Constraining effective actions via scattering amplitudes

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with

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Outline

1) Introduction

2) Supersymmetry constraints on the effective action of N=4 SYM in the Coulomb branch

3) Constraints from breaking conformal symmetry in the form of soft theorems

4) Scale invariance vs conformal symmetry

5) Conclusions and remarks

Introduction

Requirement of having a consistent S-matrix can impose highly non-trivial constraints

We apply these ideas to effective theories

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can impose highly non-trivial constraints

We apply these ideas to effective theories

four-derivative terms should have positive coefficients
Allan, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi (2006)

$$\mathcal{L}_{\text{eff}} = -\tfrac{1}{2}\partial^\mu\phi\partial_\mu\phi + \tfrac{c}{\Lambda^4}(\partial^\mu\phi\partial_\mu\phi)^2 + \ldots$$

Argument

$$\frac{c}{\Lambda^4} = \frac{2}{\pi}\int_{s_0}^{\infty} ds\frac{\mathrm{Im}\mathcal{A}_4(s,0)}{s^3} = \frac{2}{\pi}\int_{s_0}^{\infty} ds\frac{s\sigma(s)}{s^3} > 0$$

Introduction

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Positivity for the proof of the a-theorem in 4D

Komargodski, Schwimmer (2008)

$$c = \Delta a = a_{\rm UV} - a_{\rm IR} > 0$$

Effective action of N=4 SYM in the Coulomb branch

The Coulomb branch

$$\langle 0|\varphi|0\rangle = v$$

Spontaneous gauge symmetry breaking

$$U(N+1) \to U(N) \times U(1)$$

We focus on the U(1) part

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Expanding in large mw, the effective action becomes

$$S_{\mathrm{eff}} = -\frac{1}{4}F^2 + \frac{c_4^{(0)}}{m_w^4}F^4 + \frac{c_6^{(0)}}{m_w^8}F^6 + \frac{c_4^{(2)}}{m_w^8}\partial^4 F^4 + \frac{c_4^{(3)}}{m_w^{10}}\partial^6 F^4 + \dots$$

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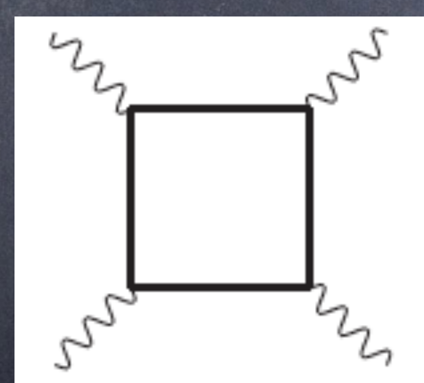

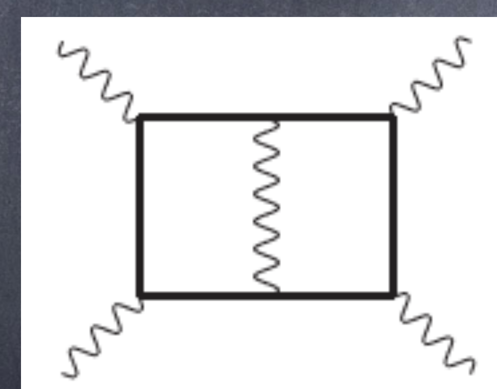

Perturbation contributions

$$A_4^{1-\text{loop}} = F^4 \int \frac{d^4\ell}{(2\pi)^4} \frac{g^4 N}{(\ell^2 + m_w^2)((\ell+p_1)^2 + m_w^2)((\ell+p_{12})^2 + m_w^2)((\ell-p_4)^2 + m_w^2)}$$

Effective action of N=4 SYM in the Coulomb branch

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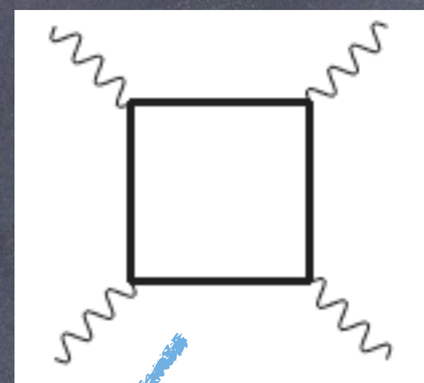

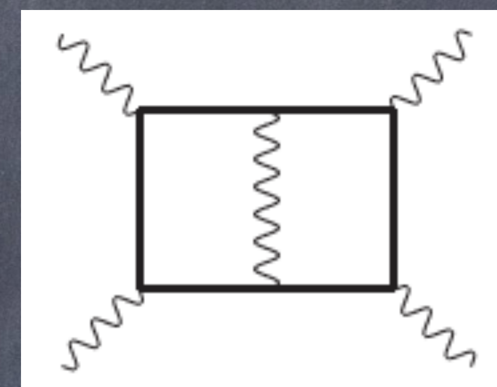

Perturbation contributions

Bianchi, Morales, Wen (2015)

$$\partial^k F^4 : \quad \frac{g^4 N}{(4\pi)^2 m^4}\left[\frac{1}{2} + \frac{1}{240 m^4}(s^2+t^2+u^2) + \cdots + \right.$$
$$\left. + \frac{1}{192192 m^{16}}\left((s^6+t^6+u^6) - \frac{7}{75}(s^2\, t^2\, u^2)\right)\right] + \cdots$$

Effective action of N=4 SYM in the Coulomb branch

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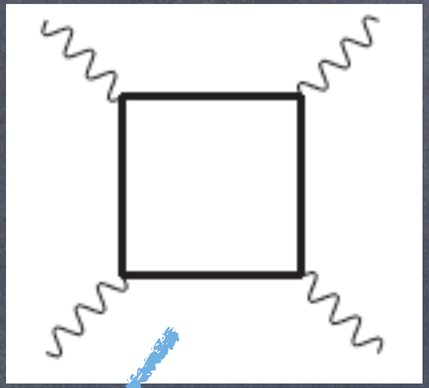

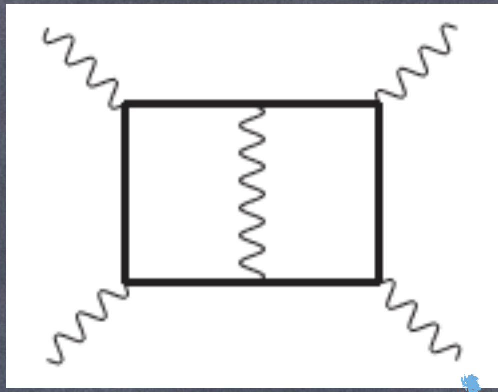

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$$\partial^k F^4 : \quad \frac{g^6 N(N+1)}{(4\pi)^4 m^4}\left[0 + \frac{1}{24 m^4}(s^2+t^2+u^2) + \cdots + \right. \\ \left. + \frac{7541}{59875200 m^{16}}\left((s^6+t^6+u^6) - \frac{249}{7541}(s^2 t^2 u^2)\right)\right] + \cdots$$

Effective action of N=4 SYM in the Coulomb branch

1-instanton effective action for Sp(2N) gauge theory

$$S_{\text{eff}}^{1-inst} = c' \, \frac{g^4}{\pi^6} \, e^{2\pi i\tau} \int \frac{d^4x \, d^8\theta \, \sqrt{\det_{4N} 2\bar{\Phi}_{Au,Bv}}}{\sqrt{\det_{2N}\left(\Phi^{AB}\bar{\Phi}_{AB} + \frac{1}{g}\bar{\mathcal{F}} + \frac{1}{\sqrt{2}g}\bar{\Lambda}_A(\Phi^{-1})^{AB}\bar{\Lambda}_B\right)_{\dot{\alpha}u,\dot{\beta}v}}}$$

Bianchi, Morales, Wen (2015)

Integrating out the Grassmann variables and expanding in 1/v

$$S_{\text{eff}}^{1-inst} = e^{2\pi i\tau} \sum_{\ell} \left(0 \times (F_-^2 F_+^{2\ell} + F_+^2 F_-^{2\ell}) + c_1 \, F_-^4 F_+^{2\ell} + c_2 \, \partial^4 F_-^2 F_+^{2\ell} + \dots \right)$$

Suggesting a non-renormalization theorem for the operator

$$F_-^2 \, F_+^{2l}$$

Effective action of N=4 SYM in the Coulomb branch

This is a known non-renormalization theorem that F^4 is one-loop exact in N=4 SYM

Dine, Seiberg (1997)

F^6 was found it is not generated at one-loop, but it appears at two-loop, and it was conjectured to be two-loop exact

Buchbinder, Petrov, Tseytlin (2001)

Indeed, exploiting the N=4 Supersymmetry it has been shown that the MHV operators

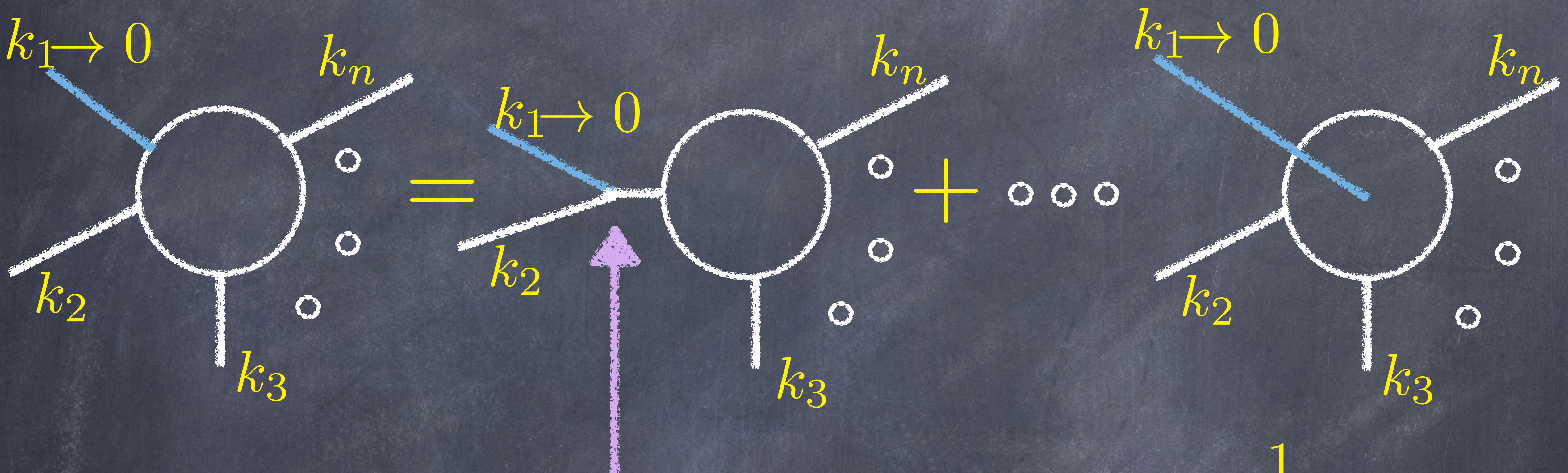
$$F_-^2 F_+^{2l}$$

is l-loop exact

Chen, Hunag, Wen (2015)

Soft theorems: QFT

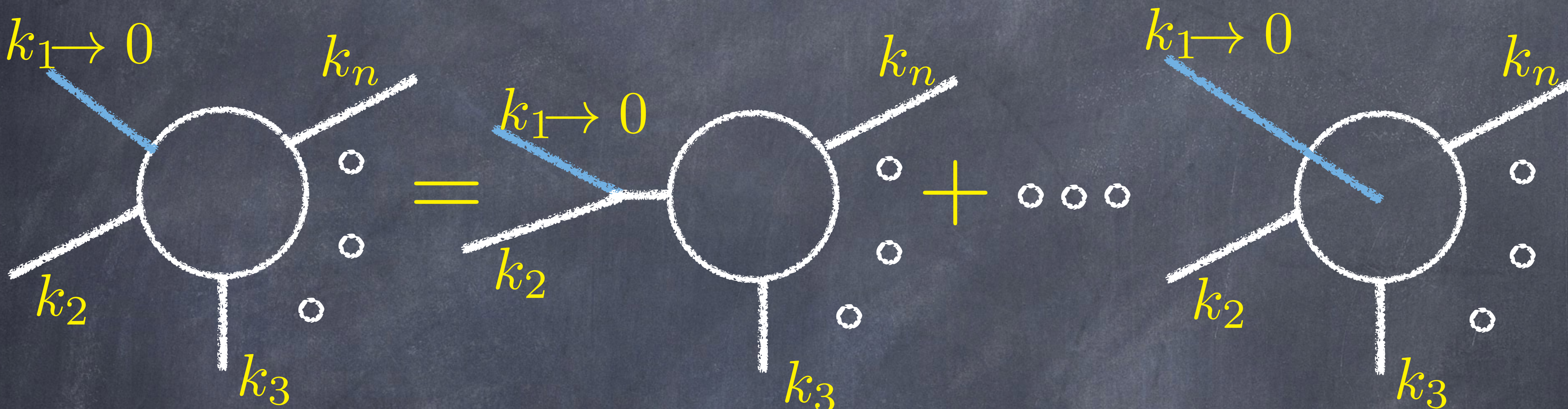
Leading order



$$vertex \times propagator = 2a_1(k_2 - k_1) \times \frac{1}{(k_1 + k_2) - m^2}$$

Soft theorems: QFT

Leading order



$$vertex \times propagator = 2a_1(k_2 - \cancel{k_1}) \times \frac{1}{(k_1 + k_2) - m^2}$$

no dependence
on k soft

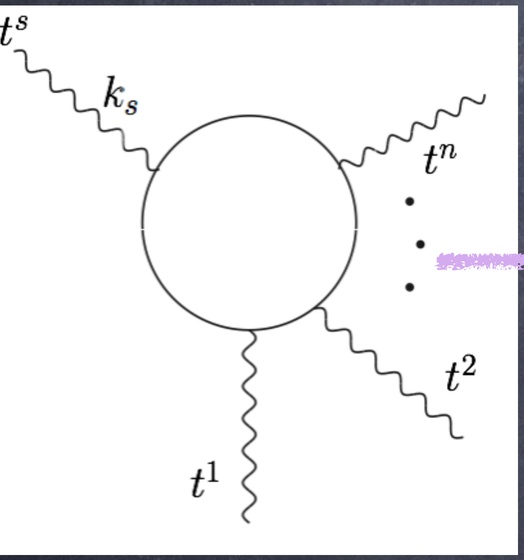
order 1/k1

The vertex interaction cannot be as general as possible!

Gauge theories: soft theorems

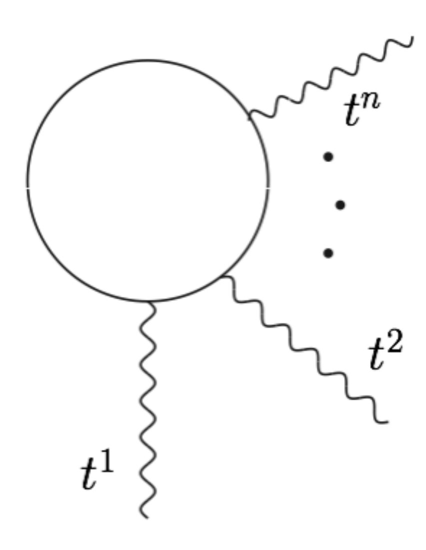
Low (1954)

1) Color-ordered gluon amplitudes in quantum field theory



$$k_s = \delta\,\hat{k}_s,\ \delta \to 0$$

$$\frac{\langle n1\rangle}{\langle ns\rangle\langle s2\rangle}\left(1+\frac{\langle sn\rangle}{\langle 1n\rangle}\bar{u}_s\frac{\partial}{\partial\bar{u}_1}+\frac{\langle s1\rangle}{\langle n1\rangle}\bar{u}_s\frac{\partial}{\partial\bar{u}_n}\right)$$



$\frac{1}{k_s}$ Leading order

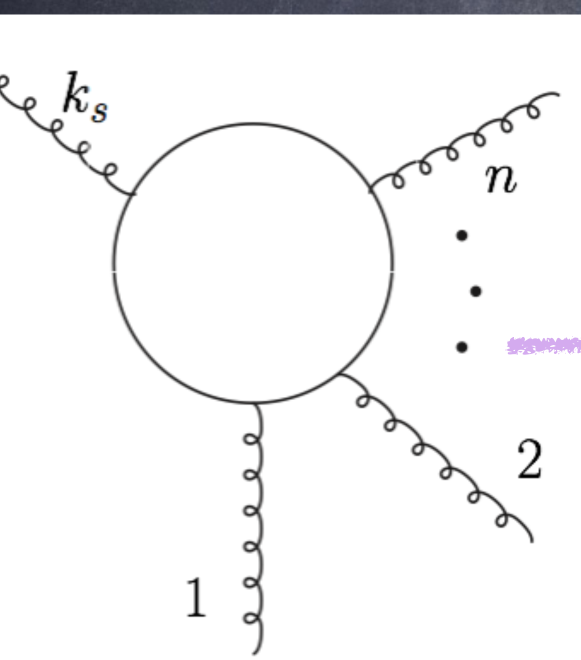
k_s^0 Sub-leading order

Gravity: soft theorems

Gross-Jackiw, Weinberg (1965)

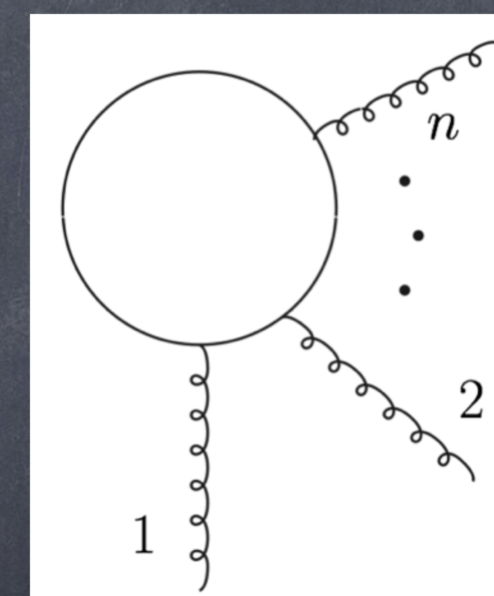
2) Graviton amplitudes in quantum field theory

$$k_s = \delta\,\hat{k}_s,\ \delta \to 0$$



$$\sum_{i\neq s}\left[\frac{k_i\cdot h_s\cdot k_i}{k_s\cdot k_i} + 2\frac{k_i\cdot h_s\cdot J_i\cdot k_s}{k_i\cdot k_s} + \boxed{\frac{k_s\cdot J_i\cdot h_s\cdot J_i\cdot k_s}{k_i\cdot k_s}}\right]$$

$$J_i = k_i^{[\mu}\frac{\partial}{\partial k_{i\nu]}} + 2h_i^{\lambda[\mu}\frac{\partial}{\partial h_{i\nu]\lambda}}$$



Universal Sub-sub-leading order

$$J_{i,\mu\nu} = k_{i[\mu}\frac{\partial}{\partial k_i^{\nu]}} + 2h_{i,\lambda[\mu}\frac{\partial}{\partial h_{i,\lambda}^{\nu]}}$$

F. Cachazo, A. Strominger
arXiv:1404.4091

Soft theorems for N=4 SYM in the Coulomb branch

We focus on the scalar sector

The 6 massless scalars coincide with the 6 Goldstone bosons

$\varphi, \phi^I \qquad I = 1, ..., 5$

Conformal symmetry breaking

R-symmetry breaking

$SO(6) \to SO(5)$

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$SO(6) \rightarrow SO(5)$

Due to the (broken) symmetries, amplitudes must satisfy

$$A_n(\phi_1, \cdots, \phi_{n-1}, \pi_n)|_{p_n \to 0} = \sum_{i=1}^{n-1} \langle \phi_1, \cdots, \delta\phi_i, \cdots, \phi_{n-1} \rangle|_{\mathrm{LSZ}}$$

Infinitesimal transformation of the hard particle

Soft theorems from spontaneously breaking of conformal and R-symmetry

Conformal symmetry breaking

Amplitudes with a soft dilation

$$\delta\phi = [\mathcal{D},\phi] = i(d + x^\mu\partial_\mu)\phi\,,$$
$$\delta_\mu\phi = [\mathcal{K}_\mu,\phi] = i\left((2x_\mu x_\nu - \eta_{\mu\nu}x^2)\partial^\nu + 2d\,x_\mu\right)\phi$$

These transformations lead to the soft-dilaton theorem

$$v\,A_n\Big|_{p_n\to\tau p_n} \to \left(\cdots + \mathcal{S}_n^{(0)} + \tau\mathcal{S}_n^{(1)}\right)A_{n-1} + \mathcal{O}(\tau^2)$$

Di Vecchia, Marotta, Mojaza, Noble (2015)

$$\mathcal{S}_n^{(0)} = \sum_{i=1}^{n-1}\left(p_i\cdot\frac{\partial}{\partial p_i} + \frac{d-2}{2}\right) - d \quad \text{From } \mathcal{D}$$
$$\mathcal{S}_n^{(1)} = p_n^\mu\sum_{i=1}^{n-1}\left[\frac{1}{2}\left(2\,p_i^\nu\frac{\partial^2}{\partial p_i^\nu\partial p_i^\mu} - p_{i\mu}\frac{\partial^2}{\partial p_{i_\nu}\partial p_i^\nu}\right) + \frac{d-2}{2}\frac{\partial}{\partial p_i^\mu}\right] \quad \text{From } \mathcal{K}_\mu$$

Soft theorems from spontaneously breaking of conformal and R-symmetry

R-symmetry breaking

$$A_n(\phi_1, \cdots, \phi_n^I)\big|_{p_n \to 0} = \sum_i A_{n-1}(\cdots, \delta_I \phi_i, \cdots) + \mathcal{O}(p_n^1)$$

Adler's zero is a particular case

In this case the hard particles are rotated by the broken generators leading to a non-zero result

We verified soft theorems by explicit computations of 1-loop amplitudes in the Coulomb branch up to 6-points and of instanton amplitudes

Soft theorems: explicit verification at 1-loop

one-loop: integrands from 10-D SYM

or 6D SYM

Mafra, Schlotterer (2014)

Brandhuber, Korres, Koschade, Travaglini (2010)

Bianchi, A.L.G. Huang, Lee, Wen (2016)

$$\mathcal{L}^{\mathrm{SU}(4)}_{1-\mathrm{loop}} = \frac{g^4 N}{32 m^4 \pi^2}\left(\mathcal{O}_{F^4} + \frac{\mathcal{O}_{D^4F^4}}{2^3\times 15 m^4} - \frac{2\mathcal{O}_{D^2F^6}}{15 m^6} + \frac{\mathcal{O}_{D^4F^6}}{2^3\times 21 m^8} - \frac{\mathcal{O}_{D^6F^6}}{2\times 15^2 m^{10}} + \ldots\right)$$

$$\mathcal{L}^{\mathrm{Sp}(4)}_{1-\mathrm{loop}} = \frac{g^4 N}{4\pi^2 m^4}\left[\frac{\partial^4\varphi^4}{4} + \frac{\partial^8\varphi^4}{240 m^4} + \frac{\partial^4\varphi^5}{m^2} + \frac{\partial^8\varphi^5}{120 m^6} - \frac{5\partial^4\varphi^6}{m^2} - \frac{\partial^8\varphi^6}{120 m^6} + \frac{\partial^{10}\varphi^6}{2^8 3^5 m^8} + \frac{\partial^{12}\varphi^6}{2^9 3^2 m^{10}} + \frac{\partial^4\varphi^2\phi^2}{2} - \frac{5\partial^4\varphi^2\phi^4}{m^2} + \frac{\partial^4\varphi^4\phi^2}{m^2}\right] + \ldots$$

All soft theorems satisfied

Soft theorems: explicit verification at 1-instanton

one-instanton: effective action for Sp(2N) gauge fields

Bianchi, Morales, Wen (2015)

$$S^{1-inst}_{\rm eff} = c' \, \frac{g^4}{\pi^6} \, e^{2\pi i \tau} \int \frac{d^4x \, d^8\theta \, \sqrt{\det_{4N} 2\bar{\Phi}_{Au,Bv}}}{\sqrt{\det_{2N}\left(\Phi^{AB}\bar{\Phi}_{AB} + \frac{1}{g}\bar{\mathcal{F}} + \frac{1}{\sqrt{2}g}\bar{\Lambda}_A(\Phi^{-1})^{AB}\bar{\Lambda}_B\right)_{\dot{\alpha}u,\dot{\beta}v}}}$$

Turning on just the scalars, the super fields become

$$\bar{\Phi}_{AB} = \bar{\phi}_{AB}\,, \quad \bar{\Lambda}_{A\dot{\alpha}} = i\,\theta^{B\alpha}\partial_{\alpha\dot{\alpha}}\bar{\phi}_{AB}\,, \quad \bar{\mathcal{F}}_{\dot{\alpha}\dot{\beta}} = \frac{1}{2}\theta^{A\alpha}\theta^{B\beta}\partial_{\alpha\dot{\alpha}}\partial_{\beta\dot{\beta}}\bar{\phi}_{AB}$$

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The final effective action produces interactions with at most 8-derivatives (dimension s^4)

$$S_{\text{eff}}^{1-inst} = c'\frac{g^4}{\pi^6}e^{2\pi i\tau}\int d^4x\, d^8\theta\,\frac{1}{1-H_{\dot{\alpha}\dot{\beta}}H^{\dot{\alpha}\dot{\beta}}} = c'\frac{g^4}{\pi^6}e^{2\pi i\tau}\int d^4x\, d^8\theta\,(H_{\dot{\alpha}\dot{\beta}}H^{\dot{\alpha}\dot{\beta}})^2$$

$$H_{\dot{\alpha}\dot{\beta}} = \frac{1}{4\,g\,\phi^2}\left(\frac{1}{2}\partial_{\alpha\dot{\alpha}}\partial_{\beta\dot{\beta}}\phi_{AB} - \frac{\phi^{CD}\partial_{\alpha\dot{\alpha}}\phi_{AC}\partial_{\beta\dot{\beta}}\phi_{DB}}{\phi^2}\right)\theta^{A\alpha}\theta^{B\beta}$$

Soft theorems: explicit verification at 1-instanton

one-instanton: effective action for Sp(2N) gauge fields

Bianchi, Morales, Wen (2015)

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The final effective action produces interactions with at most 8-derivatives (dimension s^4)

$$\mathcal{S}_{\rm dilaton} = \int d^4x\,\left[(S_{\mu\nu}S^{\mu\nu})^2 - S_{\mu\nu}S^{\nu\rho}S_{\rho\sigma}S^{\sigma\mu}\right]$$

$$S_{\mu\nu} = \frac{\partial_\mu\partial_\nu\varphi}{\varphi^2} - 2\frac{\partial_\mu\varphi\partial_\nu\varphi}{\varphi^3}$$

Traceless

Soft theorems: explicit verification at 1-instanton

one-instanton: effective action for Sp(2N) gauge fields

Bianchi, A.L.G. Huang, Lee, Wen (2016)

$$v^8 \mathcal{A}_4^{inst} = \frac{1}{32}\left(s_{12}^2 + \mathcal{P}_4\right)^2 , \quad v^9 \mathcal{A}_5^{inst} = -\frac{1}{36}\left(s_{12}^2 + \mathcal{P}_5\right)^2 ,$$

$$v^{10} \mathcal{A}_6^{inst} = -\frac{2}{3} s_{12}^4 - 6 s_{12}^2 s_{23}^2 + \frac{17}{18} s_{123}^4 + \frac{15}{2} s_{123}^2 s_{124}^2 + \mathcal{P}_6 ,$$

$$v^{11} \mathcal{A}_7^{inst} = 4 s_{12}^4 + 40 s_{12}^2 s_{23}^2 - \frac{5}{3} s_{123}^4 - 25 s_{123}^2 s_{124}^2 + \mathcal{P}_7 ,$$

$$v^{12} \mathcal{A}_8^{inst} = -\frac{809}{144} s_{12}^4 - \frac{395}{8} s_{12}^2 s_{13}^2 + \frac{1339}{576} s_{123}^4 + \frac{595}{32} s_{123}^2 s_{124}^2 + \frac{535}{32} s_{123}^2 s_{145}^2 + \mathcal{P}_8$$

$$v^{13} \mathcal{A}_9^{inst} = \frac{3935}{294} s_{12}^4 + \frac{846}{7} s_{12}^2 s_{13}^2 - \frac{475}{126} s_{123}^4 - \frac{491}{14} s_{123}^2 s_{124}^2 - \frac{535}{14} s_{123}^2 s_{145}^2 + \mathcal{P}_9 .$$

All soft theorems satisfied up to the 9-point amplitude

Systematics of soft theorems: recursion relations

How to utilize soft-theorems systematically?

Recursion relations

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Recursion relations

Standard BCFW cannot apply here!

Soft-BCFW

Cheung, Kampf, Novotny, Sheng, Trnka (2015)

$$p_{\hat{i}} = (1 - a_i z)\, p_i \qquad \sum_{i=1}^{n} a_i\, p_i = 0$$

Soft limit for the particle i for $z \to \dfrac{1}{a_i}$

Systematics of soft theorems: recursion relations

How to utilize soft-theorems systematically?

Recursion relations

Standard BCFW cannot apply here!

Soft-BCFW

Cheung, Kampf, Novotny, Sheng, Trnka (2015)

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Soft limit for the particle i for $z \to \frac{1}{a_i}$

The amplitude can be expressed as a contour integral

$$A_n = A_n(0) = \frac{1}{2\pi i} \oint_{z=0} \frac{dz}{z} \frac{A_n(z)}{F_n^{(\sigma)}(z)}$$

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$$F_n^{(\sigma)}(z) = \prod_{i=1}^{n}(1-a_i z)^{\sigma}$$

An additional function is introduced such that

for large z $\frac{A_n(z)}{F_n^{(\sigma)}(z)} \sim 0$

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This construction useful only if we know the residues of the additional poles

That's the input of the soft-theorem

Systematics of soft theorems: recursion relations

$$F_n^{(\sigma)}(z) = \prod_{i=1}^{n}(1 - a_i z)^{\sigma}$$

Sigma is determined by the order that amplitudes have universal soft behavior

For broken conformal symmetry sigma=2

$$F_n^{(2)}(z) = \prod_{i=1}^{n}(1 - a_i z)^2 \sim z^{2n}$$

Moreover

$$A_n(z) \sim z^m, \quad z \to \infty \qquad \frac{A_n(z)}{F_n^{(\sigma)}(z)} \to z^{m-n\sigma}$$

$$\downarrow$$

$$m - n\sigma < 0$$

Relation between the degree of the amplitude and the number of points

Systematics of soft theorems: recursion relations

For order s^k amplitudes with n-dilatons the recursion is valid

$$n > k$$

In other words, knowing the k-point amplitude at order s^k for k ≤ n allows us to construct all the amplitudes up to order s^n

s^k \ # of points	4	5	6	7	$\cdots$
2	×	✓	✓	✓	✓
3	×	✓	✓	✓	✓
4	×	✓	✓	✓	✓
5	✓	×	✓	✓	✓
6	✓	✓	×	✓	✓
7	✓	✓	✓	×	✓
$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

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In other words, knowing the k-point amplitude at order s^k for k ≤ n allows us to construct all the amplitudes up to order s^n

s^k \ # of points	4	5	6	7	$\cdots$
2	×	✓	✓	✓	✓
3	×	✓	✓	✓	✓
4	×	✓	✓	✓	✓
5	✓	×	✓	✓	✓
6	✓	✓	×	✓	✓
7	✓	✓	✓	×	✓
$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

Computed by other means

Single Soft

Soft-BCFW

Systematics of soft theorems: recursion relations

Example: If each scalar mostly carries one-derivative, then amplitudes at 2n, (2n+1) points mostly go as s^n

s^n \ # of points	4	5	6	7	8	$\cdots$
2	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
3	0	0	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
4	0	0	0	0	$\checkmark$	$\checkmark$
5	0	0	0	0	0	$\checkmark$
$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

Everything determined by the coefficient of the 4-point function at order s^2

For N=4 SYM, uniquely determined to be the coefficient from conformal DBI

Scale invariance vs Conformal symmetry

Scale vs Conformal in QFT

Scale invariance vs Conformal symmetry

Scale vs Conformal in QFT

Amplitudes language: to what extent does the sub-leading soft theorem due to conformal boosts follow from the leading behavior stemming from dilatation symmetry?

Scale invariance vs Conformal symmetry

Scale vs Conformal in QFT

Amplitudes language: to what extent does the sub-leading soft theorem due to conformal boosts follow from the leading behavior stemming from dilatation symmetry?

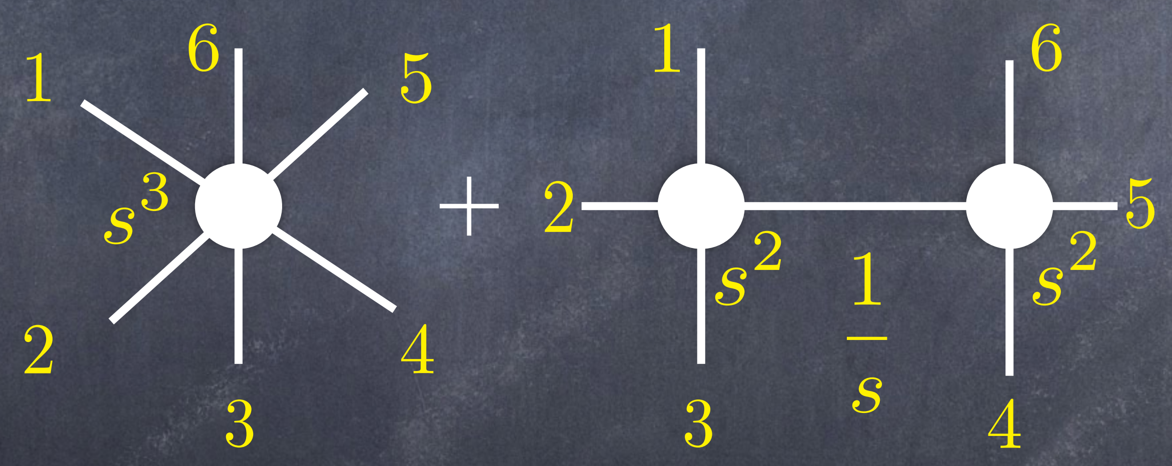
Any 5-point amplitude constrained by the leading soft theorem automatically satisfy the sub-leading soft theorem (verified up to s^11)

Scale invariance vs Conformal symmetry

A seven-point example

Using both leading and sub-leading soft theorems

$$
\begin{aligned}
A_6^{(3)} &= -c_5^{(3)}(s_{12}^3+\mathcal{P}_6) - \left(\frac{c_5^{(3)}}{2}+(c_4^{(2)})^2\right)(s_{123}^3+\mathcal{P}_6) \\
&+ (c_4^{(2)})^2\left((s_{12}^2+s_{13}^2+s_{23}^2)\frac{1}{s_{123}}(s_{45}^2+s_{46}^2+s_{56}^2)+\mathcal{P}_6\right)
\end{aligned}
$$



Scale invariance vs Conformal symmetry

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\begin{aligned}
A_6^{(3)} &= -c_5^{(3)}(s_{12}^3+\mathcal{P}_6) - \left(\frac{c_5^{(3)}}{2} + (c_4^{(2)})^2\right)(s_{123}^3+\mathcal{P}_6) \\
&+ (c_4^{(2)})^2\left((s_{12}^2+s_{13}^2+s_{23}^2)\frac{1}{s_{123}}(s_{45}^2+s_{46}^2+s_{56}^2)+\mathcal{P}_6\right)
\end{aligned}
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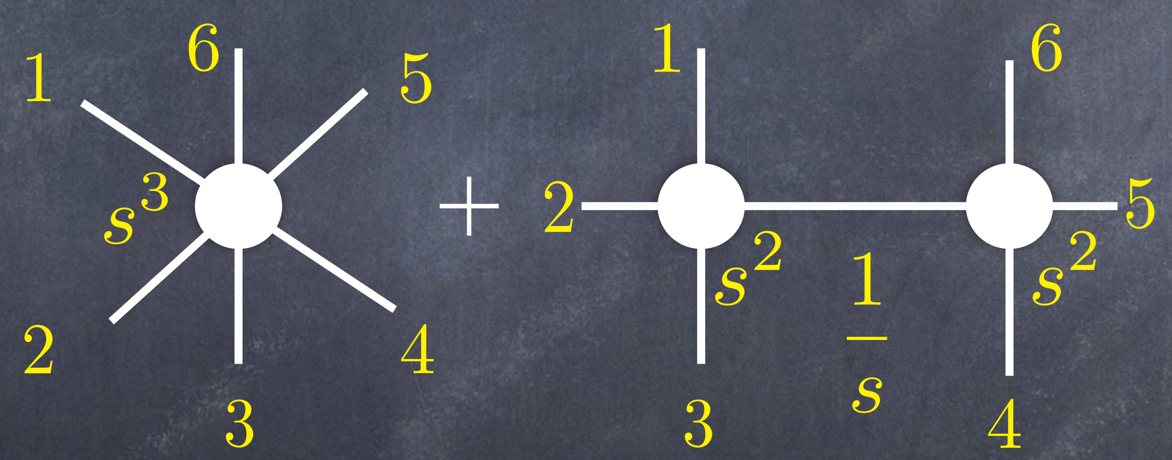


Leading soft alone allows us to determine the seven-point amplitude

$$
A_7^{(3)} = c_5^{(3)}(s_{12}^3+\mathcal{P}_7) + \left(c_5^{(3)} + 3(c_4^{(2)})^2\right)(s_{123}^3+\mathcal{P}_7) - (c_4^{(2)})^2 A_7^{\rm fac}
$$

Satisfy the sub-leading soft theorem automatically

Scale invariance vs Conformal symmetry

The s^4, s^5 case without factorization diagrams

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
4	1	0	1	1	1	1	2	1	2	2	2	2	3	2	3
5	1	0	1	1	2	2	5	4	8	9	13	15	23	24	34
6	1	0	1	2	4	6	13	19	36	58	97	149	244	364	558
7	1	0	1	2	4	8	20	36	83	169	344	680	1342	2518	4695
8	1	0	1	2	5	10	28	59	152	364	885	2093	4930	11199	25021
9	1	0	1	2	5	10	31	72	205	557	1565	4321	11942	32131	84927
10	1	0	1	2	5	11	33	81	246	722	2222	6875	21497	66299	202179

energy degree

Number of points

s^4: 9-points amplitude determined by just the leading soft

s^5: 11-points amplitude determined by just the leading soft

Conjecture: amplitudes with n>k determined only by the leading soft

Conclusions and remarks

The requirement of having a consistent S-matrix can impose highly non-trivial constraints on the theory

Both SUSY and soft theorems can strongly constrain the effective action of N=4 SYM in the Coulomb branch, and lead to new non-renormalization theorems

We observe amplitudes determined by leading soft automatically satisfy the sub-leading soft theorem

Explore other possible constraints such as SUSY at higher points as well as SL(2,Z) symmetry

Backup-slides

Spinor helicity formalism

Scattering amplitudes computed in QFT involve momenta and polarizations

$$\{k_1^{\mu_1}, k_2^{\mu_2}, \ldots, k_n^{\mu_n}\}$$

$$\{\epsilon_1^{\mu}, u_2, \bar{v}_3, \ldots, \epsilon_n^{\rho\sigma}\}$$

For massless particles

$$k^2 = 0$$

$$k_{\alpha\dot{\alpha}} = \sigma^{\mu}_{\alpha\dot{\alpha}} k_{\mu} = u_{\alpha}\bar{u}_{\dot{\alpha}}$$

Left/Right handed spinors

Spinor helicity formalism

For massless particles

$$k^2 = 0$$

$$k_{\alpha\dot{\alpha}} = \sigma^{\mu}_{\alpha\dot{\alpha}} k_{\mu} = u_{\alpha} \bar{u}_{\dot{\alpha}}$$

Example:

$$2k_1 \cdot k_2 = u_1^{\alpha} \varepsilon_{\alpha\beta} v_2^{\beta} \, \bar{u}_1^{\dot{\alpha}} \varepsilon_{\dot{\alpha}\dot{\beta}} \bar{v}_2^{\dot{\beta}}$$

$$|i\rangle = u_i \qquad \langle ij\rangle = u_i^{\alpha} \varepsilon_{\alpha\beta} u_j^{\beta}$$

$$|i] = \bar{u}_i \qquad [ji] = \bar{u}_i^{\dot{\alpha}} \varepsilon_{\dot{\alpha}\dot{\beta}} \bar{u}_j^{\dot{\beta}}$$

$$2k_1 k_2 = \langle 12\rangle [21]$$

Spinor helicity formalism

For polarizations

$$a^- \to \frac{|k\rangle[q|}{[kq]} \qquad a^+ \to \frac{|q\rangle[k|}{\langle kq\rangle} \qquad h^{--} \to \frac{|k\rangle[q||k\rangle[q|}{[kq]^2}$$

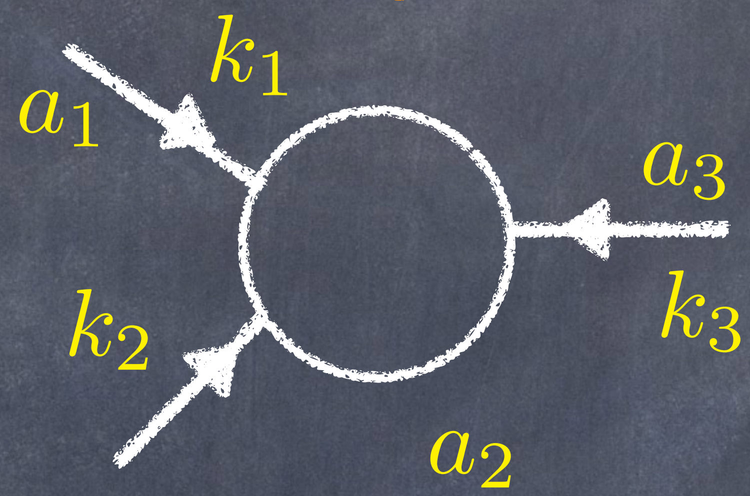
q is a reference momentum reminding the gauge dependence

$$a^\mu \to a^\mu + \alpha k^\mu$$

Every massless amplitude can be expressed using just helicity spinors

Helicity amplitudes

Ex: 3-gluons vertex

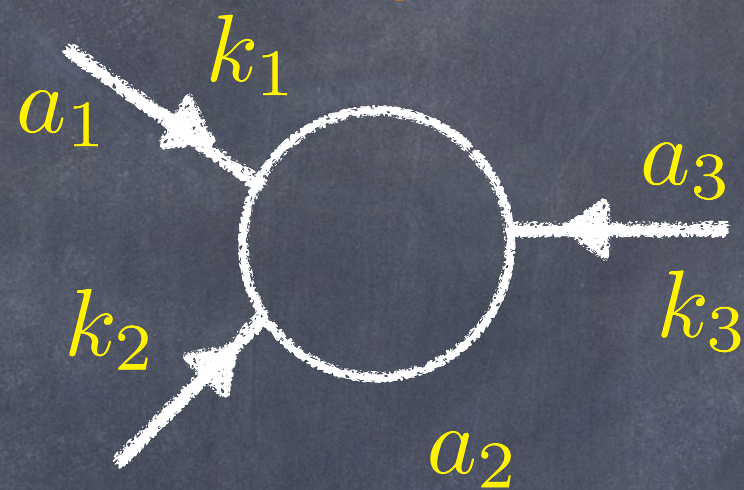


$\propto f^{c_1 c_2 c_3}$

$$a_1 a_2\, a_3(k_1 - k_2) + a_2 a_3\, a_1(k_3 - k_1) + a_3 a_1\, a_2(k_3 - k_1)$$

Helicity amplitudes

Ex: 3-gluons vertex



$$\propto f^{c_1 c_2 c_3}$$

$$a_1 a_2\, a_3(k_1 - k_2) + a_2 a_3\, a_1(k_3 - k_1) + a_3 a_1\, a_2(k_3 - k_1)$$

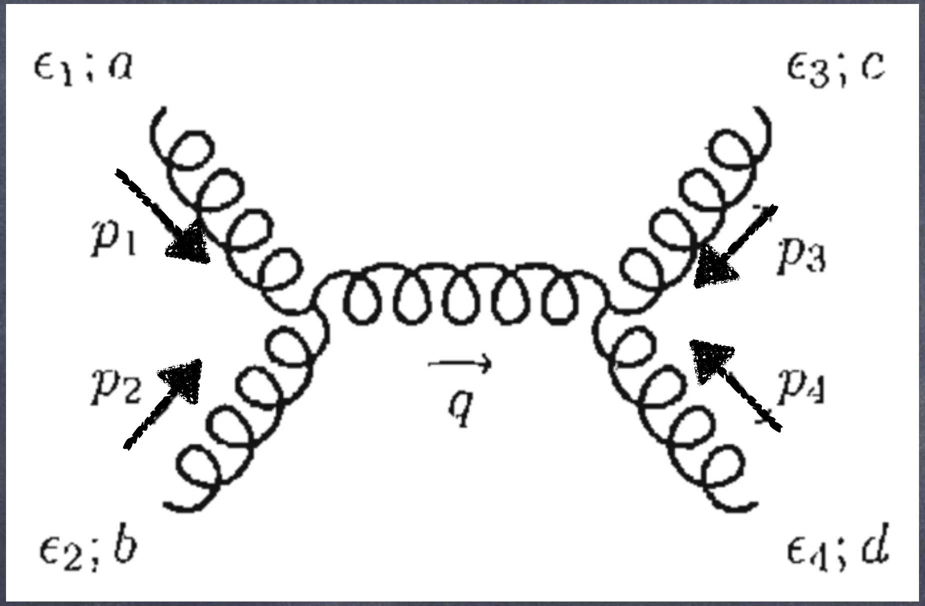
$$\mathcal{A}_3(1^- 2^- 3^-) = 0$$

$$\mathcal{A}_3(1^- 2^- 3^+) \propto \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle}$$

Helicity amplitudes

Helicity amplitudes show properties of scattering amplitudes hidden in the QFT formalism

Ex: 4-gluons amplitude



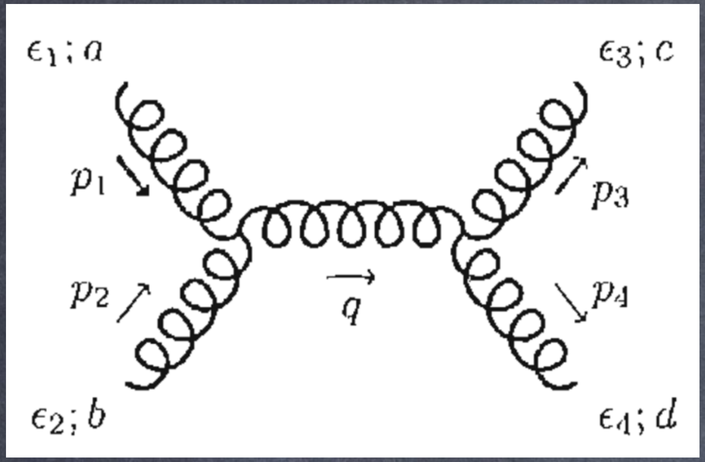
$$\mathcal{A}(1^+2^+3^+4^+) = 0$$

$$\mathcal{A}(1^-2^+3^+4^+) = 0$$

$$\mathcal{A}(1^-2^-3^+4^+) \neq 0$$

Parke-Taylor: gg -> gg scattering

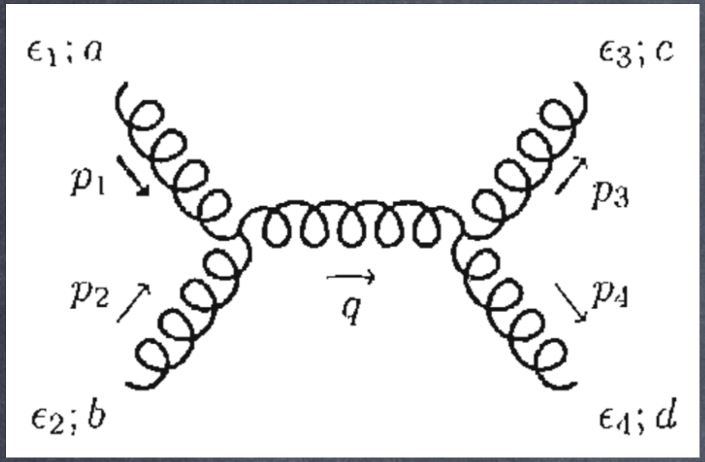
Color ordered amplitude



$$A_s(1234) = f^{c_1 c_2 x} f^{c_3 c_4 x} \widetilde{A}_s(1234)$$

Parke-Taylor: gg -> gg scattering

Color ordered amplitude



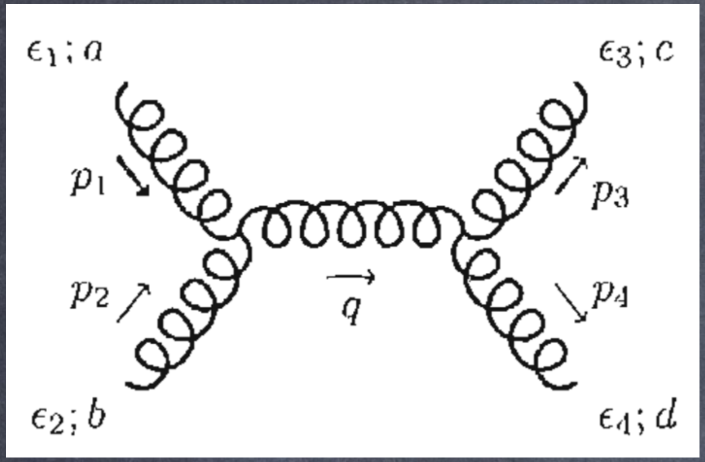
$$\mathcal{A}_s(1234) = f^{c_1 c_2 x} f^{c_3 c_4 x} \widetilde{\mathcal{A}}_s(1234)$$

$$\mathcal{A}(1234) = \sum_{\sigma \in S^4/\mathbb{Z}_4} \mathrm{tr}(\sigma(1)\sigma(2)\sigma(3)\sigma(4)) \widetilde{\mathcal{A}}(\sigma(1)\sigma(2)\sigma(3)\sigma(4))$$

$$= 2 \times (\mathrm{tr}(1234)\widetilde{\mathcal{A}}(1234) + \mathrm{tr}(1324)\widetilde{\mathcal{A}}(1324) + \mathrm{tr}(1243)\widetilde{\mathcal{A}}(1243))$$

Parke-Taylor: gg -> gg scattering

Color ordered amplitude



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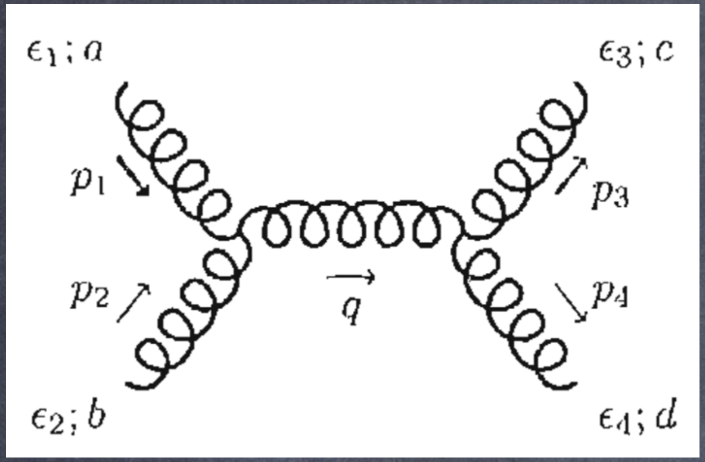
$$= 2 \times (\mathrm{tr}(1234)\widetilde{\mathcal{A}}(1234) + \mathrm{tr}(1324)\widetilde{\mathcal{A}}(1324) + \mathrm{tr}(1243)\widetilde{\mathcal{A}}(1243))$$

Parke-Taylor

$$\boxed{\widetilde{\mathcal{A}}(1^- 2^- 3^+ 4^+) = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle}}$$

Parke-Taylor: gg -> gg scattering

Color ordered amplitude



$$\mathcal{A}_s(1234) = f^{c_1 c_2 x} f^{c_3 c_4 x} \widetilde{\mathcal{A}}_s(1234)$$

$$\mathcal{A}(1234) = \sum_{\sigma \in S^4/\mathbb{Z}_4} \mathrm{tr}(\sigma(1)\sigma(2)\sigma(3)\sigma(4)) \widetilde{\mathcal{A}}(\sigma(1)\sigma(2)\sigma(3)\sigma(4))$$

$$= 2 \times (\mathrm{tr}(1234)\widetilde{\mathcal{A}}(1234) + \mathrm{tr}(1324)\widetilde{\mathcal{A}}(1324) + \mathrm{tr}(1243)\widetilde{\mathcal{A}}(1243))$$

Parke-Taylor for MHV amps

$$\boxed{\widetilde{\mathcal{A}}_n(1^-2^-3^+ \ldots n^+) = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle (n-1)n \rangle \langle n1 \rangle}}$$

BCJ relations: gravity=(YM)^2

3 gravitons vertex

$$\mathcal{M}_3(1^{--}2^{--}3^{++}) = (\mathcal{A}_3(1^{-}2^{-}3^{+}))^2 = \frac{\langle 12\rangle^8}{\langle 12\rangle^2\langle 23\rangle^2\langle 31\rangle^2}$$

BCJ double copy relations

$$\mathcal{A}_n = \sum_{i\in\text{cubic}} \frac{n_i c_i}{\prod_{\alpha_i} p_{\alpha_i}^2} \longrightarrow \mathcal{M}_n = \sum_{i\in\text{cubic}} \frac{n_i^2}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

Similar relations for the integrands at loop level!