Mesons in Ultra-Intense Magnetic Field: an Evaded Collapse

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Talk based on papers by M.Andreichikov, B.Kerbikov, V.Orlovsky and Yu.Simonov (and work in progress)

Plan

1. Motivation and Goal Setting
2. A Glimpse of the Formalism
3. Relativistic Hamiltonian and the Two Sources of Danger
4. Meson Mass Dependence on the Magnetic Field Strength

EW and QCD are expected to merge at GUT $\sim 10^{15}\ GeV$.

But!

QED and QCD have already met: $eB \sim \Lambda^2_{QCD} \sim 10^{18} - 10^{19}\ G$ in peripheral heavy-ion collisions at RHIC and LHC ($GeV^2 = 5.12 \cdot 10^{19}\ G$). In magnetars $eB \gtrsim 10^{14}\ G$.

What happens to mesons and baryons as $eB \to m^2_\pi,\ m^2_\rho$?
Different approaches, answers vary.

The interplay of $\left\{ \begin{array}{c} l_B = (eB)^{-1/2} \simeq 0.6\ fm \text{ at } eB = 10^{19}\ G \\ \sigma^{-1/2} \simeq 0.45\ fm \text{ for } \sigma = 0.2\ GeV^2 \end{array} \right.$

$l_B \simeq \sigma^{-1/2}$ at $eB \simeq 10^{19} - 10^{20}\ G$: Quark tomography inside hadrons by eB.

A talking point in literature: ρ-condensation, magnetic field collapse, fall to the center

$$m_\rho^2 + eB(1 - g_\rho) < 0$$

Without ρ-structure and $g_\rho(B)$ dependence does not work – see below.

A passing remark:

Very recent lattice result $(\sigma \| \mathbf{B}) < (\sigma \perp \mathbf{B})$ — anisotropic deconfinement at large eB? Left aside in this talk.

Two keystones:

1. Fock-Schwinger-Feynman representation of $q\bar{q}$ and qqq Green's functions. Effective relativistic Hamiltonian with einbein dynamical masses.
2. Separation of the c.m. motion: pseudomomentum for the neutral system, or for $e_1 = e_2$, $m_1 = m_2$. W. F. factorization (approximation) otherwise.

a) Fock-Schwinger propertime — everybody knows;
b) Feynman path intergral disentangling — good students know;
c) Averaging over the time fluctuations via einbein — some Phys.Rev.D referees understand.

Elaborated technique – hopeless to expose in a short talk.

- A road map: bird's-eye view

$$G(y,\, x) = \int_0^\infty ds\, e^{-sm^2} \langle y|e^{-s\hat{H}}|x\rangle \implies$$

$$\implies G_{q\bar{q}}(y,x) = \langle j(y)\, j(x)\rangle = \int\limits_0^\infty ds_1 \int\limits_0^\infty ds_2 \left(Dz^{(1)}\right)_{yx} \left(Dz^{(2)}\right)_{yx} e^{-K_1-K_2} \times$$

$$\times \langle W(A)\rangle_{\bar{A}} \exp\left[ie_1 \int\limits_x^y A_\mu^{(e)} dz_\mu^{(1)} - ie_2 \int\limits_x^y A_\mu^{(e)} dz_\mu^{(2)} + e_1 \int\limits_0^{s_1} d\tau_1\, (\boldsymbol{\sigma}\mathbf{B}) - e_2 \int\limits_0^{s_2} d\tau_2\, (\boldsymbol{\sigma}\mathbf{B})\right],$$

$$K_i = s_i m_i^2 + \frac{1}{4} \int\limits_0^{s_i} d\tau_i \left(\frac{dz_\mu^{(i)}}{d\tau_i}\right)^2 \Rightarrow \frac{m_i^2 + \omega_i^2}{2\omega_i} T + \int\limits_0^T dt_E \frac{\omega_i}{2} \left(\frac{dz^{(i)}}{dt_E}\right)^2$$

$\omega_i = \frac{T}{2s_i}$ einbein variables $\longrightarrow$ a toy example follows.

$$L_\phi = \frac{1}{2}\left|(\partial_\mu - igA_\mu)\phi\right|^2 + \frac{1}{2}m^2\phi^2,\ A_\mu = A_\mu^a(x)t^a,$$

$$G(y,x) = (m^2 - D_\mu^2)^{-1}_{yx} = \langle y|P\int_0^\infty ds\, e^{-s(m^2 - D_\mu^2)}|x\rangle \Longrightarrow$$

$$\Rightarrow \int_0^\infty ds (Dz)_{yx} e^{-K}\, P\exp\left[ig\int_x^y A_\mu(z)dz_\mu\right],$$

$$K = sm^2 + \frac{1}{4}\int_0^s d\tau\left(\frac{dz_\mu}{d\tau}\right)^2,\ z_4 \equiv t_E = 2\omega\tau,\ T = y_4 - x_4,\ s = \frac{T}{2\omega}$$

$$\left(\frac{dz_\mu}{d\tau}\right)^2 = 4\omega^2\left[1 + \left(\frac{d\mathbf{z}}{dt_E}\right)^2\right],\ \text{Let } A_\mu = 0,$$

$$K(\omega) = \int_0^T dt_E\left(\frac{\omega}{2} + \frac{m^2}{2\omega} + \frac{\omega}{2}\left(\frac{d\mathbf{z}}{dt_E}\right)^2\right),\ \int (D^3 z)_{\mathbf{yx}} e^{-K(\omega)} = \left\langle \mathbf{y}\left|e^{-H(\omega)T}\right|\mathbf{x}\right\rangle$$

$$\left.\frac{\partial H}{\partial \omega}\right|_{\omega=\omega_0} = 0 \Longrightarrow \omega_0 = \sqrt{\mathbf{p}^2 + m^2}$$

From $G_{q\bar{q}}(y, x)$ to $\hat{H}_{q\bar{q}}(\omega_1, \omega_2)$:

$$\hat{H}_{q\bar{q}} = \frac{1}{2\omega_1}\left(\mathbf{p}_1 - e_1\mathbf{A}(z_1)\right)^2 + \frac{1}{2\omega_2}\left(\mathbf{p}_2 + e_2\mathbf{A}(z_2)\right)^2 + \sigma|\mathbf{z}_1 - \mathbf{z}_2| +$$

$$+ \frac{m_1^2 + \omega_1^2 - e\boldsymbol{\sigma}_1\mathbf{B}}{2} + \frac{m_2^2 + \omega_2^2 + e\boldsymbol{\sigma}_2\mathbf{B}}{2}.$$

The spectral problem:

$$\begin{cases} \hat{H}_{q\bar{q}}\Psi_n = M_n\Psi_n, \\ \frac{\partial M_n}{\partial \omega_i} = 0. \end{cases}$$

$$M_{total} = M_n + \langle\Psi|V_{OGE}|\Psi\rangle + \langle\Psi|V_{SS}|\Psi\rangle$$

Two (almost) analytical methods.

a) Pseudomomentum – works for neutral system $Q = 0$. In eB the total momentum $\mathbf{P}$ is not a constant of motion.
Let $\mathbf{A} = \frac{1}{2}(\mathbf{B} \times \mathbf{r})$, $\boldsymbol{\eta} = \mathbf{r}_1 - \mathbf{r}_2$, $\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2$,

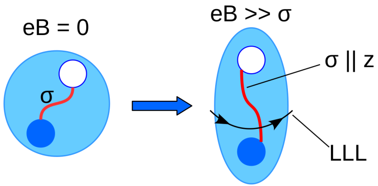
$$\hat{H} = \frac{1}{4m}\left(\mathbf{P} - \frac{e}{2}(\mathbf{B} \times \boldsymbol{\eta})\right)^2 + \frac{1}{m}\left(-i\frac{\partial}{\partial \boldsymbol{\eta}} - \frac{e}{2}(\mathbf{B} \times \mathbf{R})\right)^2,$$

$\mathbf{F} = \left(\mathbf{P} + \frac{e}{2}(\mathbf{B} \times \boldsymbol{\eta})\right)$, — *pseudomomentum*, integral of motion.

b) $Q \neq 0$. Constituent separation, or factorization

$$V(\mathbf{r}_1, \mathbf{r}_2, \ldots) \simeq \sum_{i=1,2} v(\mathbf{r}_i - \mathbf{r}_0).$$

The general picture:



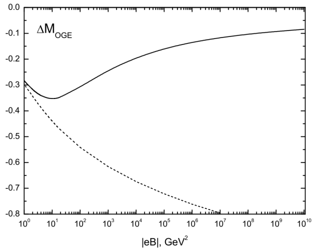
$a_\perp \simeq \sqrt{\frac{2}{eB}}$

$a_z \simeq \frac{1}{\sqrt{\sigma}}$

1. One-gluon-exchange. Hydrogen atom example:

$\varepsilon_0 \propto \ln^2 \frac{eB}{m_e^2 e^3}$ → collapse → cured by loop corrections $\varepsilon_0 \rightarrow 1.7\ KeV$

OGE ⇒ virtual $q\bar{q}$ pair at the LLL



2. Collapse due to hyperfine spin-spin interaction

$$V_{SS} = \frac{8\pi\alpha_S}{9\omega_1\omega_2}\,\delta(\mathbf{r})\,(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$$

Magnetic focusing $\delta(\mathbf{r}) \Rightarrow \psi^2(0) \sim eB \rightarrow$ boundless decrease of M. Unlawful use of the perturbation theory for $\delta^{(3)}(\mathbf{r})$. The treatment recipe: to smear at the scale of the gluon field correlation length

$$\delta(\mathbf{r}) \rightarrow \tilde{\delta}(\mathbf{r}) = \left(\frac{1}{\lambda\sqrt{\pi}}\right)^3 e^{-\frac{\mathbf{r}^2}{\lambda^2}}, \quad \lambda \sim 1\; GeV^{-1}$$

.

Total spin and isospin are not good quantum numbers at $eB \neq 0$. From $|S, S_3\rangle$ basis to $|+\rangle, |-\rangle \otimes |+\rangle, |-\rangle$, e.g. $\rho^+(S_z = 1) = (++)_{u\bar{d}}$.
The special case: both quarks belong to LLL, $e_i\sigma_i^{(z)} > 0$.

$$|eB|(2n_\perp + 1) - e_i\boldsymbol{\sigma}_i\mathbf{B} = 0 \text{ for } n_\perp = 0$$

$$\rho^+_{u\bar{d}}(S_z = +1) \text{ and } \rho^-_{d\bar{u}}(S_z = -1)$$

The threat of collapse comes from $V_{SS} \propto \delta(\mathbf{r})$, $|\psi(0)|^2 \propto eB$.

$$\delta^3(\mathbf{r}) = \left(\frac{1}{\lambda\sqrt{\pi}}\right)^3 e^{-\frac{\mathbf{r}^2}{\lambda^2}}$$

Important!

$u\bar{u}$ and $d\bar{d}$ trajectories are splitted. Next a sample of our results *v.s.* lattice (by a courtesy of E. Luschevskaya).

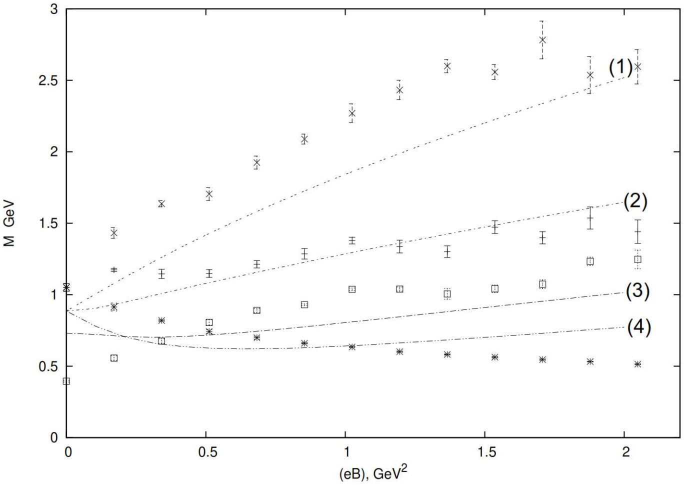


Figure 1: $\rho - \pi$ charged
(1) ρ^- $(S_z = 1)$, **(2)** ρ^- $(S_z = 0)$, **(3)** π^- $(S_z = 0)$, **(4)** ρ^- $(S_z = -1)$.

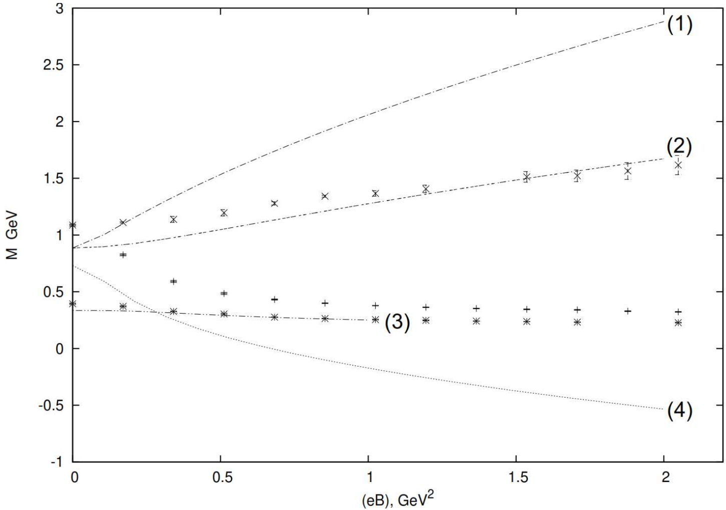


Figure 2: $\left(\rho^0 - \pi^0\right)_{u\bar{u}}$

(1) $\rho^0\,(S_z = 0)$, (2) $\rho^0\,(S_z = \pm 1)$, (3) π^0 *chiral*, (4) π^0.

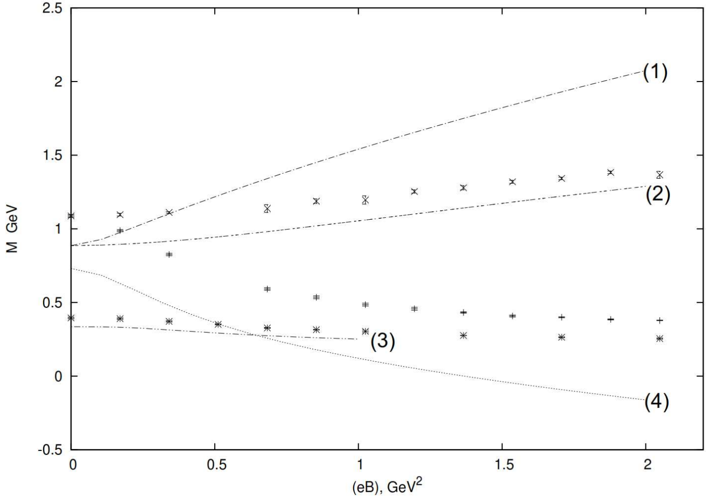


Figure 3: $\left(\rho^0 - \pi^0\right)_{d\bar{d}}$

(1) $\rho^0\,(S_z = 0)$, **(2)** $\rho^0\,(S_z = \pm 1)$, **(3)** π^0 *chiral*, **(4)** π^0.