Axion cosmology from lattice QCD

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Outline

Introduction

- Strong CP problem $\rightarrow$ axion $\rightarrow$ dark matter candidate
- Two important inputs for axion production: equation of state & topological susceptibility at high T
- Determine topological susceptibility at high T in the physical point using fixed Q integral
- Exact zero modes of the Dirac operator for $Q \neq 0$ are crucial $\rightarrow$ large discretization effects with staggered fermions
- Possible solutions
  1) eigenvalue reweighting
  2) using chiral fermions
- We use 1) for the 3 flavor theory and 2) for going down to the physical point

Strong CP problem

- CP is violated in the Standard Model
- QCD may have a CP violating $\frac{\Theta}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$ term
- Experiments constrain $\Theta \lesssim 10^{-10}$
- Possible solutions
  - $m_u = 0$ (ruled out by lattice results)
  - Spontaneous CP violation
  - Axion
- Axion: promote Θ to a dynamical field
- Elegant solution: Peccei-Quinn mechanism. Complex scalar field with a $U(1)$ symmetric Mexican hat potential.
- Spontaneous symmetry breaking $\rightarrow$ axion is the (pseudo) Goldstone-boson

Peccei-Quinn mechanism

- example: The KSVZ axion
  $\mathcal{L} = \partial_\mu \Phi^* \partial_\mu \Phi + V(\Phi^* \Phi) + \Phi \bar{\Psi}_L \Psi_R + \Phi^* \bar{\Psi}_R \Psi_L + \bar{\Psi} D(A) \Psi + \mathcal{L}_{QCD}$
  where Ψ is a heavy ($m \sim f_A$) fermion with color charge
- Φ has a vev of f_A; $\Phi = (f_A + r) e^{i\Theta}$
- Simultaneous (chiral) $U(1)$ rotations of Φ and Ψ
  $\mathcal{L} = (f_A + r)^2 \partial_\mu \Theta \partial_\mu \Theta + \partial_\mu r \partial_\mu r + V((f_A + r)^2) + (f_A + r) \bar{\Psi} \Psi +$
  $+ \bar{\Psi} D(A) \Psi + i\Theta q + \partial_\mu \Theta \bar{\Psi} \gamma_5 \gamma_\mu \Psi + \mathcal{L}_{QCD}$
- Integrating out Ψ and radial component leads to
  $\mathcal{L} = f_A^2 \partial_\mu \Theta \partial_\mu \Theta + i\Theta q + \partial_\mu \Theta \cdot (\dots) + \mathcal{L}_{QCD}$
- The axion is $a = f_A \Theta$
- If $\Theta_{QCD} \neq 0$ then $\Theta \to \Theta + \Theta_{QCD}$

- One can find the effective potential for Θ
  $e^{-\frac{V}{T}V_{eff}} = \langle e^{iQ\Theta} \rangle$ where Q is the topological charge

- At high T this leads to

$$V_{eff} = \chi(T)(1 - \cos(\Theta)) = f_A^2 m_A^2(T)(1 - \cos(\Theta))$$

- Mexican hat potential for Θ is slightly tilted by V_{eff}, minimum is at $\Theta = 0 \rightarrow$ strong CP solved

- Solve classical equations for Θ in an expanding universe $\rightarrow$ axion production

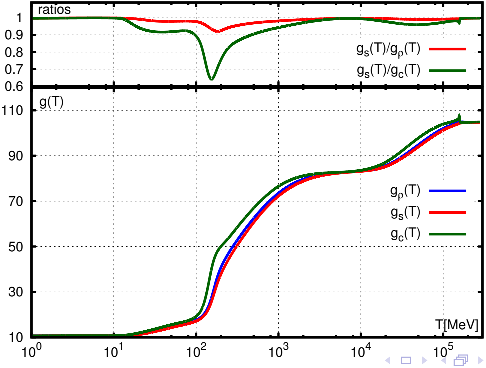
Misalignment mechanism

- We need to solve
$$\frac{d^2\theta}{dt^2} + 3H\frac{d\theta}{dt} + \frac{dV_{eff}(\theta)}{d\theta} = 0,$$
where $H(t)$ is given by the Friedmann equations
$$H^2 = \frac{8\pi}{3M_{Pl}^2}\rho$$
$$\frac{d\rho}{dt} = -3H(\rho + p) = -3HsT$$
- Two important inputs needed: equation of state (ρ, s) & $\chi(T)$
- Θ is initially frozen to Θ_0 & starts to oscillate when $3H \approx m_A \rightarrow$ axions are produced
- pre-inflation: Θ_0 uniform, post-inflation: all Θ_0 present, also (possibly large) string contribution

The equation of state

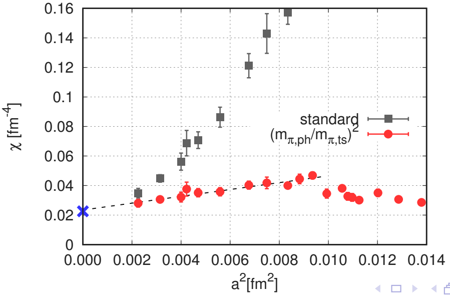
Effective number of degrees of freedom including all SM particles

$$\rho = \frac{\pi^2}{30} g_\rho T^4 \qquad s = \frac{2\pi^2}{45} g_s T^3 \qquad c = \frac{2\pi^2}{15} g_c T^3$$



The challenge of computing the susceptibility

- large autocorrelation of Q on fine lattices (algorithmic problem)
- $\chi(T)$ decreases strongly with temperature $\rightarrow$ very few $Q \neq 0$ configurations (physical problem)
  E.g. $\langle Q^2 \rangle = 10^{-6}$ means one $Q = \pm 1$ configuration per million.
  Even $\mathcal{O}(\text{million})$ configurations can lead to large statistical errors
- $\chi(T)$ has large lattice artefacts



Fixed sector integral

See also Frison,Kitano,Matsufuru,Mori,Yamada 1606.07175.

- Instead of waiting for tunneling events, we make simulations in fixed Q sectors and calculate the weight of sectors Z_i from the action difference between sectors.
- First calculate derivative of $\log Z_1/Z_0$:

$$b_1(T) \equiv -\frac{d \log Z_1/Z_0}{d \log T}$$

- Use fixed N_t-approach, ie. $T = (aN_t)^{-1}$ is changed by β:

$$b_1(T) = -\frac{d\beta}{d \log a} \left(\langle S_g \rangle_1 - \langle S_g \rangle_0 \right)$$

Fixed sector integral

- Integration gives the relative ratio:

$$Z_1/Z_0|_T = \exp\left(-\int_{T_0}^{T} d\log T'\, b_1(T')\right) Z_1/Z_0|_{T_0}$$

- Start from a temperature T_0, where standard approach works. For high temperatures, where only $Q = 0$ and 1 are contributing

$$\langle Q^2 \rangle \simeq \frac{2Z_1}{Z_0 N_t N_s^3 a^4}$$

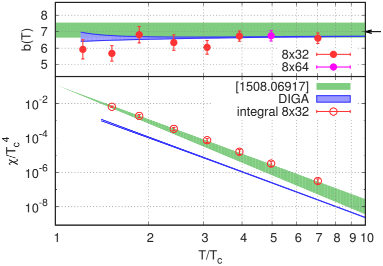
- then b_1 is directly related to the fall-off exponent:

$$b(T) = -\frac{d\log\chi}{d\log T} \simeq b_1(T) - 4$$

Fixed Q integral - quenched

Fixed Q simulation: extra acc/rej step at the end of each update, as lattice spacing decreased the acceptance gets better.

Test in quenched case: pure Wilson action upto $7 \cdot T_c$ and 8×64^3



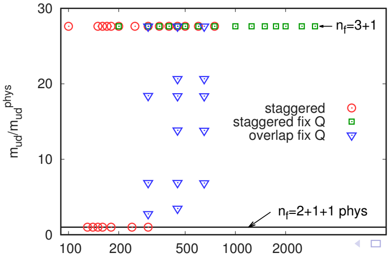
standard method: extrapolation using a fit; **integral method**; **Dilute Instanton Gas Approximation:** exponent agrees nicely, but order of magnitude difference in χ

Fixed Q integral with fermions

At high temperatures $\chi(T) \sim T^{-b}$ only $Q = 0, 1$ contribute

$$b_1 = -\frac{d\beta}{d\log a}\langle S_g\rangle_{1-0} - \sum_f \frac{d\log m_f}{d\log a} m_f \langle\overline{\psi}\psi_f\rangle_{1-0}$$

- S_g: small cutoff effects, huge statistics $\rightarrow$ staggered $N_f = 3$
- $m_f\langle\overline{\psi}\psi_f\rangle_{1-0}$: large cutoff effects $\rightarrow$ staggered reweighting for $N_f = 3$, overlap for $N_f = 2+1$



Reweighting

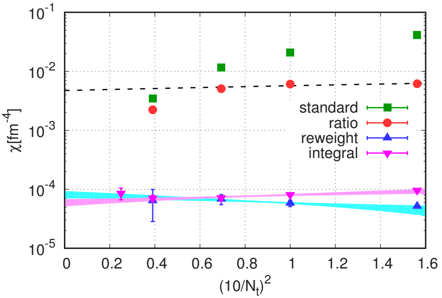
Strong cut-off effects are related to the lack of exact zero-modes.

- In the continuum non-trivial sectors are suppressed by the contribution of zero-modes to the fermion determinant, ie. by the quark mass.
- On the lattice the suppression is altered:
  $m \to m + i\lambda_0$, where λ_0 is a would be zero-mode.
  Weaker suppression $\to \chi(T)$ overestimated.
- To improve 1. identify would be zero-modes
  2. restore the continuum weight $\to$ reweight

$$w[U] \sim \frac{m}{m + i\lambda_0}$$

Reweighting - example

A direct comparison of methods @ T=300 MeV $m_{ud} = m_{ud}^{\text{phys}}$.

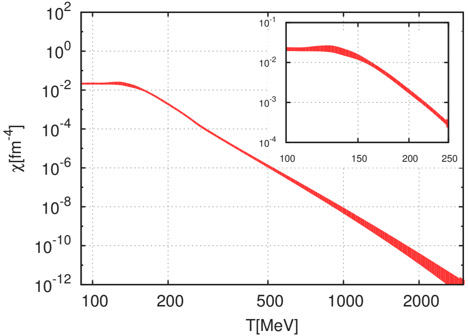


standard: huge lattice artefacts; **ratio:** $\chi(T)/\chi(T=0)$ apparent scaling is misleading; **reweight:** orders of magnitude smaller; **integral:** calculate @ $m_{ud} = m_s$ directly and integrate down in mass, consistent with reweighting

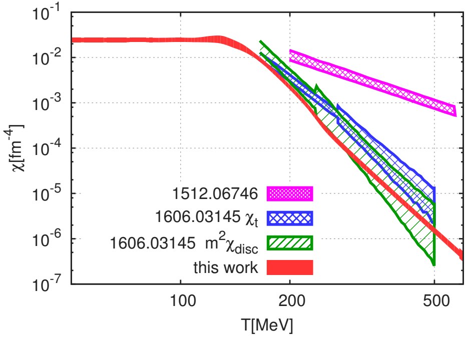
Topological susceptibility at the physical point

at $T < T_c$: $\chi \sim \frac{m_u m_d}{m_u + m_d}$ while at $T > T_c$: $\chi \sim m_u m_d$

isospin splitting in both cases results in a factor of $\frac{4 m_u m_d}{(m_u + m_d)^2} \approx 0.88$

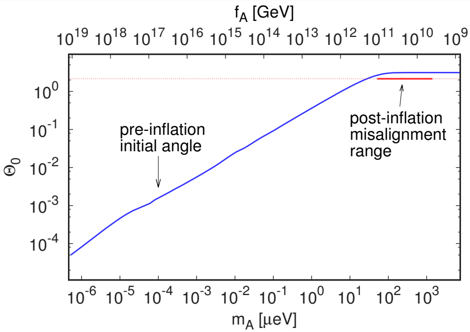


Comparison with other work



- Bonati et.al (1512.06746): smaller exponent, probably large lattice artefacts
- Petreczky, Schadler, Sharma (1606.03145): bosonic and fermionic definitions, consistent results with large errors

Constraints on the axion mass



- Pre-inflation scenario: m_A unambiguously determines the Θ_0 initial condition of our Universe
- Post-inflation: Θ_0 average equivalent to $\Theta \approx 2.15$
  absolute lower limit (all DM from misalignment): $m_A \gtrsim 28(2)\ \mu\text{eV}$
  assuming 50-99% other (e.g. strings): $m_A = 50 - 1500\ \mu\text{eV}$

Summary

- The axion offers a solution to both the strong CP and dark matter problems
- Calculating axion production in the early universe requires the EoS and $\chi(T)$
- Both were determined using lattice calculations up to high temperatures
- New techniques: fixed Q integral + eigenvalue reweighting
- Axion mass in the post-inflation scenario: $m_A = 50 - 1500\ \mu\mathrm{eV}$