Neutron Matter from Low to High Density

J. Carlson - LANL

- Very low density: relation to cold fermions
- Moderate density: nuclear symmetry energy
- 1-2 times saturation density: neutron star mass/radius
- Color superconducting quark matter
- Outlook
QCD Phase Diagram
**Very Low Density Equation of State**

Neutrons:  \( a_{nn} \approx -18 \text{ fm} \)  \( r_e \approx 2.7 \text{fm} \)

Cold atoms:  \( a \text{ from } 0 \rightarrow \infty \rightarrow 0 \)  \( r_e \rightarrow 0 \)

**BCS \rightarrow \text{BEC}**

---

**Slightly higher densities**

Gandolfi, Gezerlis, JC, ARNPS 2015

---

**Graphs:**
- **Graph 1:** Equation of state for neutron matter compared to cold atoms at very low density.
  - Neutrons: Fermi gas at the same density.
  - Cold atoms: Fermi gas at the same value of Fermi momentum times scattering length.
- **Graph 2:** Equation of state for neutron matter for very low density.
  - Neutrons: Variational upper bounds to the true energy.
  - Cold atoms: Very accurate results.
- **Graph 3:** Equation of state for neutron matter for slightly higher densities.
  - Neutrons: Results from different methods over a wider range.
  - Cold atoms: Results from experimental and theoretical approaches.
Neutron matter: largest pairing gap in nature: \( \frac{\Delta}{E_F} \approx 0.3 \)

Cold atoms at unitarity: \( \frac{\Delta}{E_F} = 0.45(0.05) \)
**Beyond very low density**

Nuclear Interactions

NN interactions fit to huge database

3N interactions fit to nuclei

Chiral EFT

and AV18, …
Beyond very low density

Method: Auxiliary Field Diffusion Monte Carlo
sampling spins/isospins with diffusion for coordinates introduced by Schmidt and Fantoni

\[ \Psi_0 = \exp[-H\tau] \Psi_T \]

\[ |RS\rangle = |r_1s_1\rangle \otimes |r_2s_2\rangle \cdots \otimes |r_n s_n\rangle. \]

\[ |s_i\rangle = | \begin{matrix} \cos(\theta_i) \exp[i\phi_i] \\ \sin(\theta_i) \exp[-i\phi_i] \end{matrix} \rangle \]

and similarly for isospin, auxiliary fields used to propagate with spin-dependent interaction

\[ \prod_{i<j} e^{-V_{ij}\delta \tau} |RS\rangle = \]

\[ \prod_n \frac{1}{(2\pi)^{3/2}} \int dx_n e^{-x_n^2/2} e^{\sqrt{-\lambda_n\delta \tau} x_n} |RS\rangle = |RS'\rangle, \]

Lattice methods also used extensively
Various approaches agree precisely at low densities, start to diverge near saturation densities

Symmetry Energy $\sim 29$-$34$ MeV

Moderate Densities

Gandolfi, Gezerlis, JC, ARNPS 2015
Neutron Matter w/ chiral 2N and 3N interactions at N2LO

- TNI fit to A=4 binding, n-alpha scattering; A=5 includes T=3/2 triplets
- Significant uncertainties from regulators, Fierz rearrangements, …
- How to reduce uncertainties?
Inhomogeneous Matter

Cold Atoms

Neutrons

Density functionals and small clusters
Inhomogeneous Systems: Neutrons vs. Cold Atoms

Figure 5: Energies for neutrons trapped in a harmonic oscillator with $\omega = 10$ MeV for different interactions and methods compared to Monte Carlo calculations for cold atoms at unitarity (open symbols) and two local density approximations (solid and dashed lines). In the right panel results for the AV8' + UIX interaction are compared to several pre-existing density functionals.

Results are also shown for no-core shell model (NCSM) calculations using chiral interactions and the JISP16 NN interaction. The chiral interactions are very similar to the AV8' results without a three-nucleon interaction; the energies for all interactions and methods agree within a few percent.

These calculations show significant shell closures, in contrast to the cold atom results. These shell closures arise because of the effective range in the neutron-neutron interaction, and the concomitant smaller size of the superfluid pairing gap. In cold atoms only superfluid pairs propagate across the whole system, while for neutrons the shell closures indicate the single-particle picture still survives to some degree. The shell closures are not as strong as in nuclei, however. Pairing gaps are also evident from the odd-even staggering, spin-orbit, and gradient corrections have been examined in many of these studies.

In the right-hand panel, results for the AV8'+UIX interaction are compared with a variety of previously existing density functionals. These functionals do not fully describe the neutron drop results, in general their energy is too low and spin-orbit and pairing are not completely correct. Modest modifications to the isovector (full neutron) gradients, pairing and spin orbit give density functional results in the full black line, labeled SLY4-adj. These do a good job of reproducing results in both 5 and 10 MeV frequency traps except for the smallest systems considered. More recently it has been shown that new density functionals, i.e. UNEDF0 and UNEDF1, can be created that simultaneously fit nuclei with accuracy comparable or better than the existing functionals, and also reproduce the neutron drop results.

In figure 6 we show results in a Wood-Saxon well. These more nearly mimic

Shell structure very different

Higher Density and the Three Neutron Interaction

Strong Correlation between S and L (symmetry energy and slope)
Experimental Constraints near Saturation Density

Also parity-violating e scattering:
PREX, CREX: neutron radius (similar to proton radius from elastic electron scattering)

\begin{align*}
\sigma \approx \frac{e^+ - Z^0}{e^- + e^-}
\end{align*}

Theory: Schwenk et al., Gandolfi et al.
Observations: Neutron Star Masses


from Lattimer
Neutron Star Mass/Radius Relations: History

For many years only ~1.4-1.5 solar mass neutron stars observed
2010: Two solar mass neutron stars
Mass-Radius Relation for Neutron Stars

By using the EOS obtained from different nuclear Hamiltonians, we can study the effect on the neutron star structure. The results of the M-R diagram of neutron stars obtained from the EOS calculated in the previous section are shown in Fig. 10. Since the radii of neutron stars are almost determined by the EOS slightly above \( \rho_c \) (98), future measurements will provide strong constraints to the nuclear Hamiltonian. In particular, radii are directly connected to the pressure of neutron matter at \( \rho_c \), and then there is a natural correlation between the symmetry energy and radii. In the figure the two bands correspond to the EOS described in the previous section (the corresponding values of the symmetry energy are also indicated in the figure). The red and black curves correspond to the EOS calculated with the AV80 two-body interaction alone, and combined with the UIX three-neutron potential. The relation between the symmetry energy and the radius is evident, as the increasing of the symmetry energy predicts a neutron star with a larger radius. In the figure, the density of the neutron matter inside the star is indicated with the orange lines. As anticipated, even at large masses the radius of the neutron star is mainly governed by the equation of state of neutron matter between 1 and 2 \( \rho_c \) (98).

As is clear from the figure, the AV80 Hamiltonian alone does not support the recent observed neutron star with a mass of 1.97(4)\( M_\odot \) and 2.01(4)\( M_\odot \). The addition of a three-body force to AV80 can provide sufficient repulsion to be consistent with all of the constraints. The results also suggest that the most modern neutron matter EOS imply a maximum neutron star radius not larger than 13.5 km, unless a drastic repulsion sets in just above the saturation density (75). This rules out EOS with large values of \( L \), typical of Walecka-type mean-field models without higher-order meson couplings which can decrease \( L \). We note that our analysis suggests it is unlikely that neutron stars have radii lower than 10 km.

Hyperons? see S. Gandolfi's talk

Gandolfi, et al, 2012
Impressive lattice QCD at $\mu = 0$, and exploratory studies at $\mu > 0$

Little work at $T \sim 0$ for large $\mu$
Fermion Sign Problem

Exponential decay in signal to noise for quantum fermi systems
Ubiquitous: electrons, cold atoms, helium, cold atoms, NP, LQCD

Decay proportional to Bose minus Fermi Energy
QCD: \( A \left( M_N - \frac{3}{2} M \right) \)
Nucleons: \( A \times \text{Fermi Energy} \)

No general solution - exponentially difficult for large systems
- believed to be NP hard

Try small \( A \) - make direct comparisons
to lattice at moderate to high densities
not necessary to go through S-matrix

Advantages: small boxes give large gaps, high excitation energies
can probe different \( N \), boundary conditions, quantum numbers,…

Can we calibrate nuclear interactions?
Can we extrapolate to matter?
Can we begin to identify the phase transition?
Small quantum systems can identify important degrees of freedom

Figure 5: Energies for neutrons trapped in a harmonic oscillator with $\omega N = 10$ MeV for different interactions and methods compared to Monte Carlo calculations for cold atoms at unitarity (open symbols) and two local density approximations (solid and dashed lines). In the right panel results for the AV8' + UIX interaction are compared to several pre-existing density functionals.

Results are also shown for no-core shell model (NCSM) calculations using chiral interactions and the JISP16 NN interaction. The chiral interactions are very similar to the AV8' results without a three-nucleon interaction; the energies for all interactions and methods agree within a few percent.

These calculations show significant shell closures, in contrast to the cold atom results. These shell closures arise because of the effective range in the neutron-neutron interaction, and the concomitant smaller size of the superfluid pairing gap. In cold atoms only superfluid pairs propagate across the whole system, while for neutrons the shell closures indicate the single-particle picture still survives to some degree. The shell closures are not as strong as in nuclei, however. Pairing gaps are also evident from the odd-even staggering, spin-orbit, and gradient corrections have been examined in many of these studies (84, 85).

In the right-hand panel, results for the AV8'+UIX interaction are compared with a variety of previously existing density functionals. These functionals do not fully describe the neutron drop results, in general their energy is too low and spin-orbit and pairing are not completely correct. Modest modifications to the isovector (full neutron) gradients, pairing and spin orbit give density functional results in the full black line, labeled SLY4-adj. These do a good job of reproducing results in both 5 and 10 MeV frequency traps except for the smallest systems considered. More recently it has been shown that new density functionals, i.e. UNEDF0 (87) and UNEDF1 (88), can be created that simultaneously fit nuclei with accuracy comparable or better than the existing functionals, and also reproduce the neutron drop results.
L = 4.4 fm, $E_F = 55$ MeV, $\Delta \sim 2E_F \sim 100$ MeV

Closed shell numbers change with boundary conditions
Neutrons: energies versus $N$ at constant density
Degrees of freedom can have a huge impact:

Comparison of neutrons to free quarks
**P-wave states for N = 4:**

original (BCS s-wave) state:

\[ \Phi = A \prod [\phi(r_{ij})(\uparrow_i \downarrow_j - \downarrow_i \uparrow_j)] \]

new (p-wave) state:

\[ \Phi = A [\uparrow_1 \downarrow_2 \sin[k_x \cdot r_{34}] + i \sin[k_y \cdot r_{34}]] \uparrow_3 \]

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>( E_s ) (MeV)</th>
<th>( E_p ) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>71.0(0.5)</td>
<td>65.0(0.5)</td>
</tr>
<tr>
<td>0.16</td>
<td>117(2)</td>
<td>92.0(1.2)</td>
</tr>
</tbody>
</table>

25 MeV difference at saturation density

\( L \sim 3 \) fm, level spacing \( \sim 100 \) MeV

very large energy differences; still exploring other states
Conclusions: Neutron Matter

Want to determine the QCD EOS at high density and low T
  Low density regime well constrained
  More precise knowledge at saturation density required
  Phase transitions - what densities in relation to neutron stars ?

Want to understand properties of neutron stars / supernovae
  EOS at zero and finite T
  Weak rates and neutron star cooling
  Neutrinos in supernovae (quantum coherence / matter)

Critical role of observations:
  x-rays
  neutrinos
  gravitational waves
  …