Axion cosmology from lattice QCD

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Outline

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Introduction

Strong CP problem $\rightarrow$ axion $\rightarrow$ dark matter candidate

Two important inputs for axion production: equation of state & topological susceptibility at high $T$

Determine topological susceptibility at high $T$ in the physical point using fixed $Q$ integral

Exact zero modes of the Dirac operator for $Q \neq 0$ are crucial $\rightarrow$ large discretization effects with staggered fermions

Possible solutions
1) eigenvalue reweighting
2) using chiral fermions

We use 1) for the 3 flavor theory and 2) for going down to the physical point
Strong CP problem

- CP is violated in the Standard Model
- QCD may have a CP violating $\frac{\Theta}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$ term
- Experiments constrain $\Theta \lesssim 10^{-10}$

Possible solutions
- $m_u = 0$ (ruled out by lattice results)
- Spontaneous CP violation
- Axion

Axion: promote $\Theta$ to a dynamical field

Elegant solution: Peccei-Quinn mechanism. Complex scalar field with a $U(1)$ symmetric Mexican hat potential.

Spontaneous symmetry breaking $\rightarrow$ axion is the (pseudo) Goldstone-boson
example: The KSVZ axion
\[ L = \partial_\mu \Phi^* \partial_\mu \Phi + V(\Phi^* \Phi) + \Phi \bar{\Psi} L \psi_R + \Phi^* \bar{\Psi} R \psi_L + \bar{\Psi} D(A) \psi + L_{QCD} \]
where \( \psi \) is a heavy \((m \sim f_A)\) fermion with color charge

- \( \Phi \) has a vev of \( f_A \);
  \( \Phi = (f_A + r)e^{i\Theta} \)

- Simultaneous (chiral) \( U(1) \) rotations of \( \Phi \) and \( \psi \)
  \[ L = (f_A + r)^2 \partial_\mu \Theta \partial_\mu \Theta + \partial_\mu r \partial_\mu r + V((f_A + r)^2) + (f_A + r) \bar{\Psi} \psi + + \bar{\Psi} D(A) \psi + i \Theta q + \partial_\mu \Theta \bar{\Psi} \gamma_5 \gamma_\mu \psi + L_{QCD} \]

- Integrating out \( \psi \) and radial component leads to
  \[ L = f_A^2 \partial_\mu \Theta \partial_\mu \Theta + i \Theta q + \partial_\mu \Theta \cdot (\ldots) + L_{QCD} \]

- The axion is \( a = f_A \Theta \)

- If \( \Theta_{QCD} \neq 0 \) then \( \Theta \rightarrow \Theta + \Theta_{QCD} \)
One can find the effective potential for $\Theta$

$$e^{-\frac{V}{T}}V_{\text{eff}} = \langle e^{iQ\Theta} \rangle$$

where $Q$ is the topological charge.

At high $T$ this leads to

$$V_{\text{eff}} = \chi(T)(1 - \cos(\Theta)) = f_A^2 m_A^2(T)(1 - \cos(\Theta))$$

Mexican hat potential for $\Theta$ is slightly tilted by $V_{\text{eff}}$, minimum is at $\Theta = 0 \rightarrow$ strong CP solved.

Solve classical equations for $\Theta$ in an expanding universe $\rightarrow$ axion production.
Misalignment mechanism

- We need to solve

\[
\frac{d^2 \theta}{dt^2} + 3H \frac{d\theta}{dt} + \frac{dV_{\text{eff}}(\theta)}{d\theta} = 0,
\]

where \( H(t) \) is given by the Friedmann equations

\[
H^2 = \frac{8\pi}{3M_{Pl}^2} \rho
\]

\[
\frac{d\rho}{dt} = -3H(\rho + p) = -3HsT
\]

- Two important inputs needed: equation of state \((\rho, s)\) & \(\chi(T)\)
- \(\Theta\) is initially frozen to \(\Theta_0\) & starts to oscillate when \(3H \approx m_A \rightarrow\) axions are produced
- pre-inflation: \(\Theta_0\) uniform, post-inflation: all \(\Theta_0\) present, also (possibly large) string contribution
The equation of state

Effective number of degrees of freedom including all SM particles

\[ \rho = \frac{\pi^2}{30} g_\rho T^4 \quad s = \frac{2\pi^2}{45} g_s T^3 \quad c = \frac{2\pi^2}{15} g_c T^3 \]
The challenge of computing the susceptibility

- large autocorrelation of $Q$ on fine lattices (algorithmic problem)
- $\chi(T)$ decreases strongly with temperature $\rightarrow$ very few $Q \neq 0$ configurations (physical problem)
  
  E.g. $\langle Q^2 \rangle = 10^{-6}$ means one $Q = \pm 1$ configuration per million.
  
  Even $\mathcal{O}(\text{million})$ configurations can lead to large statistical errors
- $\chi(T)$ has large lattice artefacts
Instead of waiting for tunneling events, we make simulations in fixed $Q$ sectors and calculate the weight of sectors $Z_i$ from the action difference between sectors.

First calculate derivative of $\log Z_1/Z_0$:

$$b_1(T) \equiv -\frac{d \log Z_1/Z_0}{d \log T}$$

Use fixed $N_t$-approach, ie. $T = (aN_t)^{-1}$ is changed by $\beta$:

$$b_1(T) = -\frac{d\beta}{d \log a} (\langle S_g \rangle_1 - \langle S_g \rangle_0)$$
Fixed sector integral

Integration gives the relative ratio:

$$Z_1/Z_0|_T = \exp \left( - \int_{T_0}^{T} d \log T' b_1(T') \right) Z_1/Z_0|_{T_0}$$

Start from a temperature $T_0$, where standard approach works. For high temperatures, where only $Q = 0$ and 1 are contributing

$$\langle Q^2 \rangle \simeq \frac{2Z_1}{Z_0 N_t N_s^3 a^4}$$

then $b_1$ is directly related to the fall-off exponent:

$$b(T) = - \frac{d \log \chi}{d \log T} \simeq b_1(T) - 4$$
Fixed $Q$ simulation: extra acc/rej step at the end of each update, as lattice spacing decreased the acceptance gets better.

Test in quenched case: pure Wilson action upto $7 \cdot T_c$ and $8 \times 64^3$

**standard method:** extrapolation using a fit; **integral method; Dilute Instanton Gas Approximation:** exponent agrees nicely, but order of magnitude difference in $\chi$
At high temperatures $\chi(T) \sim T^{-b}$ only $Q = 0, 1$ contribute

$$b_1 = -\frac{d\beta}{d \log a} \langle S_g \rangle_{1-0} - \sum_f \frac{d \log m_f}{d \log a} m_f \langle \overline{\psi}_f \psi_f \rangle_{1-0}$$

- $S_g$: small cutoff effects, huge statistics → staggered $N_f = 3$
- $m_f \langle \overline{\psi}_f \psi_f \rangle_{1-0}$: large cutoff effects → staggered reweighting for $N_f = 3$, overlap for $N_f = 2 + 1$
Reweighting

Strong cut-off effects are related to the lack of exact zero-modes.

- **In the continuum** non-trivial sectors are suppressed by the contribution of zero-modes to the fermion determinant, i.e. by the quark mass.

- **On the lattice** the suppression is altered: $m \rightarrow m + i\lambda_0$, where $\lambda_0$ is a would be zero-mode. Weaker suppression $\rightarrow \chi(T)$ overestimated.

- **To improve** 1. identify would be zero-modes
  2. restore the continuum weight $\rightarrow$ reweight

$$w[U] \sim \frac{m}{m + i\lambda_0}$$
Reweighting - example

A direct comparison of methods @ T=300 MeV \( m_{ud} = m_{ud}^{\text{phys}} \).

**standard**: huge lattice artefacts; **ratio**: \( \chi(T)/\chi(T=0) \) apparent scaling is misleading; **reweight**: orders of magnitude smaller; **integral**: calculate @ \( m_{ud} = m_s \) directly and integrate down in mass, consistent with reweighting.
Topological susceptibility at the physical point

at $T < T_c$: $\chi \sim \frac{m_u m_d}{m_u + m_d}$

while at $T > T_c$: $\chi \sim m_u m_d$

isospin splitting in both cases results in a factor of $\frac{4m_u m_d}{(m_u + m_d)^2} \approx 0.88$
Comparison with other work

- Bonati et.al (1512.06746): smaller exponent, probably large lattice artefacts
- Petreczky, Schadler, Sharma (1606.03145): bosonic and fermionic definitions, consistent results with large errors
Constraints on the axion mass

- **Pre-inflation scenario**: $m_A$ unambiguously determines the $\Theta_0$ initial condition of our Universe.
- **Post-inflation**: $\Theta_0$ average equivalent to $\Theta \approx 2.15$ absolute lower limit (all DM from misalignment): $m_A \gtrsim 28(2) \, \mu eV$
  assuming 50-99% other (e.g. strings): $m_A = 50 - 1500 \, \mu eV$
Summary

- The axion offers a solution to both the strong CP and dark matter problems
- Calculating axion production in the early universe requires the EoS and $\chi(T)$
- Both were determined using lattice calculations up to high temperatures
- New techniques: fixed Q integral + eigenvalue reweighting
- Axion mass in the post-inflation scenario: $m_A = 50 - 1500 \, \mu eV$