaussian shapes:

$$w_{P,V}(k^2) \propto \exp\left(-\frac{k^2}{2\beta_{P,V}^2}\right).$$

The decay constants $f_{P,V}$ become spectral integrals of densities $\rho_{P,V}(s)$ and the form factors $F_+^{P \rightarrow P'}(q^2)$, $V^{P \rightarrow V}(q^2)$ and $A_0^{P \rightarrow V}(q^2)$, generically labelled $\mathcal{F}(q^2)$, double dispersion integrals of double spectral densities $\Delta(s_1, s_2, q^2)$:

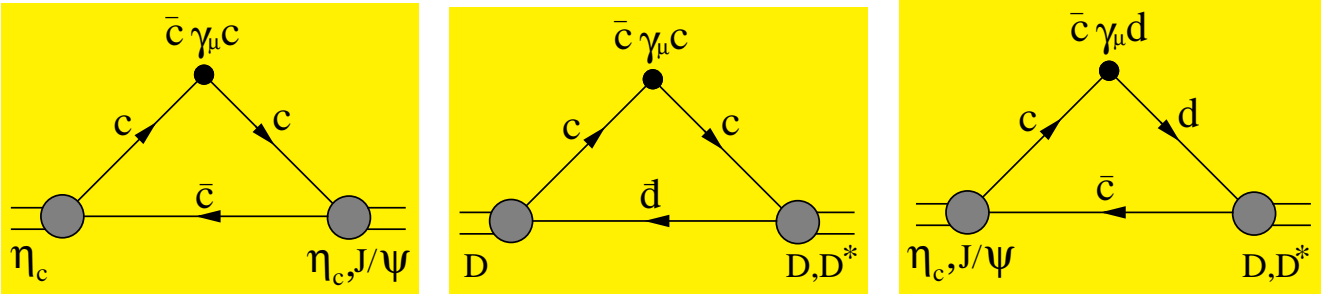
$$f_{P,V} = \int ds \phi_{P,V}(s) \rho_{P,V}(s),$$

$$\mathcal{F}(q^2) = \int ds_1 \phi_1(s_1) \int ds_2 \phi_2(s_2) \Delta(s_1, s_2, q^2).$$

$D_{(s)}^{(*)}$ and $c\bar{c}$ meson masses M , decay constants f and slope parameters β [5]:

| Meson | D | D^* | D_s | D_s^* | η_c | J/ψ |
|---------------|-------------|--------------|---------------|-------------|-----------------|-------------|
| M (GeV) | 1.87 | 2.010 | 1.97 | 2.11 | 2.980 | 3.097 |
| f (MeV) | 206 ± 8 | 260 ± 10 | 248 ± 2.5 | 311 ± 9 | 394.7 ± 2.4 | 405 ± 7 |
| β (GeV) | 0.475 | 0.48 | 0.545 | 0.54 | 0.77 | 0.68 |

Feynman graphs for transitions induced by quark vector currents $\bar{Q}_1 \gamma_\mu Q_2$:



All form factors \mathcal{F} , computed off their resonances R , can be [interpolated](#) by

$$\mathcal{F}(q^2) = \frac{\mathcal{F}(0)}{\left(1 - \frac{q^2}{M_R^2}\right) \left(1 - \frac{\sigma_1 q^2}{M_R^2} + \frac{\sigma_2 q^4}{M_R^4}\right)}, \quad \text{Res } \mathcal{F}(M_R^2) = \frac{\mathcal{F}(0)}{1 - \sigma_1 + \sigma_2}.$$

From the [residues](#), involving the meson masses, decay constants, and strong couplings, the latter are derived, by combined fits if present more than once.

Strong couplings: $\eta_c\eta_c J/\psi$ and $\eta_c J/\psi J/\psi$

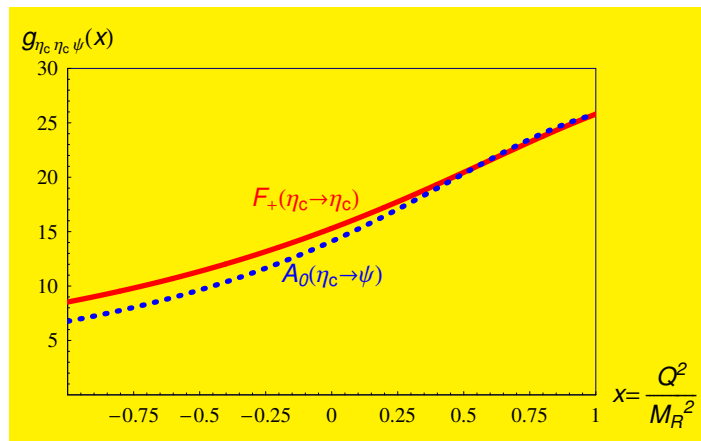
As one example representative for the general situation, consider the strong coupling $g_{\eta_c\eta_c\psi}$ of two pseudoscalar η_c mesons and one vector J/ψ meson [6] by realizing that this quantity enters in and therefore can be extracted from

- the residue $\text{Res } F_+^{\eta_c \rightarrow \eta_c}(M_\psi^2)$ of the pole at $q^2 = M_\psi^2$ of the form factor $F_+^{\eta_c \rightarrow \eta_c}(q^2)$ for the (“elastic”) transition $\eta_c \rightarrow \eta_c$ enabled by the current $\bar{c} \gamma_\mu c$ coupling with decay constant f_ψ to the vector resonance J/ψ and
- the residue $\text{Res } A_0^{\eta_c \rightarrow \psi}(M_{\eta_c}^2)$ of the pole at $q^2 = M_{\eta_c}^2$ of the form factor $A_0^{\eta_c \rightarrow \psi}(q^2)$ for the transition $\eta_c \rightarrow J/\psi$ induced by the current $\bar{c} \gamma_\mu \gamma_5 c$ that couples, with decay constant f_{η_c} , to the pseudoscalar resonance η_c :

$$\text{Res } F_+^{\eta_c \rightarrow \eta_c}(M_\psi^2) = g_{\eta_c\eta_c\psi} \frac{f_\psi}{2 M_\psi} , \quad \text{Res } A_0^{\eta_c \rightarrow \psi}(M_{\eta_c}^2) = g_{\eta_c\eta_c\psi} \frac{f_{\eta_c}}{2 M_\psi} .$$

Upon determination of the meson wave-function parameters β by requiring the dispersion representation of the decay constants $f_{P,V}$ to reproduce their known values, the strong couplings may be calculated, individually for each transition of interest, from the spectral representation of its associated form factor: our couplings’ off-resonance behaviour exhibits excellent agreement.

Behaviour of the off-shell strong coupling $g_{\eta_c\eta_c\psi}(x)$ as a function of $x \equiv \frac{q^2}{M_R^2}$, $R = J/\psi, \eta_c$, from the two transitions $\eta_c \rightarrow \eta_c$ (red) and $\eta_c \rightarrow J/\psi$ (blue):



A combined fit with the four parameters $g_{\eta_c\eta_c\psi}$, $A_0^{\eta_c \rightarrow \psi}(0)$, $\sigma_1^{\mathcal{F}}$ then yields [6]

$$g_{\eta_c\eta_c\psi} = 25.8 \pm 1.7 .$$

Along a similar route, we find [6] for the strong coupling of one pseudoscalar η_c meson and two vector J/ψ mesons, $g_{\eta_c\psi\psi}$, entering in only one transition,

$$g_{\eta_c\psi\psi} = (10.6 \pm 1.5) \text{ GeV}^{-1} .$$

Strong couplings: η_c and J/ψ to D and D^*

Allowing also for currents with a d or s quark (and merging strong-coupling multiple occurrences, all of them showing nearly perfect concord), we get [6]

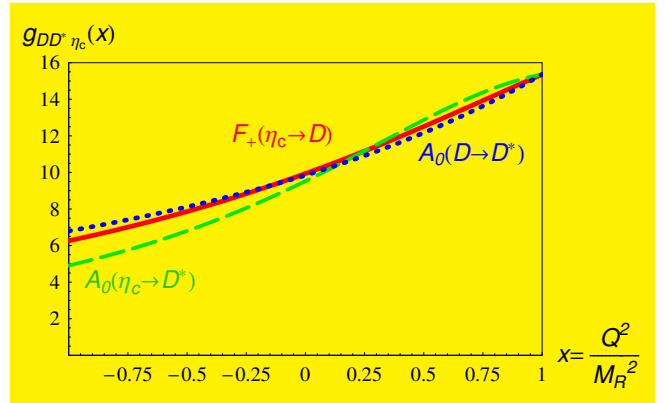
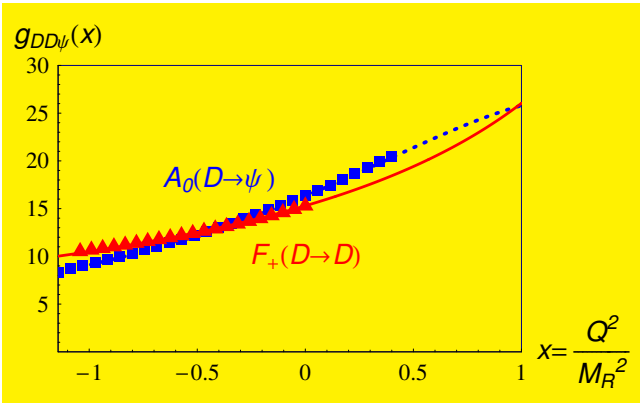
- for the strong J/ψ or η_c couplings to non-strange charmed mesons $D^{(*)}$

$$\begin{aligned} g_{DD\psi} &= 26.04 \pm 1.43 , \\ g_{DD^*\psi} &= (10.7 \pm 0.4) \text{ GeV}^{-1} , \\ g_{DD^*\eta_c} &= 15.51 \pm 0.45 , \\ g_{D^*D^*\eta_c} &= (9.76 \pm 0.32) \text{ GeV}^{-1} , \end{aligned}$$

- and for the strong J/ψ or η_c couplings to strange charmed mesons $D_s^{(*)}$

$$\begin{aligned} g_{D_s D_s \psi} &= 23.83 \pm 0.78 , \\ g_{D_s D_s^* \psi} &= (9.6 \pm 0.8) \text{ GeV}^{-1} , \\ g_{D_s D_s^* \eta_c} &= 14.15 \pm 0.52 , \\ g_{D_s^* D_s^* \eta_c} &= (8.27 \pm 0.37) \text{ GeV}^{-1} . \end{aligned}$$

Dependence of the off-shell strong couplings $g_{D\hat{D}\psi}(x) = \frac{2M_\psi}{f_D} \bar{x} A_0^{D \rightarrow \psi}(q^2)$ (left, blue), $g_{DD\hat{\psi}}(x) = \frac{2M_\psi}{f_\psi} \bar{x} F_+^{D \rightarrow D}(q^2)$ (left, red), $\bar{x} \equiv 1 - x$, $g_{D\hat{D}^*\eta_c}(x)$ (right, red), $g_{DD^*\hat{\eta}_c}(x)$ (right, blue), & $g_{\hat{D}D^*\eta_c}(x)$ (right, green), on $x \equiv \frac{Q^2}{M_R^2}$:



Observations, comparison and conclusions

- Successful extrapolations of all interpolated results for strong couplings derived at $q^2 < 0$ confirm the presence of the poles expected for $q^2 > 0$.
- Concerning $SU(3)$ breaking, the net outcome of replacing a d quark by the s quark is a reduction of the affected strong coupling by about 10%.
- Despite undeniable similarities of the approaches, our $D_{(s)}^{(*)}$ couplings [6] are more than twice as large as the results arising from QCD sum rules.

Comparison of our strong-coupling predictions with QCD sum-rule results:

| Coupling | $g_{DD\psi}$ | $g_{DD^*\psi}$ (GeV^{-1}) | $g_{D_s D_s \psi}$ | $g_{D_s D_s^* \psi}$ (GeV^{-1}) |
|-----------|--------------------|--------------------------------------|----------------------|--------------------------------------------|
| This work | 26.04 ± 1.43 | 10.7 ± 0.4 | 23.83 ± 0.78 | 9.6 ± 0.8 |
| Sum rules | 11.6 ± 1.8 [7] | 4.0 ± 0.6 [7] | 11.96 ± 1.34 [8] | 4.30 ± 1.53 [9] |

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