

Sensitivity of β -decay rates to the radial dependence of the nucleon effective mass

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Punch line: Improved description of the density of states around the Fermi energy is essential for understanding of nuclear excitations and prediction of weak interaction rates in nuclear astrophysics. We present here a simple modelling to modify the density of states.

- I. Extension of Skyrme EDF including surface peaked effective mass (SPEM)
- II. Application to beta-decay rates in double magic nuclei (^{78}Ni , $^{100,132}\text{Sn}$)

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Motivation

better description of density of state around Fermi energy

low-temperature properties
(entropy, specific heat)

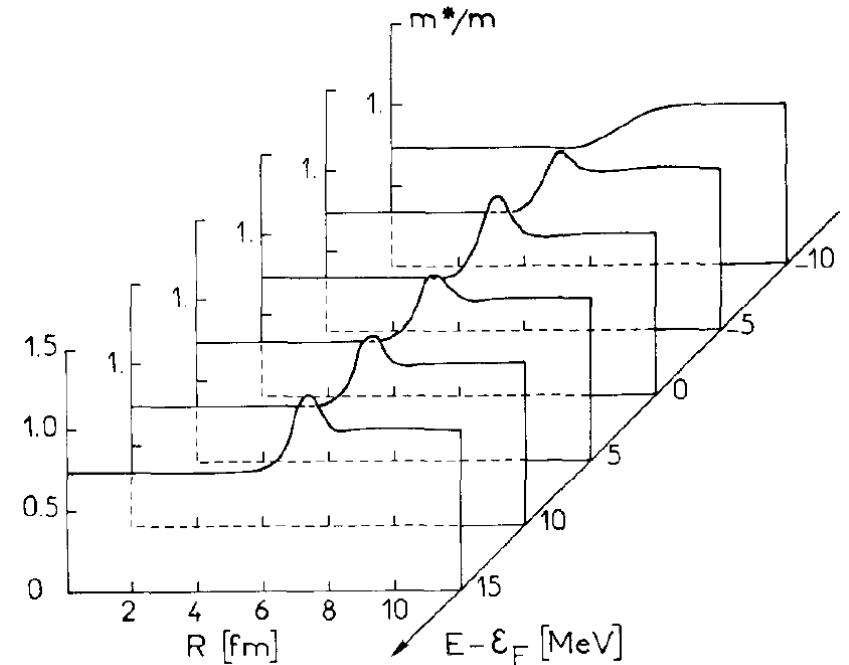
low-energy properties
(pairing, collective modes)

nuclear astrophysics
(SN core-collapse)

→ m^*/m close to 1
Brown *et al.*, NPA 46 (1963)

→ m^*/m should be energy-dependent
(particle-vibration coupling)

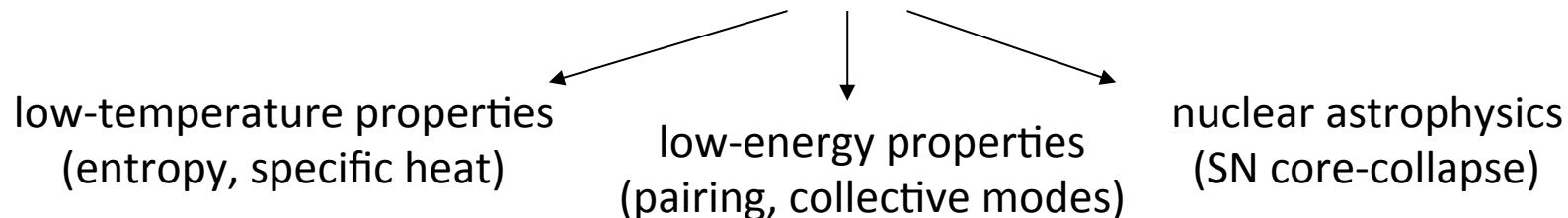
nuclei → dependence on the gradient
of the nuclear term
Ma & Wambach, NPA 402 (1983)



Van Giai & Van Thieu, PLB 126 (1983)

Motivation

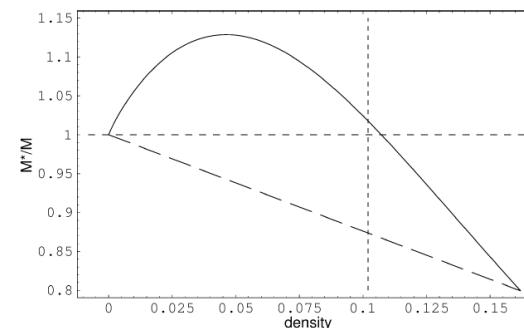
better description of density of state around Fermi energy



Attempt to modify Skyrme force:

Skyrme force with surface-peaked effective mass,
Farine, Pearson, Tondeur, Nuclear Physics A 696 (2001) 396

$$\begin{aligned}
 v_{ij} = & t_0(1 + x_0 P_\sigma) \delta(\mathbf{r}_{ij}) + t_1(1 + x_1 P_\sigma) \frac{1}{2\hbar^2} \{ p_{ij}^2 \delta(\mathbf{r}_{ij}) + \text{h.c.} \} \\
 & + t_2(1 + x_2 P_\sigma) \frac{1}{\hbar^2} \mathbf{p}_{ij} \cdot \delta(\mathbf{r}_{ij}) \mathbf{p}_{ij} + \frac{1}{6} \sum_{\mu=a}^b t_{3\mu}(1 + x_{3\mu} P_\sigma) \rho(\mathbf{r}_i)^{\gamma_\mu} \delta(\mathbf{r}_{ij}) \\
 & + \frac{1}{2\hbar^2} t_4(1 + x_4 P_\sigma) \{ p_{ij}^2 \rho(\mathbf{r}_i)^\beta \delta(\mathbf{r}_{ij}) + \text{h.c.} \} \\
 & + \frac{i}{\hbar^2} W_0(\sigma_i + \sigma_j) \cdot \mathbf{p}_{ij} \times \delta(\mathbf{r}_{ij}) \mathbf{p}_{ij},
 \end{aligned}$$



→ Not consistent
with Brueckner HF
in unif. matter.

Nuclear energy density functional

Our approach : introduce a correction to the EDF such as to get :

- a surface peaked effective mass (energy-independent)
- a moderate effect on the mean field

**Standard Skyrme
energy density :**

$$\mathcal{H}(\mathbf{r}) = \mathcal{K}(\mathbf{r}) + \sum_{T=0,1} \mathcal{H}_T(\mathbf{r}) \quad \text{Bender et al., Rev. Mod. Phys. 75 (2003)}$$

$$\mathcal{K}(\mathbf{r}) = \frac{\hbar^2}{2m} \tau(\mathbf{r}),$$

$$\begin{aligned} \mathcal{H}_T(\mathbf{r}) &= C_T^\rho \rho_T^2(\mathbf{r}) + C_T^{\Delta\rho} \rho_T(\mathbf{r}) \Delta \rho_T(\mathbf{r}) \\ &+ C_T^\tau \rho_T(\mathbf{r}) \tau_T(\mathbf{r}) \\ &+ C_T^J \mathbb{J}_T^2(\mathbf{r}) + C_T^{\nabla J} \rho_T(\mathbf{r}) \nabla \cdot \mathbf{J}_T(\mathbf{r}) \end{aligned}$$

Correction term :
(isoscalar)

$$\begin{aligned} \mathcal{H}_0^{\text{corr}}(\mathbf{r}) &= C_0^\tau (\nabla \rho)^2 \tau(\mathbf{r}) (\nabla \rho(\mathbf{r}))^2 \\ &+ C_0^{\rho^2} (\nabla \rho)^2 \rho(\mathbf{r})^2 (\nabla \rho(\mathbf{r}))^2 \end{aligned}$$

surface peaked
effective mass
Zalewski, Olbratowski,
Satula, PRC81 (2010)
mean field
compensation

Nuclear energy density functional

Effective mass :

$$\frac{\hbar^2}{2m_q^*(\mathbf{r})} \equiv \frac{\delta H}{\delta \tau_q} = \underbrace{\frac{\hbar^2}{2m} + C_q^\tau \rho_q(\mathbf{r}) + C_0^\tau (\nabla \rho)^2}_{\text{standard Skyrme } k\text{-mass}} (\nabla \rho(\mathbf{r}))^2$$

Mean field :

$$U_q(\mathbf{r}) = \underbrace{U_q^{\text{Skyrme}}(\mathbf{r}) + U^{\text{corr}}(\mathbf{r})}_{\text{standard Skyrme mean field}}$$

$$\begin{aligned} U^{\text{corr}}(\mathbf{r}) &= -2C_0^\tau (\nabla \rho)^2 \left(\tau(\mathbf{r}) \nabla^2 \rho(\mathbf{r}) + \nabla \tau(\mathbf{r}) \nabla \rho(\mathbf{r}) \right) \\ &\quad - 2C_0^\rho \rho^2 (\nabla \rho)^2 \left(\rho(\mathbf{r}) (\nabla \rho(\mathbf{r}))^2 + \rho(\mathbf{r})^2 \nabla^2 \rho(\mathbf{r}) \right) \end{aligned}$$

Nuclear energy density functional

Schrödinger equation:

$$\left[-\frac{\hbar^2}{2m} \frac{d^2}{dr^2} + \frac{\hbar^2 l(l+1)}{2m r^2} + V_q^{eq}(r, \epsilon) \right] \psi_{\lambda,q}(r) = \epsilon_{\lambda,q} \psi_{\lambda,q}(r)$$

Equivalent potential:

$$V_q^{eq}(r) = U_q^{Sky}(r) + U^{corr}(r) + U_q^{eff}(r)$$
$$U_q^{eff} = -\frac{1}{4} \frac{2m_q^*(r)}{\hbar^2} \left(\frac{\hbar^2}{2m_q^*(r)} \right)^{1/2} + \frac{1}{2} \left(\frac{\hbar^2}{2m_q^*(r)} \right)^{''}$$
$$+ \left(\frac{\hbar^2}{2m_q^*(r)} \right)' \frac{1}{r} .$$



Correction term acts on

$$\left\{ \begin{array}{l} m_q^* \\ U^{corr} \\ U_q^{eff} \end{array} \right.$$

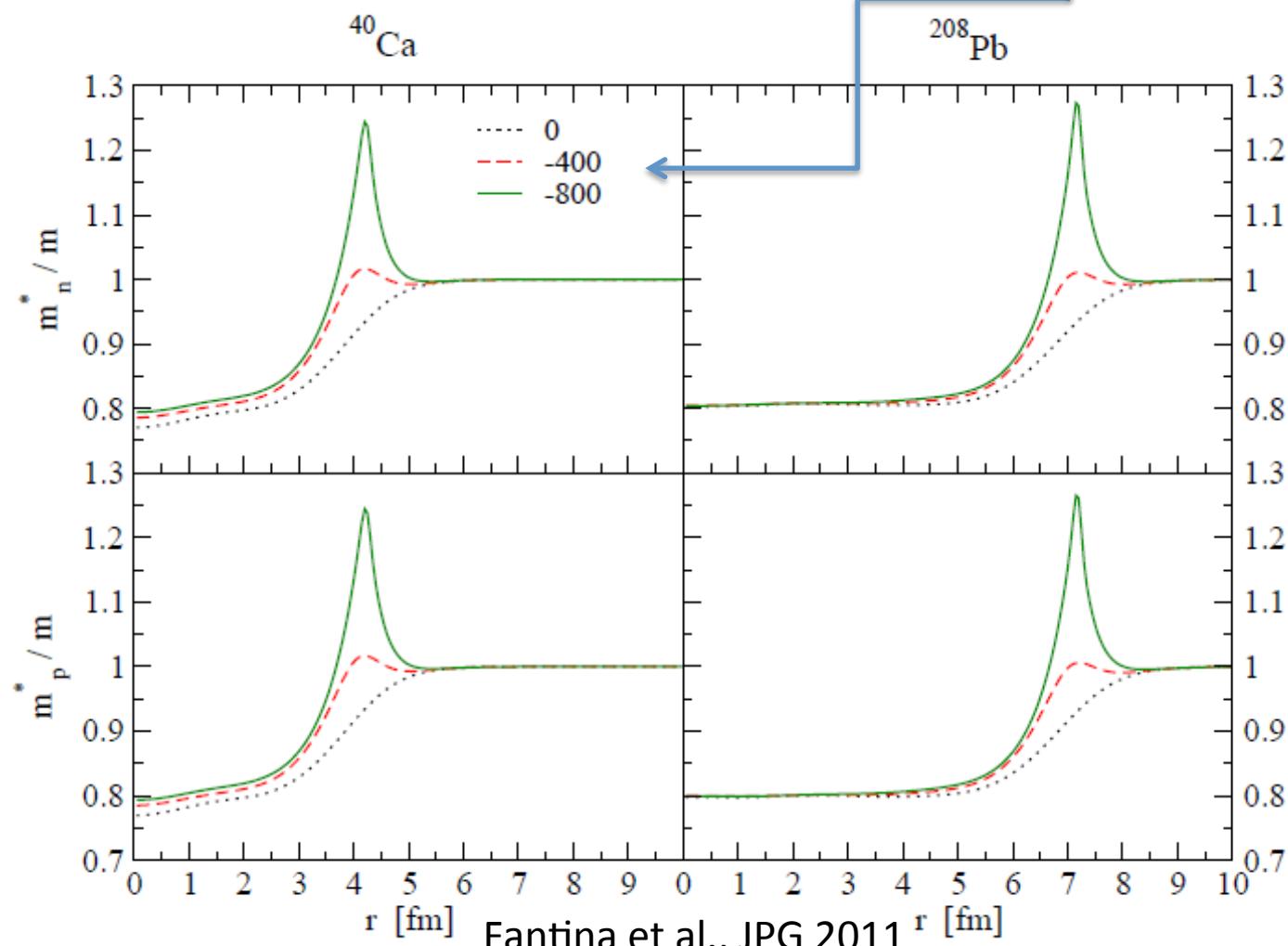


$$C_0^{\rho^2(\nabla\rho)^2} \propto C_0^\tau (\nabla\rho)^2$$

Surface peaked effective mass

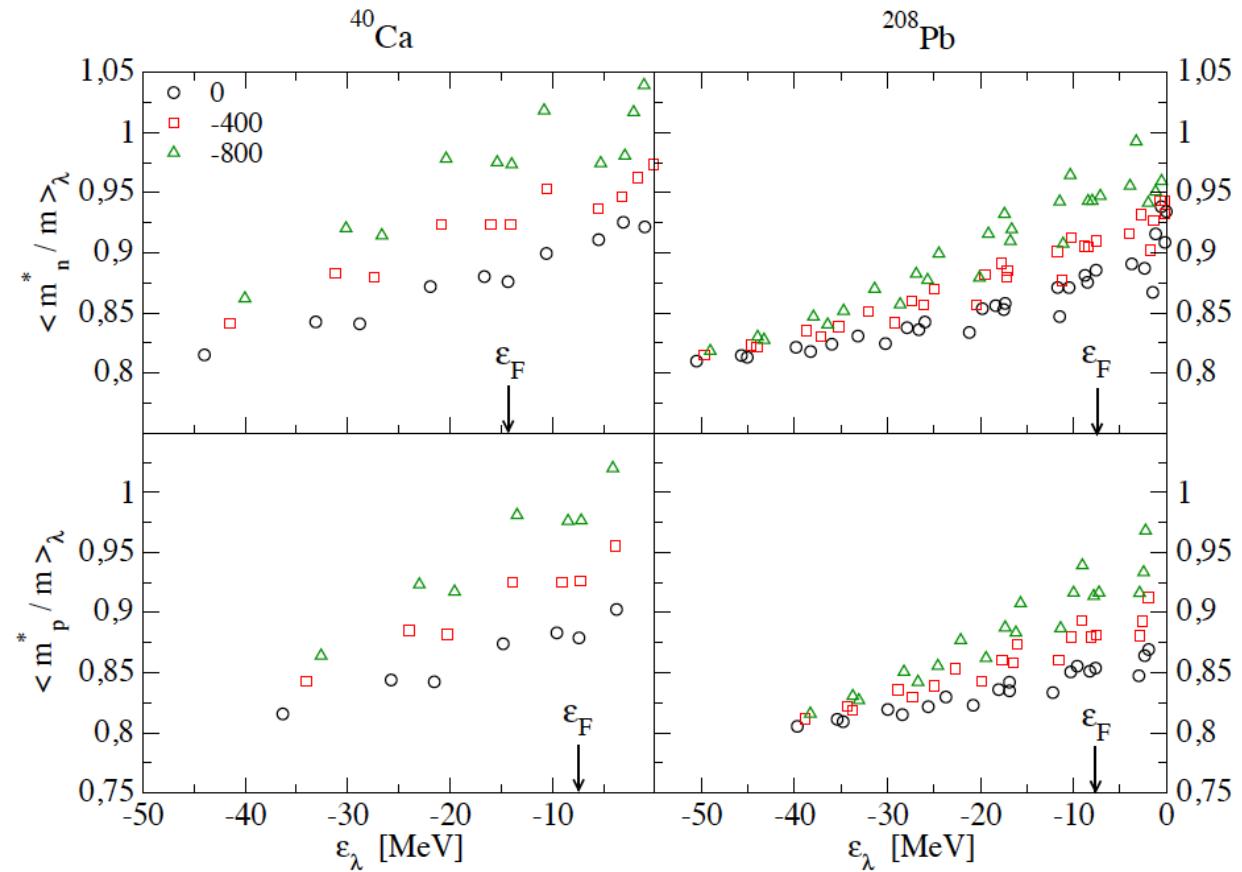
Effective mass profile:

$$\frac{\hbar^2}{2m_q^*(\mathbf{r})} \equiv \frac{\delta H}{\delta \tau_q} = \frac{\hbar^2}{2m} + C_q^\tau \rho_q(\mathbf{r}) + C_0^\tau (\nabla \rho(\mathbf{r}))^2$$



Impact on s.p. level properties

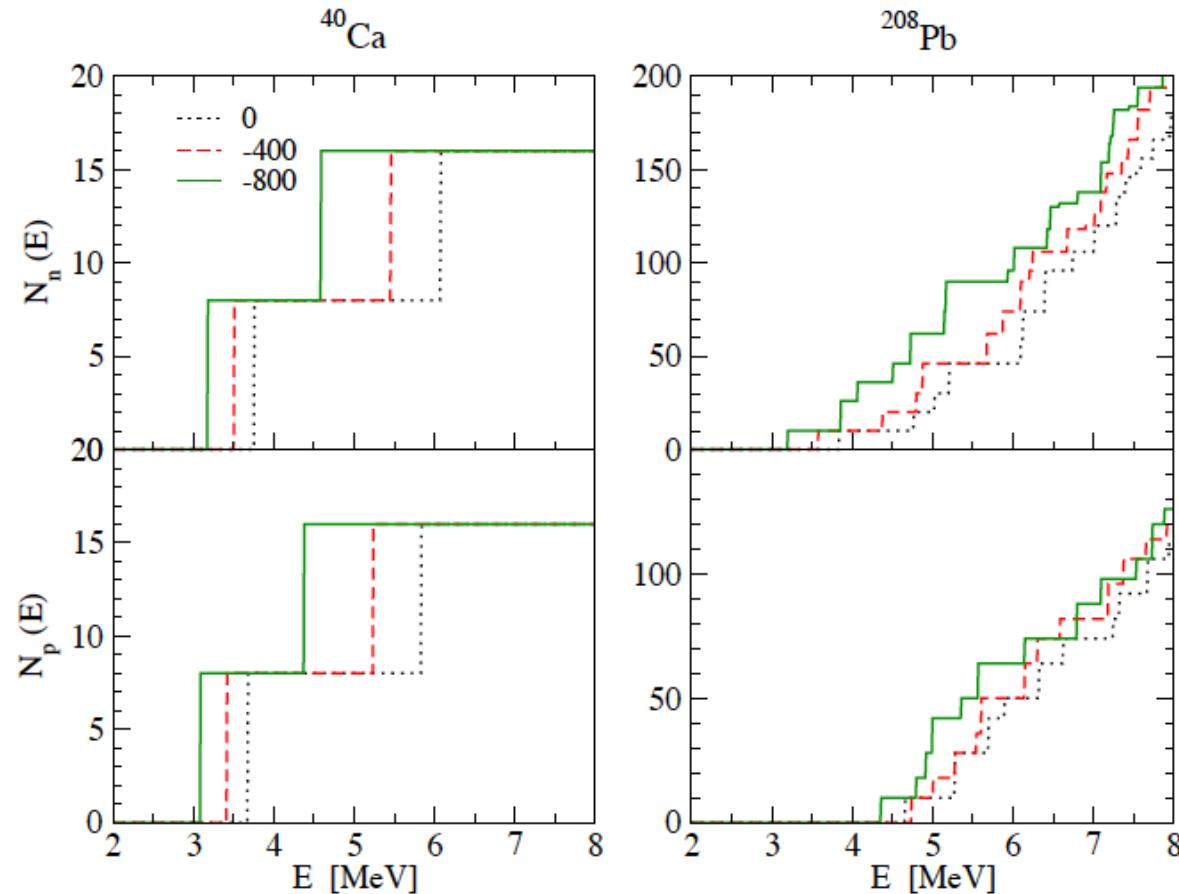
State-averaged effective mass: $\langle \frac{m_q^*}{m} \rangle_\lambda = \int dr \psi_{\lambda,q}^*(r) \frac{m_q^*(r)}{m} \psi_{\lambda,q}(r)$



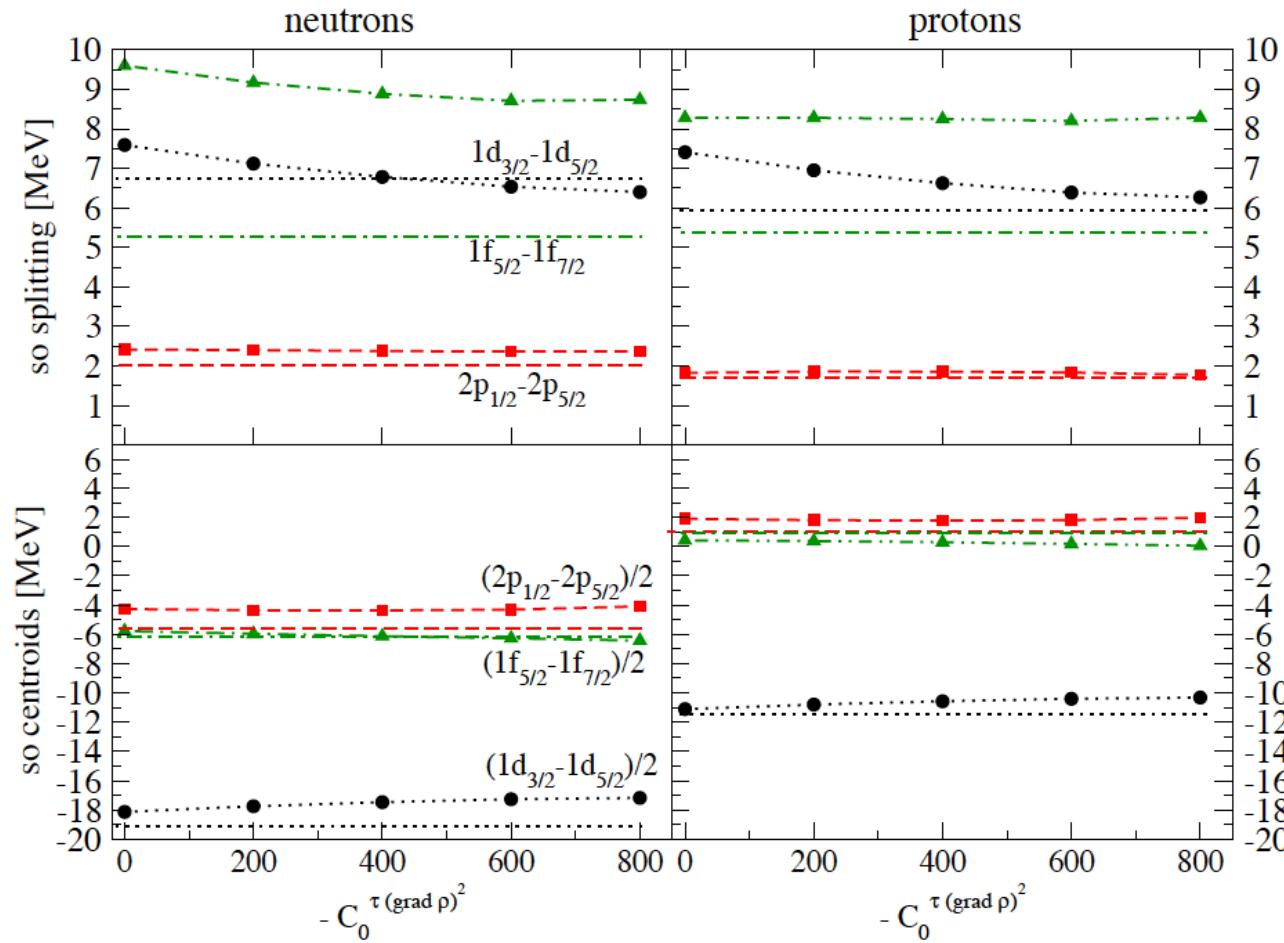
Impact on s.p. level properties

Density of states: $g(E) \equiv \frac{dN(E)}{dE} = \sum_{\substack{\lambda_1 < F \\ \lambda_2 > F}} (2j_{\lambda_2} + 1) \delta(E - (\epsilon_{\lambda_2} - \epsilon_{\lambda_1}))$

Number of states: $N(E) = \int dE g(E)$



Impact of the SPFM on S.O. splitting



Moderate effect.

Zalewski, Olbratowski,
Satula (PRC 81 2010)
found bigger effects,
but also readjusted the
functional for each

$$C_0^{\tau(\nabla \rho)^2}$$

Binding energies (SPEM added with no refitting)

$C_0^{\tau(\nabla\rho)^2}$	^{40}Ca	^{208}Pb
0	-8,781	-8,071
-200	-8,591	-7,983
-400	-8,442	-7,910
-600	-8,322	-7,849
-800	-8,227	-7,801

TABLE II: Binding energies in ^{40}Ca and ^{208}Pb (in MeV) for different values of the coefficient $C_0^{\tau(\nabla\rho)^2}$.

→ A slight readjustement of the parameters is however needed.

Application to beta-decay rates

The collapse of many Skyrme interactions

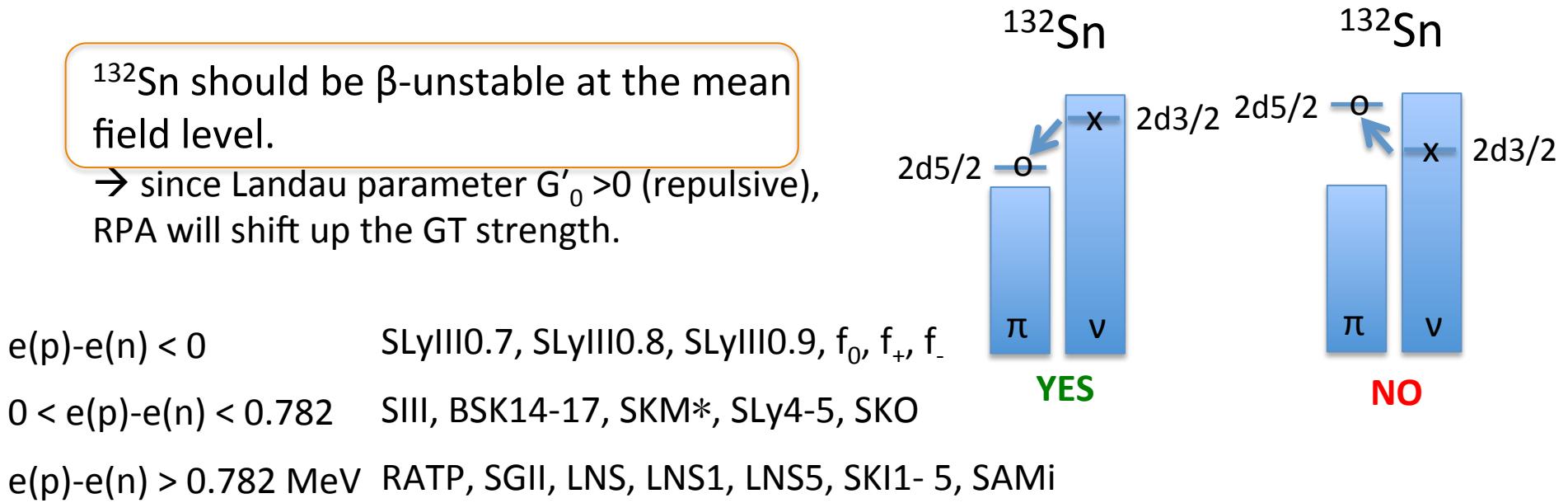
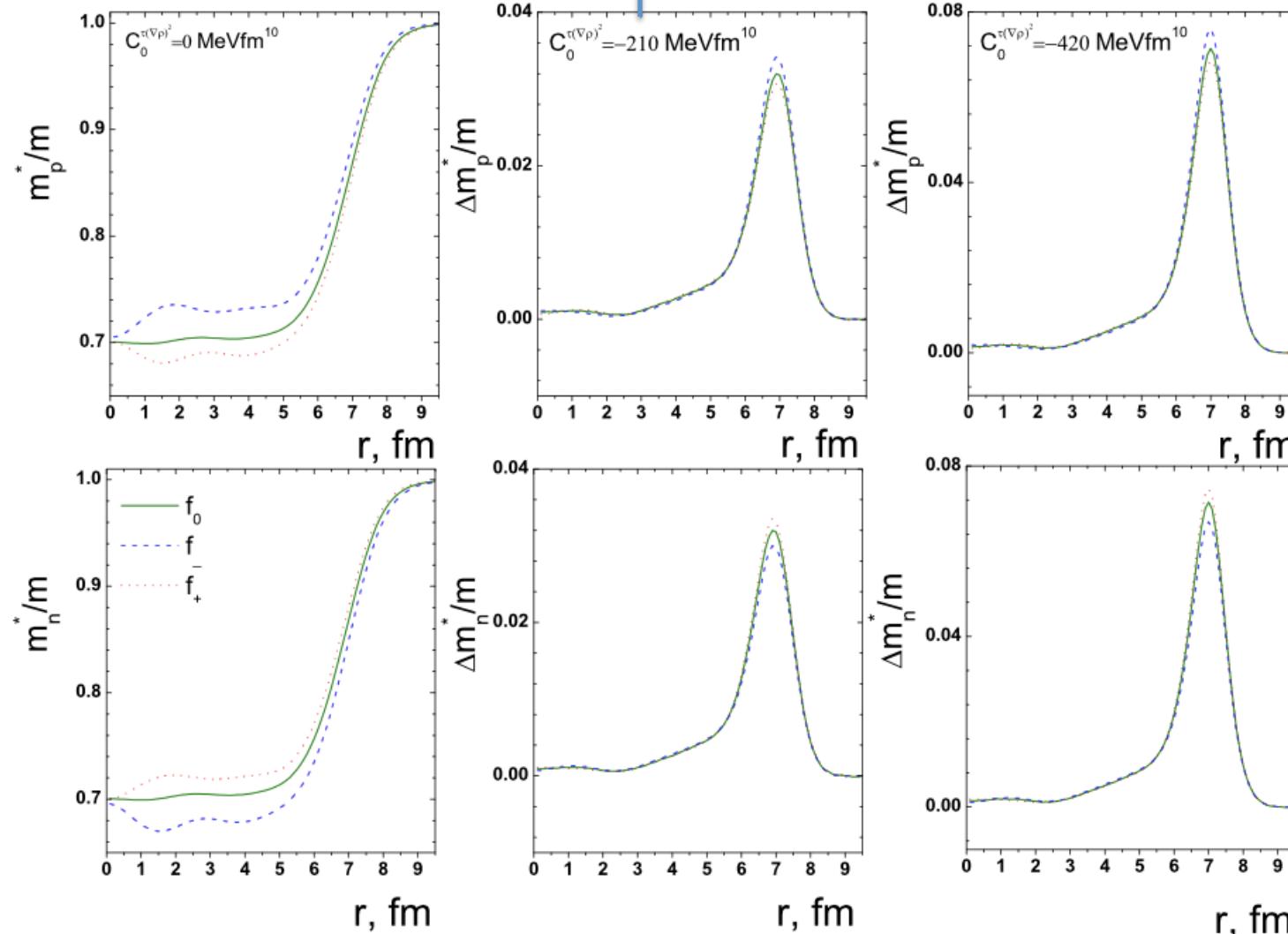


TABLE I: Bulk properties of the selected interactions.

Skyrme	$\rho_{0,sat}$ (fm $^{-3}$)	E_0 (MeV)	K_0 (MeV)	J_{sym} (MeV)	L_{sym} (MeV)	m_s^*/m (MeV)	$\Delta m^*/m$	G'_0
SLyIII0.7 [30]	0.153	-16.33	361.4	31.98	30.78	0.7	0.18	0.30
SLyIII0.8 [30]	0.153	-16.32	368.8	31.69	28.24	0.8	0.29	0.33
SLyIII0.9 [30]	0.153	-16.31	374.5	31.44	24.75	0.9	0.38	0.34
f_+ [31]	0.162	-16.04	230.0	32.00	41.53	0.7	0.17	0.08
f_0 [31]	0.162	-16.03	230.0	32.00	42.42	0.7	0	-0.01
f_- [31]	0.162	-16.02	230.0	32.00	43.79	0.7	-0.28	-0.13

SPEM in ^{208}Pb

→ 1% correction to the isoscalar quadrupole EWSR



Mean field s.p. configuration in ^{78}Ni , $^{100,132}\text{Sn}$

Mean field position of the dominant states in ^{78}Ni & ^{132}Sn correlated to J_{sym} .

Skyrme	$C_0^{\tau(\nabla\rho)^2}$ (MeV fm ¹⁰)	^{132}Sn $\epsilon_{\pi 2d\frac{5}{2}^-}$ $\epsilon_{\nu 2d\frac{3}{2}^-}$	^{100}Sn $\epsilon_{\nu 1g\frac{7}{2}^-}$ $\epsilon_{\pi 1g\frac{9}{2}^-}$	^{78}Ni $\epsilon_{\pi 2p\frac{3}{2}^-}$ $\epsilon_{\nu 2p\frac{1}{2}^-}$	J_{sym} (MeV)
SLyIII0.7	0	0.3	-7.2	-5.1	
SLyIII0.7	-210	0.2	-7.3	-5.2	
SLyIII0.7	-420	0.1	-7.3	-5.3	
SLyIII0.8	0	-0.6	-7.5	-6.2	
SLyIII0.8	-210	-0.7	-7.5	-6.3	
SLyIII0.8	-420	-0.9	-7.6	-6.4	
SLyIII0.9	0	-1.3	-7.7	-7.0	
SLyIII0.9	-210	-1.4	-7.7	-7.2	
SLyIII0.9	-420	-1.6	-7.8	-7.3	
f_+	0	-0.6	-5.9	-5.8	
f_+	-210	-0.7	-6.0	-5.9	
f_+	-420	-0.8	-6.2	-6.0	
f_0	0	-0.5	-6.0	-5.6	
f_0	-210	-0.6	-6.1	-5.7	
f_0	-420	-0.7	-6.2	-5.8	
f_-	0	-0.4	-6.2	-5.4	
f_-	-210	-0.5	-6.3	-5.5	
f_-	-420	-0.6	-6.5	-5.6	



Crucial for
beta-stability
in ^{132}Sn .

beta-decay half-lives & CERPA

For the β^- -decay case we have:

$$T_{1/2}^{\beta^-} = \frac{D}{\left(\frac{G_A}{G_V}\right)^2 \sum_k f_0(Z+1, A, E_i - E_{1_k^+}) B(GT)_k^-}, \quad (10)$$

$$E_i - E_{1_k^+} \approx \Delta M_{n-H} + \mu_n - \mu_p - E_k \quad (11)$$

while for the β^+ -decay case this becomes:

$$T_{1/2}^{\beta^+} = \frac{D}{\left(\frac{G_A}{G_V}\right)^2 \sum_k f_0(-Z+1, A, E_i - E_{1_k^+}) B(GT)_k^+}, \quad (12)$$

$$E_i - E_{1_k^+} \approx -\Delta M_{n-H} - 2m_e - \mu_n + \mu_p - E_k. \quad (13)$$

CERPA with separable residual interaction:

Severyukhin et al.
PRC 77 (2008)

- Eigen values E_k
- Wave functions: $B(GT)_k^\pm = \left| \langle N \pm 1, Z \mp 1; 1_k^+ | \hat{O}^\pm | N, Z; 0_{gs}^+ \rangle \right|^2$
 $\hat{O}_\pm = \sum_{i,m} t_\pm(i) \sigma_m(i)$

CERPA Beta-decay half-life in ^{132}Sn

Nucleus	Skyrme	$C_0^{\tau(\nabla\rho)^2}$ (MeV fm ¹⁰)	$T_{1/2}$ (s)
^{132}Sn	SLyIII0.7	0	389400
	SLyIII0.7	-210	9840
	SLyIII0.7	-420	1930
	SLyIII0.8	0	57
	SLyIII0.8	-210	33
	SLyIII0.8	-420	21
	SLyIII0.9	0	6.7
	SLyIII0.9	-210	4.7
	SLyIII0.9	-420	3.3
	f_+	0	18
	f_+	-210	12
	f_+	-420	8.5
	f_0	0	13
	f_0	-210	8.8
	f_0	-420	6.5
	f_-	0	6.4
	f_-	-210	5.0
	f_-	-420	3.6
Expt.			39.7 ± 0.8

→ Very large impact of the effective mass.

CERPA Beta-decay half-life in ^{100}Sn & ^{78}Ni

Nucleus	Skyrme	$C_0^{\tau(\nabla\rho)^2}$ (MeV fm 10)	$T_{1/2}$ (s)	^{78}Ni	SLyIII0.7	0	0.157
^{100}Sn	SLyIII0.7	0	0.232		SLyIII0.7	-210	0.140
	SLyIII0.7	-210	0.221		SLyIII0.7	-420	0.121
	SLyIII0.7	-420	0.213		SLyIII0.8	0	0.057
	SLyIII0.8	0	0.178		SLyIII0.8	-210	0.051
	SLyIII0.8	-210	0.172		SLyIII0.8	-420	0.045
	SLyIII0.8	-420	0.167		SLyIII0.9	0	0.025
	SLyIII0.9	0	0.138		SLyIII0.9	-210	0.023
	SLyIII0.9	-210	0.134		SLyIII0.9	-420	0.020
	SLyIII0.9	-420	0.131	f_+	0	0.031	
	f_+	0	0.593	f_+	-210	0.028	
	f_+	-210	0.492	f_+	-420	0.027	
	f_+	-420	0.433	f_0	0	0.020	
	f_0	0	0.381	f_0	-210	0.019	
	f_0	-210	0.323	f_0	-420	0.018	
	f_0	-420	0.290	f_-	0	0.010	
	f_-	0	0.190	f_-	-210	0.009	
	f_-	-210	0.168	f_-	-420	0.009	
	f_-	-420	0.154	Expt.		0.1222 ± 0.0051	
Expt.		1.16 ± 0.20		Expt.			

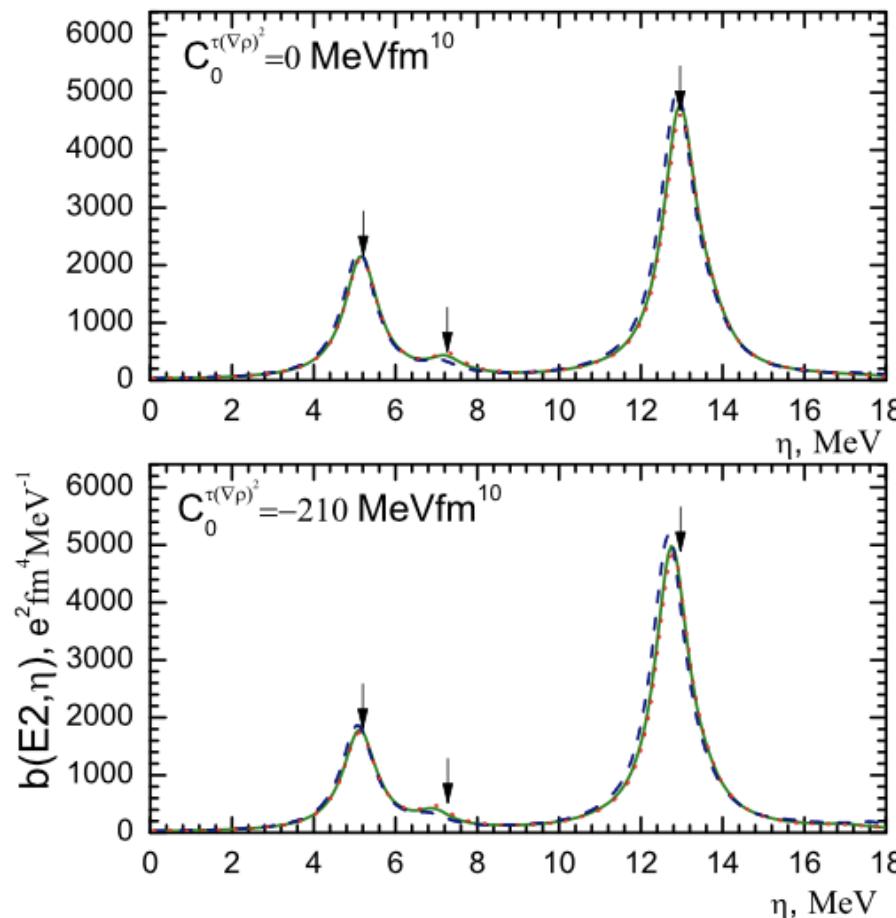
→ Poor reproduction of $T_{1/2}$ for these nuclei.
 → Small impact of the effective mass.



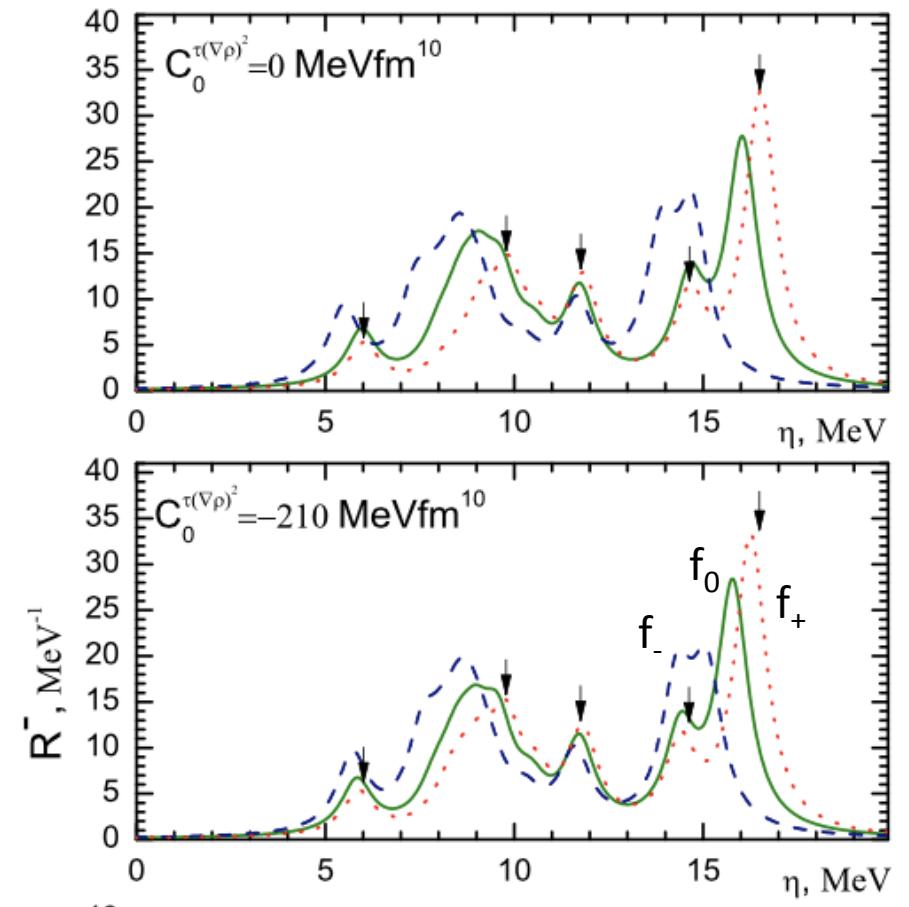
Collective modes at higher energies

Strength slightly compressed

GQR in ^{208}Pb



GTR in ^{208}Pb



Summary & conclusions

- First step in improving the Skyrme EDF for better description of nuclear data and predicting weak transition for nuclear astrophysics.
- A correction term produces a surface peaked effective mass:
 - compression of the s.p. level spacing
 - decrease of the beta-decay half-lives
 - high energy collective modes almost unmodified.
- An increase of the level density could, in principle, lead to a better description of beta-decay rates. Reality is more complex.
- From our analysis, we however have a better understanding of the impact of different terms in the EDF:
 - ^{100}Sn and ^{78}Ni only weakly impacted by the SPEM (use to calibrate G_0')
 - ^{132}Sn largely impacted by the SPEM
 - good benchmark nuclei to improve the EDF+extensions.
- Future developments: tensor force, refit of the Skyrme parameters, modif of G_0' .