The Tsallis Distribution at the LHC

J. Cleymans
University of Cape Town, South Africa

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Work done in collaboration with
M.D. Azmi
Alexandru Parvan
Oleg Teryaev

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NRF National Research Foundation
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Outline

What is being done?

STAR, PHENIX, ALICE, CMS and ATLAS use:

$$\frac{\mathrm{d}^2 N}{\mathrm{d}p_\mathrm{T}\mathrm{d}y} = p_\mathrm{T} \times \frac{\mathrm{d}N}{\mathrm{d}y}\frac{(n-1)(n-2)}{nT(nT+m_0(n-2))}\left(1+\frac{m_\mathrm{T}-m_0}{nT}\right)^{-n}$$

What is the connection with the Tsallis distribution?

Also, the physical significance of the parameters n and T has never been discussed by STAR, PHENIX, ALICE, ATLAS, CMS.

dN/dy is treated as a free parameter.

$m_0 = m_\pi$ assumed for all tracks.

- STAR Collaboration, B.I.Abelev et al.
  Phys. Rev. C75, 064901 (2007)
- PHENIX Collaboration, A.Adare et al.
  Phys. Rev. C83, 064903 (2011)
- ALICE Collaboration, K.Aamodt et al.
  Eur. Phys. J. C75, (2015) 226,
  Phys. Lett. B 736, 196; 207, 2014.
- CMS Collaboration, V.Khachatryan et al.
  JHEP 02, 041 (2010) & JHEP 05, 064 (2011)
  Eur. Phys. J. C72, 2164, 2012
- ATLAS Collaboration, G.Aad et al.
  New J. Phys. 13 (2011) 053033.

Sometimes ...

$$\left.\frac{d^2N}{dp_T dy}\right|_{y=0} = A\left[1+(q-1)\frac{p_T}{T}\right]^{-\frac{1}{q-1}}$$

A is a normalization factor
T is not related to a temperature in the thermodynamic sense.

G. Wilk, Z. Wlodarczyk, Physica, A413, 33, 2014

History

Possible Generalization of Boltzmann-Gibbs Statistics.

Constantino Tsallis
Rio de janeiro, TBPF
J. Stat. Phys. 52 (1988) 479-487.

Citations: 1389
Citations in HEP: 513

A recent example from outside HEP:

PRL **115,** 238301 (2015) PHYSICAL REVIEW LETTERS week ending 4 DECEMBER 2015

Experimental Validation of a Nonextensive Scaling Law in Confined Granular Media

Gaël Combe,[*] Vincent Richefeu, and Marta Stasiak
Université Grenoble Alpes, 3SR, F-38000 Grenoble, France and CNRS, 3SR, F-38000 Grenoble, France

Allbens P. F. Atman[†]
Departamento de Física e Matemática, National Institute of Science and Technology for Complex Systems, Centro Federal de Educação Tecnológica de Minas Gerais – CEFET-MG, Avenida Amazonas 7675, 30510-000 Belo Horizonte-MG, Brazil

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In this Letter, we address the relationship between the statistical fluctuations of grain displacements for a full quasistatic plane shear experiment, and the corresponding anomalous diffusion exponent α. We experimentally validate a particular case of the Tsallis-Bukman scaling law, $\alpha = 2/(3-q)$, where q is obtained by fitting the probability density function (PDF) of the displacement fluctuations with a q-Gaussian distribution, and the diffusion exponent is measured independently during the experiment. Applying an original technique, we are able to evince a transition from an anomalous diffusion regime to a Brownian behavior as a function of the length of the strain window used to calculate the displacements of the grains. The outstanding conformity of fitting curves to a massive amount of experimental data shows a clear broadening of the fluctuation PDFs as the length of the strain window decreases, and an increment in the value of the diffusion exponent—anomalous diffusion. Regardless of the size of the strain window considered in the measurements, we show that the Tsallis-Bukman scaling law remains valid, which is the first experimental verification of this relationship for a classical system at different diffusion regimes. We also note that the spatial correlations show marked similarities to the turbulence in fluids, a promising indication that this type of analysis can be used to explore the origins of the macroscopic friction in confined granular materials.

Tsallis Thermodynamics

The Tsallis distribution is given by

$$f(E) = \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{1}{q-1}},$$

and the thermodynamic quantities N, E, P, S, ... are integrals over this distribution.

Asymptotically

$$\lim_{E\to\infty} f(E) = \left(\frac{E}{T}\right)^{-\frac{1}{q-1}}$$

Scale is set by T.

Asymptotic behaviour is set by q.

For high energy physics a consistent form of Tsallis thermodynamics for the particle number, energy density and pressure is given by

$$\begin{aligned} N &= gV\int\frac{d^3p}{(2\pi)^3}\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}, \\ \epsilon &= g\int\frac{d^3p}{(2\pi)^3}E\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}, \\ P &= g\int\frac{d^3p}{(2\pi)^3}\frac{p^2}{3E}\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}. \end{aligned}$$

where T and μ are the temperature and the chemical potential, V is the volume and g is the degeneracy factor. This introduces only one new parameter q which for transverse momentum spectra is always close to 1.

Thermodynamic consistency

$$dE = -pdV + TdS + \mu dN$$

Inserting $E = \epsilon V$, $S = sV$ and $N = nV$ leads to

$$d\epsilon = Tds + \mu dn$$

$$dP = nd\mu + sdT$$

In particular

$$n = \left.\frac{\partial P}{\partial \mu}\right|_T, \quad s = \left.\frac{\partial P}{\partial T}\right|_\mu, \quad T = \left.\frac{\partial \epsilon}{\partial s}\right|_n, \quad \mu = \left.\frac{\partial \epsilon}{\partial n}\right|_s.$$

are satisfied.

In the Tsallis distribution the total number of particles is given by:

$$N = gV \int \frac{d^3p}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}} .$$

The corresponding momentum distribution is given by

$$E\frac{dN}{d^3p} = gVE\frac{1}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}} ,$$

which, in terms of the rapidity and transverse mass variables, $E = m_T \cosh y$, becomes (at mid-rapidity $y = 0$ and for $\mu = 0$)

$$\left.\frac{d^2N}{dp_T\, dy}\right|_{y=0} = gV\frac{p_T m_T}{(2\pi)^2} \left[1 + (q-1)\frac{m_T}{T}\right]^{-\frac{q}{q-1}} ,$$

J.C. and D. Worku, J. Phys. **G39** (2012) 025006; arXiv:1203.4343[hep-ph].

At mid-rapidity $y = 0$ and for μ = 0

$$\left.\frac{d^2N}{dp_T\,dy}\right|_{y=0} = \left.\frac{dN}{dy}\right|_{y=0}\frac{p_T m_T}{T}\left[1+(q-1)\frac{m_T}{T}\right]^{-q/(q-1)} \times \frac{(2-q)(3-2q)}{(2-q)m^2+2mT+2T^2} \times \left[1+(q-1)\frac{m}{T}\right]^{1/(q-1)}$$

or:

$$\boxed{\left.\frac{d^2N}{dp_T\,dy}\right|_{y=0} = \left.\frac{dN}{dy}\right|_{y=0} p_T\frac{m_T}{T}\left[1+(q-1)\frac{m_T}{T}\right]^{-q/(q-1)} \times(\text{factors independent of } p_T)}$$

Asymptotic Behavior

At mid-rapidity $y = 0$ and for μ = 0

$$\lim_{p_T\to\infty} \left.\frac{d^2N}{dp_T\, dy}\right|_{y=0} = p_T \left[\frac{p_T}{T}\right]^{-q/(q-1)}$$

For example, for q = 1.1 :

$$\lim_{p_T\to\infty} \left.\frac{d^2N}{dp_T\, dy}\right|_{y=0} = p_T^{-11}$$

for q = 1.2

$$\lim_{p_T\to\infty} \left.\frac{d^2N}{dp_T\, dy}\right|_{y=0} = p_T^{-6}$$

Very sensitive to small changes in q !
Upper limit on q:

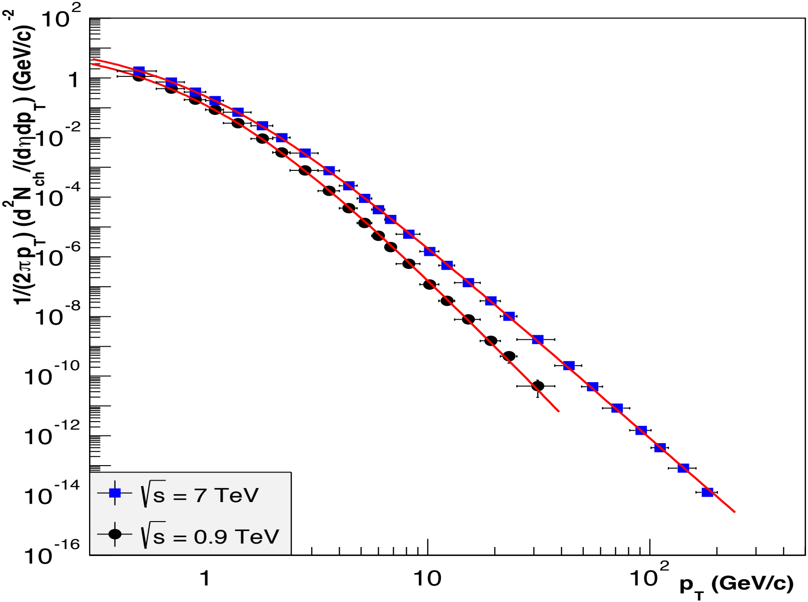
$$q < \frac{4}{3} \tag{1}$$

For charged particles use:

$$\left.\frac{d^2N(\text{charged})}{dp_T\;dy}\right|_{y=0} = \sum_{i=\pi,K,p,...} g_i V \frac{p_T m_T}{(2\pi)^2}\left[1+(q-1)\frac{m_T}{T}\right]^{-\frac{q}{q-1}},$$

M.D. Azmi and J.C. , arXiv:1501.07217v3[hep-ph].

Tsallis Distribution p-p CMS



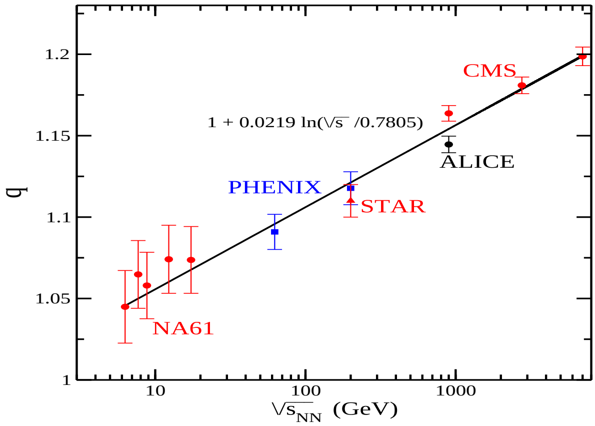
Tsallis Distribution p-p

Experiment	$\sqrt{s}$ **(TeV)**	q	T **(MeV)**
ATLAS	0.9	1.129 ± 0.005	74.21 ± 3.55
ATLAS	7	1.150 ± 0.002	75.00 ± 3.21
CMS	0.9	1.129 ± 0.003	76.00 ± 0.17
CMS	7	1.153 ± 0.002	73.00 ± 1.42

Values of the q and T parameters to fit the p_T spectra measured by the ATLAS and CMS collaborations.

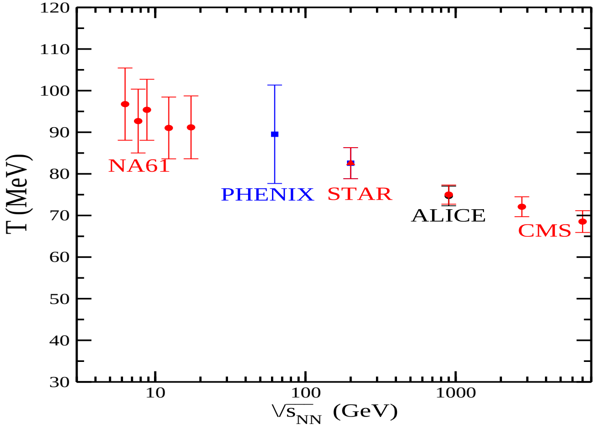
The Tsallis distribution provides an **excellent description of the transverse momentum spectra over 14 orders of magnitude** up to 200 GeV.

Energy Dependence of Tsallis Parameters



A.S. Parvan, O.V. Teryaev, J.C., arXiv hep-ph 1609

Energy Dependence of Tsallis Parameters

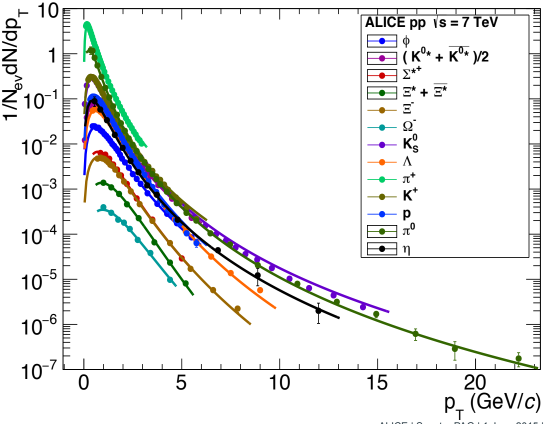


A.S. Parvan, O.V. Teryaev, J.C., arXiv hep-ph 1609

A Large Ion Collider Experiment

ALICE

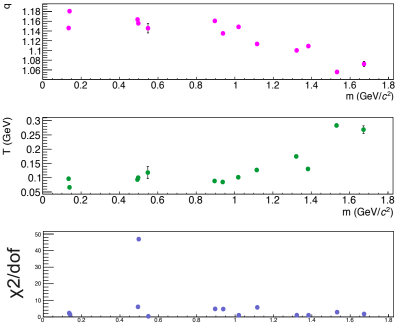
First look



A Large Ion Collider Experiment

ALICE

Extracted parameters



K^0_S very high, v. small stat. errors

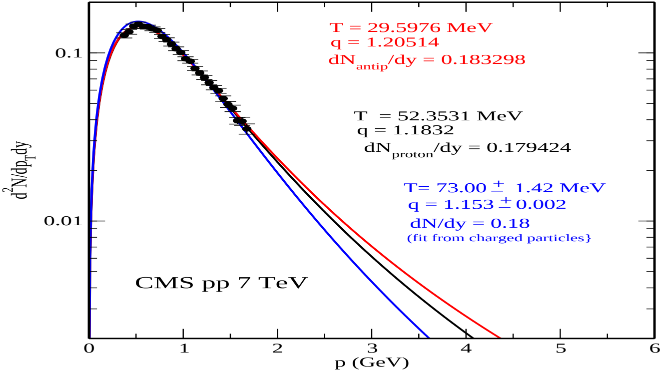
TO DO

Redo with syst. uncertainties

- **TO DO**
  - **Check whether this is due to limited p_T range**
  - **Investigate T-q 2-D correlation**

- Going to higher masses seems to get higher T?
- Anti-correlation between T and q?

Ts q


T = 29.5976 MeV
q = 1.20514
dN_antip/dy = 0.183298
T = 52.3531 MeV
q = 1.1832
dN_proton/dy = 0.179424
T= 73.00 ± 1.42 MeV
q = 1.153 ± 0.002
dN/dy = 0.18
(fit from charged particles}
CMS pp 7 TeV
d^2N/dp_T dy
0.1
0.01
0
1
2
3
4
5
6
p (GeV)

The Tsallis distribution provides an **excellent description of the transverse momentum spectra over 14 orders of magnitude** up to 200 GeV.

Use

$$\left.\frac{d^2N}{dp_T\,dy}\right|_{y=0} = \left.\frac{dN}{dy}\right|_{y=0} p_T \frac{m_T}{T}\left[1+(q-1)\frac{m_T}{T}\right]^{-q/(q-1)} \times(\text{factors independent of } p_T)$$

Advantages : thermodynamic conistency:

$$n = \frac{\partial P}{\partial \mu} \quad \text{etc...},$$

and the parameter T deserves its name since

$$T = \frac{\partial E}{\partial S} \quad \ldots$$

But ... the determination of the parameters needs to be resolved.