Quark Confinement and the Hadron Spectrum
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Multiplicities from black-hole formation in heavy-ion collisions

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Introduction

• Holography uses string-theory/gravity as an alternative description of strong-coupling physics at large N.

• In this language a heavy-ion collisions leads to the formation of a localized unstable black hole.

• This black holes expands adiabatically and finally decays through a process that is similar but may be different from Hawking evaporation.

KIritsis+Taliotis

• Although QCD is not exactly a near-semiclassical theory, it is close: therefore heavy ion collisions may help understand some aspects of black hole physics that so far were considered far from experiments.

• So far, numerical gravitational calculations are not fully in line to describe the details of such a collision but we are getting there.

• We may use however techniques first due to Penrose (trapped surfaces) to get quantitative information on heavy-ion collision multiplicities.

• The idea is that a minimal trapped surface is always hidden behind a horizon.

• Therefore its area is always smaller that the horizon area

• Such trapped surfaces can be calculated rather straightforwardly, by solving a classical boundary value problem.

• From Bekenstein's formula the area of the horizon determines the entropy of the black hole and this can be converted to the total multiplicity in a heavy ion collision.

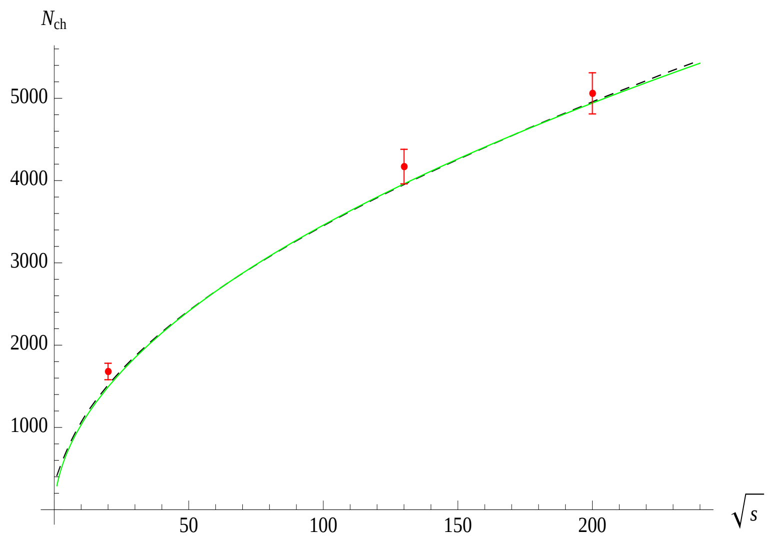
• Although this gives a lower bound on the total multiplicity as a function of the cm energy per nucleon, s we can use properties of AdS space and numerical data to argue that the leading s-dependence is the same.

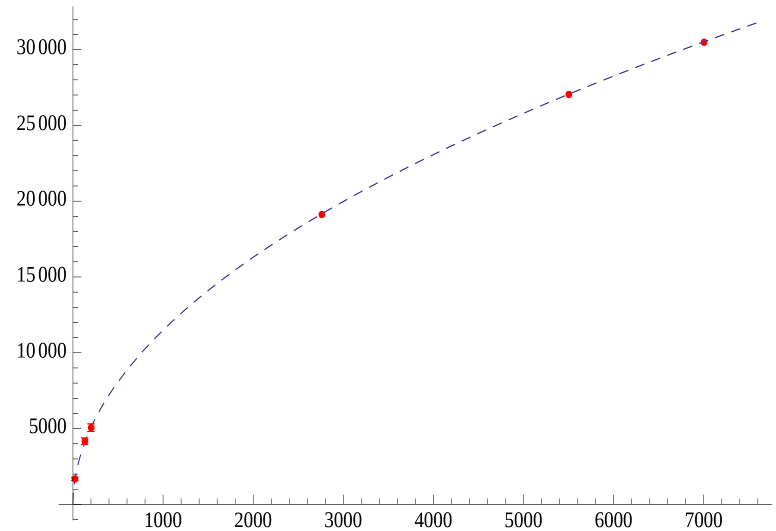
• One can therefore produce a multiplicity function for high-energies that is known up to a multiplicative constant that can be fit to experiment.

The IHQCD multiplicities

- There is no UV cutoff involved here.

$$N_{ch} = 78.05 \left(\frac{A}{A_{au}} \, \frac{\sqrt{s}}{1 \text{ GeV}} \right)^{0.451} \left[\log \left(535 \frac{A}{A_{au}} \, \frac{\sqrt{s}}{1 \text{ GeV}} \right) \right]^{0.718}$$





PHOBOS, Arxiv:0210015

- Predictions for PbPb (A=207) at LHC:

$Nch = 19100, 27000, 30500$ for $2.76, 5.5$ and 7 TeV respectively.

Bibliography

Based on work with Tassos Taliotis

Multiplicities from black-hole formation in heavy-ion collisions.
arXiv:1111.1931 [hep-ph]

Mini-Black-Hole production at RHIC and LHC.
arXiv:1110.5642 [hep-ph]

The string/gravity description of strong-coupling QCD

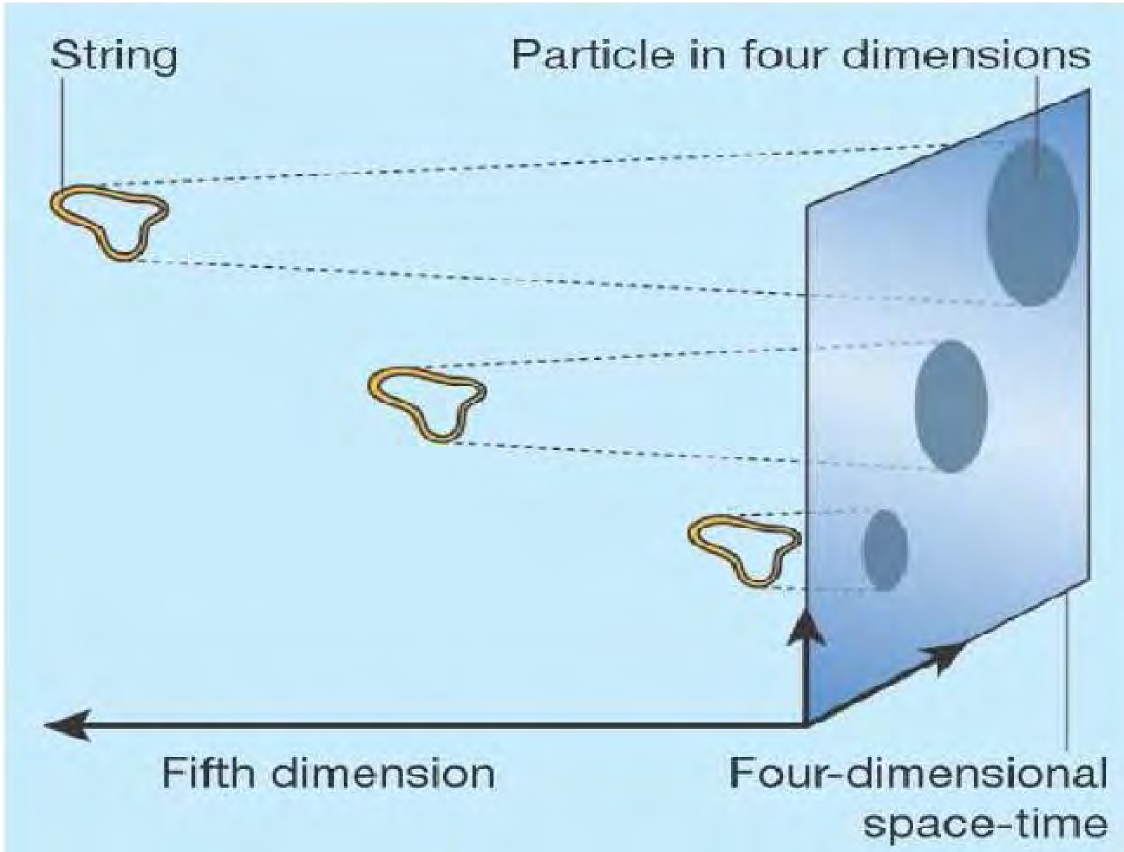
• As there are no generic techniques to control strong coupling physics, any related tool is important.

• The correspondence between strongly-coupled gauge theories and string theories have provided tools, in order to calculate at strong coupling.

• This correspondence works in a large-N_c, and strong ('t Hooft) coupling regime. It is complementary to pQCD.

• The simplest and most controlable example involves a highly symmetric, and scale invariant theory, $\mathcal{N} = 4$ sYM. Its dual description involves string theory (and gravity) in AdS_5 space.

The gauge-theory/gravity correspondence



• There is one-to-one correspondence between on-shell string states $\Phi(r, x^\mu)$ and gauge-invariant (single-trace) operators $O(x^\mu)$ in the sYM theory

• In the string theory we can compute the "S-matrix" , $S(\phi(x^\mu))$ by studying the response of the system to boundary conditions $\Phi(r = 0, x^\mu) = \phi(x^\mu)$

• The correspondence states that this is equivalent to the generating function of correlators of O

$$\langle e^{\int d^4x \;\phi(x)\; O(x)} \rangle \;=\; e^{-S(\phi(x))}$$

Maldacena 1997, Gubser+Klebanov+Polyakov, Witten, 1998

• One of the most remarkable facts of the correspondence is that thermalization in the QFT corresponds to the formation of a BH in the bulk.

• The thermal gauge theory ensemble maps to a large BH filling the AdS_5 space.

• The laws of BH thermodynamics now find their explanation: they correspond to the thermodynamics of the dual gauge theory.

• Therefore a heavy-ion collision with a thermalized final state must correspond to the formation (and decay) of a black hole in the dual language.

• I should also stress that in the gravitational language, we have seen an extra dimension that is "infinite". Its KK states, are known since 30-50 years: they are the radial excitations of glueballs, mesons and baryons. It is a "fuzy" extra dimension: all but the few ligher KK states have large widths and are unobservable.

• The context however is different in many respects from the "popular" bhs of "large extra dimensions" (that are not visible at LHC).

• What is the dual gravity/string theory that describes YM?

A model for Holographic YM

• We know that for $N_c \to \infty$ QCD should be described by a (soft) string theory in the UV.

• But there should be some gravity description in the IR, as the coupling there is strong.

• The most important bulk fields are expected to be the metric $g_{\mu\nu}$ dual to $T_{\mu\nu}$, and a scalar ϕ (the "dilaton") dual to $tr[F^2]$.

• A good guess is an action of the form

$$\mathcal{S}_\mathrm{g} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3}(\partial\phi)^2 + V_g(\phi) \right]$$

Gursoy+Kiritsis+Nitti, 2007, Gubser+Nelore, 2008

• The potential $V_g \leftrightarrow$ QCD β-function

• The scale factor of the bulk metric corresponds to the YM energy scale.

• $e^\phi \to \lambda$ 't Hooft coupling

YM Entropy

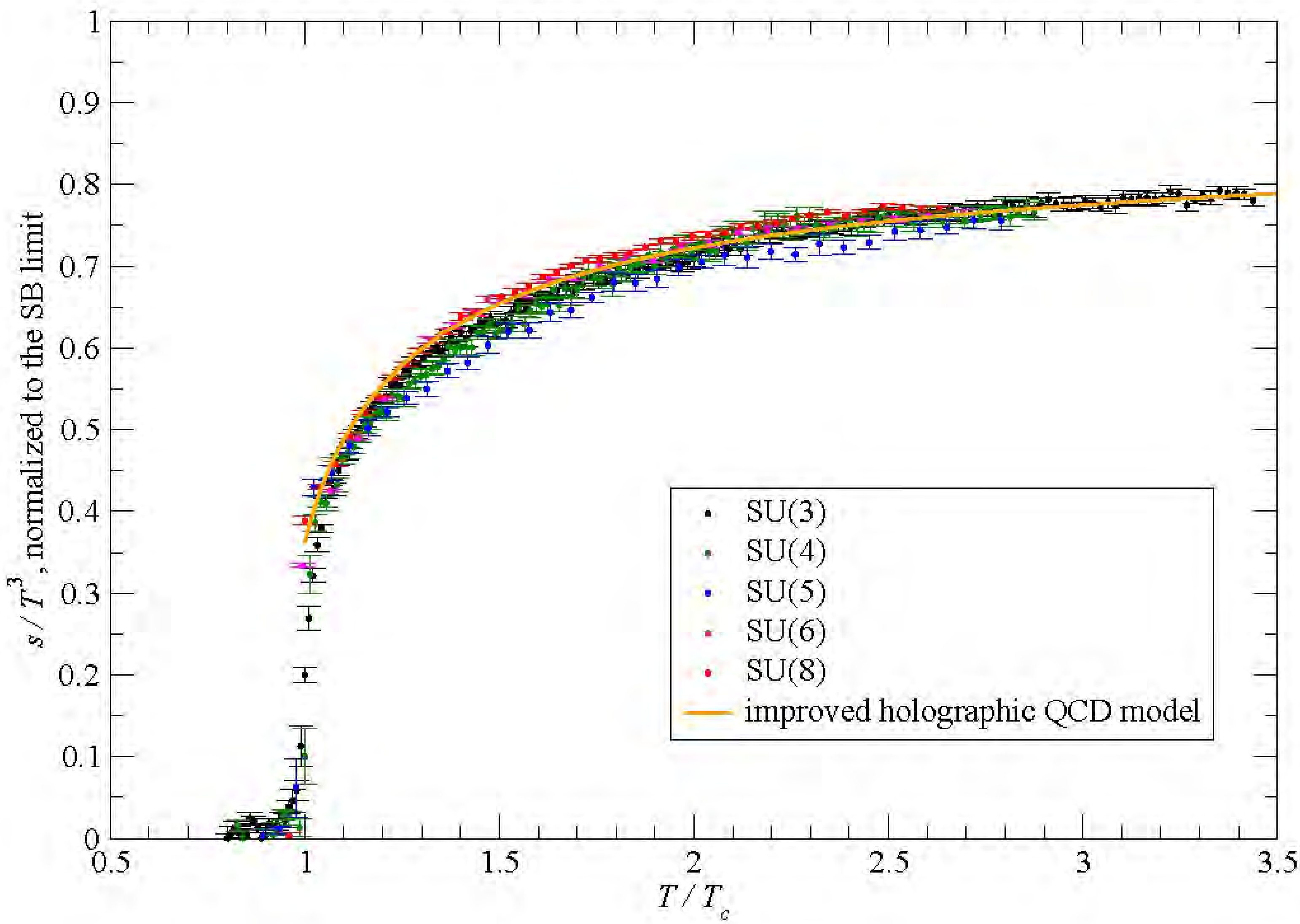


Figure 4: (Color online) Same as in fig. 1, but for the s/T^3 ratio, normalized to the SB limit.

From M. Panero, arXiv:0907.3719

Equation of state

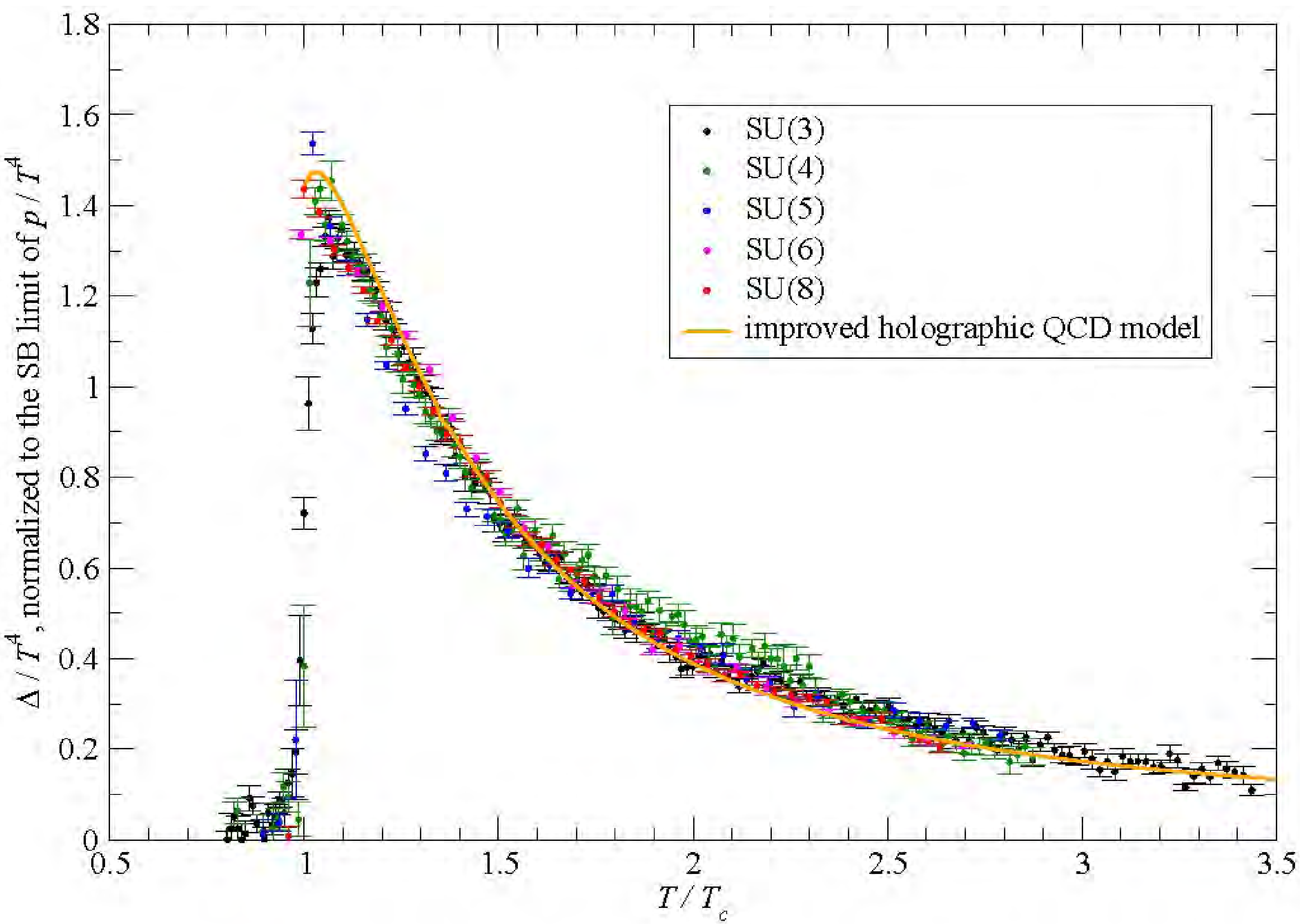
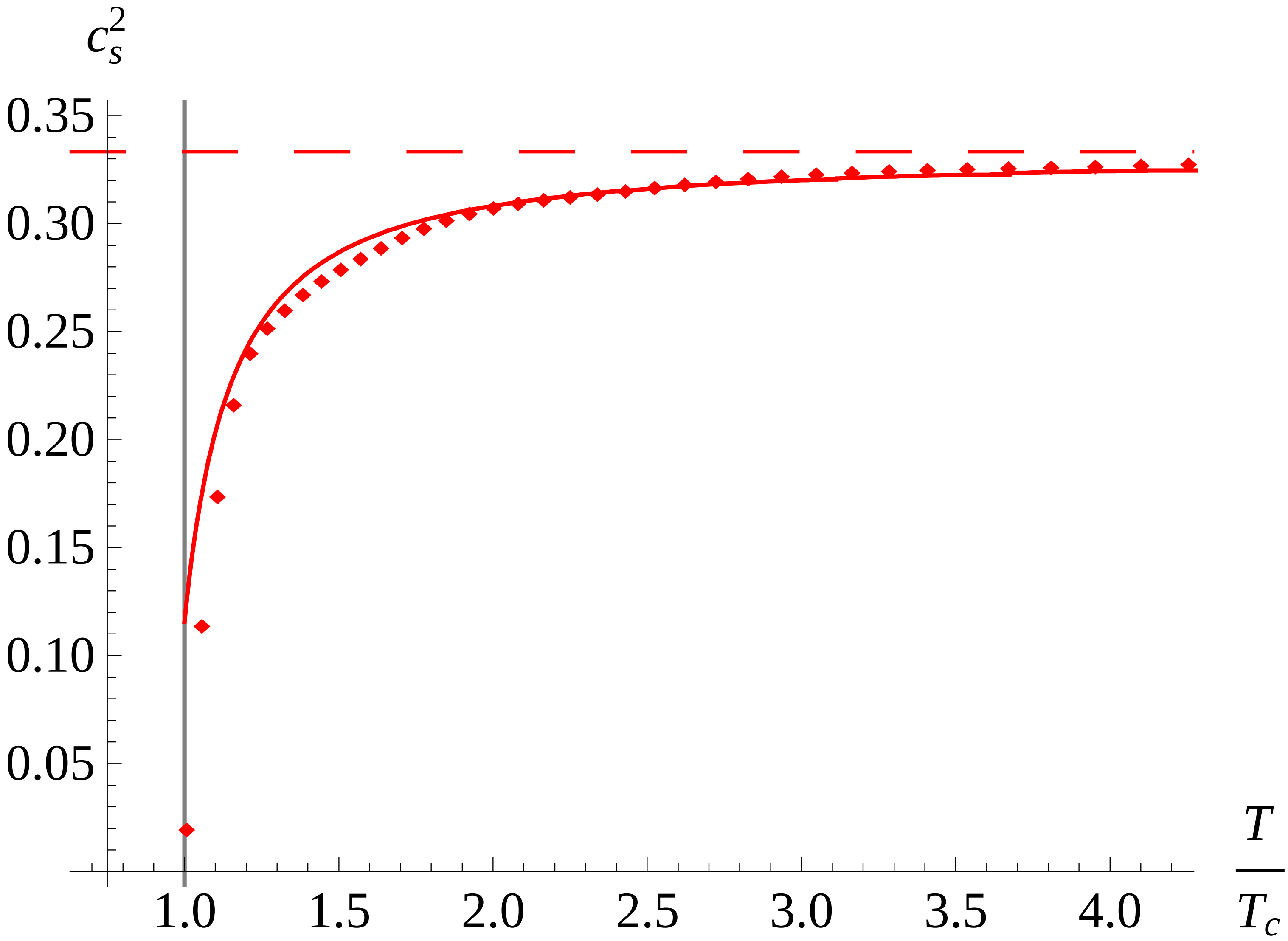


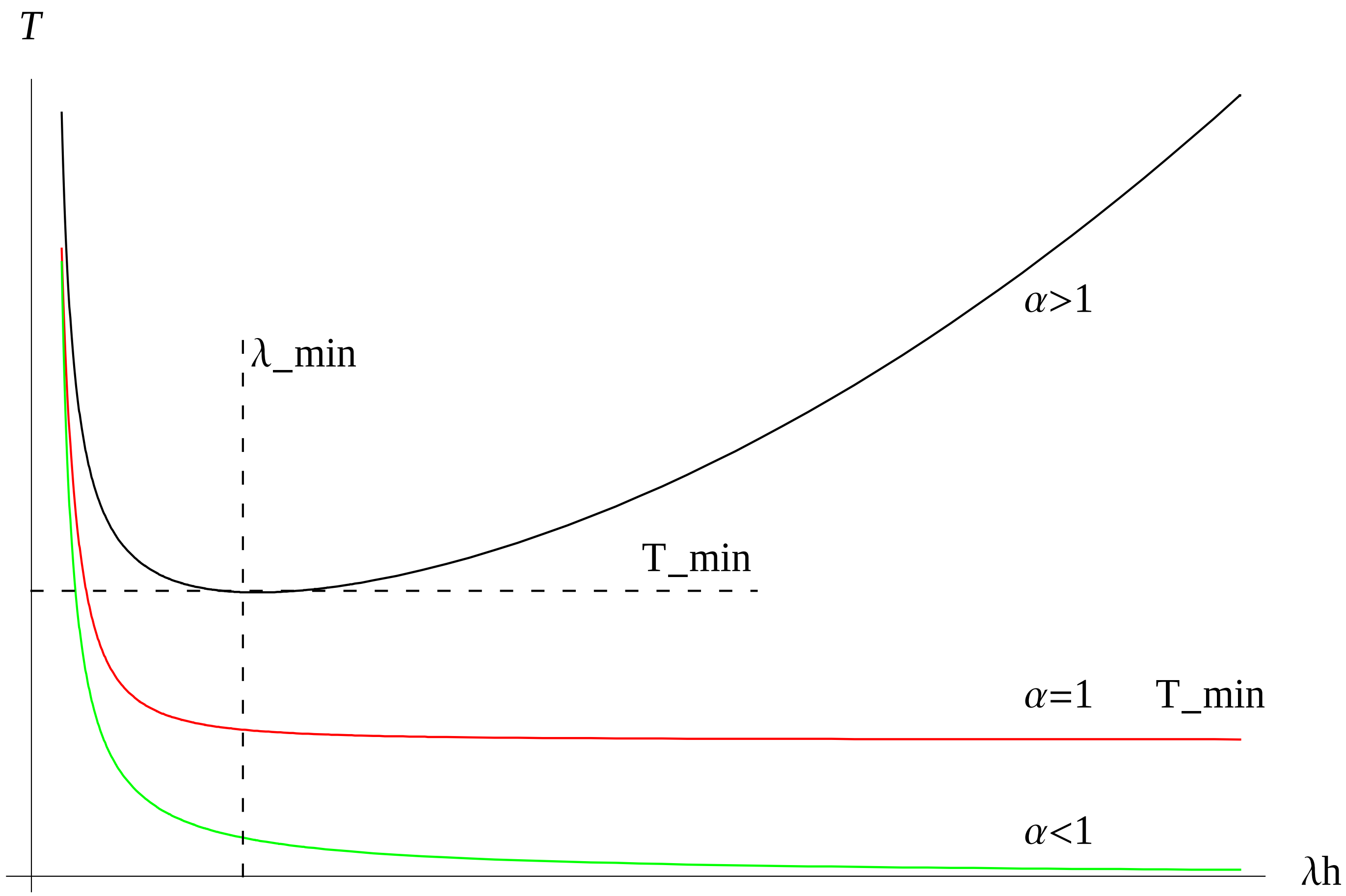
Figure 2: (Color online) Same as in fig. 1, but for the Δ/T^4 ratio, normalized to the SB limit of p/T^4.

From M. Panero, arXiv:0907.3719

The sound speed



Holographic YM Black holes



- There are small uniform black-holes that are thermodynamically unstable
- There are Large uniform black holes that thermodynamically stable.

• The large black holes have $E \sim T^4$, $S \sim T^3$ as $T \to \infty$. The horizon position is $r_h^{\text{large}} \sim \frac{1}{T} \to 0$ at large temperatures, where $r = 0$ is the position of the AdS boundary. The specific heat is positive.

• The small black holes are unstable, with negative specific heat. They are however nowhere near Schwartzschild black holes.

• As $T \to \infty$ their horizon shrinks to zero size as $r_h^{\text{small}} \sim \frac{T}{\Lambda_{QCD}^2}$.

$$S \simeq V_3 \ \exp\left[-3\frac{T^2}{\Lambda_{QCD}^2}\right] \quad , \quad E \simeq V_3 M_P^3 \ T \ \exp\left[-3\frac{T^2}{\Lambda_{QCD}^2}\right] \quad , \quad T \to \infty$$

• At large T, the small and large black holes satisfy the duality relation

$$(\Lambda_{QCD} \ r_h^{\text{small}})(\Lambda_{QCD} \ r_h^{\text{large}}) \simeq 1$$

• We also have for any T_i, T_j

$$E_{\text{large}}(T_i) > E_{\text{small}}(T_j) \quad , \quad S_{\text{large}}(T_i) > S_{\text{small}}(T_j)$$

• These suggest that during a collision it is an unstable analogue of the large black holes that will be created.

• Studying however, time dependent unstable black holes is very difficult analytically.

Collisions of shock waves

- We would like to model the heavy ion collision in the dual gravitational 5d language.

- In the dual gravitational theory to QCD, heavy ions can be modeled as localised energy distributions collided at high relative velocities.

- Energy sources in gravitational theories generate gravitational fields described by generalizations of the Schwartzschild solution.

- Two such energy sources colliding at high energy pose a formidable problem even for the classical theory as their gravitational fields start interacting long before the sources collide.

- However at high energies, things simplify a bit: at ultrarelativistic speeds, the gravitational fields squeeze into a cone, that becomes narrower with energy. In the limit $E \to \infty, v \to c$, the field is squeezed on the light cone $x^+ = 0$, or $x^- = 0$.

• The metric around flat space is

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + \phi(x_\perp)\delta(x^+)(dx^+)^2$$

• $\phi(x_\perp)$ describes the transverse profile of the wave.

• It can be obtained :

(a) By solving the Einstein equations with an appropriate ansatz

(b) By infinitely boosting the Schwartzschild solution.

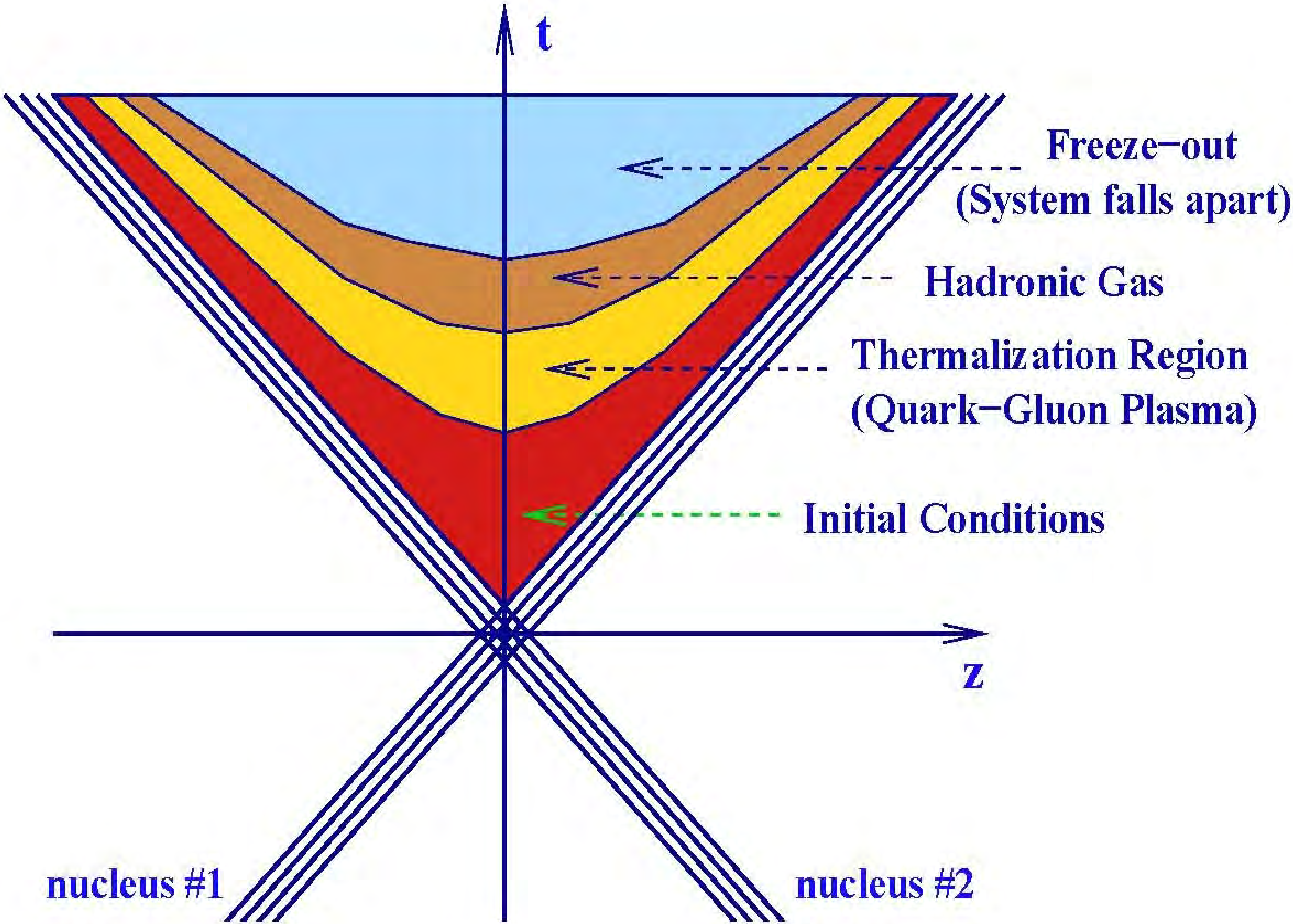
Aichelburg+Sexl, 1971

• As the gravitational field of shock waves is squeezed, one can superpose two such solutions that describe two particles in a head-on collision

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + \left[\phi_1(x_\perp)\delta(x^+)(dx^+)^2 + \phi_2(x_\perp)\delta(x^-)(dx^-)^2\right]$$

• The metric is valid in the three quadrants due to causality: $x^+ < 0$ or $x^- < 0$.

• The metric changes in $x^+ > 0$ and $x^- > 0$ and it is a non-trivial task to determine it.


t
Freeze−out
(System falls apart)
Hadronic Gas
Thermalization Region
(Quark−Gluon Plasma)
Initial Conditions
z
nucleus #1
nucleus #2

Horizons and trapped surfaces

• A defining property of a black-hole formation during the collisions of energy packets is the appearance of an event horizon.

• This is a difficult property to verify, as it is global, and the full metric in the whole of spacetime is needed.

• There are however hints of the formation of a horizon that may appear earlier during the dynamical process.

• Such hints are the trapped surfaces: Surfaces whose null normals are inward.

• In plain words: they are surfaces that due to the attractiveness of gravity and the focusing of geodesics they will evolve inside an event horizon (that may form later).

• Such (codimension 2) surfaces are much easier to find, and sometimes (as with Penrose-type surfaces) the shock-wave geometry before the collision is enough to determine them.

• A limit form of a trapped surface is a "marginal trapped surface": in plain words it is a surface that is only barely trapped.
Mathematically it has vanishing "expansion": $\nabla \cdot \ell = 0$. It is also known as the apparent horizon.

• In the gravitational theory the area of a horizon is interpreted as entropy according to the Bekenstein postulate.

• It can be shown, using the singularity thms, that the area of a marginally trapped surface, is always smaller or equal to that of an event horizon that will eventually form.

Penrose

• As the marginally trapped surface has the largest area, we obtain a lower bound on the entropy:

$$S \geq S_{\text{marginally trapped}}$$

• This is a central ingredient in our subsequent calculations.

Entropy and multiplicity

• Entropy can be translated into the total multiplicity N_{ch} measured in heavy ion collisions.

• 1 Charged particle is accompanied by approximately $\frac{1}{2}$ Neutral particle (isospin symmetry). Therefore:

$$N_{tot} = N_{ch} + N_{neutral} \simeq \frac{3}{2}\, N_{ch}$$

• Estimate of total entropy:

Heinz

$$S \simeq 5 \quad \times \quad \frac{3}{2} \quad \times \quad N_{ch} \simeq 7.5\; N_{ch}$$

• We will use N_{ch}, N_{tot}, and S interchangeably as they are proportional.

Shock waves in Einstein-Dilaton gravity

$$S_5 = -M^3 \int d^5x\sqrt{g}\left[R - \frac{4}{3}(\partial\Phi_s)^2 + V(\Phi_s)\right]$$

- We first find shock wave solutions in this theory of the form

$$ds^2 = b(r)^2\left[dr^2 + dx^i dx^i - 2dx^+dx^- + \phi(r, x^1, x^2)\delta(x^+)(dx^+)^2\right]$$

and $\Phi_s = \Phi_s(r, x^+)$ with the asymptotically AdS boundary at $r = 0$.

- The shock wave profile ϕ must satisfy

$$\left(\nabla_\perp^2 + 3\frac{b'}{b}\partial_r + \partial_r^2\right)\phi = -M^3 J_{++}, \quad , \quad \partial_- J_{++} = 0$$

- This theory for $\Phi_s = V$ =constant provides the AdS solution $b(r) = \frac{\ell}{r}$. For a constant transverse profile the solution is

$$\phi_{AdS_5} = E\ r^4$$

Non-trivial transverse profiles

- The simplest profile, is the uniform one in transverse space

$$\phi \sim \int \frac{dr}{b(r)^3}$$

- For non-trivial profiles we separate variables $\phi_k \sim f_k(x_\perp) g_k(r)$

$$\left(\partial^2_{x_\perp} + \frac{1}{x_\perp}\partial_{x_\perp} - k^2\right) f_k(x_\perp) = 0 \quad \left(\partial^2_r + 3\frac{b'(r)}{b(r)}\partial_r + k^2\right) g_k(r) = 0.$$

- The first equation yields

$$f_k(x_\perp) = C_1 K_0(k x_\perp) + C_2 I_0(k x_\perp)$$

- The solution to $g(r)$ depends on the scale factor $b(r)$. The equation for g is the same as that for 2^{++} glueballs with mass $m^2 = k^2$. For discrete spectra, the transverse radii are quantized, $\frac{1}{m_n}$ following the spectrum.

- There are two classes of transverse distributions:

♠ Exponentially localised ones corresponding to a single k: $K_0 \sim e^{-k|x_\perp|}$.

♠ Power like ones corresponding to integrals over k: $\frac{1}{(|x_\perp|^2+L^2)^a}$

A host of non-AdS backrounds

• non-AdS scale factors: (not everything is allowed).

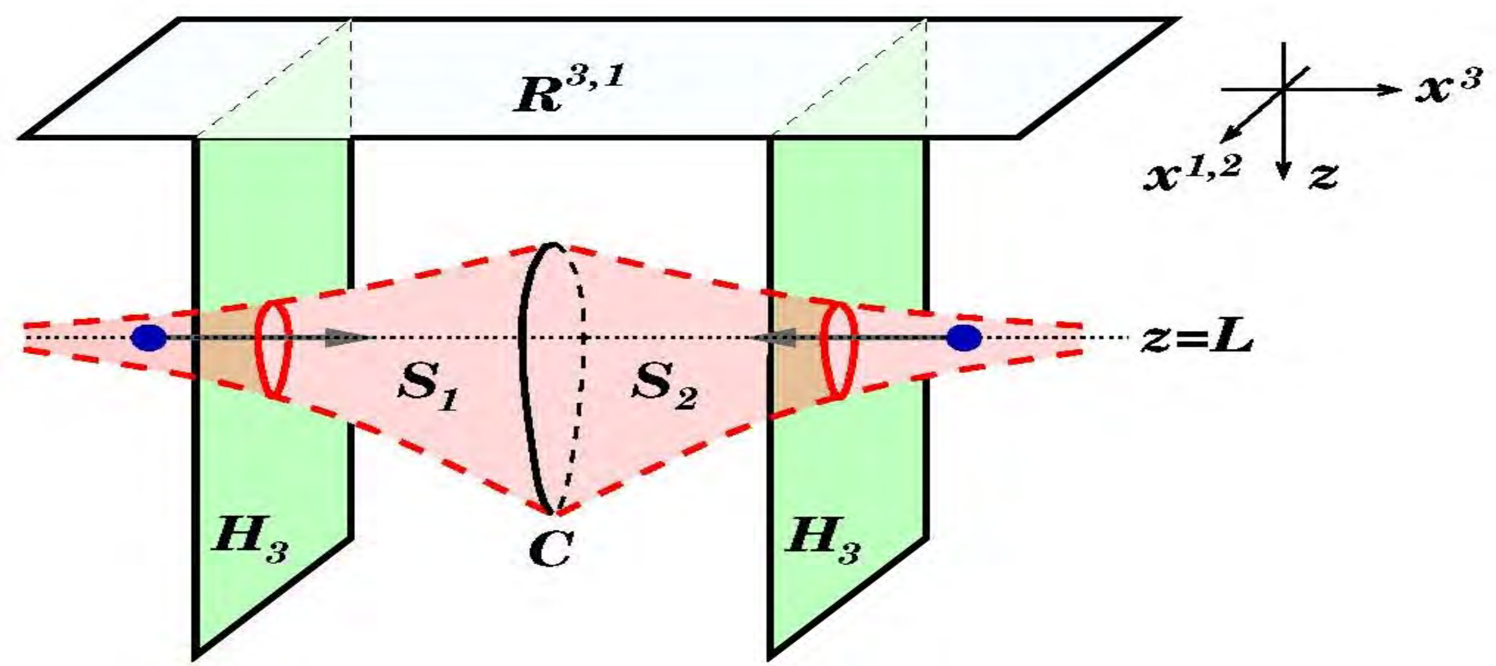
♠ $b \sim r^a$, with $a \leq -1$. This corresponds to quasiconformal geometries, with no confinement, continuous spectrum and no mass gap, with potential asymptotics as $\Phi_s \to \infty$, $V \sim e^{Q\Phi_s}$, $Q < \frac{4}{3}$.

♠ Confining backgrounds that are scale invariant in the IR, with $b(r) \sim (r_0 - r)^a$, $a > \frac{1}{3}$. They have a discrete spectrum of glueballs and a mass gap. The potential asymptotics as $\Phi_s \to \infty$ are $V \sim e^{Q\Phi_s}$, $Q > \frac{4}{3}$.

♠ Confining backgrounds with $b(r) \sim e^{-(\Lambda r)^a}$, $a > 0$. They have a discrete spectrum and a mass gap. The potential asymptotics as $\Phi_s \to \infty$ are $V \sim e^{\frac{4}{3}\Phi_s} \, \Phi_s^{\frac{a-1}{a}}$. $a = 2$, corresponds to IHQCD

♠ Confining backgrounds with $b(r) \sim e^{-\left(\frac{\Lambda}{r-r_0}\right)^a}$, $a > 0$. They have a discrete spectrum and a mass gap. The potential asymptotics as $\Phi_s \to \infty$ are $V \sim e^{\frac{4}{3}\Phi_s} \, \Phi_s^{\frac{a+1}{a}}$.

The Penrose-type marginal trapped surface



• S_1 is defined by $x^+ = 0$, $x^- + \frac{1}{2}\psi_1(r, x_1, x_2)$. S_2 is defined by $x^- = 0$, $x^+ + \frac{1}{2}\psi_2(r, x_1, x_2)$.

• They are glued along a two-surface C: $\psi_1|_C = \psi_2|_C = 0$, with a continuity condition on the normals on C.

$$S_{\text{trapped}} = \frac{M^3}{4}(Vol(S_1) + Vol(S_2))$$

Review of the equations

• For two shock wave profiles $\phi_{1,2}(r, x_\perp)$ solving the appropriate equations

$$\nabla^2 \phi_{1,2} = -M^3 J_{++} \quad , \quad \nabla^2 \equiv \left(\nabla_\perp^2 + 3\frac{b'}{b}\partial_r + \partial_r^2 \right)$$

• The two pieces of the marginal trapped surface, determined by $\psi_{1,2}(r, x_\perp)$ must satisfy:

$$\nabla^2 \left(\phi_{1,2} - \psi_{1,2} \right) = 0$$

Their boundary surface C is determined as

$$\psi_1|_C = \psi_2|C = 0$$

and the continuity of the normals along C gives the final equation

$$\sum_{i=r,1,2} \partial_i \psi_1 \ \partial_i \psi_2|_C = 8b(r)^2$$

Then the entropy bound is

$$S_{\text{trapped}} = \frac{M^3}{4}(Vol(S_1) + Vol(S_2))$$

• What is the use of a lower bound $S_{trapped}$?

• Assume that at large energy $s \to \infty$, we find that $S_{trapped} = A\ s^a +$ subleading.

• The ratio $\frac{S}{S_{\text{trapped}}}$, if the trapped surface ends in the UV part of the geometry (that is expected to be close to AdS_5) is to a good accuracy s-independent due to scale invariance.

Romatschke, 2011

• Therefore $S = \tilde{A} s^a\ +$subleading. We can predict the leading high energy dependence.

Known results

- We will only consider head-on collisions

- The entropy production was calculated in AdS_5 from shock waves uniform in transverse space . The result is $S_{\text{trapped}} \sim s^{\frac{1}{3}}$.

  Gubser+Pufu+Yarom

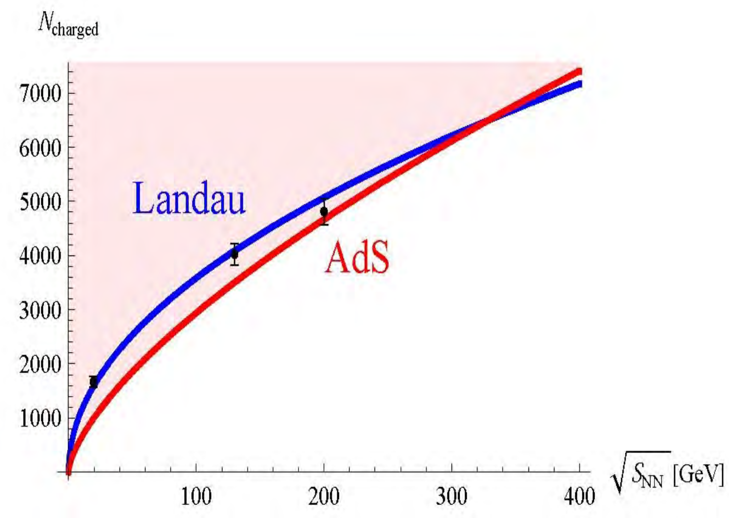
- The entropy production was calculated in AdS_5 from shock waves with a $(x_\perp^2 + L^2)^{-3}$ transverse profile. The result is again $S_{\text{trapped}} \sim s^{\frac{1}{3}}$.

  Gubser+Pufu+Yarom

- Some more general distributions were explored in the same context and a maximal size was found for the formation of the trapped surface.

  Alvarez-Gaumé+Gomez+Vera+Tavanfar+Vazquez-Mozo

- These results seem to overestimate the pre-LHC data



• We would like to test other backgrounds beyond AdS_5. In particular test the different 5d backgrounds as a function of their ability to confine, and have discrete spectra.

• We would like to test different transverse profiles, in particular well localized (exponential) ones in transverse space, that look more like the energy distribution of nuclei.

• We would like to somehow accommodate the fact that QCD is weakly coupled in the UV and this suppresses multiplicities at large momentum transfers.

What are we missing?

- AdS_5 is a VERY crude model for the QCD dynamics.

- Improved Holographic QCD is much better but contains only glue. What we scatter are very energetic 2^{++} glueballs. No quarks, no nuclei.

- The holographic theories are not properly weakly coupled in the UV.

- This description is valid as $N_c \gg 1$.

- The geometry is not always well defined during the collision.

- There may be other marginal trapped surfaces that are NOT of the Penrose type.

Uniform transverse glueballs

$$b^3(r_H) = \frac{E}{\sqrt{8}} \quad , \quad S_{\text{trapped}} \sim \int_{\infty}^{r_H} b^3(r)dr$$

- Non-confining

$$b(r) \sim \left(\frac{r}{\ell}\right)^a \quad , (a \leq -1) \quad , \quad S_{\text{trapped}} \sim s^{\frac{1}{2}-\frac{1}{6a}} \quad , \quad \frac{1}{3} \leq \frac{1}{2} - \frac{1}{6a} \leq \frac{1}{2}$$

- Confining (scaling)

$$b(r) \sim \left(\frac{r-r_0}{\ell}\right)^a \quad , \left(a \geq \frac{1}{3}\right) \quad , \quad S_{\text{trapped}} \sim s^{\frac{1}{2}+\frac{1}{6a}} \quad , \quad \frac{1}{2} \leq \frac{1}{2} + \frac{1}{6a} \leq 1$$

- Confining (IR of IHQCD)

$$b(r) \sim e^{-\left(\frac{r}{\ell}\right)^a} \quad , (a > 0) \quad , \quad S_{\text{trapped}} \sim s^{\frac{1}{2}} (\log \ s)^{\frac{1+a}{a}}$$

- Confining II

$$b(r) \sim e^{-\left(\frac{\ell}{r-r_0}\right)^a} \quad , (a > 0) \quad , \quad S_{\text{trapped}} \sim s^{\frac{1}{2}} (\log \ s)^{\frac{1-a}{a}}$$

Non-uniform transverse glueballs

- Powerlike distribution

$$\phi = \left(\frac{r-r_0}{L}\right)^{-\frac{1+3a}{2}} \frac{P^{\frac{1}{2}}_{\frac{3a}{2}-1}(1+2q)}{(q(1+q))^{\frac{1}{4}}} \quad , \quad q = \frac{x_\perp^2 + (r-r')^2}{4(r-r_0)(r'-r_0)}$$

- Confining $b(r) \sim \left(\frac{r}{\ell}\right)^a$, $a > \frac{1}{3}$

$$S_{\text{trapped}} \sim s^{\frac{1}{2}\frac{3a+3}{3a+2}}$$

- Non-Confining , $a < -1$

$$S_{\text{trapped}} \sim s^{\frac{3a+1}{6a}}$$

- Exponential scale factor,

$$\phi = e^{\frac{3r}{2R}} \frac{e^{-\frac{3}{2}\sqrt{u}}}{\sqrt{u}} \quad , \quad u = \frac{(x_\perp - x'_\perp)^2 + (r-r')^2}{R^2}$$

$$b \sim e^{-\frac{r}{R}} \quad , \quad S_{\text{trapped}} \sim s^{1.66}(\log s)^{1.17}$$

IHQCD-like geometry

- We call this (by abuse of language): "IHQCD model"

$$b(r) = \frac{\ell}{r} \quad e^{-\frac{r^2}{R^2}}$$

- The geometry is asymptotically AdS$_5$ in the UV, $(r \to 0)$, and confining in the IR.

$$\phi_k = \frac{E}{8\pi(M\ell)^3 k^2}\, g_n(kr)K_0(kx_\perp) \quad , \quad g_n = \frac{r^4}{R^4} L_n^{(2)}\left(\frac{3r^2}{R^2}\right)$$

- The size is quantized : $k_n^2 = (n+2)\frac{12}{R^2}$ (linear glueball trajectory).

- Trapped surface area can be computed only numerically.

General lessons

• If the UV geometry is asymptotically AdS_5 most of the "trapped entropy" originates in the UV part of the geometry. Multiplicity comes from the high-energy part of phase space.

• For uniform transverse distributions only, the AdS geometry produces the less entropy from the rest.

• A general trend in non-trivial transverse energy distributions is that at equal total energy the most dilute energy distribution produces the most entropy.

• Sometimes we do not find a trapped surface. In IHQCD there is no trapped surface for the lowest lying 2^{++} gluball.

• An AdS geometry produces more entropy than an asymptotically AdS confining geometry.

	$b(r)$	$\frac{\ell}{r}$	$\frac{\ell}{r}\exp[-\frac{r^2}{R^2}]$
Transverse profile			
Uniform		S^{AdS}_{unif}	S^{IHQCD}_{unif}
GYP		S^{AdS}_{GPY}	Not studied
Exponential		S^{AdS}_{exp}	S^{IHQCD}_{exp}

$$S^{AdS}_{unif} > S^{AdS}_{GPY} > S^{AdS}_{exp} \quad , \quad S^{IHQCD}_{unif} > S^{IHQCD}_{exp}$$

$$S^{AdS}_{unif} > S^{IHQCD}_{unif} \quad , \quad S^{AdS}_{GPY} > S^{IHQCD}_{exp} \quad , \quad S^{AdS}_{exp} > S^{IHQCD}_{exp}$$

The perturbative UV-fix

• How do we encode the weak coupling dynamics in the UV?

• Use a UV cutoff in AdS, and assume that above it the entropy production vanishes. This changes $S \sim s^{\frac{1}{3}}$ to $S \sim s^{\frac{1}{6}}$.

Gubser+Pufu+Yarom

• A more natural place to cutoff the strong coupling regime is the "saturation" scale Q_s.

McLerran,Venugopalan,Khrazeev......

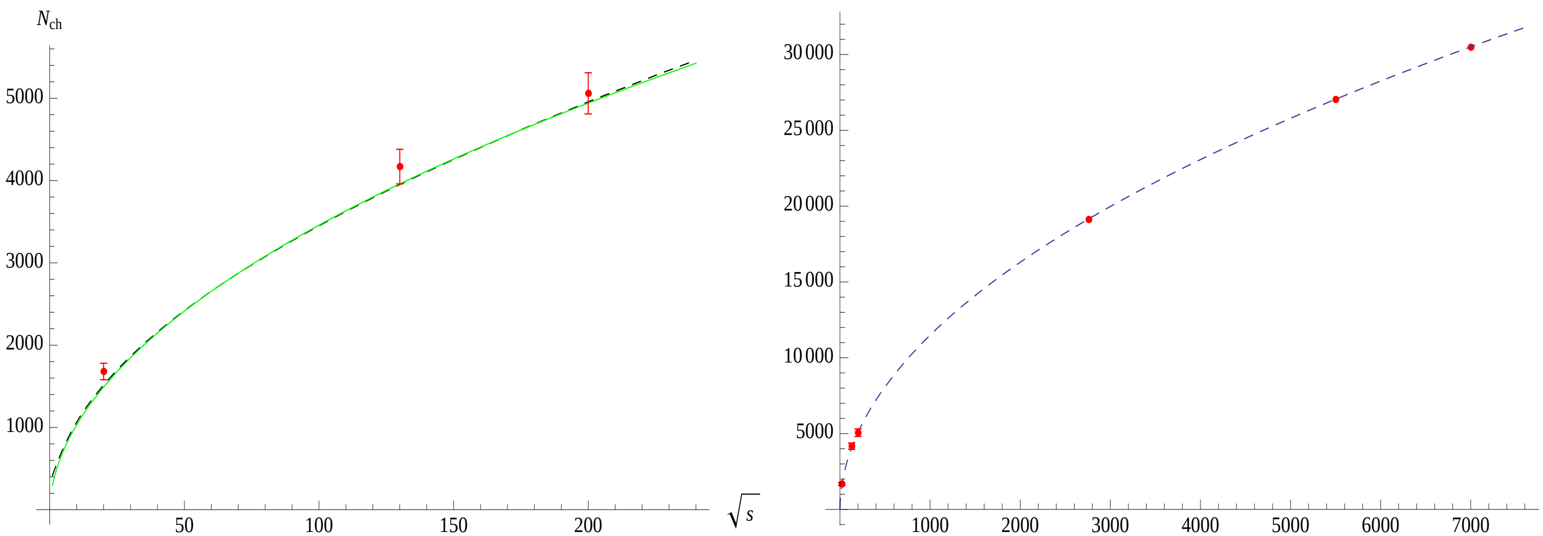
$$Q_s^2 \simeq (0.2\ GeV)^2 \times A^{\frac{1}{3}} \times (\sqrt{s_{NN}})^{2\lambda} \quad , \quad \lambda \in [0.1, 0.15]$$

• This defines the $AdS - Q_s$ "model"

The IHQCD multiplicities

- There is no UV cutoff involved here.

$$N_{ch} = 78.05 \left(\frac{A}{A_{au}} \frac{\sqrt{s}}{1 \text{ GeV}} \right)^{0.451} \left[\log \left(535 \frac{A}{A_{au}} \frac{\sqrt{s}}{1 \text{ GeV}} \right) \right]^{0.718}$$



PHOBOS, Arxiv:0210015

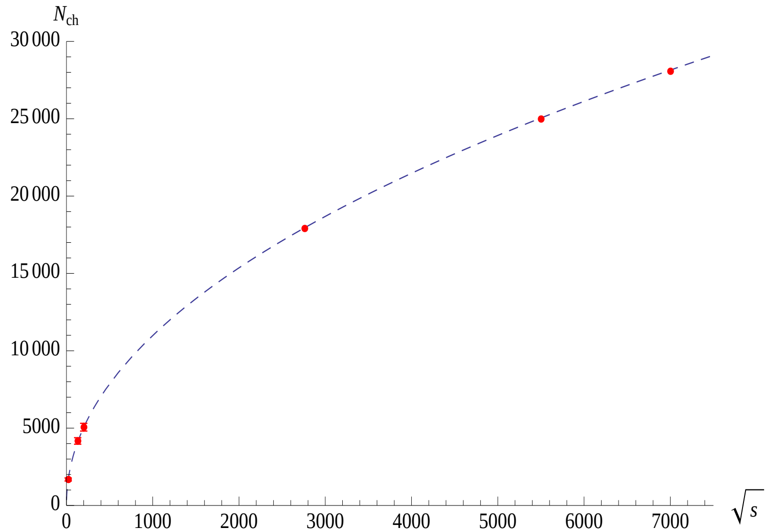
- Predictions for PbPb (A=207) at LHC:

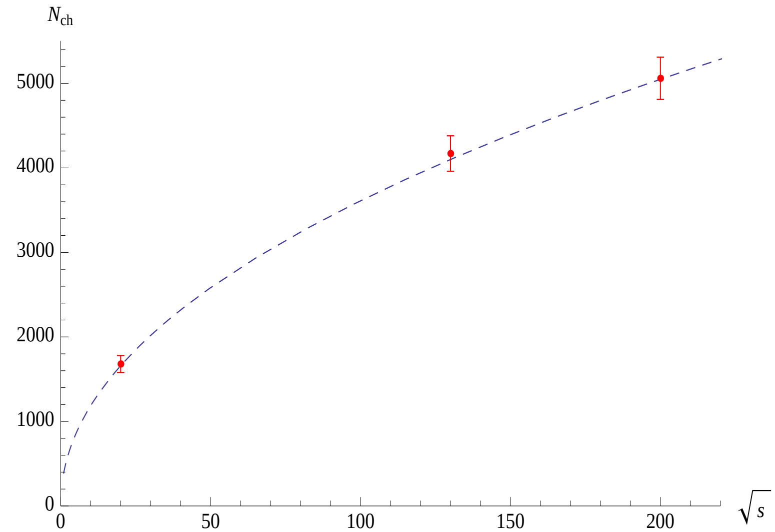
$Nch = 19100, 27000, 30500$ for $2.76, 5.5$ and 7 TeV respectively.

The AdS-Q_s multiplicities

- There is a UV cutoff at Q_s.

$$N_{ch} = 1.54 \left(\frac{A}{A_{au}}\right)^{\frac{17}{18}} \left(\frac{\sqrt{s}}{1\ \text{GeV}}\right)^{0.483}$$



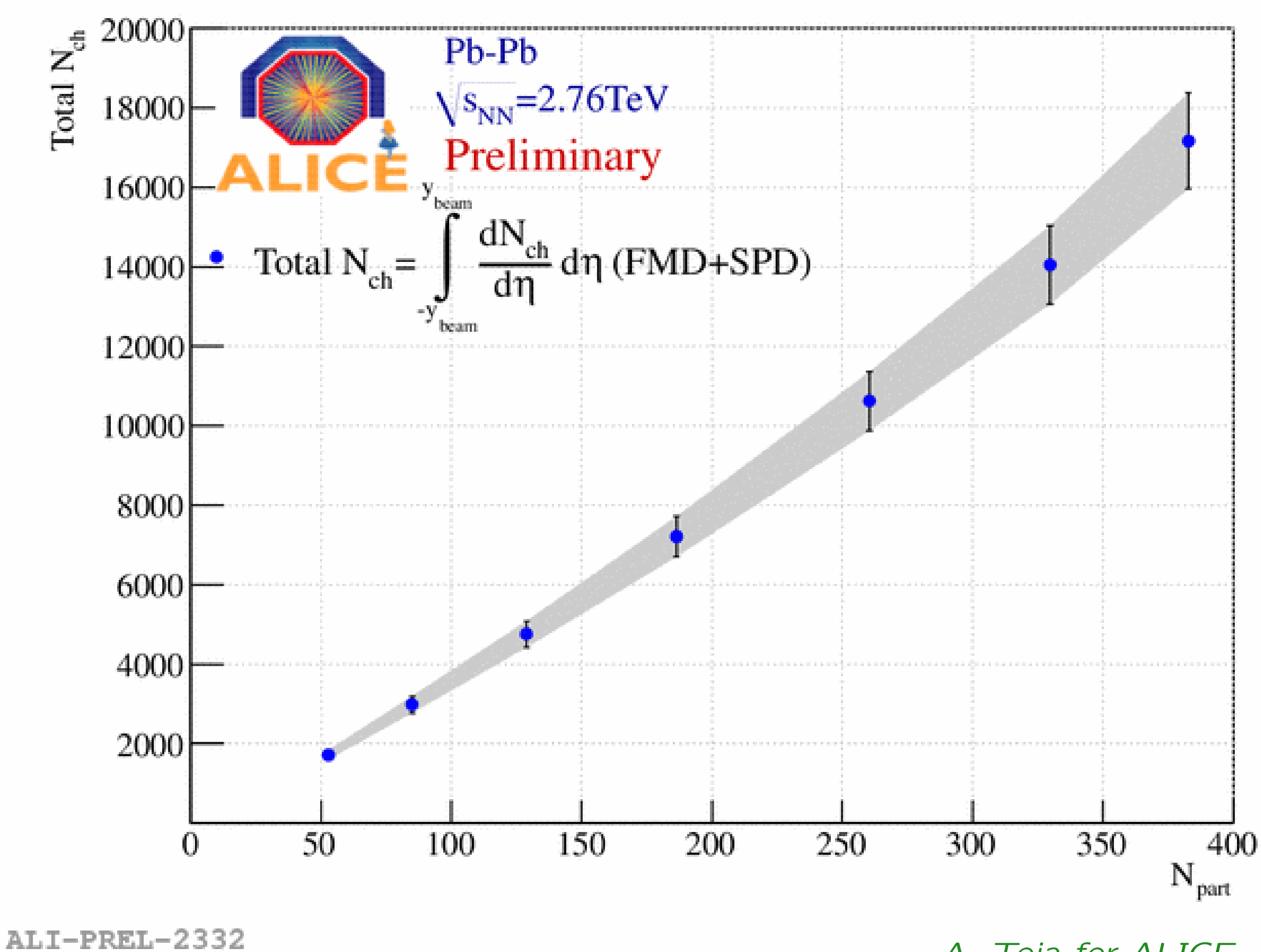


PHOBOS, Arxiv:0210015

- Predictions for pp (A=1) at LHC: $N_{ch} = 70, 110, 190, 260$ for $0.9, 2.36, 7$ and 14 TeV respectively.

- Predictions for PbPb (A=207): $N_{ch} = 18750, 261800, 29400$ for $2.76, 5.5$ and 7 TeV respectively.

ALICE multiplicities



A. Toia for ALICE, Arxiv:1107.1973

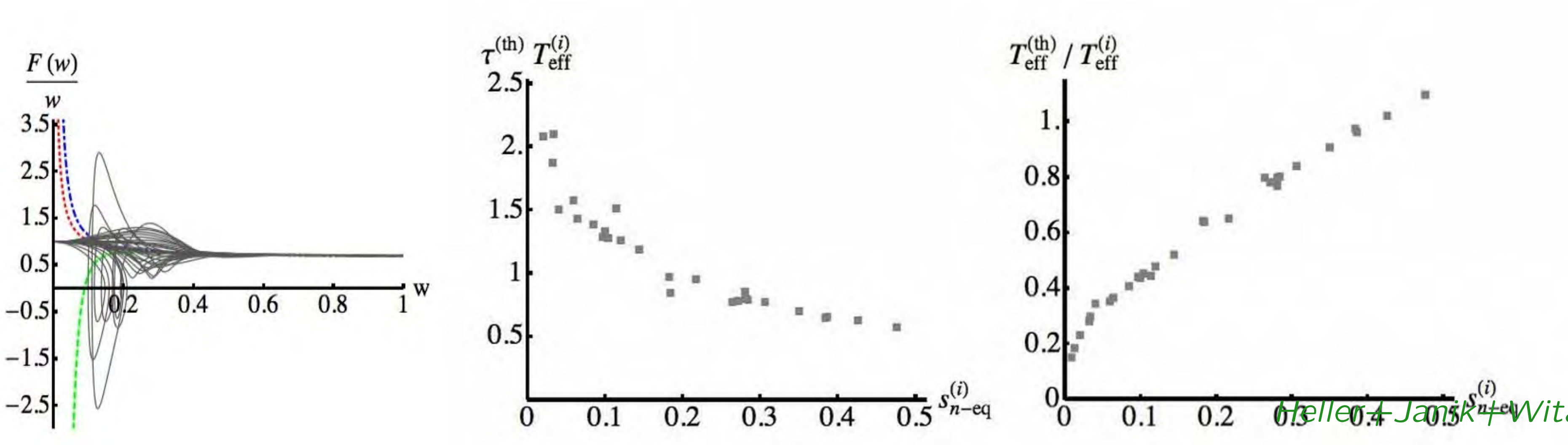
Outlook

• Our approach is of restricted validity, it seems to give good numbers, but cannot distinguish between the two models.

• It can be improved in several ways.

• Obtaining the differential multiplicities is preferable, but this requires full scale PDE evolution of the BH geometry

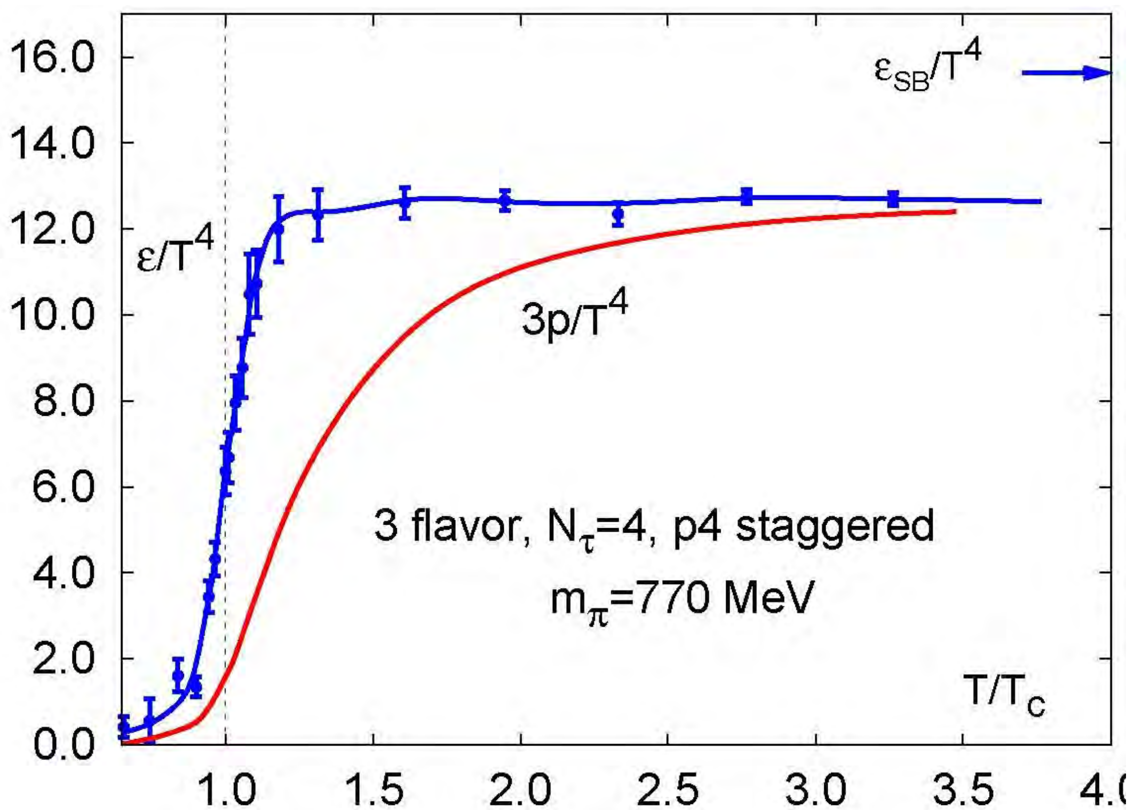
• In the last year this seems to becoming possible



Heller+Janik+Witaszczyk

THANK YOU

Phase transition vs crossover

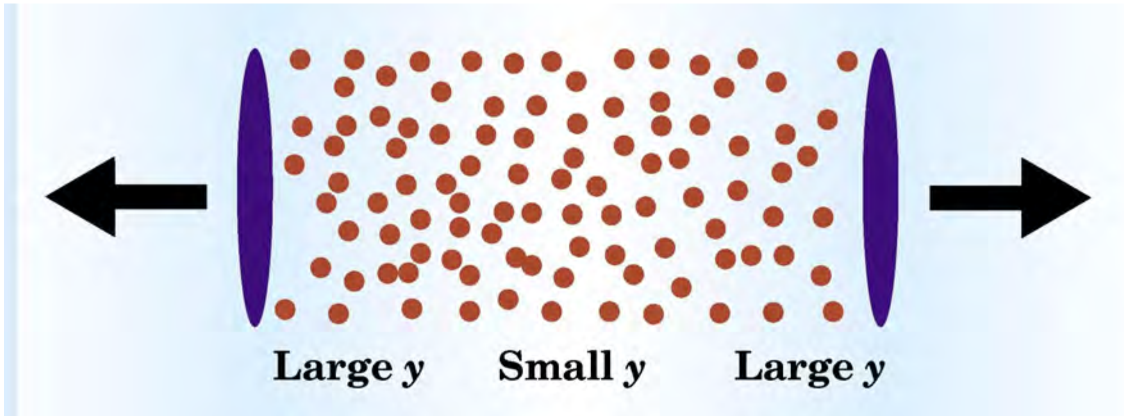


• The pure gauge theory (first-order) critical temperature is $T_c \simeq 240 \pm 15$ MeV.

• It is interesting that the lightest bound state (glueball) in the pure gauge theory has a mass 1700 MeV so that $\frac{T_c}{M_{0^{++}}} \simeq 0.14$

• The crossover with almost physical quarks is at $T_c \simeq 175 \pm 15$ MeV $\simeq 10^{12}$ ^{0}K. $\rightarrow$ 10^{-6} sec

The mid-rapidity range



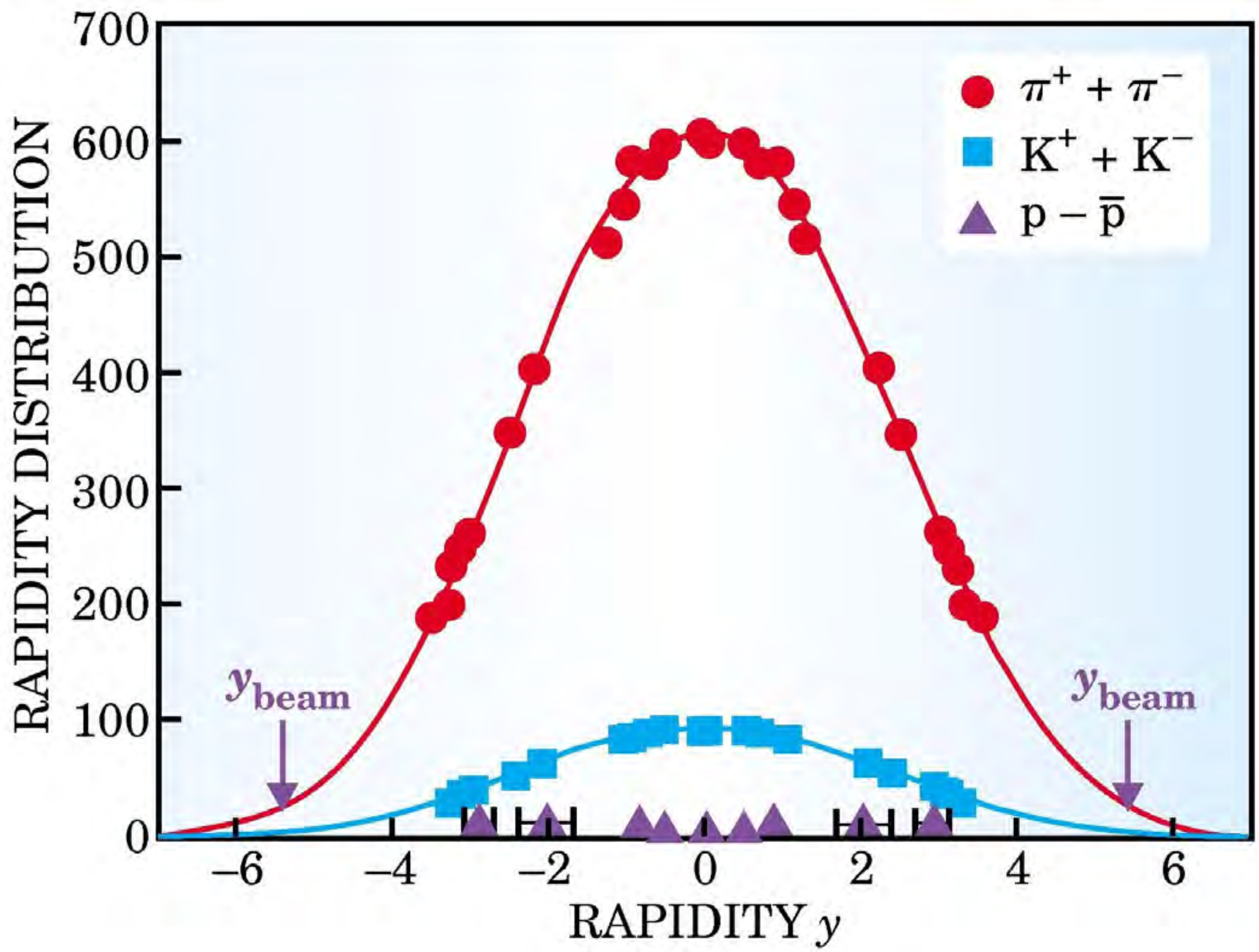
- The crossing time for Au nuclei (with radius 8 fm) is $\sim$ 0.1fm/c $\simeq$ 3×10^{-25} seconds.

- The particles with small v_L are produced after 1 fm/c $\simeq$ 3×10^{-24} seconds. Those with higher v_L are produced later due to time dilation.

- Use the rapidity variable $y = \frac{1}{2} \log \left[\frac{1 + \frac{v_L}{c}}{1 - \frac{v_L}{c}} \right]$. Δy is Lorentz invariant.

- The "new matter" (free of fragments) is produced near $y \simeq 0$. This is what we are looking for.

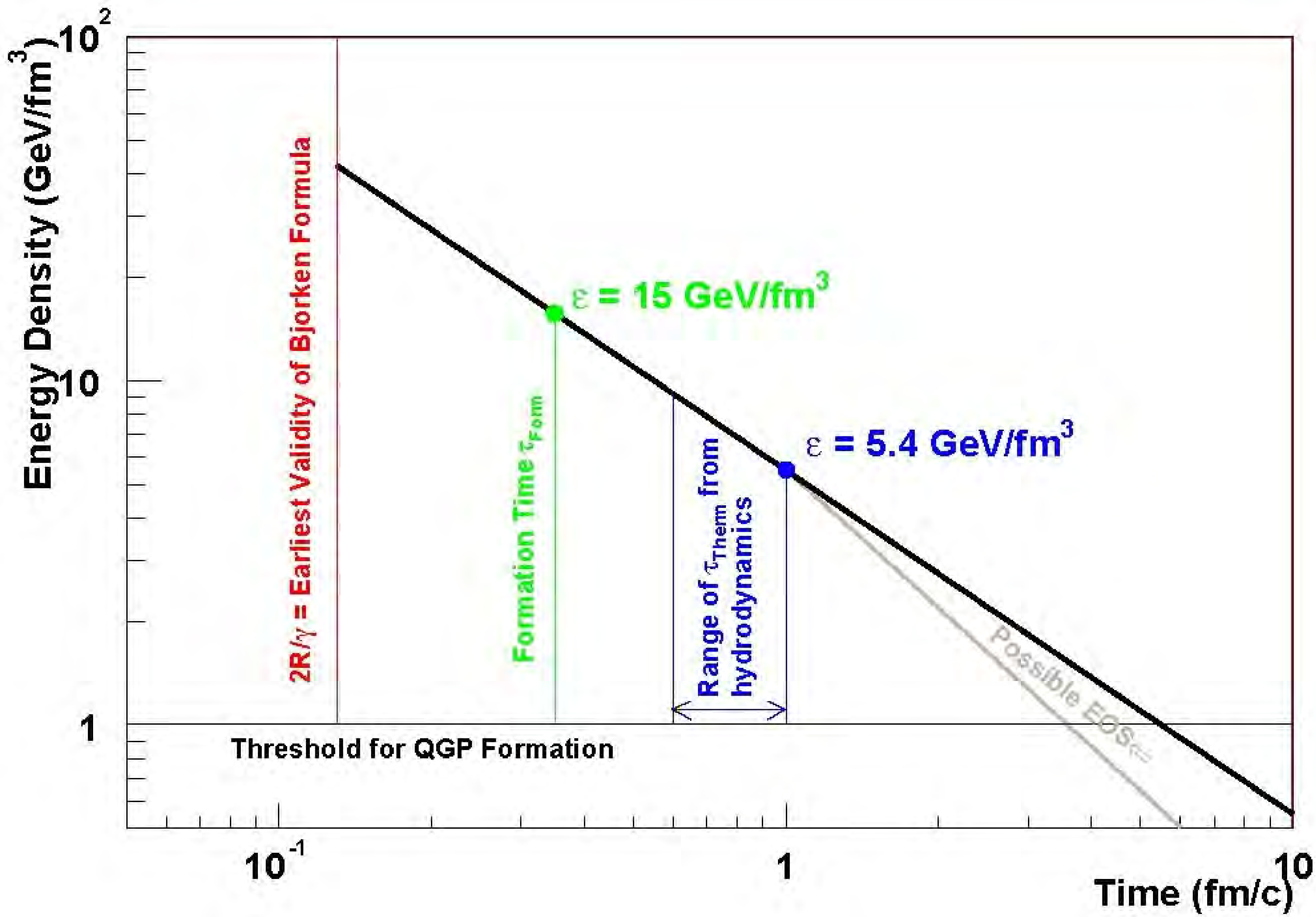
- This can be tested by looking at how much "baryon" number is at mid-rapidity



• Distribution of zero baryon number and net baryon number particles as a function of rapidity (from BRAHMS)

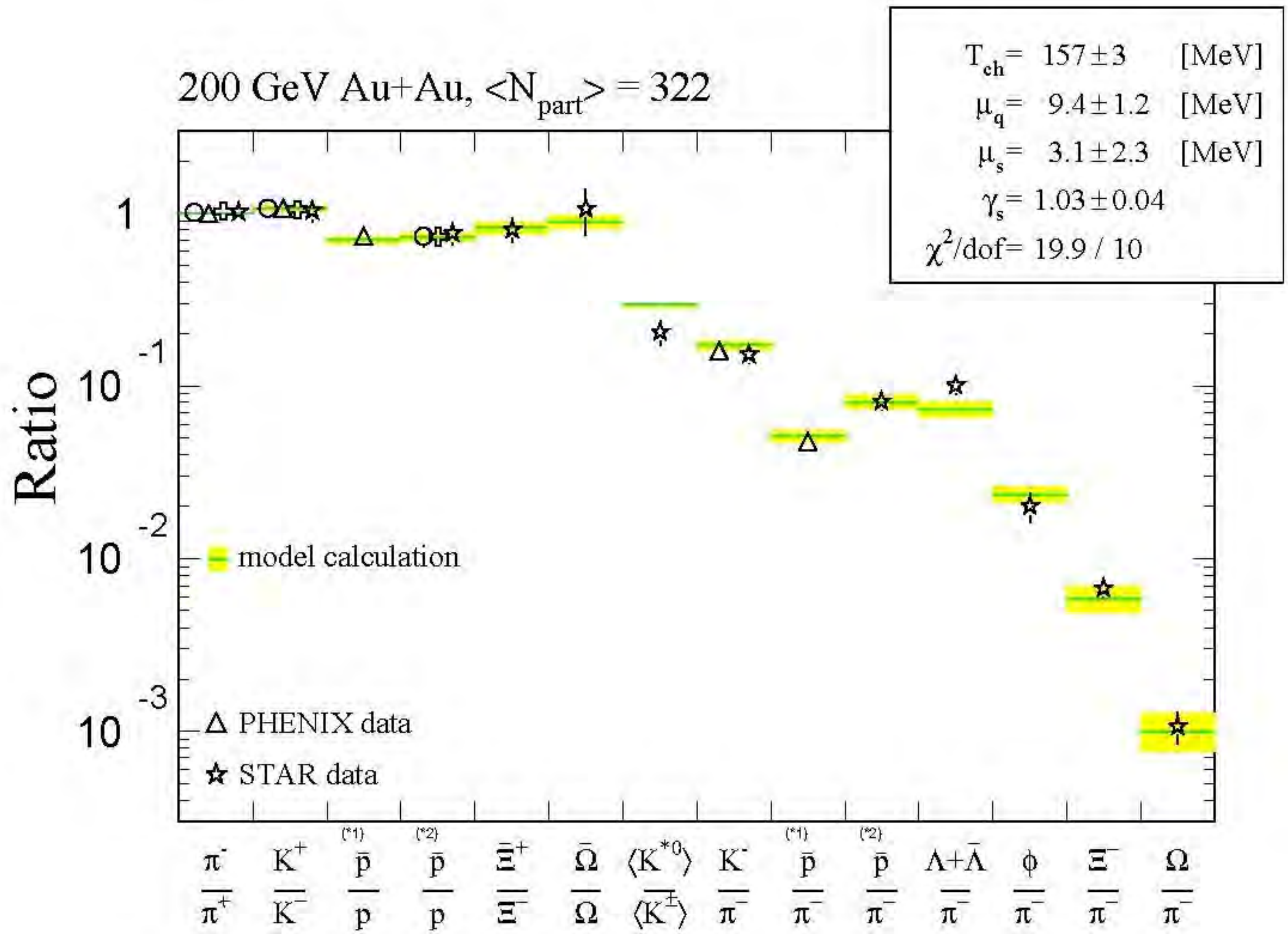
• Each beam nucleon looses 73 ± 6 GeV on the average that goes into creating new particles. Therefore there is 26 TeV worth of energy available for particle production.

Phases of a collision



The “initial” energy density is given by the Bjorken formula

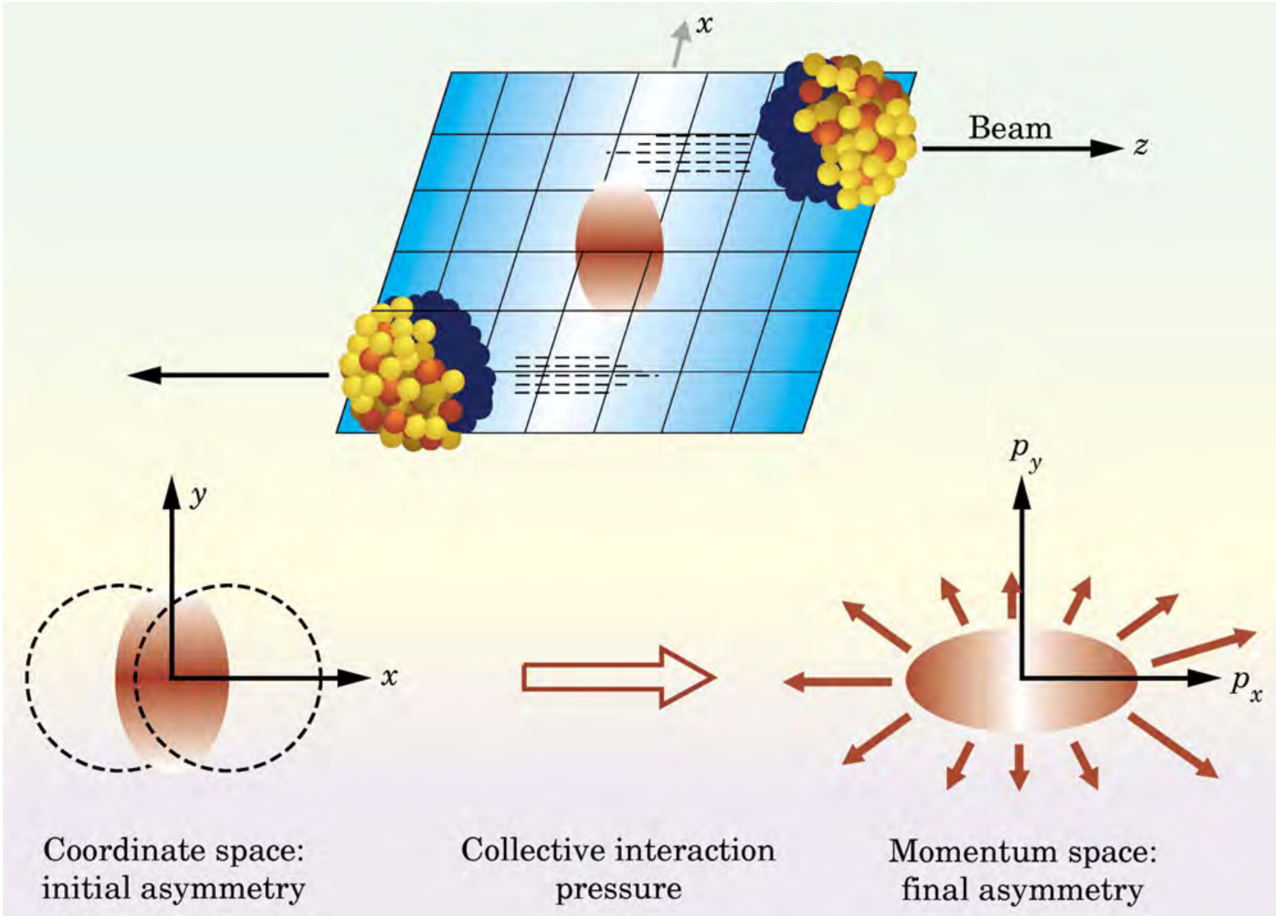
Is there thermal equilibrium?



PHENIX (triangles), STAR(stars), BRAHMS (circles) PHOBOS (crosses) particle ratios, at Au+Au (s=200 GeV) at mid-rapidity vs thermal ensemble predictions.

Ellipticity

Ollitrault, 1992



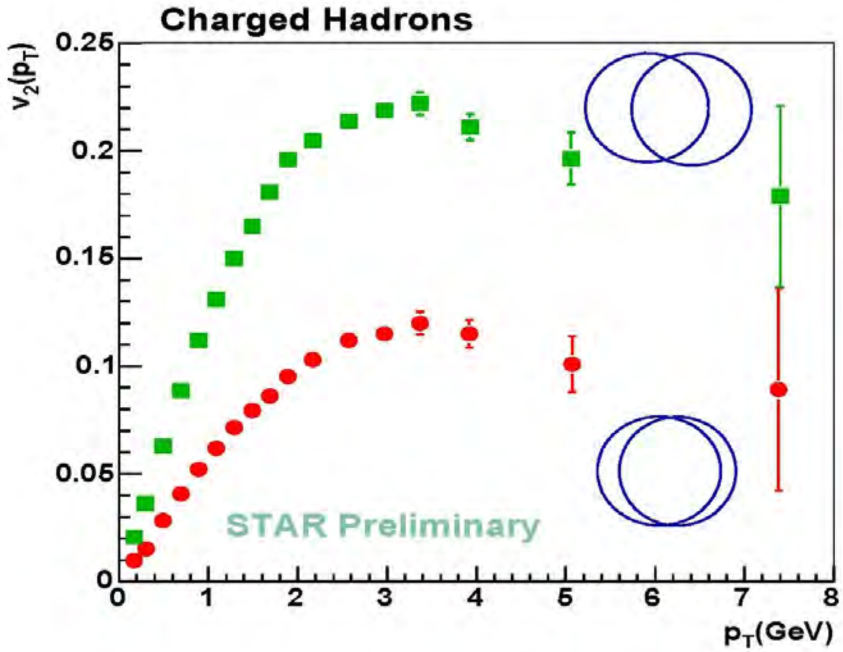
• In an off-center collision, an initial elliptic pattern is produced.

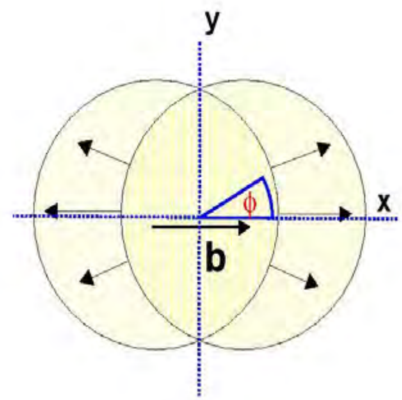
• If the subsequent interactions are weak particles are free streaming and this elliptic pattern is wiped-out

• If the interactions are strong, this pattern persists and is visible in the detectors.

Elliptic flow

$$\frac{1}{p_T}\frac{dN}{dp_T\, d\phi} = \frac{1}{p_T}\frac{dN}{dp_T}\left(1 + 2\, v_2(p_T)\cos(2\,\phi) + ...\right)$$



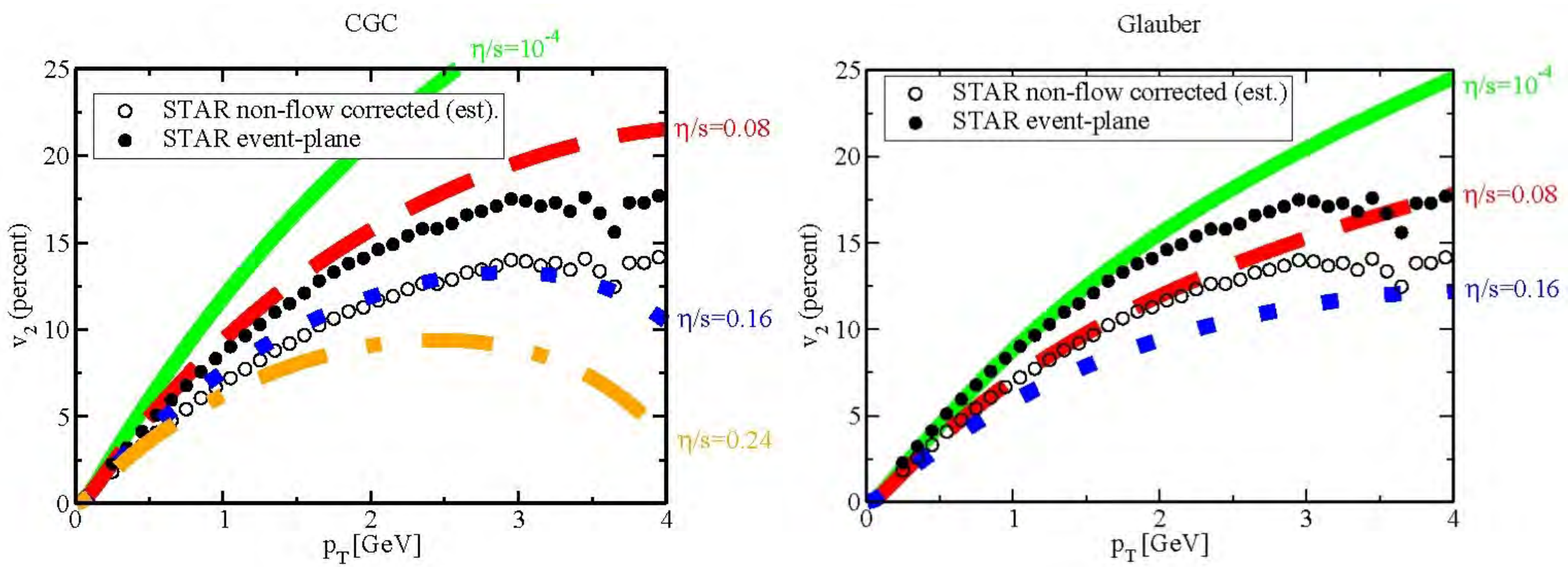


$$X{:}Y = (1 + \underbrace{2v_2}_{\sim 0.4} : 1 - \underbrace{2v_2}_{\sim 0.4})$$

Elliptic flow is large X:Y $\sim 2.0 : 1$

- Such Elliptic flow has been observed recently in strongly coupled cold gases.

Hydrodynamic elliptic flow



Elliptic flow data from STAR as a function of p_T (right) compared to relativistic hydrodynamics calculations with non-zero shear viscosity, from Luzum+Romanschke (2008).

Boost-invariant expansion and AdS/CFT

• Bjorken has guessed correctly in 1983 that a heavy-ion collision will be described in its later stages as a boost invariant expansion of a relativistic fluid: densities will depend only on $\tau^2 = (x^0)^2 - (x^3)^2$.

• Moreover, from scale invariance $\rho = T_{00} \sim \tau^{-\frac{4}{3}}$, instead of the free-streaming option $\rho = T_{00} \sim \tau^{-1}$. This implies that $T \sim \tau^{-\frac{1}{3}}$ and that the entropy remains approximately constant.

• This symmetric late time behavior was first justified by finding it as a (non-singular) solution of the dual gravitational equations corresponding to a bulk black hole where the horizon position shrinks with time.

Janik+Peschanski, 2005

Subleading terms in this gravitational solution indicated the presence of small viscosity.

• Finally it was shown in general that large-wavelength solutions to the AdS Einstein equations generate the relativistic Navier-Stokes equation with an infinite series of "viscous" corrections that generate dissipation.

Bhattacharya+Hubeny+Minwalla+Rangamani, 2008

The Bjorken Relation

• Consider that after the collision of the nuclear pancakes a lot of particles are produced at $t=\tau$. These are confined in a slice of longitudinal width dz and transverse area A.

• The longitudinal velocities have a spread $dv_L = \frac{dz}{\tau}$.

• Near the middle region $v_L \to 0$

$$\frac{dy}{dv_L} = \frac{d}{dv_L}\left[\frac{1}{2}\log\frac{1+v_L}{1-v_L}\right] = \frac{1}{1-v_L^2} \simeq 1$$

• We may now write

$$dN = dv_L\,\frac{dN}{dv_L} \simeq \frac{dz}{\tau}\,\frac{dN}{dy} \quad \to \quad \frac{dN}{dz} \simeq \frac{1}{\tau}\,\frac{dN}{dy}$$

• If $\langle E_T\rangle \simeq \langle m_T\rangle$ is the average energy per particle then the energy density in this area at $t=\tau$ is given by the Bjorken formula:

$$\langle\epsilon(\tau)\rangle \simeq \frac{dN\langle m_T\rangle}{dz\;\;A} = \frac{1}{\tau}\frac{dN}{dy}\frac{\langle m_T\rangle}{A} = \frac{1}{\tau\;\;A}\frac{dE_T^{\text{total}}}{dy}$$

• It is valid if (1) τ can be defined meaningfully (2) The crossing time $\ll \tau$.

RETURN

Glauber initial conditions

- We model the nucleus with the Woods-Saxon density distribution

$$\rho_A(\vec{x}) = \frac{\rho_0}{1 + \exp\left[\frac{(|\vec{x}|-R)}{\chi}\right]}$$

For Au, $A = 197$, $R \simeq 6.4$ fm, $\chi =\simeq 0.54$ fm, and $\int d^3x\rho(\vec{x}) = A$.

- The nuclear thickness function is defined as

$$T_a(x_\perp) = \int_{-\infty}^{\infty} dz \; \rho(\vec{x})$$

- We can calculate the number density of nucleons participating in the collision as

$$n_{\rm part}(x,y,b) = T_A\left(x+\frac{b}{2},y\right)[1-P(x,y)]+(b\to -b) \quad , \quad P(x,y) = 1-\left(1-\frac{\sigma T_A\left(x-\frac{b}{2},y\right)}{A}\right)^A$$

P is the probability of finding at least one nucleon of the second nucleus in position (x,y) and σ is the nucleon-nucleon cross-section. The number density of binary collisions

$$n_{\rm coll}(x,y,b) = \sigma T_A\left(x+\frac{b}{2},y\right) T_A\left(x-\frac{b}{2},y\right)$$

(The two nuclei are at $(b/2,0)$ and $(-b/2,0)$.)

- The centrality is determined by the total number of participating nucleons, $N_{\rm part}(b) = \int d^2x \; n_{\rm part}$ and the initial energy density from

$$\epsilon(\tau=\tau_0,x,y,b) = \text{constant}\cdot n_{\rm coll}(x,y,b)$$

- The constant is fitted to the data.

The Color Glass Condensate initial conditions

- The number density of gluons, produced during the collision of two nuclei is given by

$$\frac{dN}{d^2x_T dY} = \mathcal{N} \int \frac{d^2p_T}{p_T^2} \int^{p_T} d^2k_T \; \alpha_s(k_T)\phi_A(x_1, (\vec{p}_T + \vec{k}_T)^2/4, \vec{x}_T)\phi_A(x_1, (\vec{p}_T - \vec{k}_T)^2/4, \vec{x}_T)$$

$\vec{P}_T$ and Y are the transverse momentum and rapidity of produced gluons, $x_{1,2} = p_T \; e^{\pm Y}/\sqrt{s}$ is the momentum fraction of colliding gluon ladders. $\mathcal{N}$ is fitted to data.

- The gluon distribution function is

$$\phi_A(x, k_T^2, \vec{y}) = \frac{1}{\alpha_s(Q_s^2)} \frac{Q_s^2}{max[Q_s^2, k_T^2]} \; P_T(\vec{y}) \; (1-x)^4$$

and P is the probability of finding at least one nucleon in position $\vec{y}$

$$P_T(\vec{y}) = 1 - \left(1 - \frac{\sigma T_A(\vec{y})}{A}\right)^A$$

- The saturation scale is taken to be

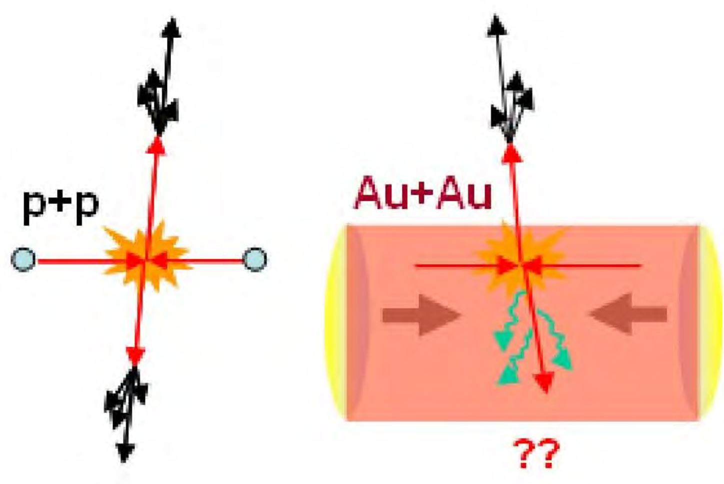
$$Q_s^2(x, \vec{y}) = 2 \text{ GeV}^2 \; \left(\frac{T_A(\vec{y})/P_T(\vec{y})}{1.53/fm^2}\right) \left(\frac{0.01}{x}\right)^\lambda$$

with $\lambda \simeq 0.288$

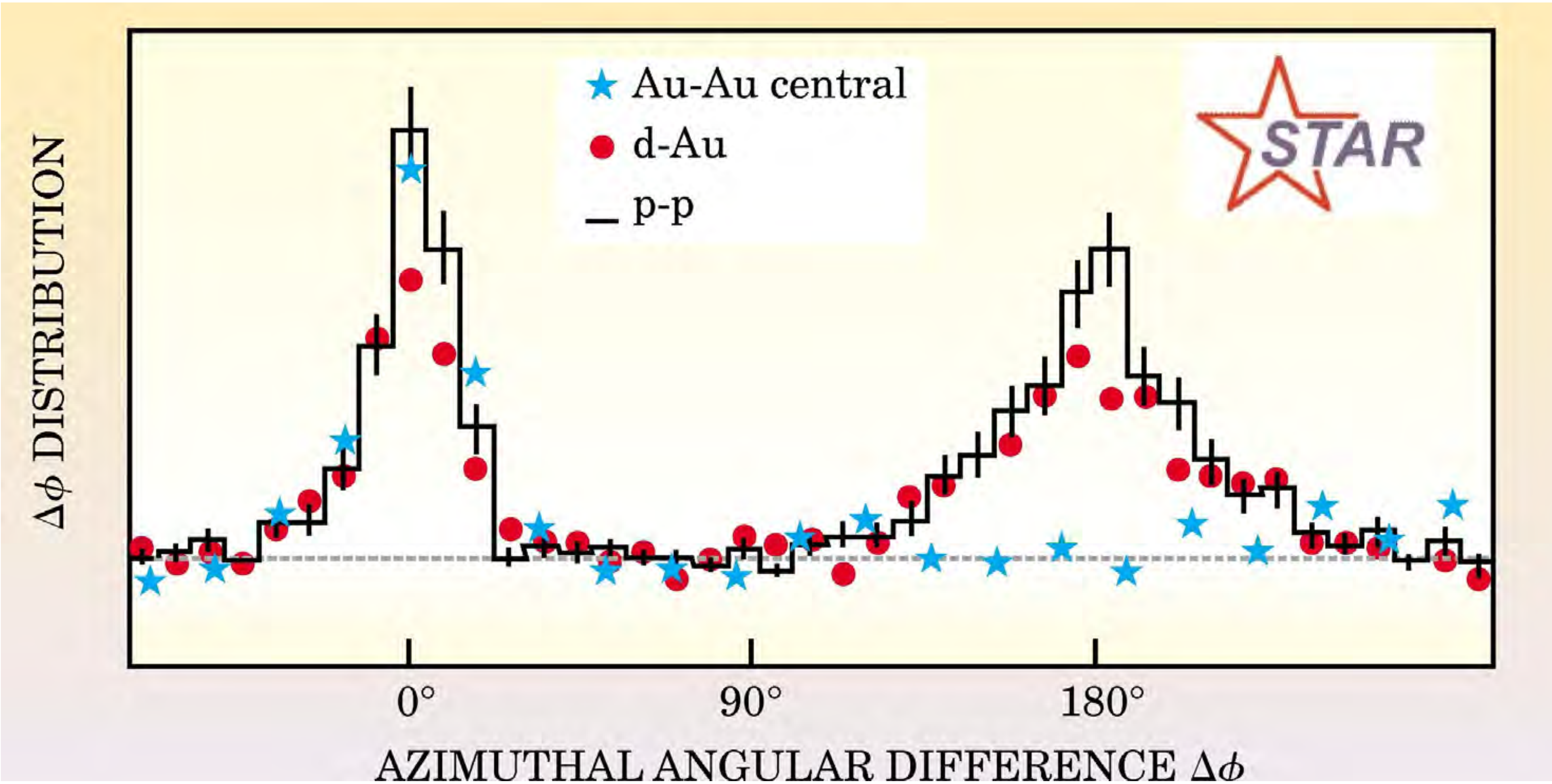
- The initial energy density is given by

$$\epsilon(\tau = \tau_0, \vec{y}, b) = \text{constant} \times \left[\frac{dN}{d^2x_T dY}\right]^{\frac{4}{3}}$$

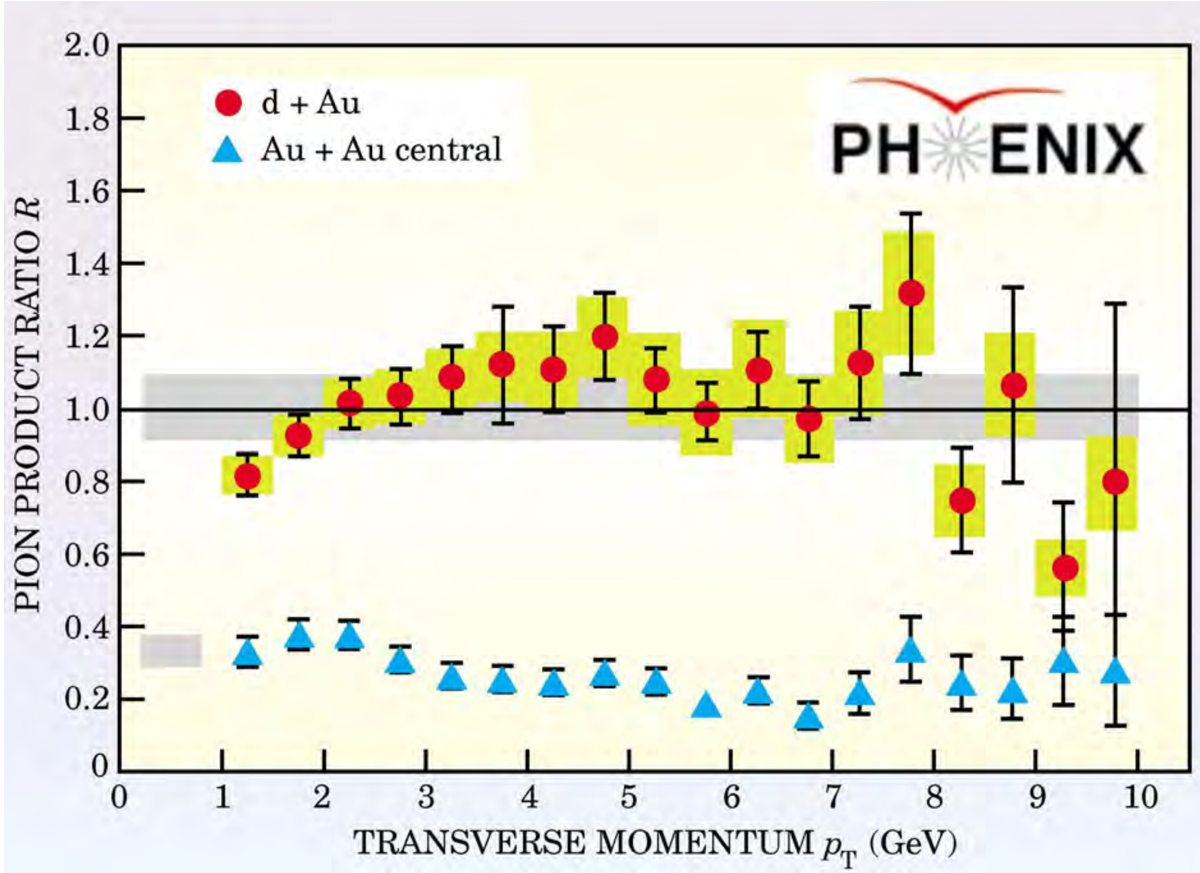
Jet-quenching



- In p-p and in d-Au collisions high-p_T jets appear back-to-back.



- This is not the case in Au-Au central collisions
- This is strong evidence for jet-quenching



- R_{AA} is the ratio of π^0 cross section at mid-rapidity in Au+Au central or d-Au collisions to that in p-p collisions corrected for the multiplicity.

- R_{AA} is small in Au+Au because the medium strongly interacts and reduces the rate of production of pions for the same momentum.

Detailed plan of the presentation

- Meson Spectra 101 minutes
- Mass dependence of f_π 102 minutes
- Linear Regge trajectories 103 minutes
- Fit to data 109 minutes
- Steps Forward 110 minutes
- Numerical solutions :$T = 0$ 112 minutes
- Numerical solutions: Massless with $x < x_c$ 117 minutes
- Comparison to N=1 sQCD 120 minutes
- BKT scaling 125 minutes