Transport at low and high opacity and the elliptic flow

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Li, He, Lin, DM, Wang & Xie, PRC93, 051901 (2016) [arXiv:1601.05390v2]
Li, He, Lin, DM, Wang & Xie, 1604.07387
I. Elliptic flow in heavy-ion collisions

II. Hydrodynamics vs covariant transport

III. Anisotropic escape from AMPT and MPC/Cascade

IV. Mass ordering for $v_2$ in AMPT

V. Conclusions, open questions
Heavy ion physics

goal: study quark-gluon plasma (many-body QCD)

initial nuclei  quark-gluon plasma  hadron gas

The bulk (> 99%): → relativistic, dissipative hydrodynamics

- many-body QCD system, collectivity  ~ statistical physics, hydro

Rare probes: → perturbative QCD

- evolution inevitably reflects presence of medium ⇒ Tomography
Elliptic flow

initial spatial anisotropy converts to final momentum space anisotropy

\[ \varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle} \]

\[ v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} \equiv \langle \cos 2\phi_p \rangle \]

flow harmonics: \[ \frac{dN}{p_T dp_T d\phi} \equiv \frac{1}{2\pi p_T dp_T} \left[ 1 + \sum_{n=1}^{\infty} 2v_n \cos n(\phi - \psi_n) \right] \]
- quite large: $v_2(p_T) \sim 20\%$ at high $p_T$ means $\sim 2:1$ anisotropy

- “mass ordering”: heavier particles $v_2$ smaller at low $p_T$ ($\pi > K > p > ..$)

  e.g., Au+Au at RHIC PHENIX, PRL91, 182301 ('03)
Hydrodynamics paradigm

assumption of local thermalization quite successful in explaining observables

conserved currents:

\[ \partial_\mu T^{\mu\nu}(x) = 0 \quad , \quad \partial_\mu N_B^{\mu}(x) = 0 \]

if dissipation - extra fields and equations of motion:

\[ T^{\mu\nu}(x) = T_{\text{ideal}}^{\mu\nu}(x) + \delta T^{\mu\nu}(x) \]
\[ N^{\mu}(x) = N_{\text{ideal}}^{\mu}(x) + \delta N^{\mu}(x) \]

\[ \delta T^{\mu\nu} = A^{\mu\nu}(\{T^{\alpha\beta}, N^{\gamma}\}) \quad , \quad \delta N^{\mu} = B^{\mu}(\{T^{\alpha\beta}, N^{\gamma}\}) \]

= causal 2nd-order dissipative hydrodynamics \((\neq \text{Navier-Stokes})\)

Israel & Stewart ’79; Muronga ’04; Baier, Romatschke et al ’06; Heinz, Song et al; Teaney et al
DM, Huovinen et al; Koide, Kodama et al; Schenke, Jeon et al; Denicol, Rischke et al; Strickland et al...

utilizes equation of state \((p(e, n_B), T(e, n_B))\), transport coeffs \((\eta, \zeta, \kappa_B)\),
relaxation times \((\tau_\eta, \tau_\zeta, \tau_\kappa)\)

open questions remain (thermalization, freezeout, small systems,...)
- hydrodynamic evolution does generate $v_2$ e.g., Ollitrault, PRD46 ('92)

- mass ordering is natural in hydro (at fixed $p_T$, velocities differ)
  e.g., Huovinen, Kolb et al PLB 503 ('01)

- $v_2$ can be used to measure viscosity

Romatschke & Luzum, PRC78 ('08):

\[ \eta/s = 10^{-4} \]

\[ \eta/s = 0.08 \]

\[ \eta/s = 0.16 \]

\[ \eta/s = 0.24 \]

→ hydro implies opaque system


(on-shell) phase-space density \( f(x, \vec{p}) \equiv \frac{dN(\vec{x},\vec{p},t)}{d^3x d^3p} \)

transport equation:
\[
p^\mu \partial_\mu f_i(x, p) = C^i_{2\rightarrow 2}[\{f_j\}](x, p) + C^i_{2\leftrightarrow 3}[\{f_j\}](x, p) + \cdots
\]

with, e.g.,
\[
C^i_{2\rightarrow 2} = \frac{1}{2} \sum_{jkl} \int_{234} (f_k^3 f_l^4 - f_1^3 f_2^4) W^{ij}_{12\rightarrow 34} \left( \int_j \equiv \int \frac{d^3p_j}{2E_j}, \quad f_a^k \equiv f^k(x, p_a) \right)
\]

thermalizes (in box), fully causal and stable \(\rightarrow\) can derive hydro eqns
e.g., Denicol, Rischke et al
handles both high or low opacities \(\rightarrow\) usable for fluid-to-particle conversion
e.g., Teaney, Moore & Dusling; DM & Wolff, ...

high opacity (hydro) limit: transport coeffs & relaxation times
\[
\eta \approx 1.2T/\sigma, \quad \tau_\pi \approx 1.2\lambda_{tr}
\]
$v_2$ builds up early

both in transport

Zhang, Gyulassy & Ko ('99):

\[ v_2 \]

and in hydro

Kolb & Heinz, nucl-th/0305084v2

\[ \tau (\text{fm/c}) \]

as system expands, spatial eccentricity reduces $\rightarrow v_2$ “self-quenches” itself
DM & Gyulassy, NPA 697 ('02): $v_2(p_T, \chi)$

![Graph showing $v_2$ vs. $p_T$ for different $\chi$ values.]

**parton transport model MPC**

- $2 \rightarrow 2$ only, forward-peaked
- $\sigma_{tr} \approx 0.3\sigma_{tot}$

**Au+Au @ 130 GeV, $b = 8$ fm**

- minijet initconds
- 1 parton $\rightarrow$ 1 $\pi$ hadronization

$$\langle N_{coll} \rangle \sim 40 - 70: \text{ i.e., } 10 - 15 \times \text{ higher opacity than from perturbative } 2 \rightarrow 2$$
One way out: include perturbative $ggg \leftrightarrow gg$

e.g., Frankfurt/BAMPS (2005-), or MPC/Grid (2015-)

$\tau_{eq} \sim 1\text{ fm}$

Xu & Greiner, (’08): significant $v_2$

some challenges remain: e.g., treatment of LPM and sensitivity to screening
A Multi-Phase Transport
Lin, Ko et al, PRC72 ('05)

also quite successful with observables

\[ \text{AMPT} \approx \text{Lund string model (HIJING)} \]
\[ + 2 \rightarrow 2 \text{ parton cascade (ZPC)} \]
\[ + \text{hadron transport (ART)} \]

version with “string melting”:

- energy density in strings converted to quanta (quarks/antiquarks)
- \(\rightarrow\) fluctuating initial geometry (random nucleon positions)
- hadronization via coalescence

\[\rightarrow\] higher parton density, enhanced collectivity

HERE: use it to study elliptic flow in 200-GeV Au+Au arXiv:1502.05572
AMPT also gets low-\(p_T\) v2

Z.-W. Lin, PRC90 (’14)
ideal hydro needs $\sigma \to \infty$ (hard) though viscous hydro could be reachable

DM & Huovinen, PRL94 ('05)

Huovinen & DM, JPG35 ('08)

$\tau_0 = 0.1 \text{ fm/c}$

$\nu_2$ at $b = 8 \text{ fm}$

$\sqrt{7 \text{ mb}}$, $\sqrt{20 \text{ mb}}$, $\sqrt{47 \text{ mb}}$

$\nu_2$ vs $p_{\perp} \text{ [GeV]}$

$\nu_2$ vs $p_T \text{ (GeV)}$

- Transp. $\sigma = \text{const.}$
- Transp. $\sigma \propto \tau^{2/3}$

Is AMPT just hydro?? $\rightarrow$ No.
$v_2$ from AMPT - vs $N_{\text{coll}}$

He et al, PLB 753 ('15)

\[
\frac{d\sigma}{dt} \propto \frac{1}{(t - \mu_D^2)^2}
\]

$\mu_D^2 = 0.2 \text{ GeV}^2$

$\sigma_{\text{TOT}} = \frac{9\pi\alpha_s^2}{2\mu_D^2} = 3 \text{ mb}$

$\Rightarrow \langle N_{\text{coll}} \rangle \approx 4.5$

**solid:** all - all partons right after their $N_{\text{coll}}$-th collision

**dashed:** frozen out - partons that do not interact after the $N_{\text{coll}}$-th collision

**dotted:** non-freezeout - these will have more than $N_{\text{coll}}$ collisions in future
DM et al (’15): $v_2$ vs $N_{coll}$

yellow: same partons as green curve but their $v_2$ plotted right before $N_{coll}$
$v_2$ from AMPT - vs (proper) time

DM et al (’15):

\[ \tau = \sqrt{t^2 - z^2} \] [fm]

- **all**: all partons at the given $\tau = \sqrt{t^2 - z^2}$
- **frozen out**: partons that had their last collisions before given $\tau$
- **active**: partons that will have collisions after $\tau$ (still interacting part)
DM et al ('15): frozen/active/all fraction vs time

\[ \tau = \sqrt{t^2 - z^2} \text{ [fm]} \]

AMPT \( \langle N_{\text{coll}} \rangle = 4.5 \)
So far:

- interacting “hydro” component has low $v_2$ during most of the flow buildup
- large $O(1)$ change in $v_2$ near the very last collision

→ is it the collision (change in momenta)?
  or that there will be no further collisions (escape probability)?

Test: randomize azimuth after each $2 \rightarrow 2$ collision (rotate outgoing momenta by random angle uniformly sampled over $[0, 2\pi]$)
thick lines: standard AMPT, thin lines: AMPT with $\varphi$-randomization

dashed: frozen out, solid: all

$\rightarrow$ escape $v_2$ largely survives randomization $\Rightarrow$ azimuthal escape bias
It is general, not just AMPT

Recheck with MPC/Cascade DM & Gyulassy, PRC62 ('00)

- parameters like in DM & Huovinen, PRL 94 ('05):

  \[ \text{Au+Au, } b = 8 \text{ fm, smooth binary collision profile,} \]
  \[ \text{uniform } \frac{dN}{d\eta} \text{ for } |\eta| < 5, \text{ with } \left. \frac{dN}{d\eta} \right|_{b=0} = 1000 \]
  \[ \tau_0 = 0.6 \text{ fm, } T_0 = 385 \text{ MeV} \]
  \[ \sigma = \text{const}, \text{ isotropic} \]

  study 3 different opacities \( \langle N_{\text{coll}} \rangle \approx 4.5, 17, \text{ and } 35 \)
DM ('15): \textbf{v2 vs Ncoll comparison}

**AMPT**

\begin{align*}
\text{AMPT} \quad \langle N_{\text{coll}} \rangle &= 4.5
\end{align*}

\begin{align*}
\text{MPC,} \quad \langle N_{\text{coll}} \rangle &= 4.68
\end{align*}

very similar at the same opacity $\langle N_{\text{coll}} \rangle \sim 4.5$
AMPT

\[ \tau = \sqrt{t^2 - z^2} [\text{fm}] \]

\[ \langle N_{\text{coll}} \rangle \sim 4.5 \]

very similar at the same opacity

MPC

\[ \tau = \sqrt{t^2 - z^2} [\text{fm}] \]

\[ \text{all, } N_{\text{coll}}=4.7 \]
\[ \text{frozen, } N_{\text{coll}}=4.7 \]
\[ \text{active, } N_{\text{coll}}=4.7 \]
\[ \text{Ncoll}=17.5 \]
DM ('15): standard vs phi-randomized MPC/Cascade

qualitative picture unaffected by randomization (like for AMPT)
DM ('15): $v_2$ vs tau at lowest and highest opacity

Low opacity: escape dominates; high opacity: active part carries most of $v_2$
DM ('15): \textbf{v2 vs Ncoll at highest opacity}

\begin{equation*}
\text{MPC} \quad \langle N_{\text{coll}} \rangle = 35.2
\end{equation*}
DM ('15): active/frozen/all fraction vs tau

\[ \tau = \sqrt{t^2 - z^2} \text{ [fm]} \]

MPC \( <N_{\text{coll}} > = 35.2 \)
$\nu_4$ also similar...

$v_4$ vs tau DM (‘15)

$$\tau = \sqrt{t^2 - z^2} \text{ [fm]}$$

MPC $<N_{\text{coll}}>$=4.7
**v4 vs Ncoll** DM ('15)

Anisotropic escape is a generic feature of transport models, as long as opacities are not very large

- not captured by standard hydro + Cooper-Frye
- Fokker-Planck also lacks chance for no interaction

- revive continuous emission hydro that weights by $P_{\text{escape}} = e^{-\int dz \sigma \rho}$

Socolowski, Grassi, Hama & Kodama PRL 93, 182301 (2004)
Alright, escape gives $v_2$ - but a mass-ordered one??

**standard hydro explanation: flow boost effect** Huovinen, Kolb et al PLB 503 (’01)

**AMPT:** parton cascade + hadronization + rescatterings (+ decays)
AMPT - partonic flow (AA, dA, pA)

Li et al, 1601.05390v2:

weak mass ordering at low pT, even with \( \phi \)-randomization
Hadronization (coalescence)

in AMPT, meson and baryons form via $q\bar{q} \rightarrow M$, $qqq \rightarrow B$

whereas in hydro hadronization built into EoS ($p(e,n_B)$)

fusion nearby in phase space $\Rightarrow$ helps “amplify” elliptic flow

naively $v_{2,n}(p_T) \approx n v_{2,q}(p_T/n)$

while in AMPT

DM & Voloshin, PRL91 ('03)

Chen & Ko, PRC73 ('06)
AMPT - flow after coalescence

Li et al, 1601.05390v2:

clear mass ordering, mainly from coalescence
quark momenta and opening angles in coalescence depend on species

Li et al., 1601.05390v2:

Parton pT for given hadron pT

hadron-parton momentum opening angle

\[ \text{opening angle varies within same meson/baryon group} \]
Li et al, 1601.05390v2:

$\rightarrow$ decays reduce $v2$ but effect varies with species
AMPT - hadronic rescatterings

Li et al, 1601.05390v2:

→ enhancement/reduction based on species, system size also matters
Summary

Anisotropic escape is a generic feature of transport models. At modest $N_{\text{coll}} \sim 5$, most of the $v_2$ in mid-peripheral Au+Au is built up via escape and carried by already frozen out partons in both AMPT and MPC/Cascade, while the still interacting component has zero or even negative $v_2$. At very high opacities $N_{\text{coll}} \sim 30$, the interacting (“hydrodynamic”) part does dominate the evolution of $v_2$.

Therefore, application of hydrodynamics to extract medium properties from elliptic flow (such as shear viscosity) gets entangled with decoupling physics. Ignoring escape can lead to an underestimate of $v_2$ from the theory, and thus a systematic bias towards smaller shear viscosities inferred from data.

Flow mass ordering arises in AMPT as an interplay among hadronization via coalescence, hadronic rescatterings, and decays. Most mass difference is generated by rescatterings and coalescence.

Open questions remain, such as how to properly incorporate escape into hydrodynamics or Fokker-Plank theory, and how anisotropic escape affects heavy quark $v_2$. 
Backup slides
spacetime picture looks like evaporation - “outside-in”

He et al, PLB 753 (’15)

\( N_{\text{coll}} \) distribution

\( \text{position distribution vs } N_{\text{coll}} \)
i) for all $N_{coll}$, the frozen-out component carries most of the $v_2$

ii) even escapees with $N_{coll} = 0$ show sizable “corona” $v_2$

- confirms DM, nucl-th/0503051
DM ('15): Ncoll distribution from MPC/Cascade

\[ \langle N_{\text{coll}} \rangle = 4.41, \text{Poiss} \]
\[ \langle N_{\text{coll}} \rangle = 4.68, |\eta_{\text{fin}}| < 2 \]
\[ \langle N_{\text{coll}} \rangle = 16.39 \]
\[ \langle N_{\text{coll}} \rangle = 17.47, |\eta_{\text{fin}}| < 2 \]
\[ \langle N_{\text{coll}} \rangle = 33.1 \]
\[ \langle N_{\text{coll}} \rangle = 35.2, |\eta_{\text{fin}}| < 2 \]
AMPT - mass ordering evolution

Li et al, 1601.05390v2, soon in PRC

(a) Au+Au

(b) d+Au

$p_L = (0.7-0.8) \text{ GeV/c}$
Cooper-Frye freezeout

Not a very satisfactory solution (still open problem)

Assume sudden transition to a gas on a 3D hypersurface (typically $T = \text{const}$ or $\varepsilon = \text{const}$)

\[ E \, dN = p^\mu d\sigma_\mu(x) \, d^3p \, f_{\text{gas}}(x, \vec{p}) \]

(covariant analog of $t = \text{const}$ freezeout $dN/d^3xd^3p = f(\vec{x}, \vec{p}, t_{\text{fo}})$)

Good: - conserves energy-momentum and charges locally

Bad: - negative contributions possible $p \cdot d\sigma < 0$
  - arbitrariness in choice of HS & self-consistency problem
For panel discussion on small systems
Re: hydro/kinetic theory applied to systems of $\mathcal{O}(1 \text{ fm})$ size with sub-fermi structures...

one intriguing question: how/when quantum mechanics (wave physics) enters - e.g., cannot localize both in $x$ and $p$

$\Rightarrow$ inherent momentum anisotropies e.g. DM, Wang & Greene 1404.4119

Also, if these systems are opaque enough for hydro, there should be energy loss (jet quenching) signatures. E.g., $p+Pb$ @ 5.02 TeV (top 3.4% cent):

DM & Sun at QM2015