FROM RESEARCH TO INDUSTRY

Calculation of beta decays

The most common assumptions and how to go beyond

CEA Saclay – LNHB
CEA Saclay – LNHB

M.-M. Bé, C. Dulieu, M. A. Kellett, X. Mougeot
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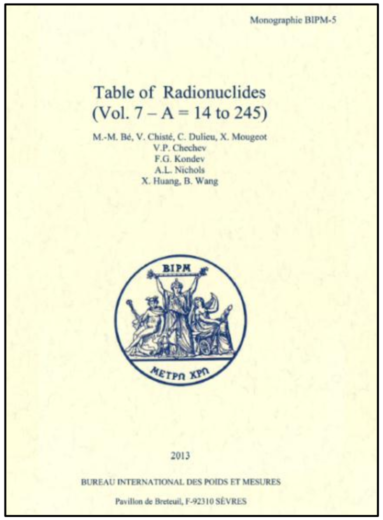
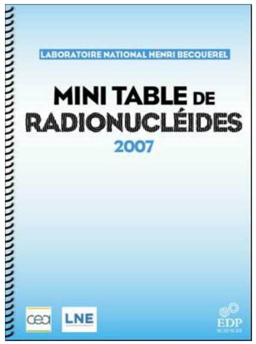
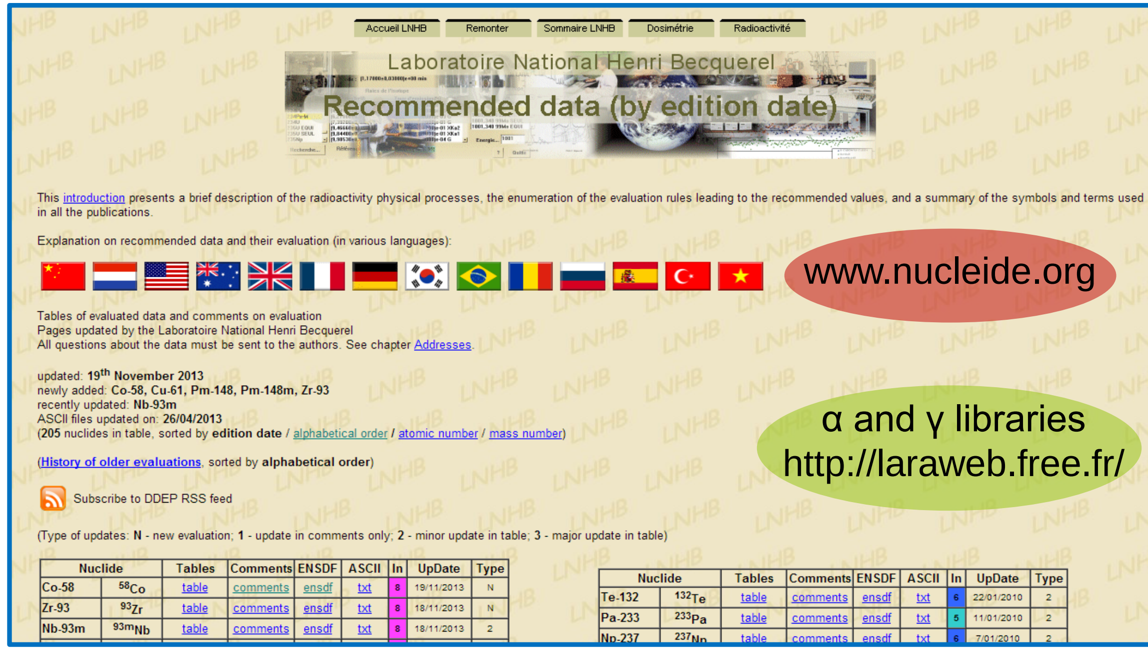
Laboratoire National Henri Becquerel
LNE-LNHB

Atomic and nuclear data evaluations

Decay Data Evaluation Project (**DDEP**)
→ **Recommended** data by **BIPM**

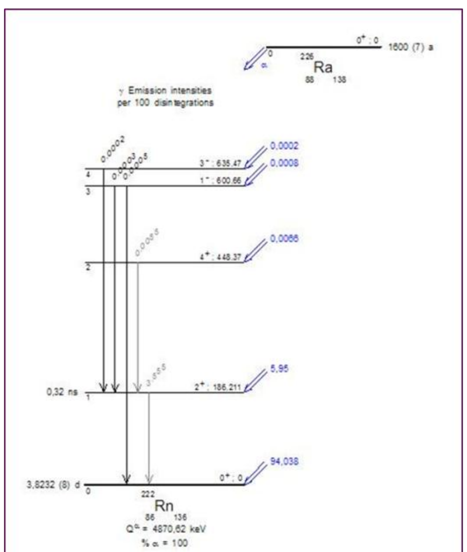






Nuclide		Tables	Comments	ENSDF	ASCII	In	UpDate	Type
Co-58	^{58}Co	table	comments	ensdf	txt	8	19/11/2013	N
Zr-93	^{93}Zr	table	comments	ensdf	txt	8	18/11/2013	N
Nb-93m	^{93m}Nb	table	comments	ensdf	txt	8	18/11/2013	2

Nuclide		Tables	Comments	ENSDF	ASCII	In	UpDate	Type
Te-132	^{132}Te	table	comments	ensdf	txt	6	22/01/2010	2
Pa-233	^{233}Pa	table	comments	ensdf	txt	5	11/01/2010	2
Np-237	^{237}Np	table	comments	ensdf	txt	6	7/01/2010	2



All published data are considered and carefully analyzed *(provided they exist!)*.

Values may be rejected, uncertainties may be expanded.

The decay scheme will be based *as much as possible* **on the available measured data.**

Otherwise, calculation codes are used.

→ log *ft* for β and electron capture processes.

Laboratoire National Henri Becquerel
LNE-LNHB

Our users

Ionizing radiation metrology

Activity measurements by Liquid Scintillation, MetroMRT, MetroRWM

Better knowledge of the β spectra → **better uncertainties**

Medical uses, nuclear fuel cycle

Residual power of nuclear reactors, nuclear waste, internal dosimetry, internal radiotherapy, imaging, etc.

Scientific research

New detectors ($BrLa_3$), monitoring and safeguards applications, fundamental physics (CeLAND, etc.)

IAEA
International Atomic Energy Agency

AIEA (Nuclear Data Section), **DDEP**, **NNDC**

Atomic and nuclear data

> Our users need **a precise knowledge of β spectra**, coupled with **well-established uncertainties**.

FROM RESEARCH TO INDUSTRY

Evaluation of beta spectra shapes

Purpose

Evaluation of the **shapes** of beta spectra

Calculations at LNHB

Short half-lives, multiple beta decays, etc.

BetaShape

+ improvements

X. Mougeot

Measurements at LNHB

Low energies

≲ 700 keV

Metallic magnetic calorimeters

M. Loidl

Medium energies

15 keV – 3 MeV

Si PIPS, Si(Li)

C. Bisch (PhD 2014)
X. Mougeot

Laboratoire National Henri Becquerel

LNE-LNHB

Outlines

Basics of beta decay, the **most common assumptions**

→ **Systematic comparison** with 130 **experimental shape factors**

Recent precise measurements of **^{63}Ni** and **^{241}Pu** beta spectra

→ **Improvements** of the calculation to include **atomic effects**

Basics of beta decay

Behrens & Bühring

Similarly we obtain for the space components

$$\langle p|\mathbf{V}+\mathbf{A}|n\rangle = iu_p^+\gamma_4\gamma_\mu(1+\lambda\gamma_5)u_n = \sqrt{\frac{(W_n+M_n)}{2W_n}}\sqrt{\frac{(W_p+M_p)}{2W_p}}\begin{pmatrix}0 & i\sigma\\ i\sigma & 0\end{pmatrix}\lambda\begin{pmatrix}0&1\\1&0\end{pmatrix}$$

$$\times\left\{\left(\frac{\sigma p}{W_p+M_p}\chi_p^{m'}\right)^+\sigma\chi_n^m+(\chi_p^{m'})^+\sigma\frac{\sigma p}{W_n+M_n}\chi_n^m-\lambda(\chi_p^{m'})^+\sigma\chi_n^m -\lambda\left[\left(\frac{\sigma p}{W_p+M_p}\chi_p^{m'}\right)^+\sigma\frac{\sigma p}{W_n+M_n}\chi_n^m\right]\right\}. \quad (6.38)$$

This equals to

$$\langle p|\mathbf{V}+\mathbf{A}|n\rangle = \sqrt{\frac{(W_n+M_n)}{2W_n}}\sqrt{\frac{(W_p+M_p)}{2W_p}}\left\{(\chi_p^{m'})^+\underbrace{\frac{\sigma\mathbf{p}_p}{W_p+M_p}\sigma}_{\frac{\mathbf{p}_p+i(\sigma\times\mathbf{p}_p)}{W_p+M_p}}\chi_n^m + (\chi_p^{m'})^+\underbrace{\sigma\frac{\sigma\mathbf{p}_n}{W_n+M_n}}_{\frac{\mathbf{p}_n-i(\sigma\times\mathbf{p}_n)}{W_n+M_n}}\chi_n^m-\lambda(\chi_p^{m'})^+\sigma\chi_n^m -\lambda\left[(\chi_p^{m'})^+\underbrace{\frac{\sigma\mathbf{p}_p}{W_p+M_p}\sigma\frac{\sigma\mathbf{p}_n}{W_n+M_n}}_{\frac{-(\mathbf{p}_p\mathbf{p}_n)\sigma+(\sigma\mathbf{p}_p)\mathbf{p}_n+\mathbf{p}_p(\sigma\mathbf{p}_n)-i(\mathbf{p}_p\times\mathbf{p}_n)}{(W_p+M_p)(W_n+M_n)}}\chi_n^m\right]\right\}. \quad (6.39)$$

Finally we obtain for the space components

$$\langle p|\mathbf{V}(0)+\mathbf{A}(0)|n\rangle = \sqrt{\frac{(W_n+M_n)}{2W_n}}\sqrt{\frac{(W_p+M_p)}{2W_p}}\times\left\{\left[\frac{\mathbf{p}_p}{W_p+M_p}+\frac{\mathbf{p}_n}{W_n+M_n}\right](\chi_p^{m'})^+\chi_n^m+(\chi_p^{m'})^+\times\left[\frac{i(\sigma\times\mathbf{p}_p)}{W_p+M_p}-\frac{i(\sigma\times\mathbf{p}_n)}{W_n+M_n}\right]\chi_n^m-\lambda(\chi_p^{m'})^+\sigma\chi_n^m +\lambda\frac{\mathbf{p}_p\mathbf{p}_n}{(W_p+M_p)(W_n+M_n)}\{(\chi_p^{m'})^+\sigma\chi_n^m\}+\lambda\frac{i(\mathbf{p}_p\times\mathbf{p}_n)}{(W_p+M_p)(W_n+M_n)}\times(\chi_p^{m'})^+\chi_n^m-\lambda\left[(\chi_p^{m'})^+\frac{(\sigma\mathbf{p}_p)\mathbf{p}_n+\mathbf{p}_p(\sigma\mathbf{p}_n)}{(W_p+M_p)(W_n+M_n)}\chi_n^m\right]\right\}. \quad (6.40)$$

$$-\frac{i}{2M_A}F_M(q^2)(\mathbf{P}\times\mathbf{q})\sigma-F_S(q^2)q_0+\frac{1}{4(2M_A)^2}F_S(q^2)q_0(\mathbf{P}^2-\mathbf{q}^2) -\frac{i}{2}\frac{1}{(2M_A)^2}F_S(q^2)q_0(\mathbf{P}\times\mathbf{q})\sigma\Big\}\chi^{M_i} \quad (9.15)$$

$$\langle\phi_f(p_f)|A_0(0)|\phi_i(p_i)\rangle = N(\chi^{M_f})^+\Big\{-\frac{1}{2M_A}F_A(q^2)(\mathbf{P}\sigma) -\frac{q_0}{2M_A}F_P(q^2)(\mathbf{q}\sigma)-F_T(q^2)(\mathbf{q}\sigma)+\frac{1}{4}\frac{1}{(2M_A)^2}F_T(q^2) \times[(\mathbf{Pq})(\sigma\mathbf{P}+\sigma\mathbf{q})-(\sigma\mathbf{q})(\mathbf{P}^2-\mathbf{q}^2)]\Big\}\chi^{M_i} \quad (9.16)$$

$$\langle\phi_f(p_f)|\mathbf{V}(0)|\phi_i(p_i)\rangle = N(\chi^{M_f})^+\Big\{\frac{1}{2M_A}F_V(q^2)\mathbf{P}+\frac{i}{2M_A}F_V(q^2)(\sigma\times\mathbf{q}) +iF_M(q^2)(\sigma\times\mathbf{q})-\frac{1}{2M_A}F_M(q^2)q_0\mathbf{q}-\frac{i}{4M_A}F_M(q^2)q_0(\sigma\times\mathbf{P}) -F_S(q^2)\mathbf{q}+\frac{1}{4(2M_A)^2}F_S(q^2)\mathbf{q}(\mathbf{P}^2-\mathbf{q}^2)-\frac{i}{2(2M_A)^2}F_S(q^2)\mathbf{q} \times((\mathbf{P}\times\mathbf{q})\sigma)-\frac{i}{2(2M_A)^2}F_M(q^2)\mathbf{P}((\mathbf{P}\times\mathbf{q})\sigma)-\frac{i}{4(2M_A)^2} \times F_M(q^2)(\mathbf{P}^2+\mathbf{q}^2)(\sigma\times\mathbf{q})+\frac{i}{2(2M_A)^2}F_M(q^2)(\mathbf{Pq})(\sigma\times\mathbf{P})\Big\}\chi^{M_i} \quad (9.17)$$

$$\langle\phi_f(p_f)|\mathbf{A}(0)|\phi_i(p_i)\rangle = N(\chi^{M_f})^+\Big\{-F_A(q^2)\sigma+\frac{1}{2(2M_A)^2} \times F_A(q^2)\mathbf{P}^2\sigma-\frac{1}{4(2M_A)^2}F_A(q^2)(\mathbf{P}^2+\mathbf{q}^2)\sigma-\frac{i}{2(2M_A)^2} \times F_A(q^2)(\mathbf{P}\times\mathbf{q})-\frac{1}{2(2M_A)^2}F_A(q^2)[(\sigma\mathbf{P})\mathbf{P}-(\sigma\mathbf{q})\mathbf{q}] +\frac{1}{2M_A}F_T(q^2)[(\mathbf{Pp})\sigma-\mathbf{q}(\sigma\mathbf{P})]-F_T(q^2)q_0\sigma +\frac{1}{2(2M_A)^2}F_T(q^2)q_0\mathbf{P}^2\sigma-\frac{1}{4(2M_A)^2}F_T(q^2)q_0(\mathbf{P}^2+\mathbf{q}^2)\sigma -\frac{i}{2(2M_A)^2}F_T(q^2)q_0(\mathbf{P}\times\mathbf{q})-\frac{1}{2(2M_A)^2}F_T(q^2)q_0[(\sigma\mathbf{P})\mathbf{P} -(\sigma\mathbf{q})\mathbf{q}]-\frac{1}{2M_A}F_P(q^2)(\sigma\mathbf{q})\mathbf{q}\Big\}\chi^{M_i}. \quad (9.18)$$

$$+\sqrt{\tfrac{2}{3}}\left(\int rI'(r)\beta\gamma_5T_{121}\right)\Big\} \mp\frac{f_P}{R}(W_0R\pm\tfrac{6}{5}\alpha Z)\,{}^D\mathfrak{N}^{(0)}_{110}(1,1,1,1) \quad (14.101)$$

$${}^AF^{(0)}_{121}=\mp\lambda\,{}^A\mathfrak{M}^{(0)}_{121}-\frac{f_T}{R}\left[\frac{5}{\sqrt3}\,{}^C\mathfrak{N}^{(0)}_{111}-(W_0R\pm\tfrac65\alpha Z)\,{}^A\mathfrak{M}^{(0)}_{121}\right]\mp\frac{f_P}{R}5\sqrt{\tfrac23}\,{}^D\mathfrak{N}^{(0)}_{110} \quad (14.102)$$

$${}^AF^{(0)}_{121}(1,1,1,1)=\mp\lambda\,{}^A\mathfrak{M}^{(0)}_{121}(1,1,1,1) -\frac{f_T}{R}\left\{\sqrt{\tfrac13}\left(\int\left(\frac rR\right)[5I(r)+rI'(r)]\beta T_{111}\right) -(W_0R\pm\tfrac65\alpha Z)\,{}^A\mathfrak{M}^{(0)}_{121}(1,1,1,1)\right\} \mp\frac{f_P}{R}\sqrt{\tfrac23}\left(\int\left(\frac rR\right)[5I(r)+rI'(r)]\beta\gamma_5T_{110}\right) \quad (14.103)$$

$${}^VF^{(0)}_{211}=-{}^V\mathfrak{M}^{(0)}_{211}-\frac{f_M}{R}(W_0R\pm\tfrac65\alpha Z)\,{}^C\mathfrak{N}^{(0)}_{211} \quad (14.104)$$

$${}^VF^{(0)}_{220}={}^V\mathfrak{M}^{(0)}_{220}+\frac{f_M}{R}\sqrt{(10)}\,{}^C\mathfrak{N}^{(0)}_{211}\pm\frac{f_S}{R}(W_0R\pm\tfrac65\alpha Z)\,{}^V\mathfrak{M}^{(0)}_{220} \quad (14.105)$$

$${}^VF^{(0)}_{220}(1,1,1,1)={}^V\mathfrak{M}^{(0)}_{220}(1,1,1,1) +\frac{f_M}{R}\left\{\sqrt{\tfrac25}\left(\int\left(\frac rR\right)[5I(r)+rI'(r)]\beta T_{211}\right) +\sqrt{\tfrac35}\left(\int\left(\frac rR\right)rI'(r)\beta T_{231}\right)\right\} \pm\frac{f_S}{R}(W_0R\pm\tfrac65\alpha Z)\,{}^V\mathfrak{M}^{(0)}_{220}(1,1,1,1) \quad (14.106)$$

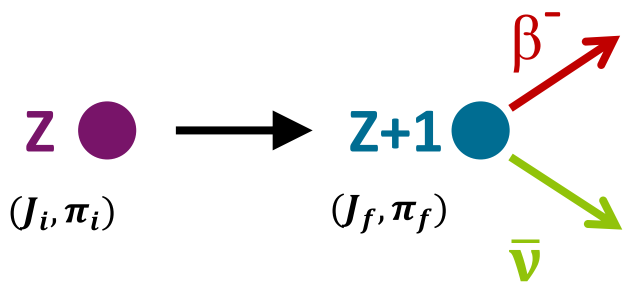
$${}^AF^{(0)}_{221}=\pm\lambda\,{}^A\mathfrak{M}^{(0)}_{221}+\frac{f_T}{R}[\sqrt{(15)}\,{}^C\mathfrak{N}^{(0)}_{211}-(W_0R\pm\tfrac65\alpha Z)\,{}^A\mathfrak{M}^{(0)}_{221}] \quad (14.107)$$

$${}^AF^{(0)}_{221}(1,1,1,1)=\pm\lambda\,{}^A\mathfrak{M}^{(0)}_{221}(1,1,1,1) +\frac{f_T}{R}\left\{\sqrt{\tfrac35}\left(\int\left(\frac rR\right)[5I(r)+rI'(r)]\beta T_{211}\right) -\sqrt{\tfrac25}\left(\int\left(\frac rR\right)rI'(r)\beta T_{231}\right) -(W_0R\pm\tfrac65\alpha Z)\,{}^A\mathfrak{M}^{(0)}_{221}(1,1,1,1)\right\} \quad (14.108)$$

H. Behrens, W. Bühring, *Electron Radial Wave functions and Nuclear Beta Decay*, Oxford Science Publications (1982)

More than 600 p.!

Basics of beta decay



ΔJ	$\pi_i \pi_f$	Classification
$0, 1$	1	Allowed
$0, 1$	-1	1st fnu
> 1	$(-1)^{\lvert\Delta J\rvert}$	$\lvert\Delta J\rvert^{th}$ fnu
> 1	$(-1)^{\lvert\Delta J\rvert - 1}$	$(\lvert\Delta J\rvert - 1)^{th}$ fu

$\Delta J = \lvert J_f - J_i \rvert$

fnu: forbidden non-unique

fu: forbidden unique

Electroweak interaction

$$M_{W+,W-,Z0} \sim 80 \text{ GeV}$$
$$E_{\max}(\beta) \lesssim 50 \text{ MeV}$$

Fermi: **4 particles interacting at one vertex**

Free neutron decay

$$H_\beta = \frac{G_\beta}{\sqrt{2}} \left[\overline{\psi}_p \gamma_\mu (1 + \lambda \gamma_5) \psi_n \right.$$ nuclear current

$$\left. \times \overline{\psi}_e \gamma_\mu (1 + \gamma_5) \psi_\nu + \text{h.c.} \right]$$ leptonic current

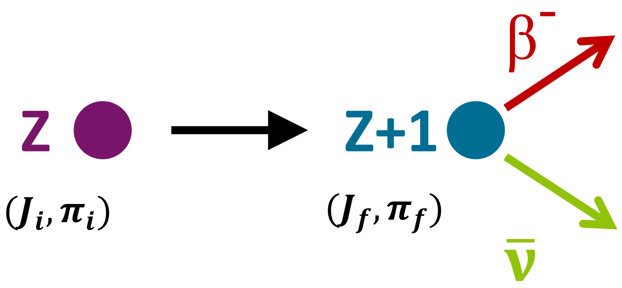
Neutrino

$m_{\bar{\nu}} \sim 0 \rightarrow$ β spectrum very poorly modified, in the endpoint region

Nucleus

- **Point charge**, **spherical symmetry** → no deformation of the nucleus
- $M_{\text{nucleus}} \sim \infty \rightarrow E_{\text{recoil}} \sim 0$

BetaShape (analytical)



Beta spectrum $\frac{dN}{dW} \propto \boxed{p\ W\ q^2}\ \boxed{F_0 L_0}\ \boxed{C(W)}$

Coulomb part (Fermi function) — $F_0 L_0$

Phase space — $p\ W\ q^2$

Shape factor — $C(W)$

Nuclear current can be **factored out** for **allowed** and **forbidden unique** transitions

$$C(W) = (2L-1)! \sum_{k=1}^{L} \lambda_k \frac{p^{2(k-1)}\ q^{2(L-k)}}{(2k-1)![2(L-k)+1]!}$$

$$L = 1 \text{ if } \Delta J = 0$$
$$L = \Delta J \text{ otherwise}$$

$\lambda_k = 1?$

Forbidden **non-unique** transitions calculated according to the **ξ approximation**

if $2\xi = \alpha Z/R \gg E_{max}$

1st fnu	$\rightarrow$	allowed
2nd fnu	$\rightarrow$	1st fu
3rd fnu	$\rightarrow$	2nd fu

Assumptions → Corrections

- **Screening**: $W \rightarrow W \pm V_0(\beta^\pm)$, Thomas-Fermi potential such as $V_0(Z)$ only
- **Finite nuclear size**
- **Radiative corrections** (virtual photons, internal bremsstrahlung)

X. Mougeot *et al.*, Proceedings of the LSC2010 International Conference, Paris, France, p. 249 (2010)

Systematic comparison

Small database of **130 experimental shape factors**

- Allowed: 36
- Forbidden unique: 25 (1^{st}), 4 (2^{nd}), 1 (3^{rd})
- Forbidden non-unique: 53 (1^{st}), 9 (2^{nd}), 1 (3^{rd}), 1(4^{th})

But almost comprehensive!

→ Very few measurements below 50 keV (7)

→ Very few transitions of high forbidding order

→ 10 published shape factors since 1976!

This study will be published within the proceedings of the ICRM 2015 conference.

Results

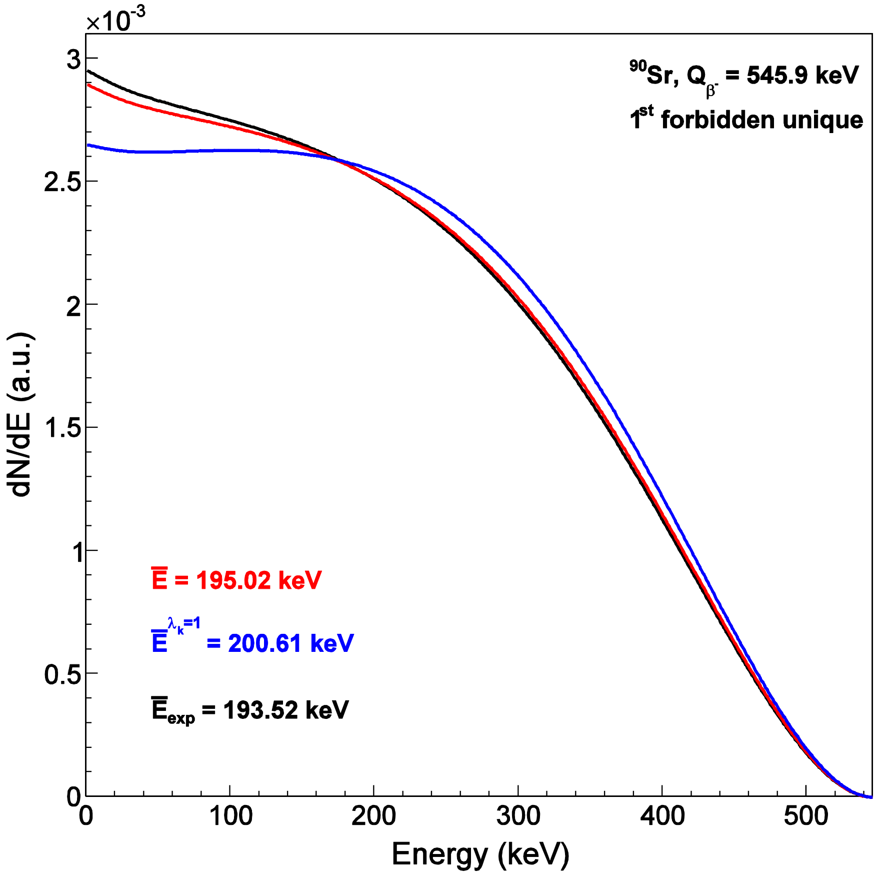
→ $\lambda_k = 1$ is generally a bad approximation

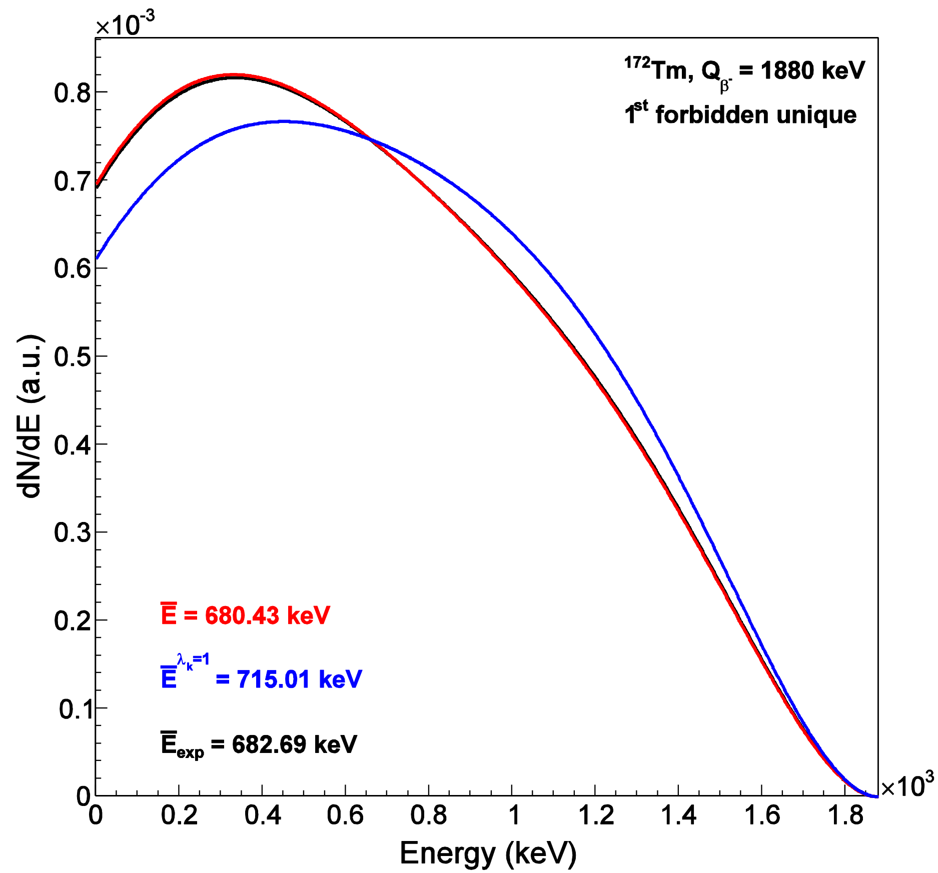
→ Allowed and forbidden unique spectra are generally reproduced well

→ ξ approximation is correct **only** for ~ 50 % of the 1^{st} forbidden non-unique transitions, and **incorrect** for all other non-unique transitions

New measurements are needed to test the theoretical predictions

$\lambda_k = 1$ approximation





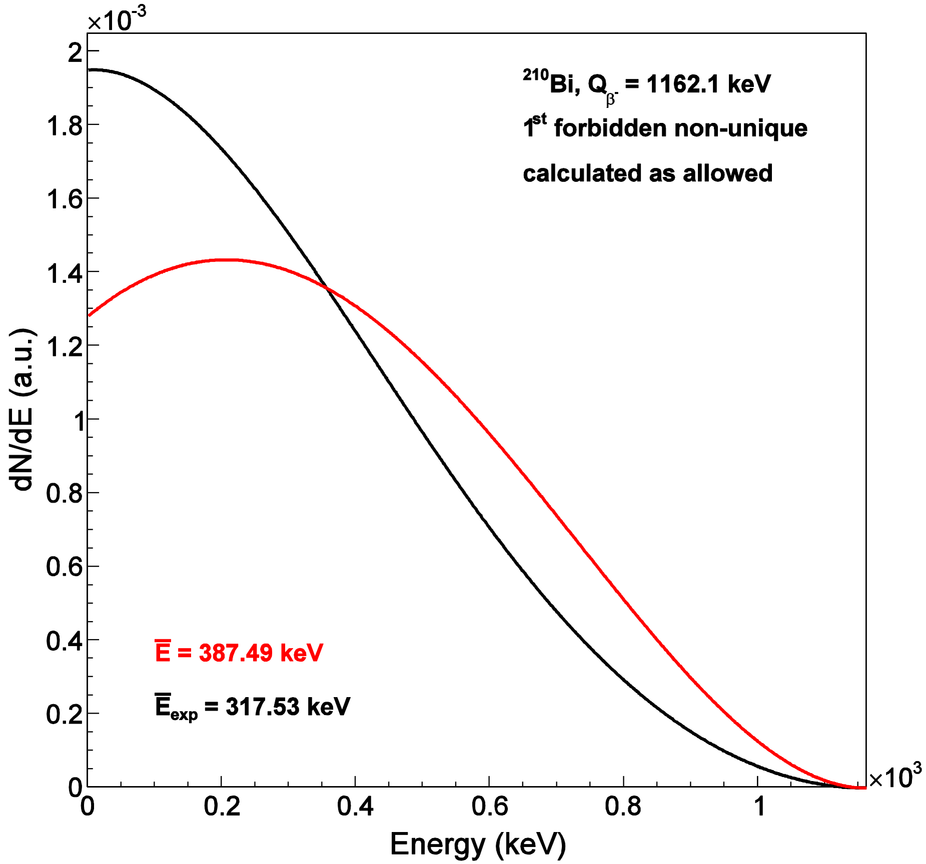
Mean energy disagrees by **3.6 %**

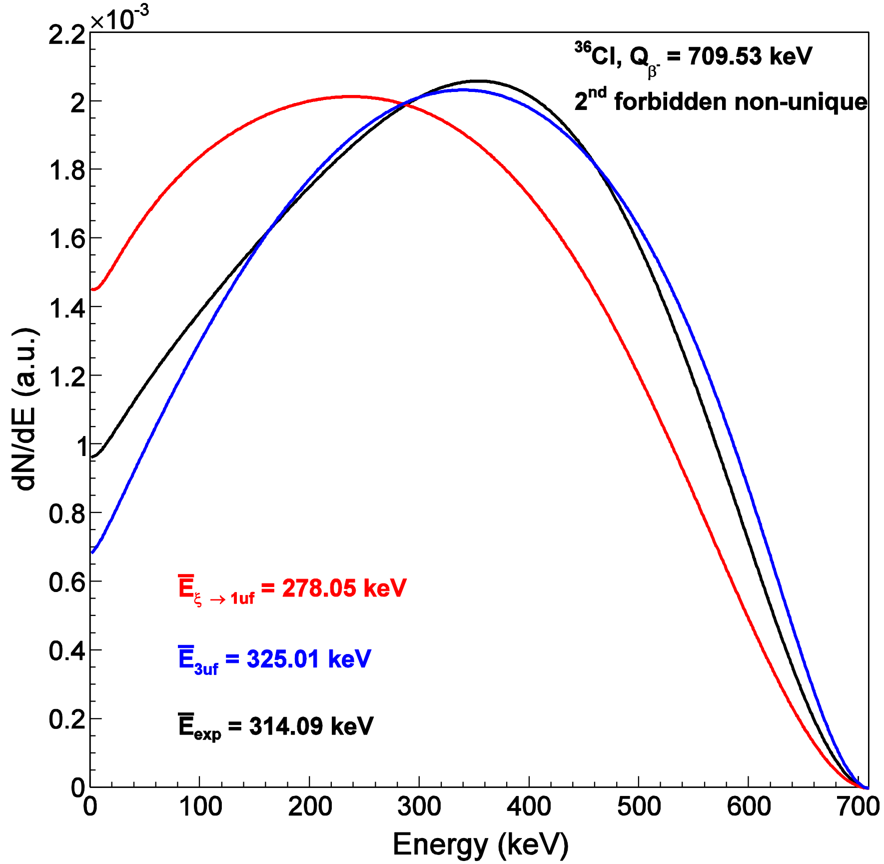
High influence at low energy

Mean energy disagrees by **4.6 %**

High influence at low energy and on the overall shape of the spectrum

ξ approximation





Calculated as **allowed**, this spectrum **is not correct**

Mean energy disagrees by **20 %** (!)

Calculated as **1st forbidden unique**, this spectrum **is not correct**

Mean energy disagrees by **14 %** (!)

Better as **3rd forbidden unique** → **justification?**

Atomic effects

Impact of low energy electrons

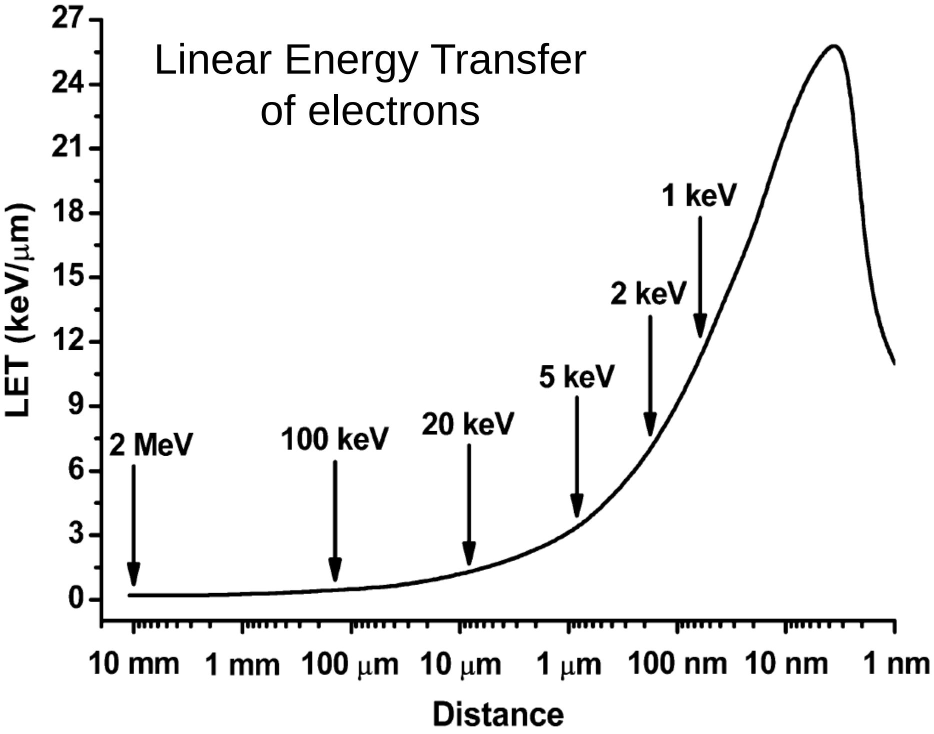
- **Activity measurements** using **Liquid Scintillation Counting** technique

R. Broda, P. Cassette, K. Kossert, Metrologia 44, S36-S52 (2007)

- Evaluation of the **dose deposited** in patient's cells

M. Bardiès, J.-F. Chatal, Phys. Med. Biol. 39, 961-981 (1994)

→ *e.g.* 1 electron of 2 keV
≠
2 electrons of 1 keV



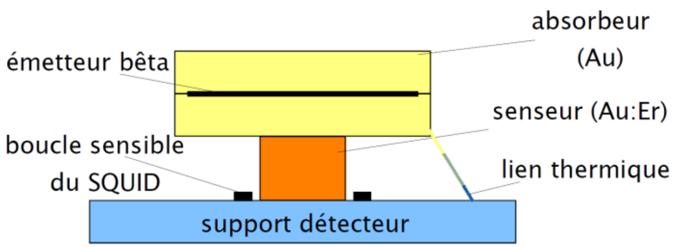
A.I. Kassis, Int. J. Radiat. Biol. 80, 789 (2004)

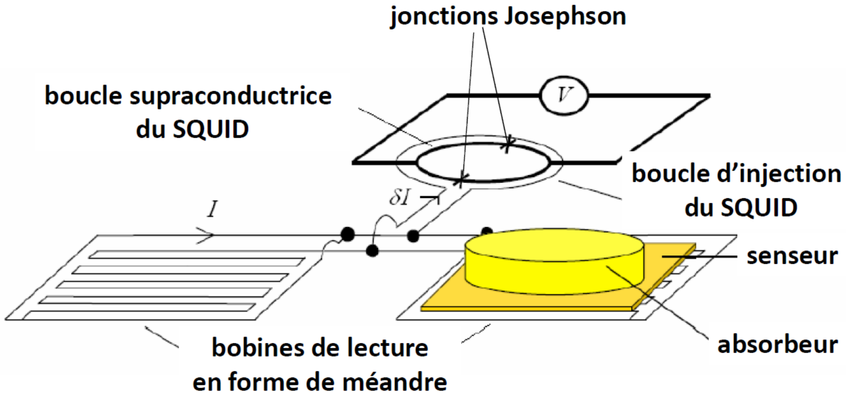
Precise knowledge of beta spectra shapes is needed at low energy

Laboratoire National Henri Becquerel
LNE-LNHB

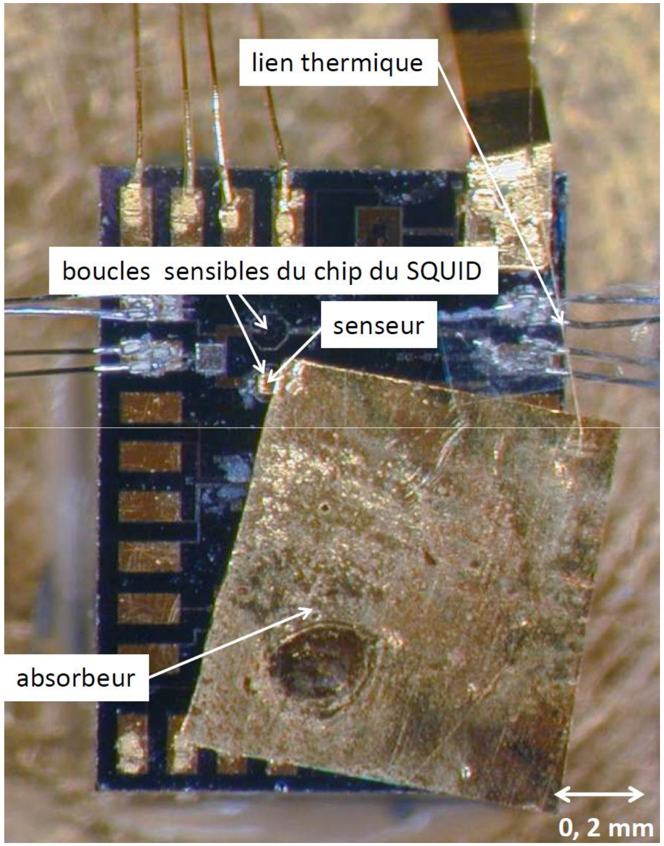
Metallic magnetic calorimeters

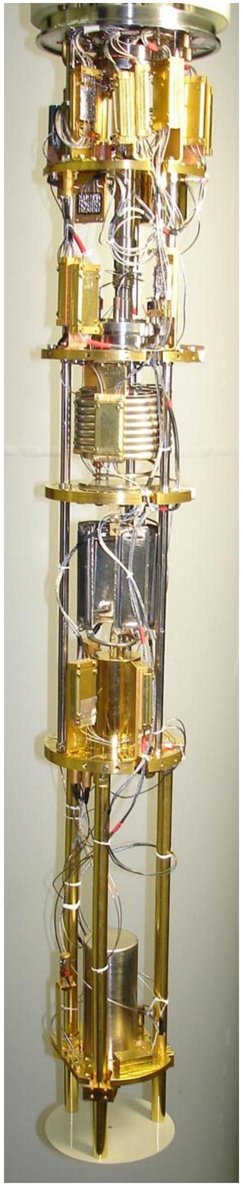
Direct magnetic coupling





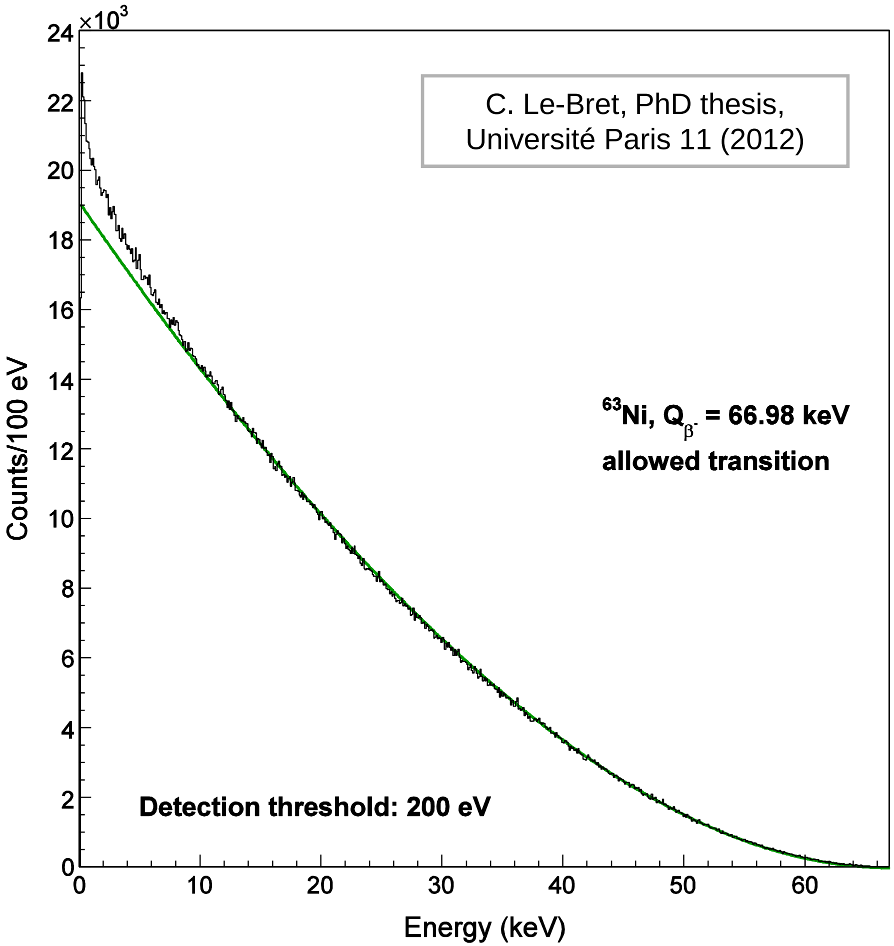
Indirect magnetic coupling

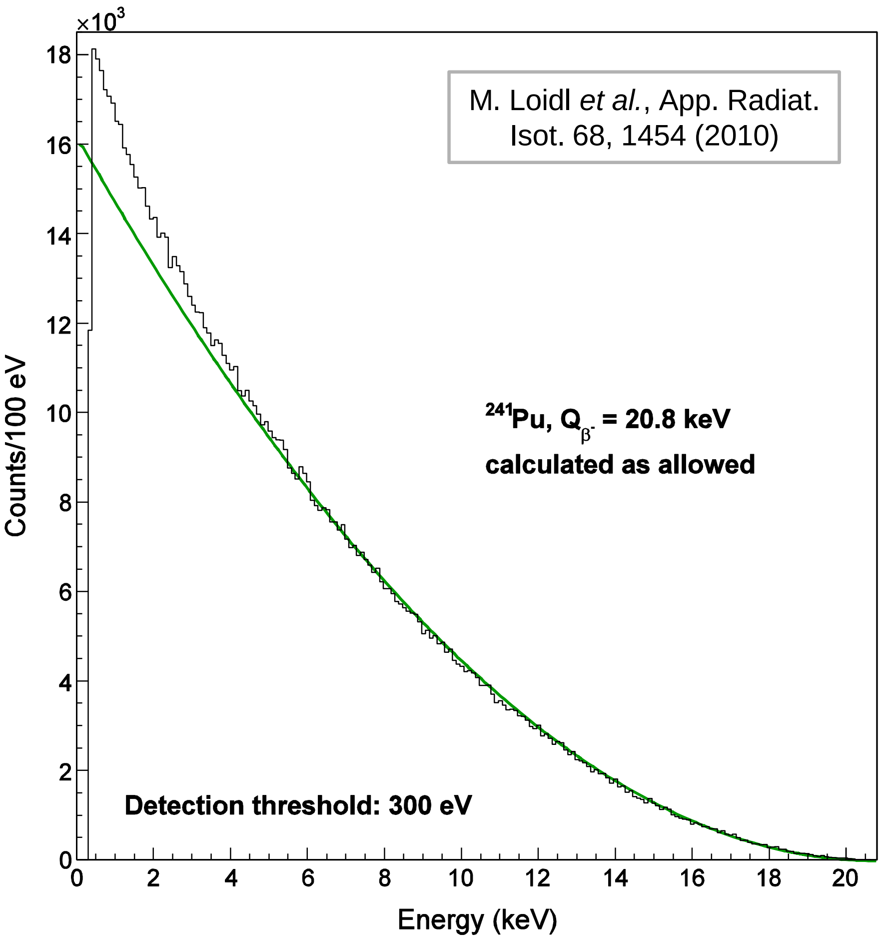




System cooled down to 10 mK

^{63}Ni and ^{241}Pu beta spectra from MMC





Classical beta calculations fail to reproduce these “simple” spectra

1st forbidden non-unique transition calculated as **allowed**

$2\xi = \alpha Z/R \gg E_0$ = 20.8 keV ≪ 19.8 MeV

The atomic exchange effect

N.C. Pyper, M.R. Harston, Proc. Roy. Soc. Lond. A 420, 277 (1988)

X. Mougeot *et al.*, Phys. Rev. A 86, 042506 (2012)

First work using analytical wave functions

Atomic exchange effect

→ Indistinguishable from the direct decay to a final continuum state

→ Depends on the **overlap of the continuum and bound electron wave functions**

→ **Allowed transitions**: only the ***ns* orbitals** are reachable

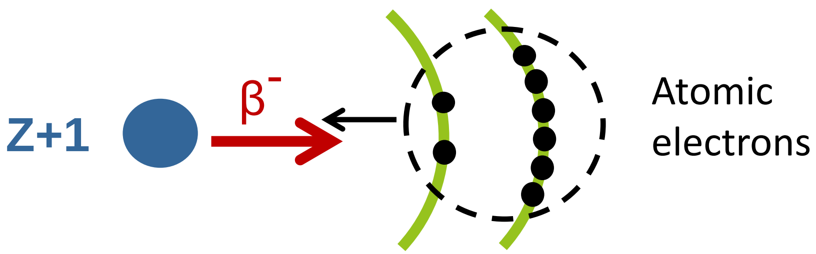
Spectrum correction factor $\left[1 + \eta_{ex}^{T}(E)\right]$

Total exchange factor $$\eta_{ex}^{T}(E) = \sum_{n} \eta_{ex}^{ns}(E) + \sum_{\substack{m,n \\ (m \neq n)}} \mu_m \mu_n$$

Subshell contribution $$\eta_{ex}^{ns}(E) = f\left(\mu_n^2 - 2\mu_n\right)$$

with $$\mu_n = \langle Es' | ns \rangle \frac{g_{n,\kappa}^{b}(R)}{g_{\kappa}^{c}(R)}, \quad f = \frac{g_{\kappa}^{c}(R)^2}{g_{\kappa}^{c}(R)^2 + f_{\kappa}^{c}(R)^2}$$

A new screening correction



H. Behrens, W. Bühring, *Electron Radial Wave functions and Nuclear Beta Decay*, Oxford Science Publications (1982)

Screening

Generally corrected for using a constant Thomas-Fermi potential, which creates a **non physical discontinuity** in the spectrum.

Evaluating the **wave functions at the nuclear surface** cannot provide a good result because of the **weakness of the screened potentials** in this region.

→ Implementation of a **new screening correction** which:

- avoids complete calculation of leptonic and nuclear matrix elements
- is available only for allowed transitions up-to-now

$$C_{sc} = 1 - \frac{\Delta R_{unsc}}{\Delta R_{sc}} \cdot \left(1 - \frac{f_{sc}}{f_{unsc}}\right)$$

exchange formalism
→ f factor

$$f^{-1} = \frac{\overline{g^2_{-1}}}{\overline{g^2_{-1}} + \overline{f^2_{-1}}}$$

mean value
→ spatial extension

$$\overline{g^2_\kappa} = \frac{1}{\Delta R} \int_{\Delta R} g^2_\kappa(r)\,dr$$

Electron wave functions

Electron wave function

→ spherical symmetry

$$\Psi(\vec{r}) = \begin{pmatrix} S_\kappa f_\kappa(r)\chi^\mu_{-\kappa} \\ g_\kappa(r)\chi^\mu_\kappa \end{pmatrix}$$

Radial component

Spin-angular functions → spherical harmonics expansion

Dirac equation

→ system of coupled differential equations

$$\begin{cases} \dfrac{\mathrm{d}f_\kappa}{\mathrm{d}r} = \dfrac{(\kappa-1)}{r} f_\kappa - [W - 1 - V(r)] g_\kappa \\ \dfrac{\mathrm{d}g_\kappa}{\mathrm{d}r} = [W + 1 - V(r)] f_\kappa - \dfrac{(\kappa+1)}{r} g_\kappa \end{cases}$$

Power series expansion (exact solutions)

regular singular $r = 0$ ⟶ ordinary $r = r_0$ ⟶ irregular singular $r = \infty$

$$\begin{Bmatrix} f(r) \\ g(r) \end{Bmatrix} = \frac{(pr)^{k-1}}{(2k-1)!!} \sum_{n=0}^{\infty} \begin{Bmatrix} a_n \\ b_n \end{Bmatrix} r^n$$

$$\begin{Bmatrix} rf(r) \\ rg(r) \end{Bmatrix} = \sum_{n=0}^{\infty} \begin{Bmatrix} a_n \\ b_n \end{Bmatrix} (r - r_0)^n$$

$$\begin{Bmatrix} f_{\infty j}(r) \\ g_{\infty j}(r) \end{Bmatrix} = \frac{r^{-1+yt_0/p}}{W(W+1)} e^{t_0 r} \sum_{n=0}^{\infty} \begin{Bmatrix} a_n \\ b_n \end{Bmatrix} r^{-n}$$

H. Behrens, W. Bühring, *Electron Radial Wave functions and Nuclear Beta Decay*, Oxford Science Publications (1982)

Coulomb potential

Point charge | **Exchange potential**

$$V_{ex}(r) = K\,\frac{3\alpha}{2}\left(\frac{3Z}{4\pi^2}\right)^{1/3}\left[\frac{1}{r}\frac{\mathrm{d}^2\phi(r)}{\mathrm{d}r^2}\right]^{1/3}$$

for atomic electrons (**fermions**)

Global potential

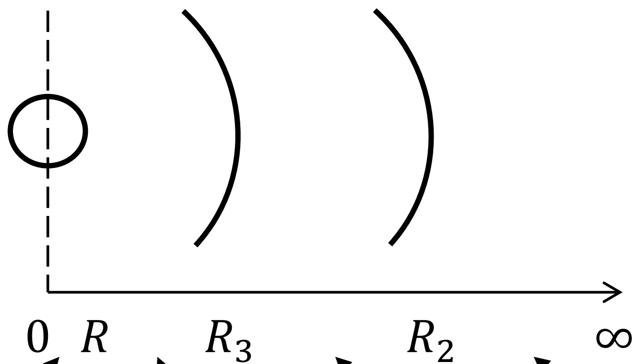
$$V(r) = -\frac{\alpha Z}{r}\phi(r) - V_{ex}(r)$$

Screened potential

$$\phi(r) = \sum_{i=1}^{N} a_i \mathrm{e}^{-\beta_i r}$$

F. Salvat *et al.*, Phys. Rev. A 36, 467 (1987)

Space subdivision



Finite nuclear size, quadratic behavior

Global potential

2^{nd} order polynomial for reconnection

Asymptotic solutions

A power series expansion of the global potential is required.

Numerical procedure

- For bound states, the **orbital energy** is **not known** in advance
  → **iterative procedure**
- **Orbital energy** → Oscillation frequency of the wave functions
  → **Accuracy** of the **overlap**

⟹ **Adjustment** of V_{ex} to reach the "good" energies in

J.P. Desclaux, At. Data Nucl. Data Tab. 12, 311 (1973)

Inspection

Useful **tabulated parameters** for β spectra, electron capture, electron polarization, β-γ angular correlation, etc.

Tabulated **screened** parameters $F_0L_0^*/F_0L_0$ and λ_2^*/λ_2 **only**, but for **very few energies**

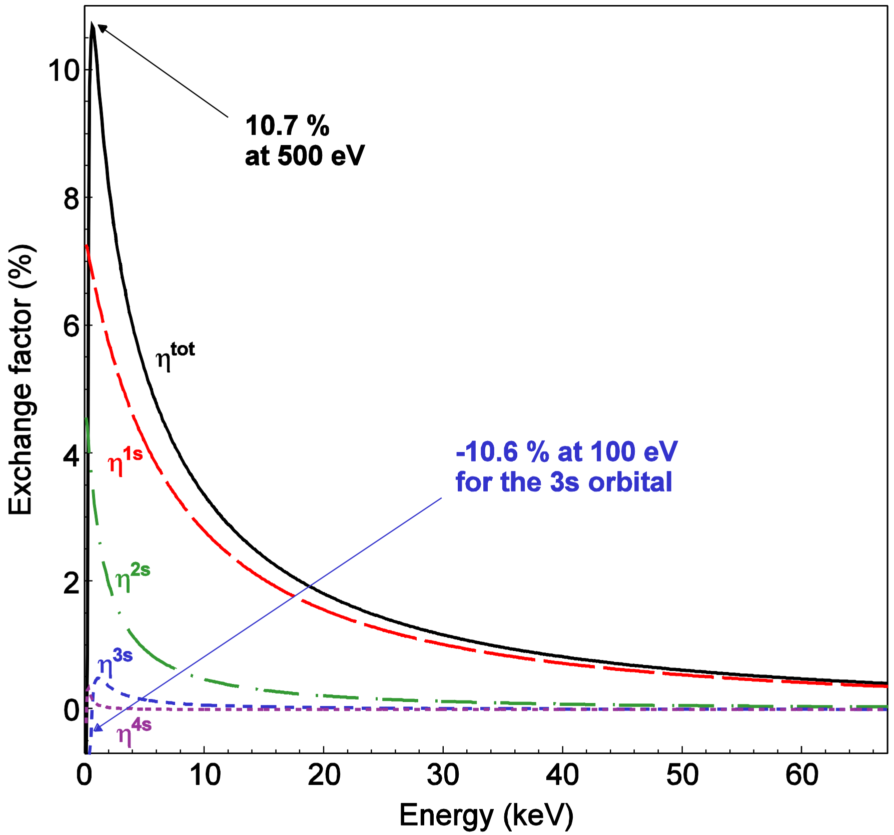
H. Behrens, J. Jänecke, Landolt-Börnstein, New Series, Group I, vol. 4, Springer Verlag, Berlin (1969)

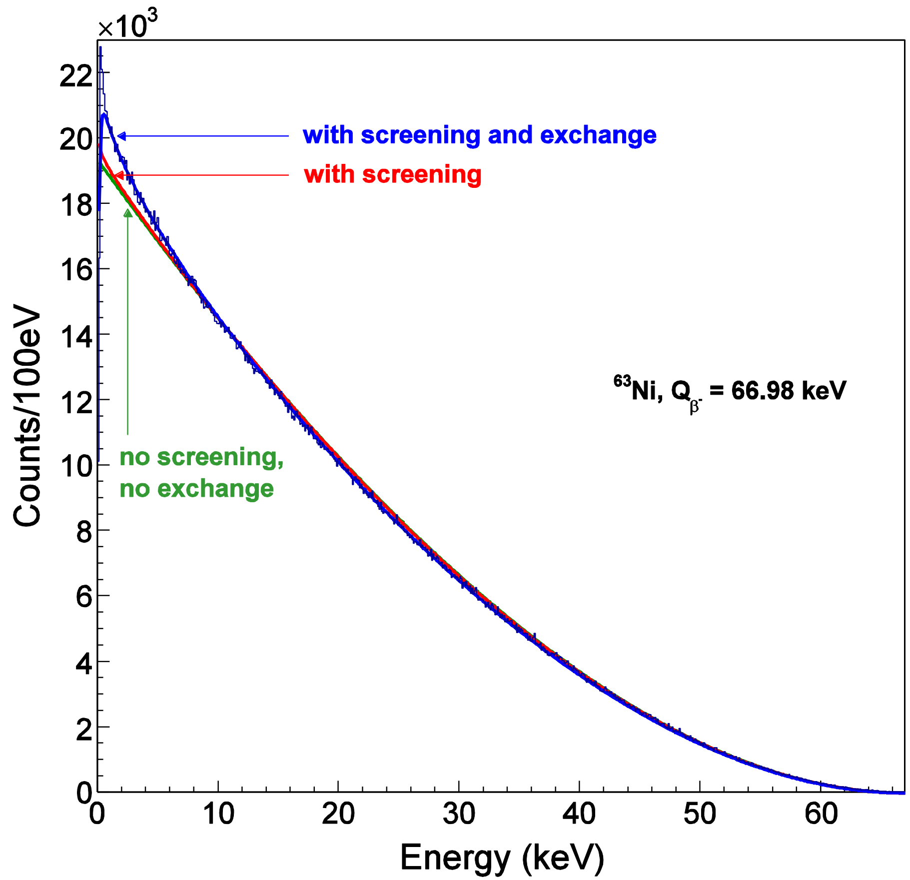
For both the continuum and bound wave functions,

Without screening: parameters perfectly reproduced

With screening: parameters in excellent agreement, despite different potentials

The allowed ^{63}Ni beta spectrum





Analytic: $\bar{E} = 17.45\ \mathrm{keV}$

With screening: $\bar{E} = 17.40\ \mathrm{keV}$

With screening and exchange: $\bar{E} = 17.14\ \mathrm{keV}$

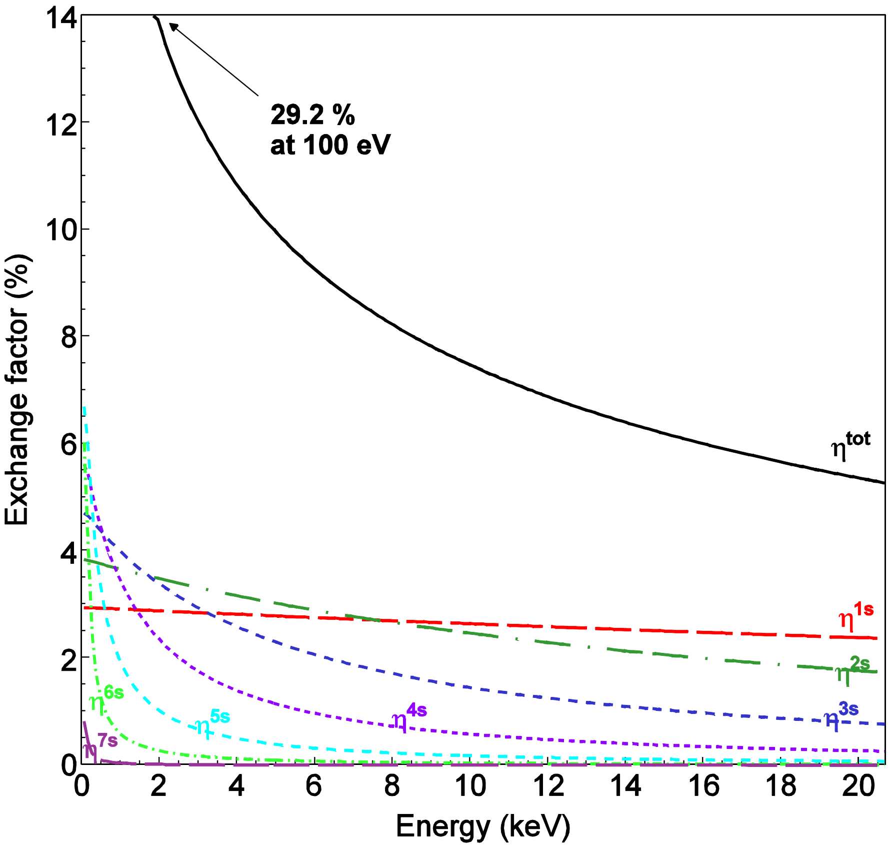
Mean energy of the spectrum decreased by **1.8 %**

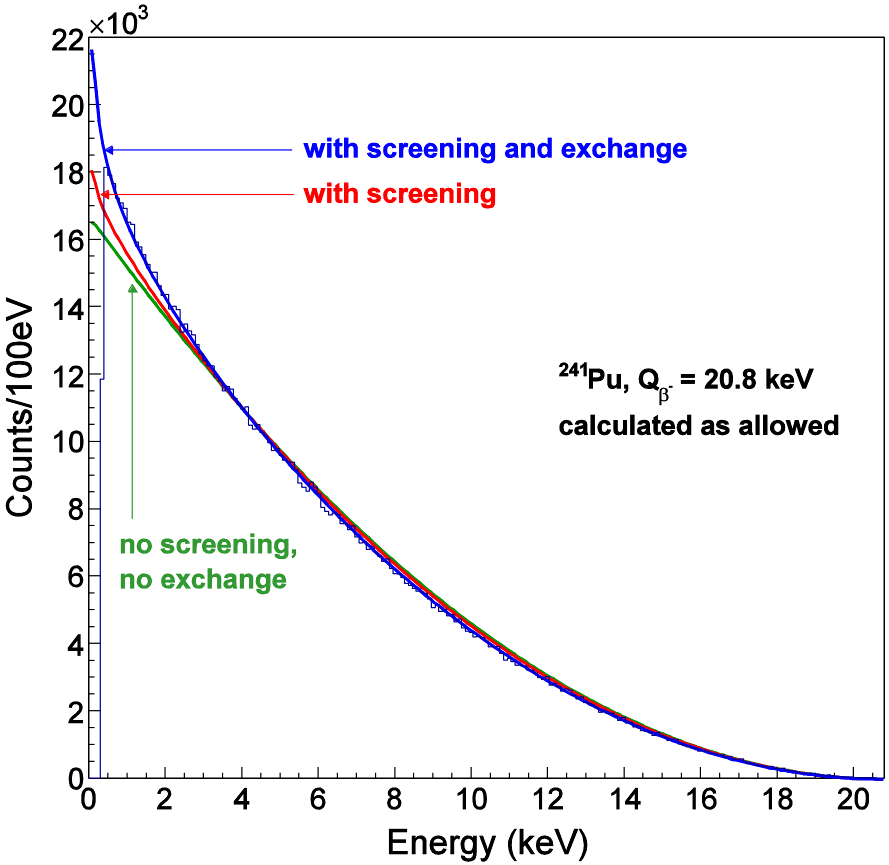
Allowed transition

Experimental spectrum

C. Le-Bret, PhD thesis, Université Paris 11 (2012)

The almost allowed ^{241}Pu beta spectrum





Analytic: $\bar{E} = 5.24 \text{ keV}$

With screening: $\bar{E} = 5.18 \text{ keV}$

With screening and exchange: $\bar{E} = 5.03 \text{ keV}$

Mean energy of the spectrum decreased by **4** %

Calculated as **allowed**

Experimental spectrum

M. Loidl *et al.*, App. Radiat. Isot. 68, 1454 (2010)

A simple statistical analysis

Modelisation error $\hat{e}_i = y_i - y_i^{th}$

Fit quality $R^2 = 1 - \frac{\mathrm{var}(\hat{e}_i)}{\mathrm{var}(y_i)}$ → $(1 - R^2)$ is the **disagreement**

Here, 2 parameters: endpoint energy, global normalization on data

Standardized residuals $r_i = \frac{\hat{e}_i}{\sqrt{\mathrm{var}(\hat{e}_i)}}$

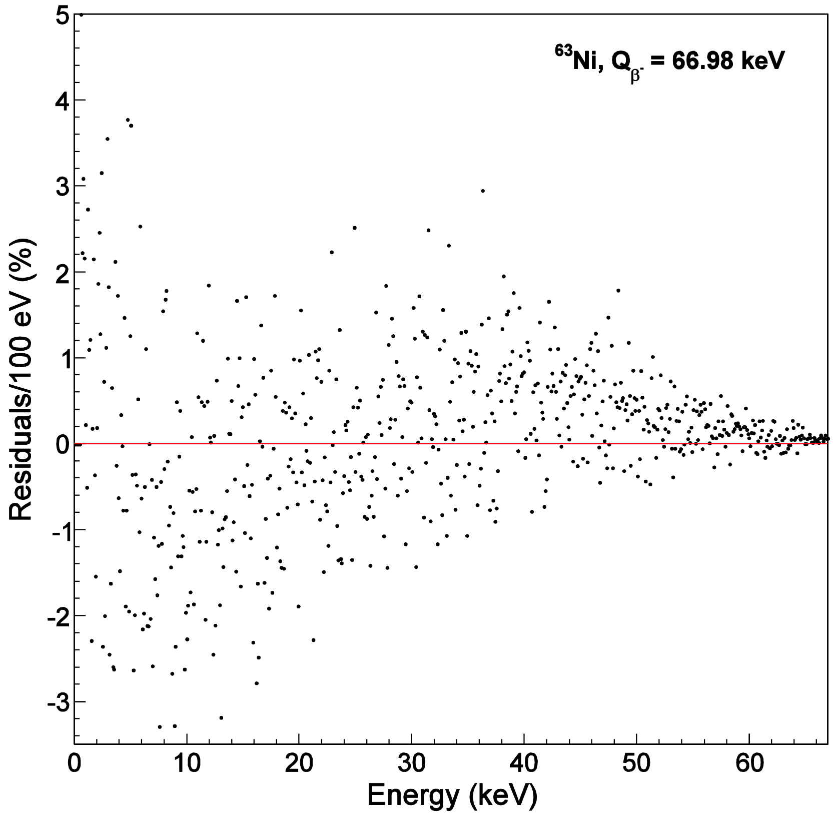
If almost equally distributed around zero → gaussian

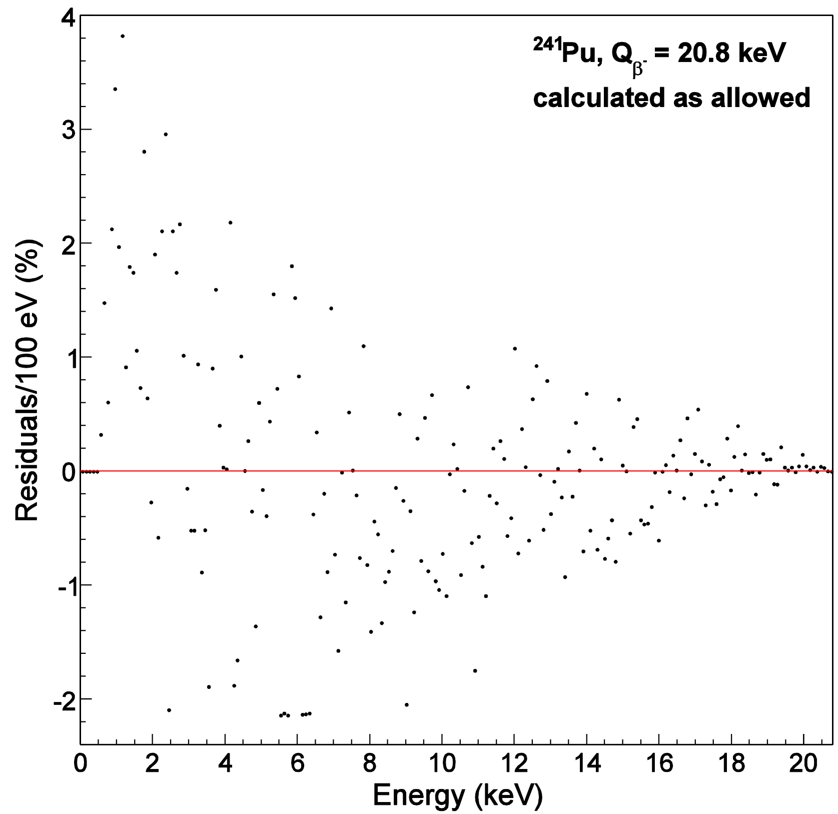
$$\sigma_{r_i} = \sqrt{\mathrm{var}(r_i)}$$

is an estimate of the **overall uncertainty** of the calculated spectrum

In $[500\ \mathrm{eV}, E_{\mathrm{max}}]$ for both ^{63}Ni and ^{241}Pu beta spectra

Quality of calculations for ^{63}Ni and ^{241}Pu





^{63}Ni		^{241}Pu
$\bar{r_i} = 0.093\ \%$	**Residuals mean**	$\bar{r_i} = 0.0019\ \%$
$(1 - R^2) = 0.028\ \%$	**Disagreement**	$(1 - R^2) = 0.040\ \%$
$\sigma_{r_i} = 1.03\ \%$	**Global uncertainty**	$\sigma_{r_i} = 0.99\ \%$

X. Mougeot, C. Bisch, Phys. Rev. A 90, 012501 (2014)

Laboratoire National Henri Becquerel

LNE-LNHB

Conclusion

Conclusion

Exchange and **screening** effects have been demonstrated to have a **great influence** on the **spectrum shape at low energy**.

→ Explicit calculation of **exchange** and **screening** for **forbidden unique** transitions is needed and **must be compared to new measurements**.

$\boldsymbol{\lambda_k = 1}$ is generally a bad approximation.

ξ **approximation** is correct **only** for ~ 50 % of the **1st forbidden non-unique** transitions, and **incorrect** for **all** other **non-unique** transitions.

Within 2 – 3 years (hopefully)

Collaboration with nuclear theorists from IPHC Strasbourg to evaluate the **influence** of the **nuclear matrix elements** in order to **calculate specifically** the **forbidden non-unique** transitions.

→ We aim for a code that accounts **consistently** for **the atomic and nuclear structure effects**.

Thank you for your attention

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