H-dibaryon in Holographic QCD

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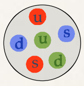
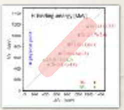
The aim of this study is to investigate the properties of H-dibaryon in chiral limit using a recent new method of non-perturbative QCD.



- In particular, H-dibaryon mass is interesting from the viewpoint of "existence" of H-dibaryon.

- H-dibaryon: B=2 flavor singlet state (uuddss)
  ($J^\pi = 0^+$, $I = 0$, $B = 2$, $S = -2$, $Y = 0$)
  - In 1977, Jaffe first predicted the existence of H-dibaryon using MIT bag model.
  - In this model, the H-dibaryon mass is $M_H = 2150$ MeV and may be smaller than the $\Lambda\Lambda$ threshold. [R. L. Jaffe, Phys. Rev. Lett. 38 (1977) 195]
    cf. $\Lambda\Lambda$(2231MeV)
- However, the prediction of the low-mass H-dibaryon was experimentally denied in 1991.
  Instead, the double hyper nuclei $_{\Lambda\Lambda}{}^6$He was found. [K. Imai, Nucl. Phys. A527, 181(1991)] [H. Takahashi et al., Phys. Rev. Lett. 87, 212502(2001)]
  - There is no-deeply-bound H particle at least at the physical point.
  - How about general cases such as flavor SU(3) symmetric case ($m_u=m_d=m_s$) ?
- Lattice QCD calculation indicates the existence of H-dibaryon at "unphysical points":
  - H is stable at the flavor SU(3) symmetric ($m_u=m_d=m_s$) and large quark-mass region.

[NPLQCD (S. Beane et al.), Phys. Rev. Lett. 106 (2011)162001]
[HAL QCD (T. Inoue et al.), Phys. Rev. Lett. 106 (2011) 162002]
[HAL QCD (T. Inoue et al.), Nucl. Phys. A881, 28 (2012)]



- Then, how about in chiral limit?
  - Some model approach such as Skyrme model would be useful instead of lattice QCD. (It is difficult for the lattice QCD calculation to take the chiral limit.)
  - It is desirable to use some QCD-based model for the calculation.

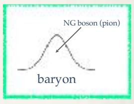
Skyrme Model

- Skyrme model [T. H. R. Skyrme, Nucl. Phys. 31 (1962) 556]

$$\mathcal{L} = \underbrace{\frac{f_\pi^2}{4}\mathrm{tr}(L_\mu L^\mu)}_{\text{kinetic term}} + \underbrace{\frac{1}{32e^2}\mathrm{tr}[L_\mu, L_\nu]^2}_{\text{"Skyrme term" (introduced by hand)}} \quad (+\text{ WZW term}) \qquad L_\mu \equiv \frac{1}{i}U^\dagger\partial_\mu U$$

chiral field: $U(x) = e^{i\pi(x)/f_\pi} \in SU(N_f)$

- The B=2 Skyrmion corresponding to H-dibaryon can be obtained using hedgehog Ansatz of SO(3) subalgebra of SU(3) flavor:



Hedgehog Ansatz $N_f = 3$ case (uds flavor) SU(3) ⊃ SU(2), SO(3)

[A. P. Balachandran et al., Nucl. Phys. B256 (1985) 525]
[R. L. Jaffe et al., Nucl. Phys. B258 (1985) 468]

- SO(3) subalgebra:

$$\Lambda_1 = \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \Lambda_2 = -\lambda_5 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix} \quad \Lambda_3 = \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad [\Lambda_i, \Lambda_j] = i\epsilon_{ijk}\Lambda_k$$

- SO(3) hedgehog Ansatz:

$$U(\mathbf{x}) = e^{i[(\mathbf{\Lambda}\cdot\hat{\mathbf{x}})F(r) + ((\mathbf{\Lambda}\cdot\hat{\mathbf{x}})^2 - 2/3)\varphi(r)]} \in SU(3)$$

- B = 2 ⇒ dibaryon (stable) ← with

boundary condition

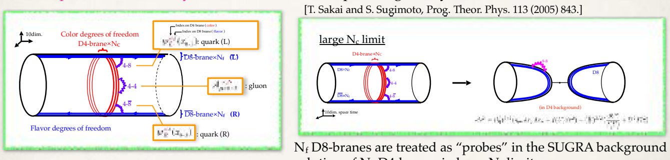
$\cos F(\infty) = 1$	$\cos F(0) = -1$
$e^{i\varphi(\infty)/3} = 1$	$e^{i\varphi(0)/3} = -e^{i2\pi k/3}$
$G(\infty) = 0$	$G(0) = 0$

Holographic QCD

Sakai-Sugimoto Model

- QCD-equivalent D-brane system (D4/D8/$\overline{\text{D8}}$) in superstring theory.

[T. Sakai and S. Sugimoto, Prog. Theor. Phys. 113 (2005) 843.]



N_f D8-branes are treated as "probes" in the SUGRA background solution of N_c D4-branes in large N_c limit.

- Holographic QCD leads to the 4-dimensional meson effective theory in terms of pion & vector meson fields:
  - D8-brane action in large N_c limit: $S^{\rm DBI}_{\rm D8} = T_8 \int d^9x\, e^{-\phi}\sqrt{-\det(g_{MN} + 2\pi\alpha' F_{MN})}$

↓ integrate over S^4 and expand according to $1/N_c$, $1/\lambda$

$$S_{\rm HQCD} \sim \kappa \int d^4x\,dz\,\mathrm{tr}\left(\frac{1}{2}K(z)^{-1/3}F_{\mu\nu}F_{\mu\nu} + K(z)F_{\mu z}F_{\mu z}\right) + O(F^4)$$

↓ mode expansion of gauge field $A_\mu(x, z)$ along z-direction $A_\mu(x_\nu, z) \simeq l_\mu(x_\nu)\psi_+(z) + r_\mu(x_\nu)\psi_-(z) + \rho_\mu(x_\nu)\psi_1(z)$

pion (NG mode) pion (NG mode) ρ-meson

$S_{\rm HQCD}$

[K. Nawa, H. Suganuma, T. Kojo, Phys. Rev. D75, 086003 (2007)]

H-dibaryon in Holographic QCD

The Soliton Solution of the Meson Effective Theory $S_{\rm HQCD}$

- The B=1 SU(2) case was investigated. [K. Nawa, H. Suganuma, T. Kojo, Phys. Rev. D75, 086003 (2007)] [H. Hata, T. Sakai, S. Sugimoto, S. Yamato, PTP 117, 1157 (2007)]

- We consider the SU(3) flavor case to describe H-dibaryon in the chiral limit in holographic QCD for the first time.

- We study H-dibaryon in holographic QCD in the following manner:
  - Action (Euclid. metric)

$$S_{\rm HQCD} = \int d^4x\left\{\frac{f_\pi^2}{4}\mathrm{tr}(L_\mu L_\mu) - \frac{1}{32e^2}\mathrm{tr}[L_\mu, L_\nu]^2\right\} + \frac{1}{2}\mathrm{tr}(\partial_\mu\rho_\nu - \partial_\nu\rho_\mu)^2 + m_\rho^2\mathrm{tr}(\rho_\mu\rho_\mu) + (\text{int. terms})\}$$

  - Meson Field ☞ "SO(3)" Hedgehog Ansatz ($N_f = 3$)
    - Chiral Field: $U(\mathbf{x}) = e^{i[(\mathbf{\Lambda}\cdot\hat{\mathbf{x}})F(r) + ((\mathbf{\Lambda}\cdot\hat{\mathbf{x}})^2 - 2/3)\varphi(r)]} \in SU(3)$
    - lowest SO(3) vector-meson field "ρ-meson":

$$\rho_0(\mathbf{x}) = 0 \qquad \rho_i(\mathbf{x}) = \{\epsilon_{ijk}\hat{x}_j G(r)\}\Lambda_k$$

"Wu-Yang-'t Hooft-Polyakov Ansatz"

⇒ Energy (static soliton mass): $E[F(r), \varphi(r), G(r)] \equiv S_{\rm HQCD}|_{\rm hedgehog}$

Effective Action on SU(3) Flavor

We derive the effective action for H-dibaryon as SO(3)-type hedgehog soliton in terms of profile functions $F(r)$, $\varphi(r)$, $G(r)$ from holographic QCD as follows:

$$S_{\rm HQCD} \sim \kappa \int d^4x\,dz\,\mathrm{tr}\left(\frac{1}{2}K(z)^{-1/3}F_{\mu\nu}F_{\mu\nu} + K(z)F_{\mu z}F_{\mu z}\right) + O(F^4)$$

$$\sim \int d^4x\Big\{\frac{f_\pi^2}{4}[2F'^2 + \frac{2}{3}\varphi'^2 + \frac{8}{r^2}(1 - \cos F\cos\varphi)] \quad \text{(chiral term)}$$

$$+ \frac{1}{32e^2}\frac{16}{r^2}[(\varphi'^2 + F'^2)(1 - \cos F\cos\varphi) + 2\varphi' F'\sin F\sin\varphi + \frac{1}{r^2}\{(1 - \cos F\cos\varphi)^2 + 3\sin^2 F\sin^2\varphi\}] \quad \text{(Skyrme term)}$$

$$-\frac{1}{2}[8(\frac{3}{r^2}G^2 + \frac{2}{r}GG' + G'^2)] + m_\rho^2[4G^2] \quad (\varrho\text{-kinetic term / }\varrho\text{-mass term})$$

$$+ g_{3\rho}[8\frac{G^3}{r}] + \frac{1}{2}g_{4\rho}[4G^4] \quad (3\varrho \text{ coupling / } 4\varrho \text{ coupling})$$

$$-g_1[\frac{16}{r}\{(\frac{1}{r}G + G')(F'\sin\frac{F}{2}\cos\frac{\varphi}{2} + \varphi'\cos\frac{F}{2}\sin\frac{\varphi}{2}) + \frac{1}{r^2}G(1 - \cos F\cos\varphi)\}] \quad (\partial\varrho\text{-}2\alpha \text{ coupling})$$

$$-g_2[\frac{8}{r^2}G^2(1 - \cos F\cos\varphi)] \quad (2\varrho\text{-}2\alpha \text{ coupling})$$

$$+g_3[\frac{16}{r^3}G\{3\sin F\sin\frac{F}{2}\sin\varphi\sin\frac{\varphi}{2} + (1 - \cos\frac{F}{2}\cos\frac{\varphi}{2})(1 - \cos F\cos\varphi)\}] \quad (\varrho\text{-}2\alpha\text{-}\beta \text{ coupling})$$

$$-g_4[\frac{16}{r^2}G^2(1 - \cos\frac{F}{2}\cos\frac{\varphi}{2})] \quad (\varrho\text{-}\partial\varrho\text{-}\beta \text{ coupling})$$

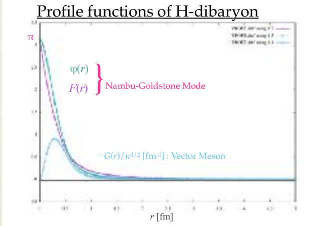
$$-g_5[\frac{8}{r}G^3(1 - \cos\frac{F}{2}\cos\frac{\varphi}{2})] \quad (3\varrho\text{-}\beta \text{ coupling})$$

$$+g_6[4G^2(F'^2 + \varphi'^2)] \quad (2\varrho\text{-}2\alpha \text{ coupling})$$

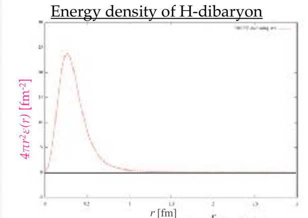
$$+g_7[\frac{8}{r^2}G^2\{3\sin^2\frac{F}{2}\sin^2\frac{\varphi}{2} + (1 - \cos\frac{F}{2}\cos\frac{\varphi}{2})^2\}]\Big\} \quad (2\varrho\text{-}2\beta \text{ coupling})$$

Numerical Results

Profile functions of H-dibaryon



Energy density of H-dibaryon



$E[F(r), \varphi(r), G(r)] = 4\pi\int dr\, r^2\varepsilon(r)$

Mass & Radius of H-dibaryon

Mass: $M_H \simeq 1673$ MeV , Radius: $\sqrt{\langle r^2\rangle} \simeq 0.413$ fm (in chiral limit)

input parameters
$f_\pi = 92.4$ MeV
$m_\rho = 776.0$ MeV

- The mass and radius of B=1 hedgehog baryon in holographic QCD (in chiral limit) is $M_{B=1} \simeq 834.0\mathrm{MeV}$, $\sqrt{\langle r^2\rangle} \simeq 0.37\mathrm{fm}$ [K. Nawa, H. Suganuma, T. Kojo, Phys. Rev. D75, 086003 (2007)]

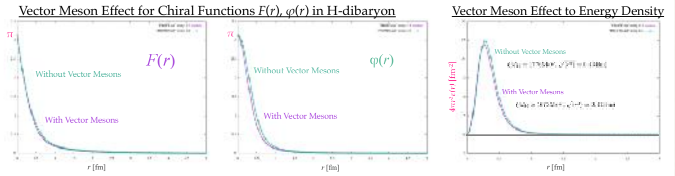
We thus find $M_H \simeq$ 2.006 $M_{B=1}$ (in chiral limit)

- Then, H mass is expected to be smaller than two nucleon mass in the chiral limit:
  - In fact, nucleon (flavor-octet baryon) mass M_N is larger than hedgehog mass $M_{B=1}$ by rotational energy: $M_N = M_{B=1}$ + (rotational energy), and satisfies $M_H < 2M_N$.

Vector Meson Effect

Vector Meson Effect for Chiral Functions $F(r)$, $\varphi(r)$ in H-dibaryon

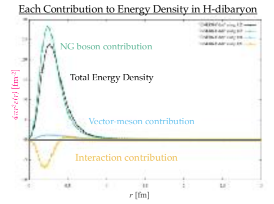
Vector Meson Effect to Energy Density



- Chiral profile functions F(r), $\varphi(r)$ and energy density slightly shrink by the vector-meson effect.
- About 100MeV (6%) mass reduction is occurred by the vector-meson effect.

Details of Energy Density

Each Contribution to Energy Density in H-dibaryon



Each Contribution to the Energy
Total Energy of Soliton H
1673MeV (100)
NG-boson Contribution
1795MeV (107.3)
Vector-meson Contribution
119MeV (7.1)
Interaction Contribution
-241MeV (-14.4)

- The H-dibaryon mass is lowered by the interaction between NG bosons and Vector mesons in the interior region of the H-dibaryon.

Summary and Conclusion

- We have formulated H-dibaryon (uuddss) in holographic QCD for the first time.
- We have investigated H-dibaryon as an SO(3) hedgehog soliton solution in holographic QCD, and have found that
  - the H-dibaryon mass is about two times B=1 hedgehog mass in the chiral limit, which can be smaller than the two octet-baryon mass.
  - chiral profile functions F(r), $\varphi(r)$ and energy density shrink slightly by the vector-meson effect.