Low-energy dipole excitations, Gamow-Teller and beta decay within the QRPA and the Gogny force

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Formalism

$$H|\nu\rangle = E_\nu|\nu\rangle \qquad Q_\nu^\dagger|0\rangle = |\nu\rangle \qquad Q_\nu|0\rangle = 0$$

Particle-hole excitations: RPA

$$Q_\nu^\dagger = \sum_{ph} X_{ph}^\nu \, a_p^\dagger a_h - Y_{ph}^\nu \, a_h^\dagger a_p$$

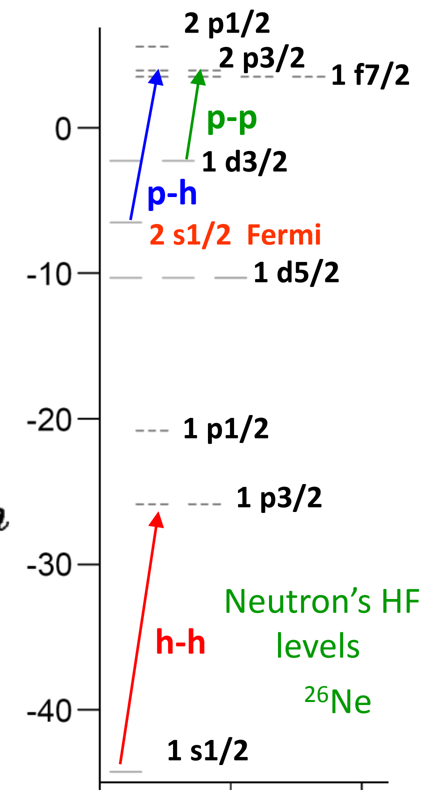
2 quasi-particles excitations: QRPA

$$\eta_i^+ = \sum_\alpha u_{i\alpha} a_\alpha^+ - v_{i\alpha} a_\alpha \qquad Q_\nu^+ = \sum_{ij} X_{ij}^\nu \eta_i^+ \eta_j^+ + Y_{ij}^\nu \eta_j \eta_i$$

$$\begin{pmatrix} A & B \\ B & A \end{pmatrix} \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} X^\nu \\ -Y^\nu \end{pmatrix}$$

$$A_{minj} = (\epsilon_m + \epsilon_i)\delta_{ji}\delta_{nm} + \overline{V}_{mjin}$$

$$B_{minj} = \overline{V}_{mnij}$$



Hartree-Fock Bogoliubov: ε, u, v ⟶ Ground state properties

QRPA: ω, X, Y ⟶ Excited states properties

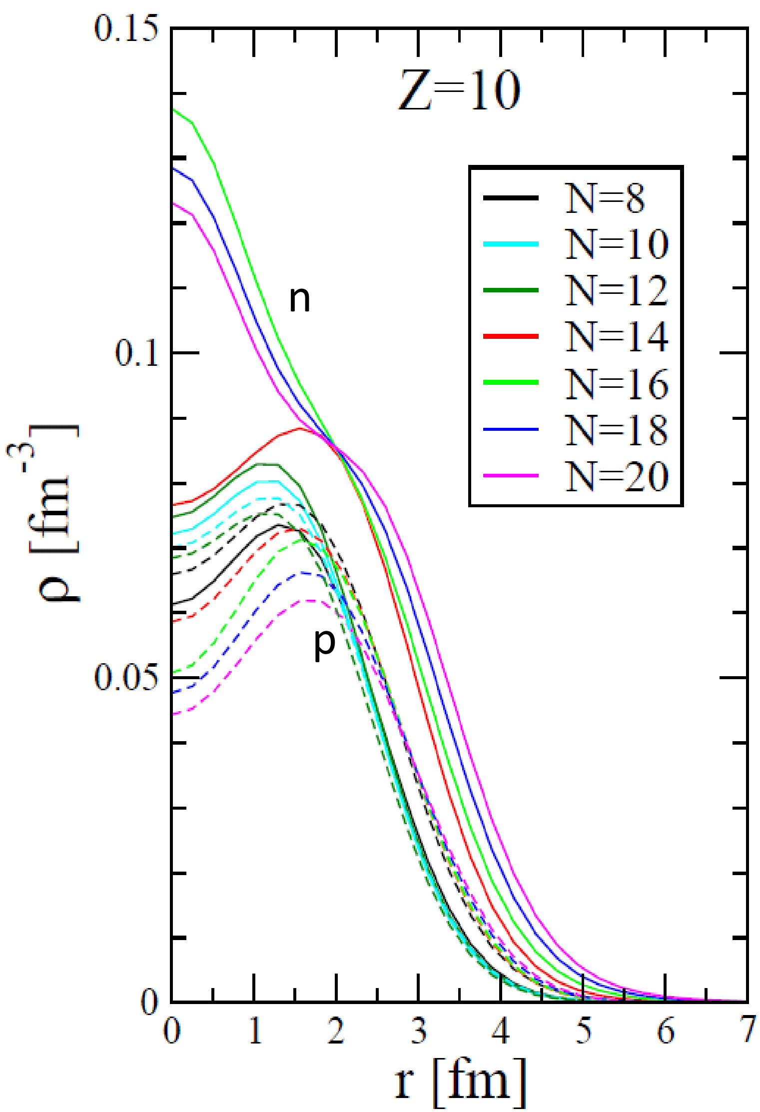
Same interaction (Gogny) in HFB and QRPA

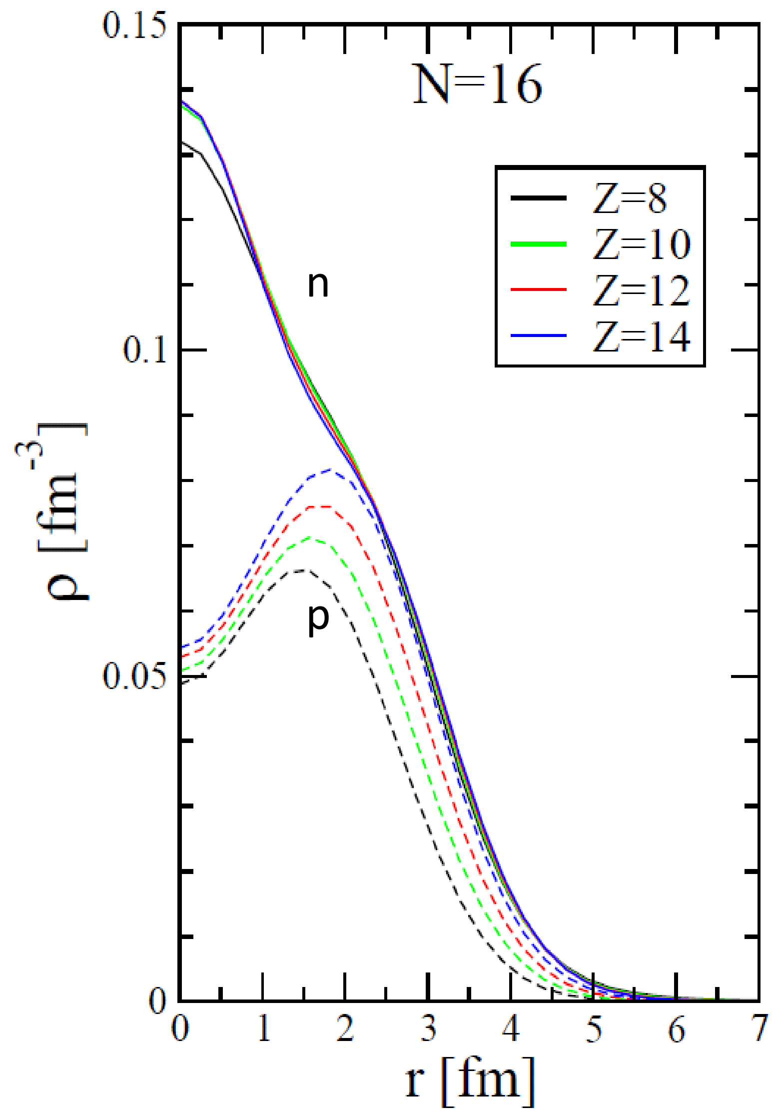
Low-energy dipole excitations

M. Martini, S. Péru and M. Dupuis

Phys. Rev. C 83, 034309 (2011)

Neon isotopes N=16 isotones

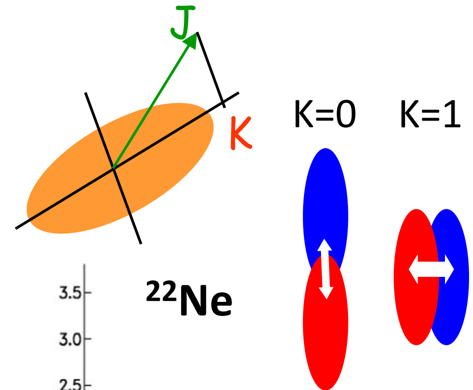


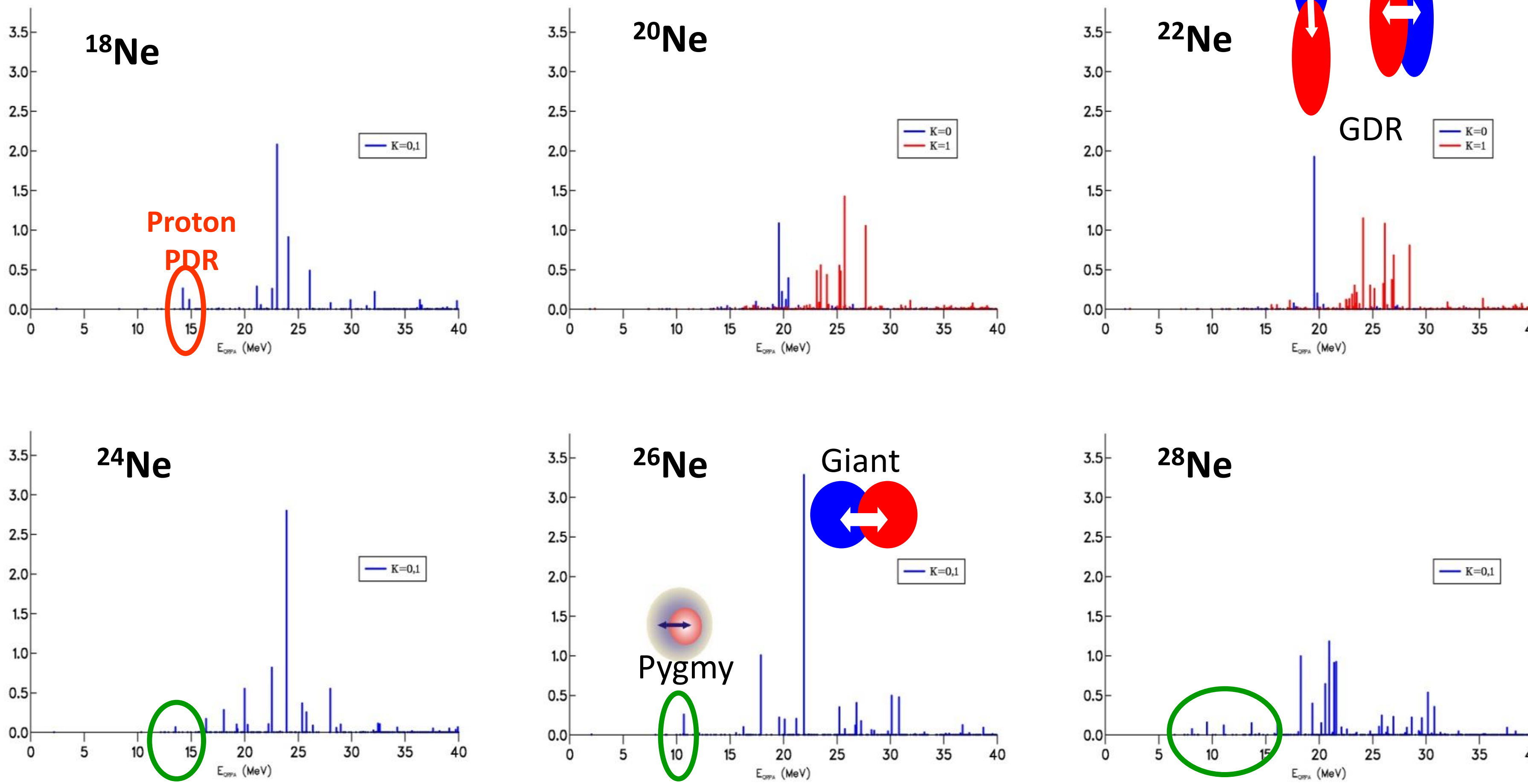


densities through axially-symmetric deformed HFB

B(E1) QRPA distributions for Ne isotopes

$$B(EJ, J_i \rightarrow J_f) = \frac{1}{2J_i+1} |\langle f || \hat{F}_J || i \rangle|^2 \qquad \hat{F}_{JM}(\mathbf{r}) = \sum_{i=1}^{A} r_i^J Y_{JM}(\hat{r}_i)$$

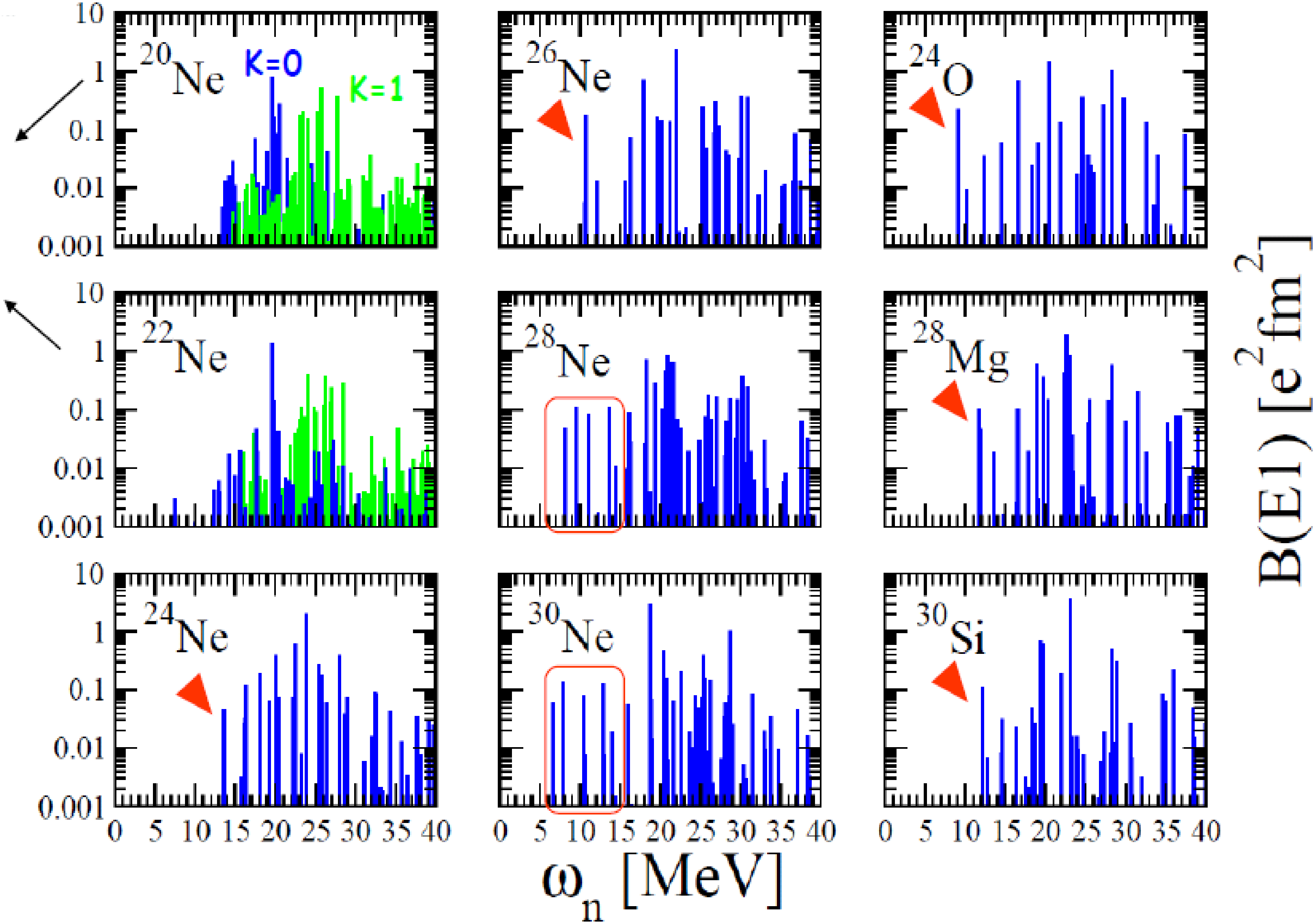




M. Martini, S. Péru and M. Dupuis, Phys. Rev. C 83, 034309 (2011)

Logarithmic scale

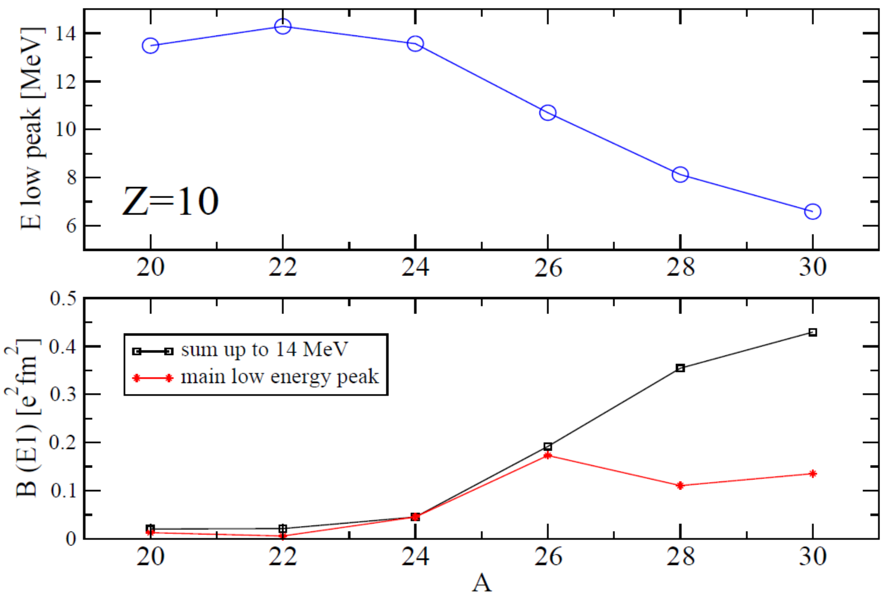
Only 2 not spherical: prolate



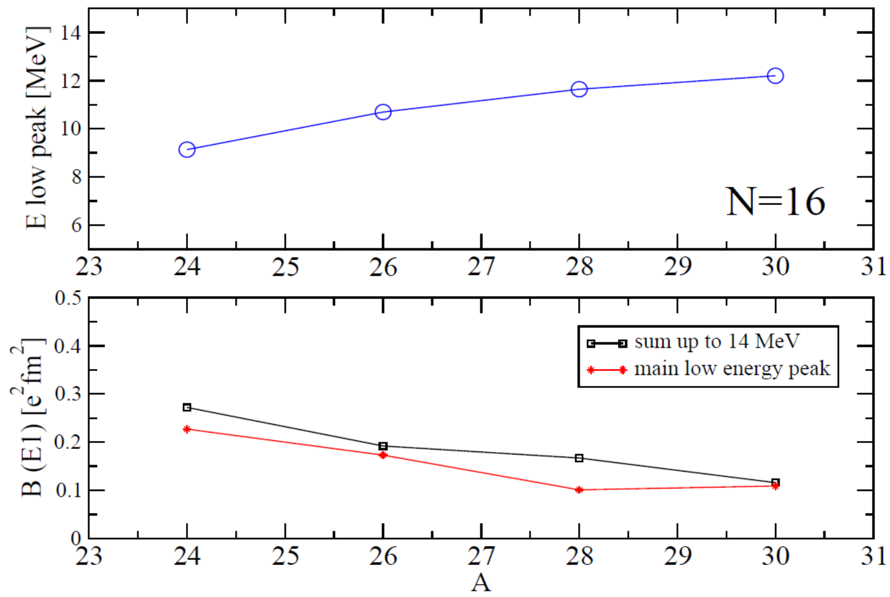
Ne isotopes and N=16 isotones

A dependence of low-lying excitations

Neon isotopes

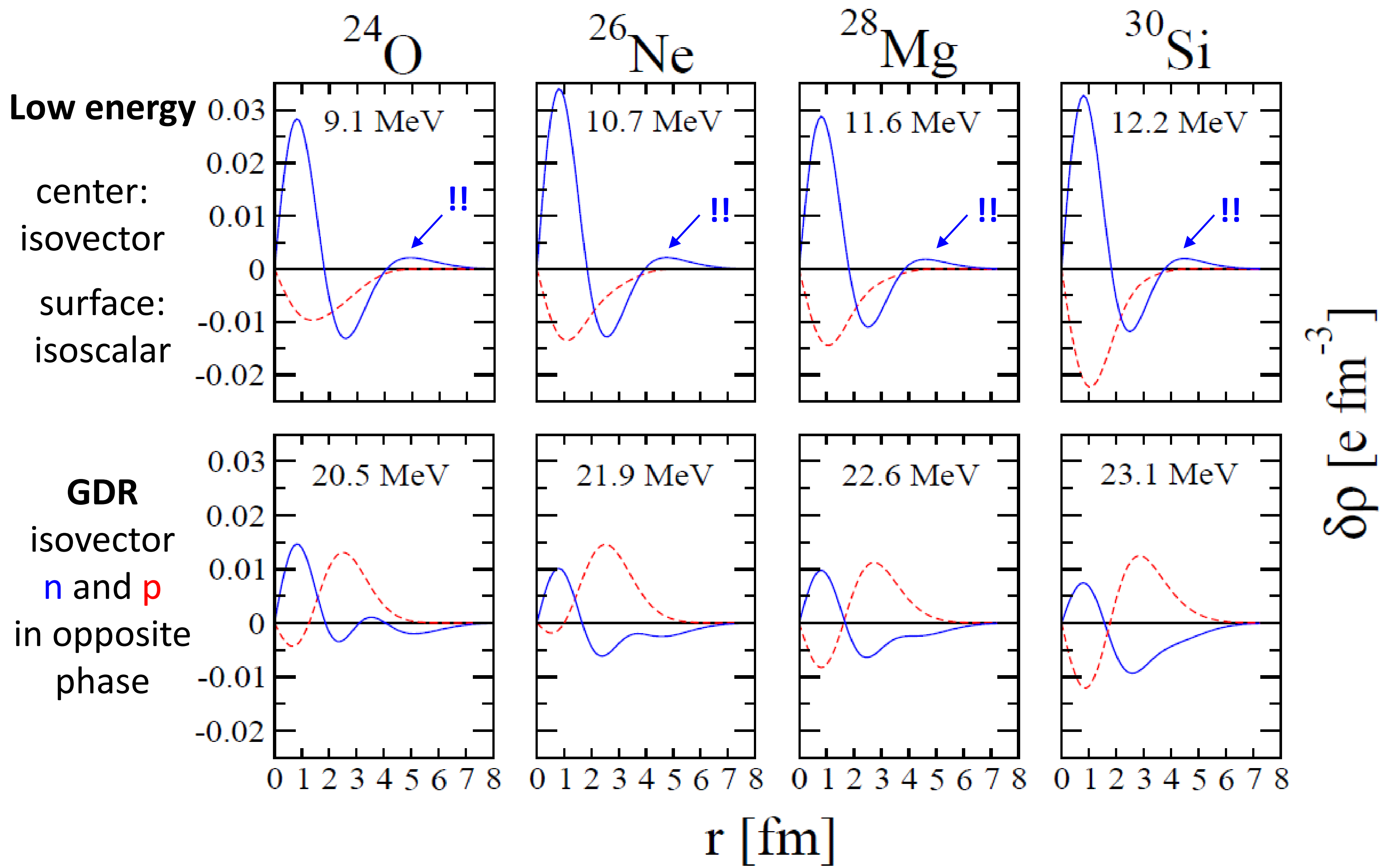


N=16 isotones



- The energy of the low peak decreases with the isospin asymmetry
- The B(E1) strength increases with the isospin asymmetry
- The fragmentation increases with the neutron number

Neutron and proton transition densities

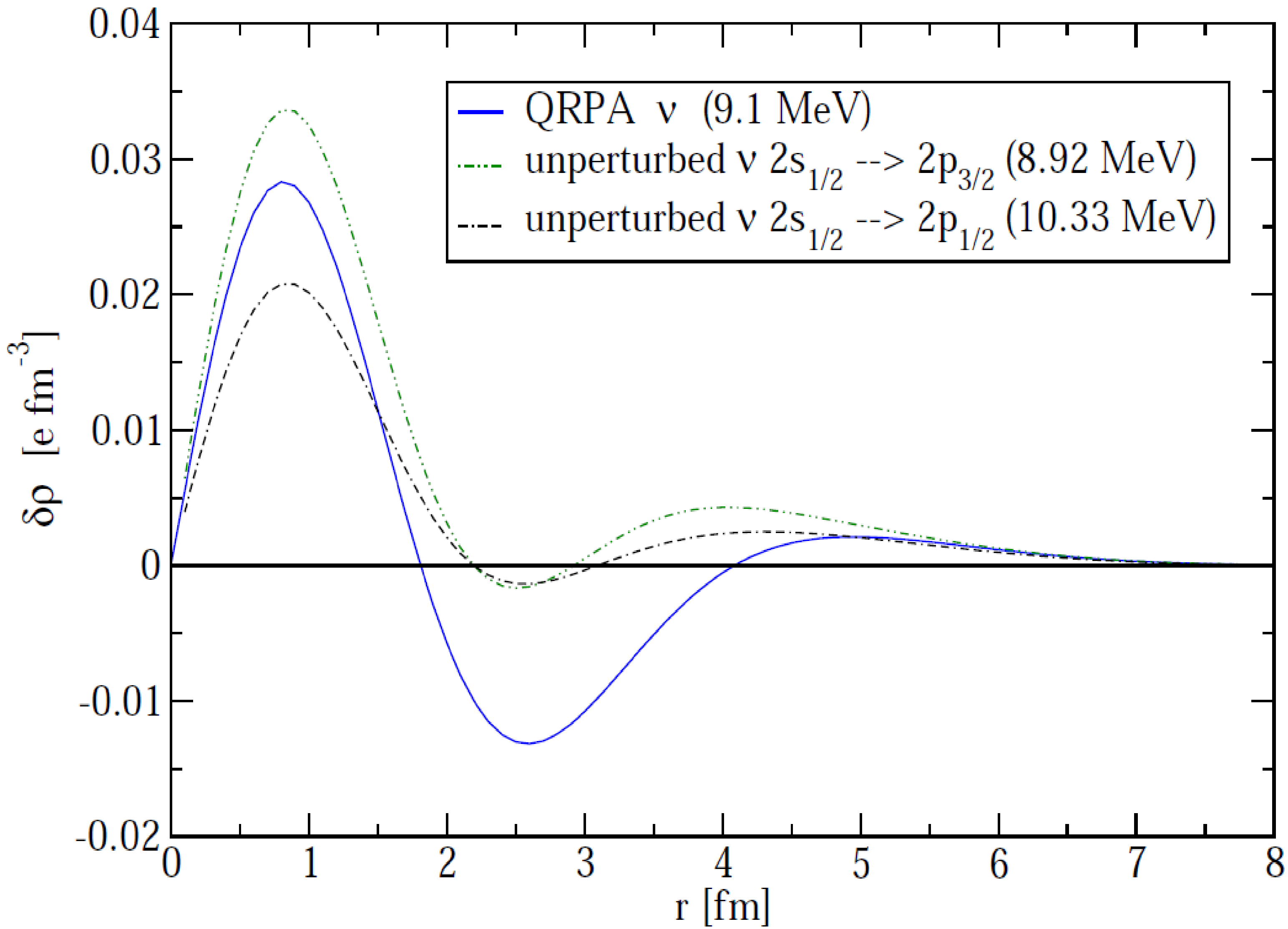


Main p-h (and p-p) configurations in N=16 isotones

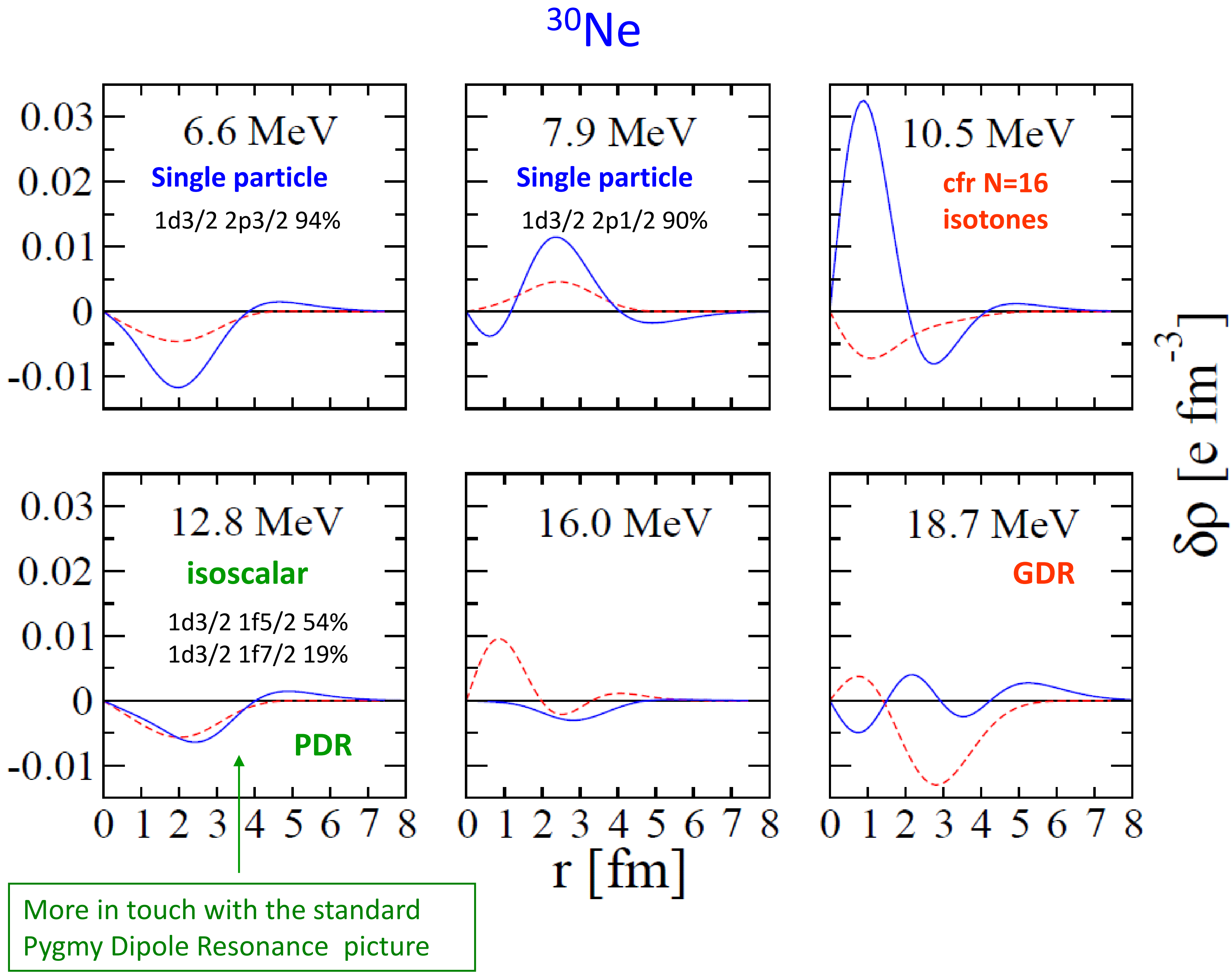
	^{24}O	^{26}Ne	^{28}Mg	^{30}Si
First peak	$\omega_n = 9.1$ MeV	$\omega_n = 10.7$ MeV	$\omega_n = 11.6$ MeV	$\omega_n = 12.2$ MeV
$\nu\ 2s_{1/2} \rightarrow 2p_{3/2}$	73.5% (8.92 MeV)	67.6% (10.52 MeV)	39.7% (11.68 MeV)	43.8% (12.61 MeV)
$\nu\ 1d_{3/2} \rightarrow 2p_{3/2}$	0.0% (9.26 MeV)	2.8% (10.82 MeV)	32.7% (11.93 MeV)	7.0% (12.79 MeV)
$\nu\ 2s_{1/2} \rightarrow 2p_{1/2}$	10.0% (10.33 MeV)	9.5% (12.44 MeV)	5.8% (13.98 MeV)	6.6% (15.23 MeV)
$\nu\ 1d_{5/2} \rightarrow 1f_{7/2}$	8.7% (12.87 MeV)	8.7% (13.68 MeV)	8.2% (14.18 MeV)	11.1% (14.55 MeV)
$\nu\ 1d_{3/2} \rightarrow 2p_{1/2}$	0.0% (10.67 MeV)	0.1% (12.74 MeV)	2.7% (14.22 MeV)	1.1% (15.41 MeV)
$\nu\ 1p_{1/2} \rightarrow 1d_{3/2}$	1.2% (17.70 MeV)	1.1% (18.50 MeV)	0.5% (19.10 MeV)	0.3% (19.60 MeV)
ν total contribution	94.6%	91.9%	90.6%	70.9%
$\pi\ 1p_{1/2} \rightarrow 2s_{1/2}$	2.0% (15.19 MeV)	4.1% (15.97 MeV)	5.6% (16.22 MeV)	18.5% (15.92 MeV)
$\pi\ p_{3/2}1d_{5/2}$	2.7% (15.89 MeV)	2.0% (17.60 MeV)	0.8% (18.97 MeV)	0.9% (17.15 MeV)
	$(1p_{3/2} \rightarrow 1d_{5/2})$	$(1p_{3/2} \rightarrow 1d_{5/2})$	$(1p_{3/2} \rightarrow 1d_{5/2})$	$(1d_{5/2} \rightarrow 2p_{3/2})$
$\pi\ 1d_{5/2} \rightarrow 1f_{7/2}$	0.0% (24.80 MeV)	0.9% (17.48 MeV)	1.9% (16.02 MeV)	7.7% (14.33 MeV)
π total contribution	5.4%	8.1%	9.4%	29.1%
Main peak (GDR)	$\omega_n = 20.5$ MeV	$\omega_n = 21.9$ MeV	$\omega_n = 22.6$ MeV	$\omega_n = 23.1$ MeV
ν total contribution	80.5%	64.5%	78.5%	54.4%
π total contribution	19.5%	35.5%	21.5%	45.6%
Peaks with	$B(E1) \geqslant 0.5\ e^2\text{fm}^2$	and	15 MeV $\leqslant \omega_n \leqslant$ 30 MeV	
ν total contribution	81.6%	78.0%	82.9%	52.7%
π total contribution	18.4%	22.0%	17.1%	47.3%
N/A	66.7%	61.5%	57.1%	53.3%

Low energy states present a small but finite collective behavior:
also other transitions with respect to the dominant neutron 2s1/2→2p3/2 contribute

Neutron transition density of ^{24}O



in the surface region the two unperturbed transition densities differ
from the one obtained in QRPA



^{30}Ne
6.6 MeV
Single particle
1d3/2 2p3/2 94%
7.9 MeV
Single particle
1d3/2 2p1/2 90%
10.5 MeV
cfr N=16
isotones
12.8 MeV
isoscalar
1d3/2 1f5/2 54%
1d3/2 1f7/2 19%
PDR
16.0 MeV
18.7 MeV
GDR
δρ [e fm^{-3}]
r [fm]
0.03
0.02
0.01
0
-0.01
0 1 2 3 4 5 6 7 8
More in touch with the standard
Pygmy Dipole Resonance picture

N.B.

This kind of analysis in the QRPA approach can be performed for any axially-symmetric deformed nucleus as well as in the charge exchange sector

Charge exchange:
Gamow-Teller excitations and β decay

M. Martini, S. Péru and S. Goriely
Phys. Rev. C 89, 044306 (2014)

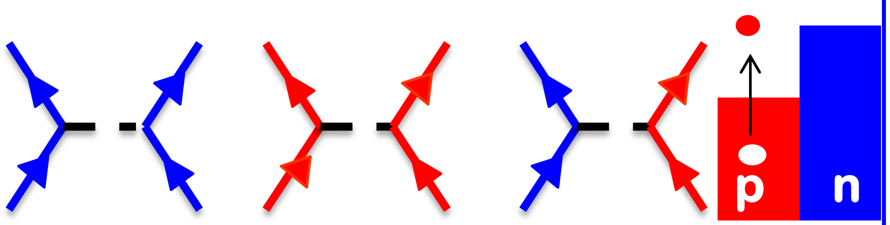
From QRPA to pnQRPA

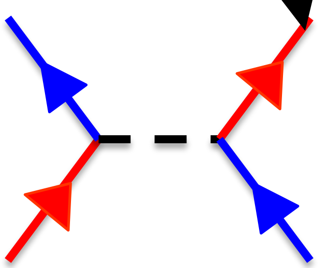
General expression $$\begin{pmatrix} A & B \\ B & A \end{pmatrix} \begin{pmatrix} X_{\alpha,K} \\ Y_{\alpha,K} \end{pmatrix} = \omega_{\alpha,K} \begin{pmatrix} X_{\alpha,K} \\ -Y_{\alpha,K} \end{pmatrix}$$

$$\theta^+_{n,K} = \sum_{i<j} X^{ij}_{n,K} \eta^+_{i,k_i} \eta^+_{j,k_j} - (-)^K Y^{ij}_{n,K} \eta_{j,-k_j} \eta_{i,-k_i}$$

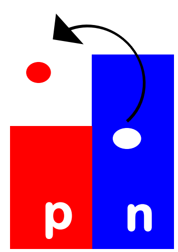
QRPA

$$\begin{aligned} \mathbf{A}_{ij,kl} = & (\epsilon_i + \epsilon_j)\delta_{ik}\delta_{jl} \\ & + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} (1+\delta_{\alpha\beta})(1+\delta_{\gamma\delta}) < \alpha\beta|\mathcal{V}|\widetilde{\gamma\delta} > \\ & \Big(\tilde{U}_{i\alpha}\tilde{V}_{j\gamma}U_{\delta k}V_{\beta l} - \tilde{U}_{i\alpha}\tilde{V}_{j\gamma}V_{\beta k}U_{\delta l} \\ & - \tilde{V}_{i\gamma}\tilde{U}_{j\alpha}U_{\delta k}V_{\beta l} + \tilde{V}_{i\gamma}\tilde{U}_{j\alpha}V_{\beta k}U_{\delta l} \\ & + \tilde{U}_{i\alpha}\tilde{U}_{j\beta}U_{\gamma k}U_{\delta l} + V_{\gamma i}V_{\delta j}\tilde{V}_{k\alpha}\tilde{V}_{l\beta} \Big) \end{aligned}$$





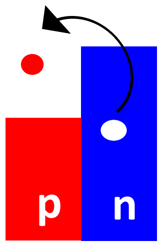
pnQRPA



$$\begin{aligned} A_{pn,p'n'} = & (\epsilon_p + \epsilon_n)\delta_{pp'}\delta_{nn'} \\ & + u_{p\pi}v_{n\nu}u_{p'\pi'}v_{n'\nu'}(\langle\pi\nu'|V|\nu\pi'\rangle - \langle\pi\nu'|V|\pi'\nu\rangle) \\ & + v_{p\pi}u_{n\nu}v_{p'\pi'}u_{n'\nu'}(\langle\nu\pi'|V|\pi\nu'\rangle - \langle\nu\pi'|V|\nu'\pi\rangle) \\ & + u_{p\pi}u_{n\nu}u_{p'\pi'}u_{n'\nu'}(\langle\pi\nu|V|\pi'\nu'\rangle - \langle\pi\nu|V|\nu'\pi'\rangle) \\ & + v_{p\pi}v_{n\nu}v_{p'\pi'}v_{n'\nu'}(\langle\pi'\nu'|V|\pi\nu\rangle - \langle\pi'\nu'|V|\nu\pi\rangle) \end{aligned}$$

$$\eta^+_p = u_{p\pi}c^+_\pi - v_{p\pi}c_\pi$$

$$\theta^+_{\alpha,K} = \sum_{pn} X^{pn}_{\alpha,K}\eta^+_p\eta^+_n - (-)^K Y^{pn}_{\alpha,K}\eta_n\eta_p$$



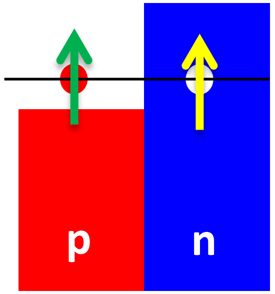
Main charge exchange excitations

Isobaric Analog Resonance (IAR)

$$\hat{O}_{IAR} = \sum_{i=1}^{A} \tau_-(i)$$

isospin flip τ

S=0 T=1 $J^\pi=0^+$



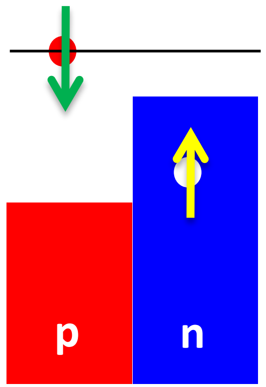
Gamow Teller (GT)

$$\hat{O}_{GT} = \sum_{i=1}^{A} \vec{\sigma}(i)\ \tau_-(i)$$

isospin flip τ

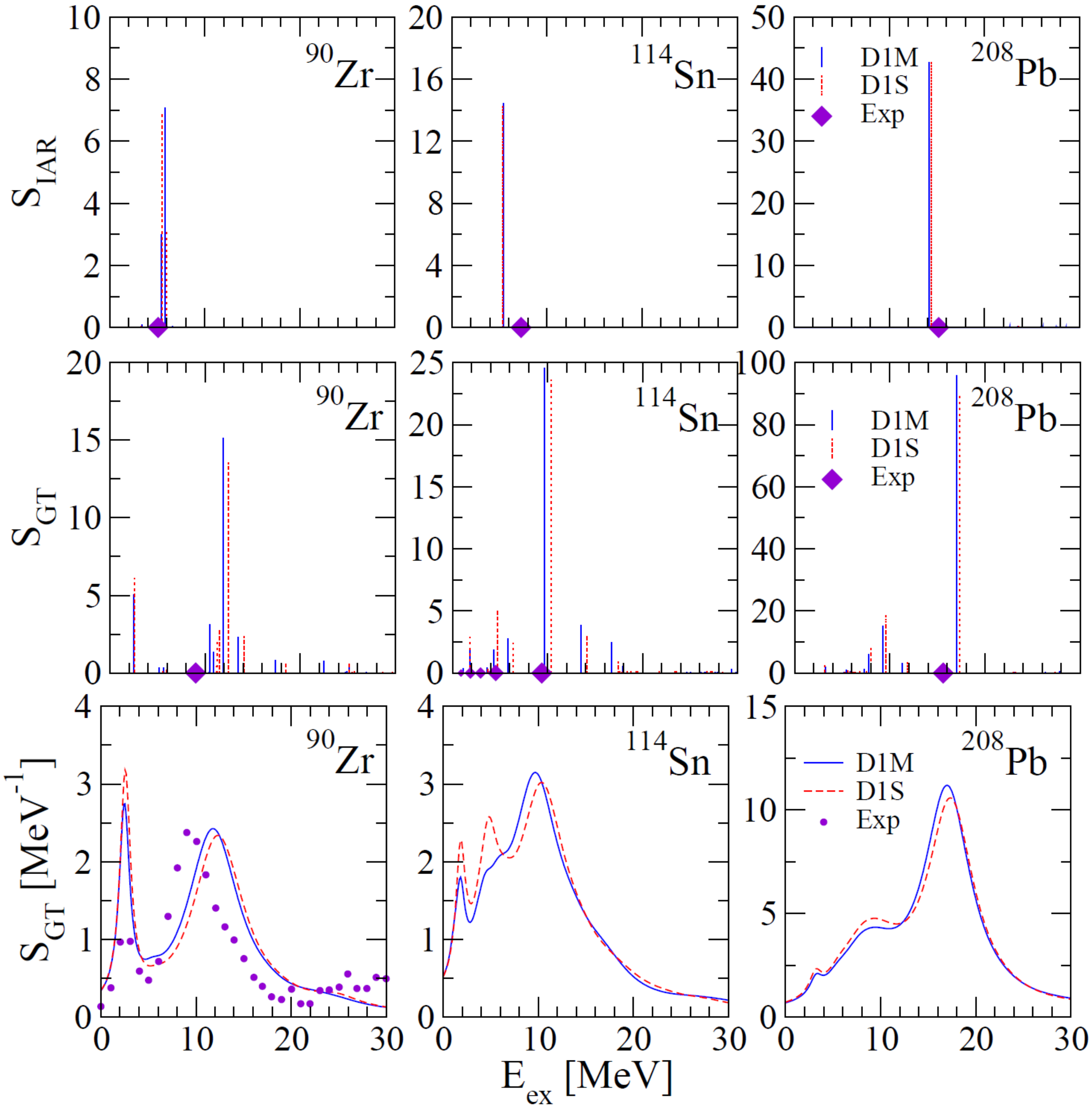
spin flip σ

S=1 T=1 $J^\pi=1^+$



pnQRPA Strength Distributions with Gogny (D1M and D1S) force

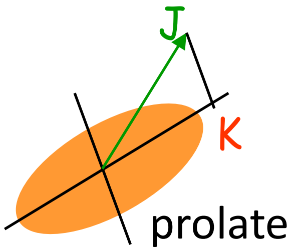
$$\left|\langle JK | \hat{O} | 0 \rangle\right|^2$$

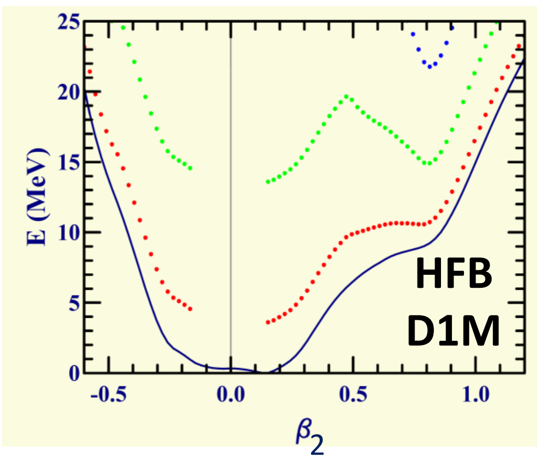


M. Martini, S. Péru and S. Goriely, Phys. Rev. C **89**, 044306 (2014)

An example of deformed nucleus : ^{76}Ge

GT $J^{\pi}=1^{+}$ distributions obtained by adding twice the $K^{\pi}=1^{+}$ result to the $K^{\pi}=0^{+}$ one

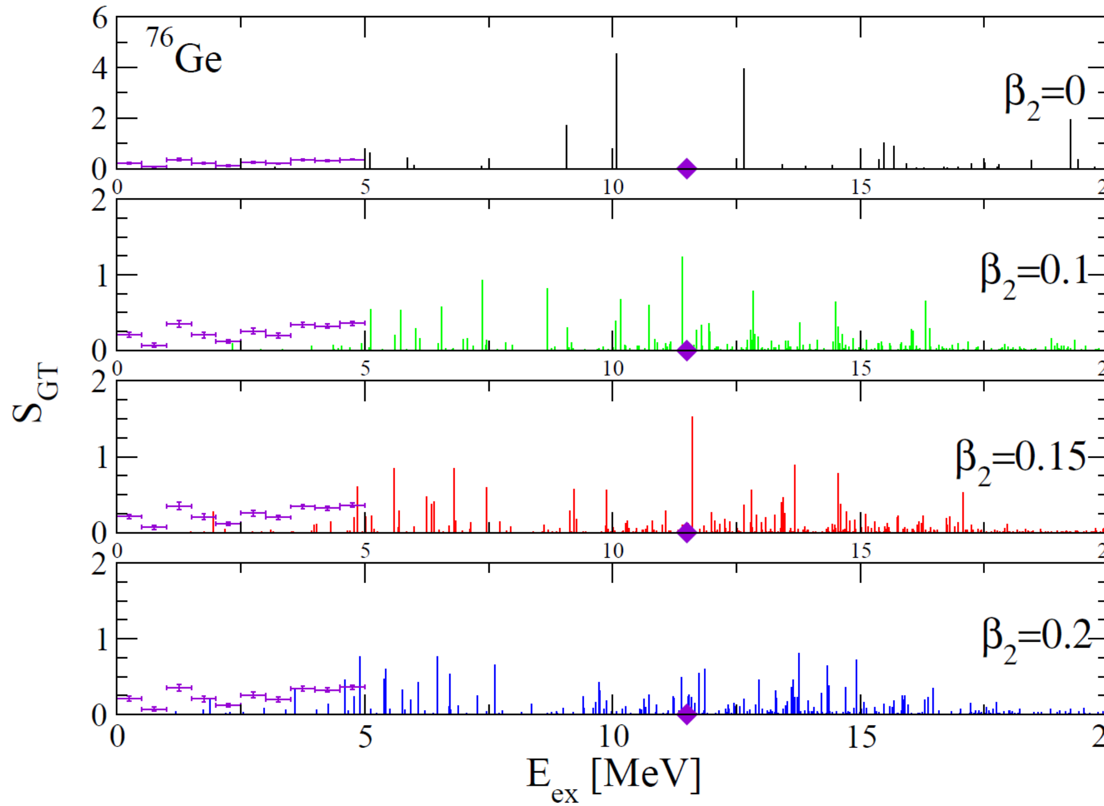




Quadrupole deformation parameter

$$\beta_2 \propto \langle HFB|3z^2 - r^2|HFB\rangle$$

pnQRPA-D1M



- The deformation tends to increase the fragmentation
- Displacements of the peaks
- Deformation effects also influence the low energy strength

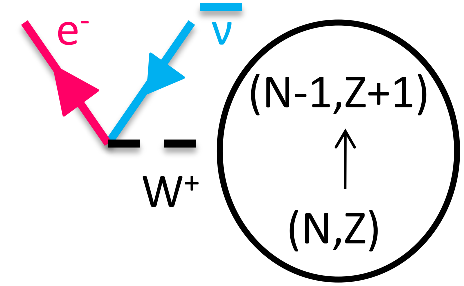
Folding of GT strength

The pnQRPA calculation provides, as shown in previous figures, a discrete strength distribution. In order to derive a smooth continuous strength function, the pnQRPA GT strength is folded with a Lorentz function $L(E,\omega)$ of width Γ

$$S_{GT}^{fold}(E) = \int_{-\infty}^{+\infty} L(E,\omega) S_{GT}(\omega) d\omega \qquad L(E,\omega) = \frac{1}{\pi} \frac{\Gamma/2}{(E-\omega)^2 + \Gamma^2/4}$$

The spreading width Γ is the only parameter of our calculation.
For all the calculations of β decay half-life we use S^{fold} and we take Γ=2 MeV.

β^- decay half-life $T_{1/2}$



In the allowed GT decay approximation the β^- decay half-life $T_{1/2}$ can be expressed in terms of the **GT strength function S_{GT}**

$$\boxed{\frac{\ln 2}{T_{1/2}} = \frac{(g_A/g_V)^2_{\rm eff}}{D} \sum_{E_{ex}=0}^{Q_\beta} f_0(Z, A, Q_\beta - E_{ex}) S_{GT}(E_{ex})}$$

$$D = 6163.4 \pm 3.8 \;\; \mathrm{s} \qquad g_A/g_V = 1.26 \qquad (g_A/g_V)_{\rm eff} = 1$$

Lepton phase-space volume: $f_0(Z,A,\omega) = \int_{m_e c^2}^{\omega} p_e E_e (\omega - E_e)^2 F_0(Z,A,E_e) dE_e$

$$E_{ex} = \omega_{QRPA} - E_0$$

The reference energy E_0 corresponds to the lowest 2-qp excitation associated with the ground state of the odd-odd daughter nucleus in which the quantum numbers of the single quasi-proton and neutron states are obtained from the self-consistent HFB calculation of the odd-odd system.

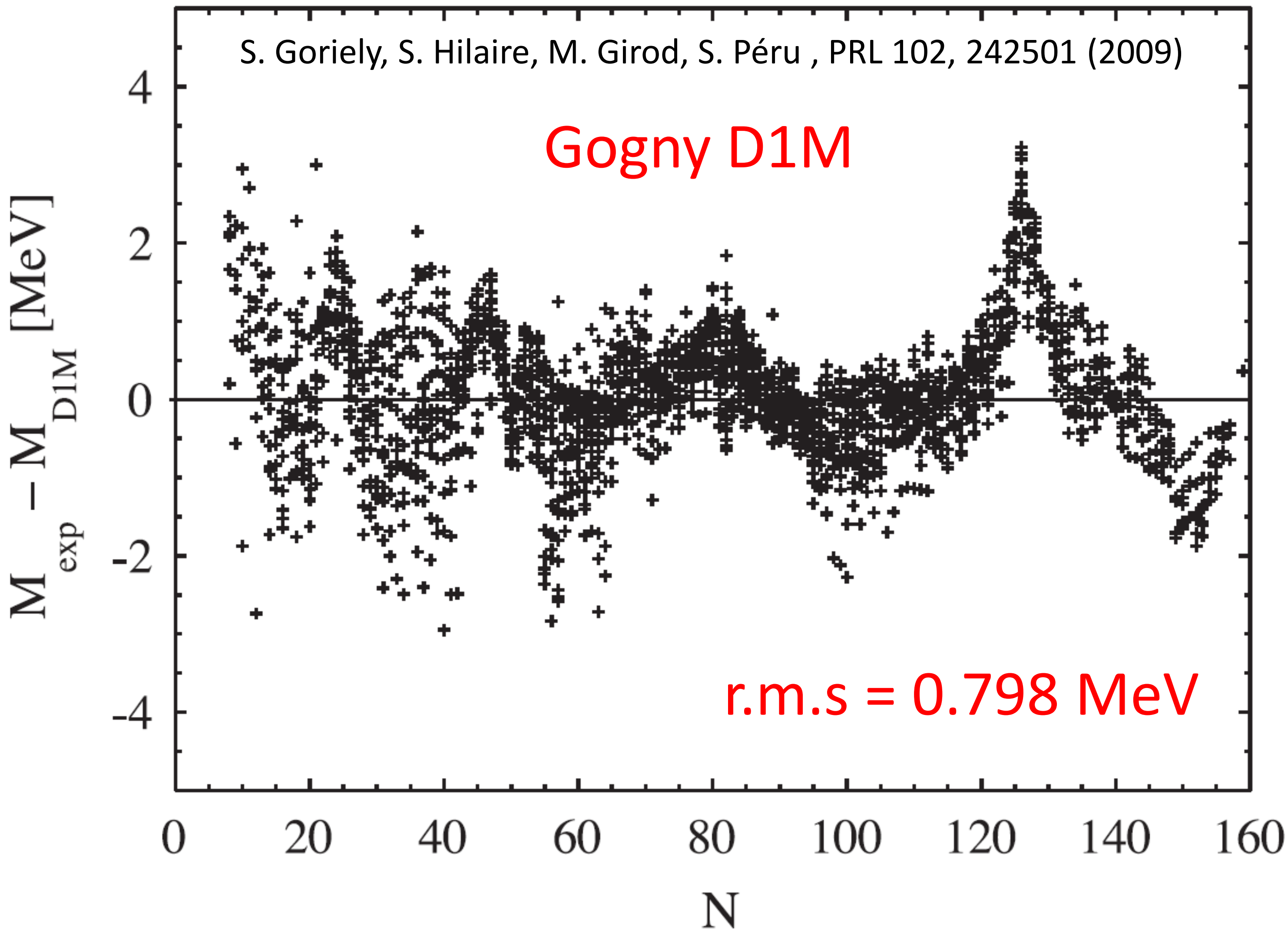
In most cases E_0 is equal to the lowest energy of the 2-qp excitation of ph type.

$$\Delta M_{Z,Z+1} = Q_\beta = B_{\rm nucl}(Z,A) - B_{\rm nucl}(Z+1,A) + m(nH)$$

For the Q_β mass differences, we take experimental (and recommended) masses when available or the D1M mass predictions , otherwise.

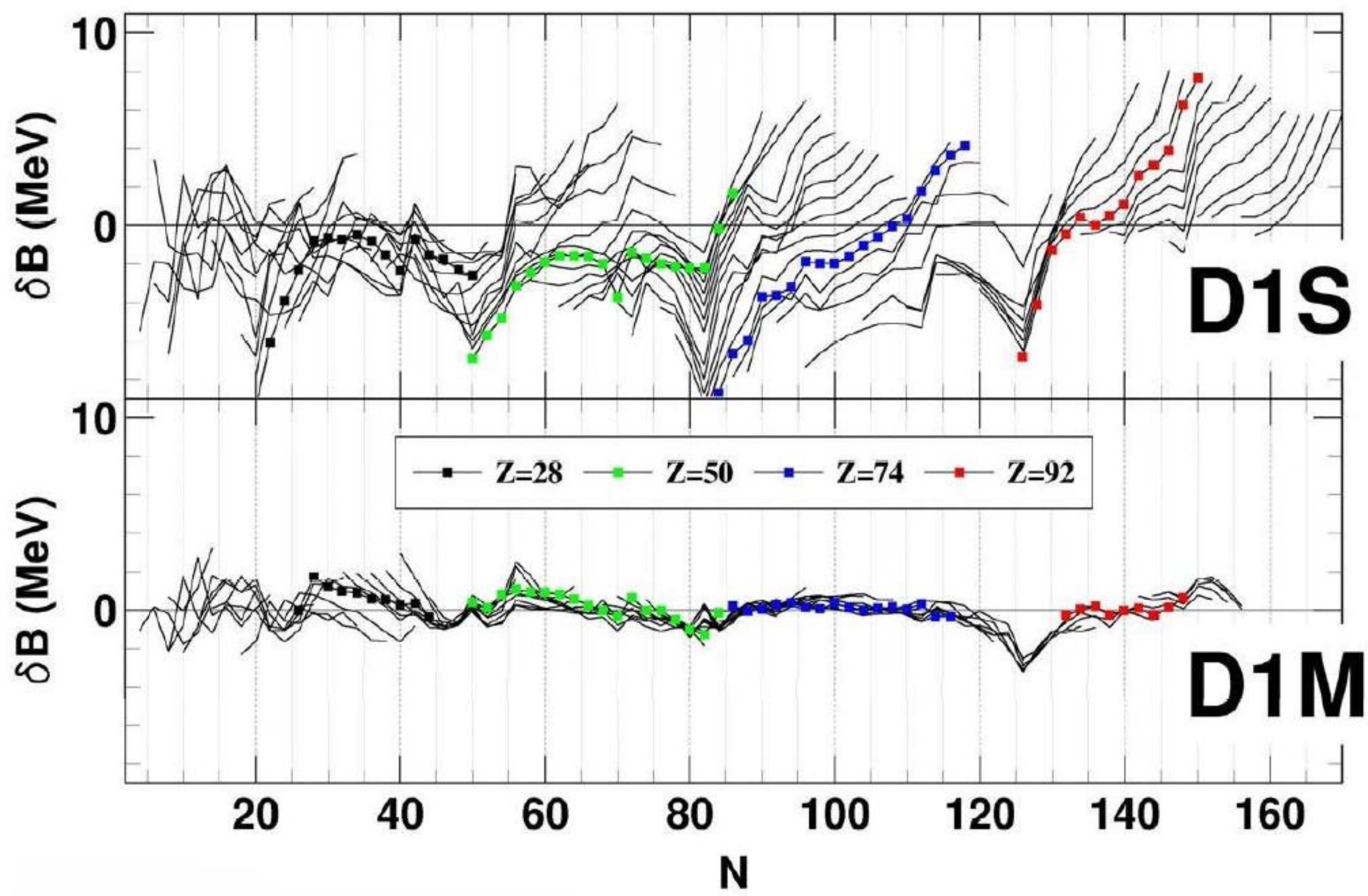
Nuclear Masses

Comparison with experimental data
(2149 nuclei: Audi, Wapstra & Thibault 2003)



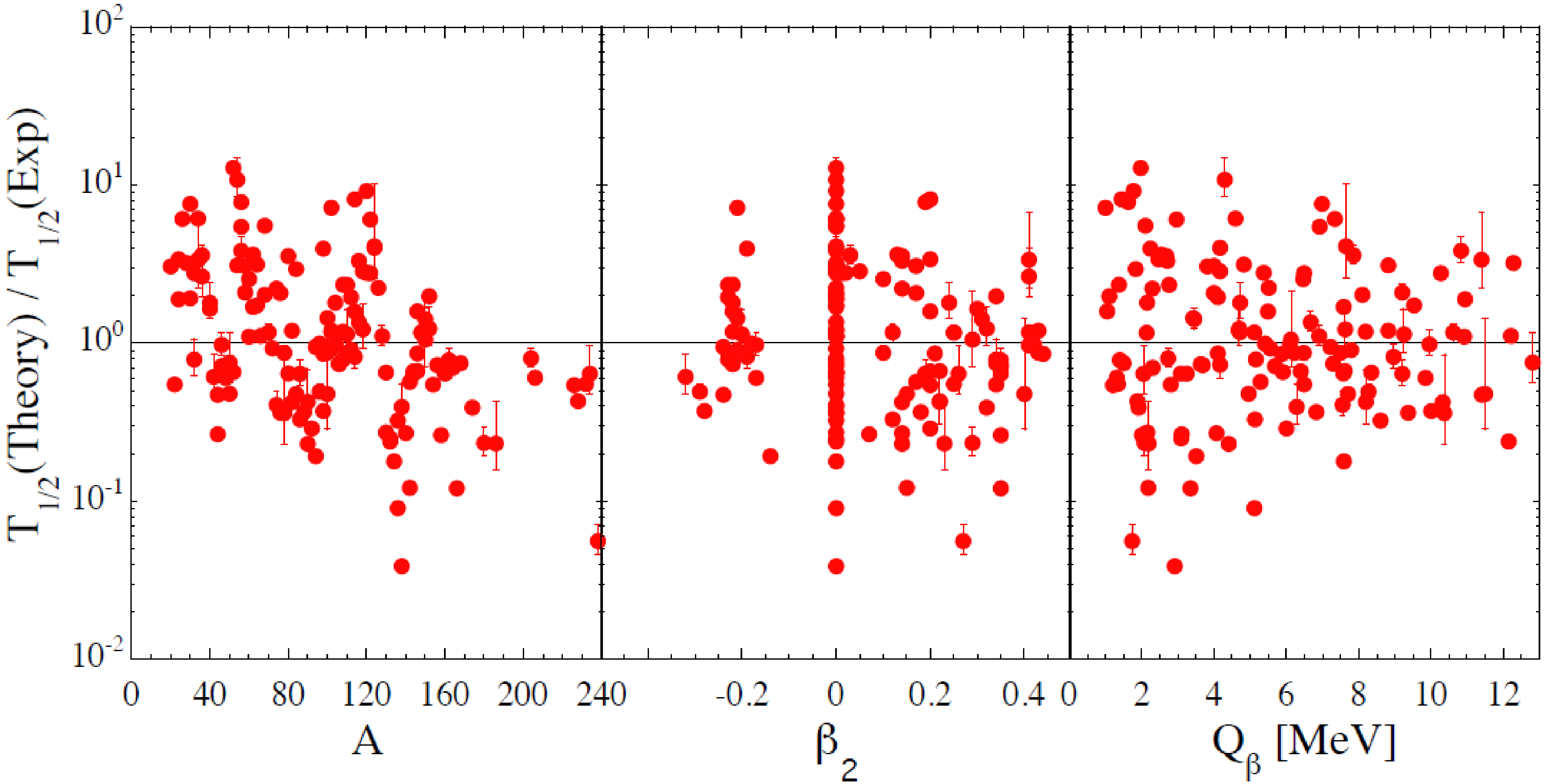
r.m.s. deviation at the same level of some Skyrme microscopic or macroscopic-microscopic approaches

Comparison between D1M and D1S (from 1991)

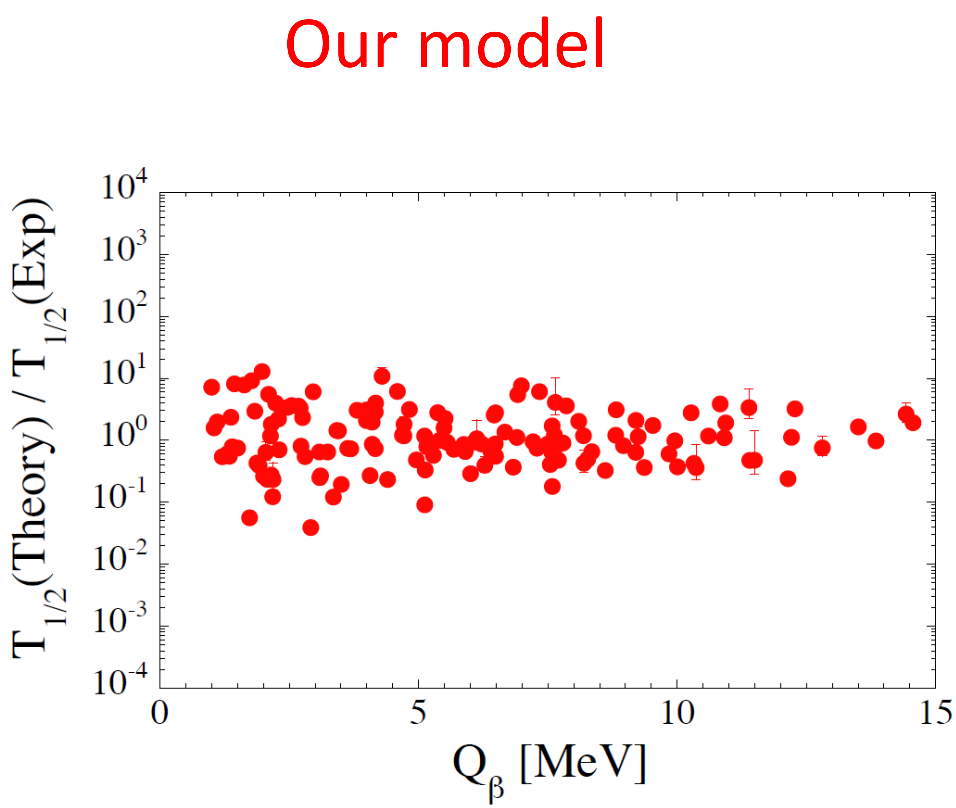


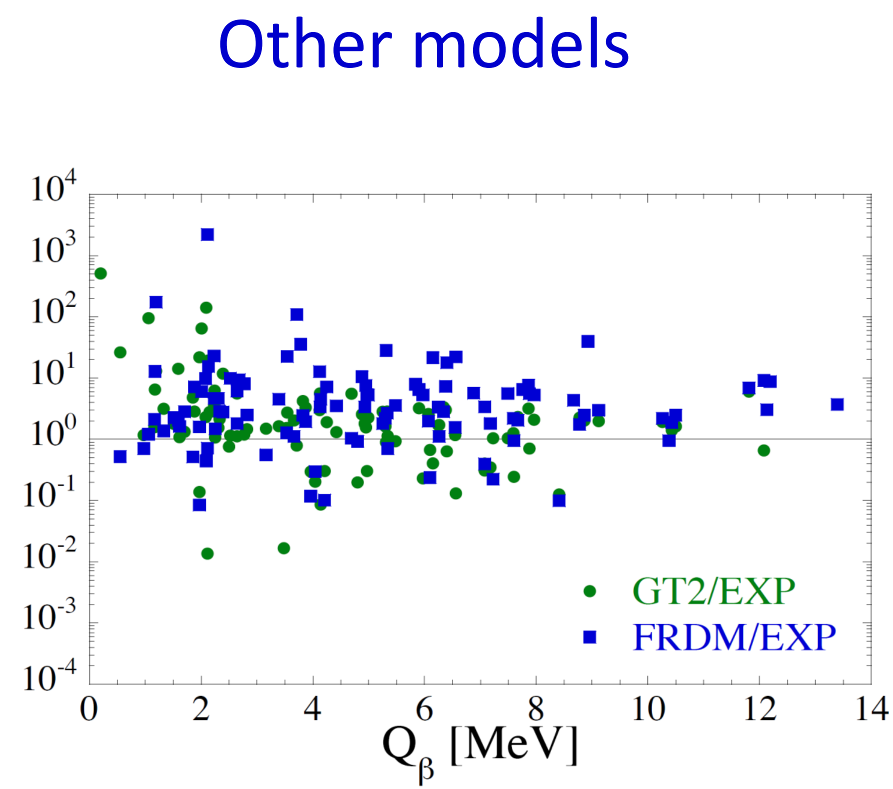
β^- decay half-life $T_{1/2}$

Comparison with experimental data for 145 even-even nuclei



- Deviation with respect to data rarely exceeds one order of magnitude
- Larger deviations for nuclei close to the valley of β-stability, as found in most models

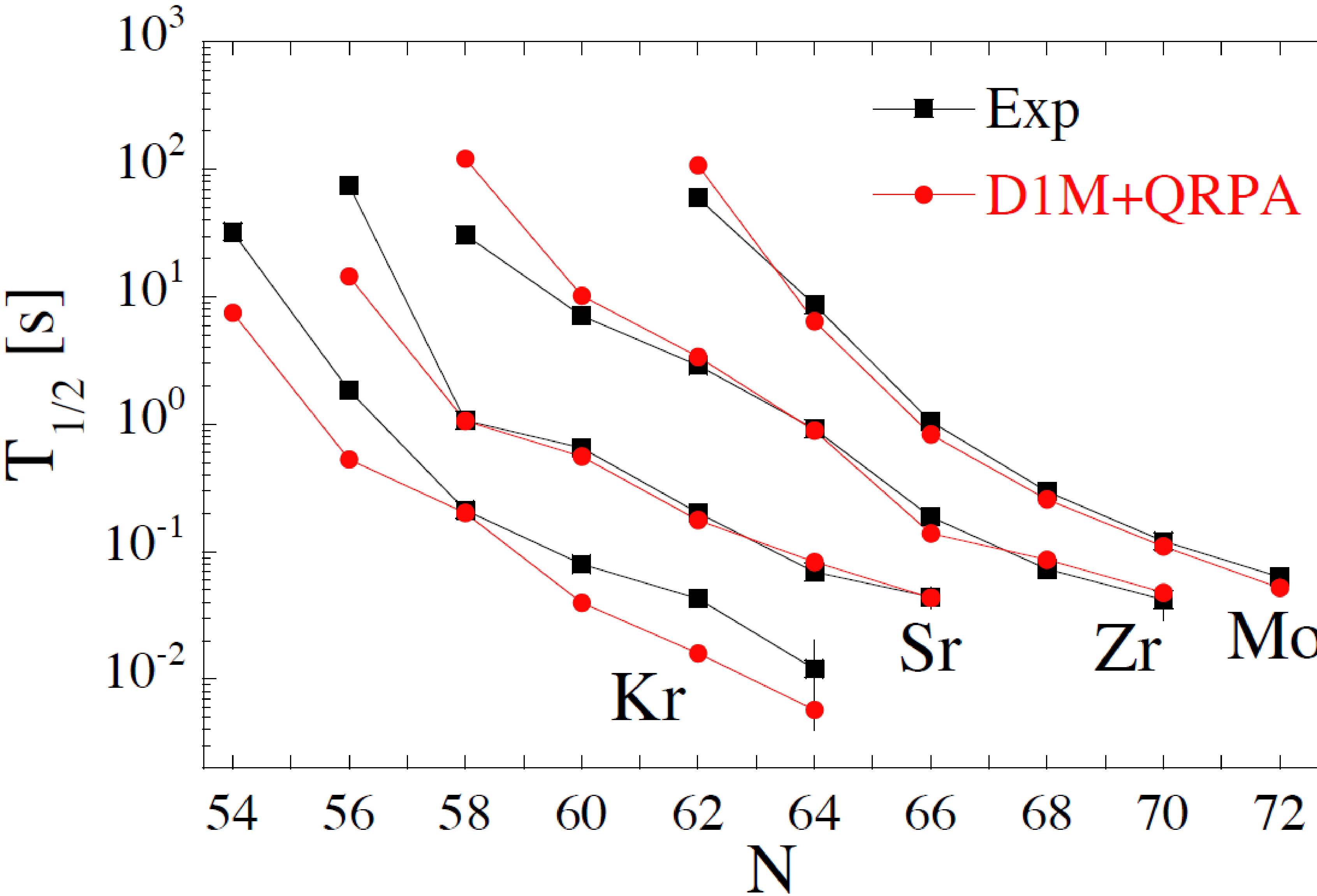




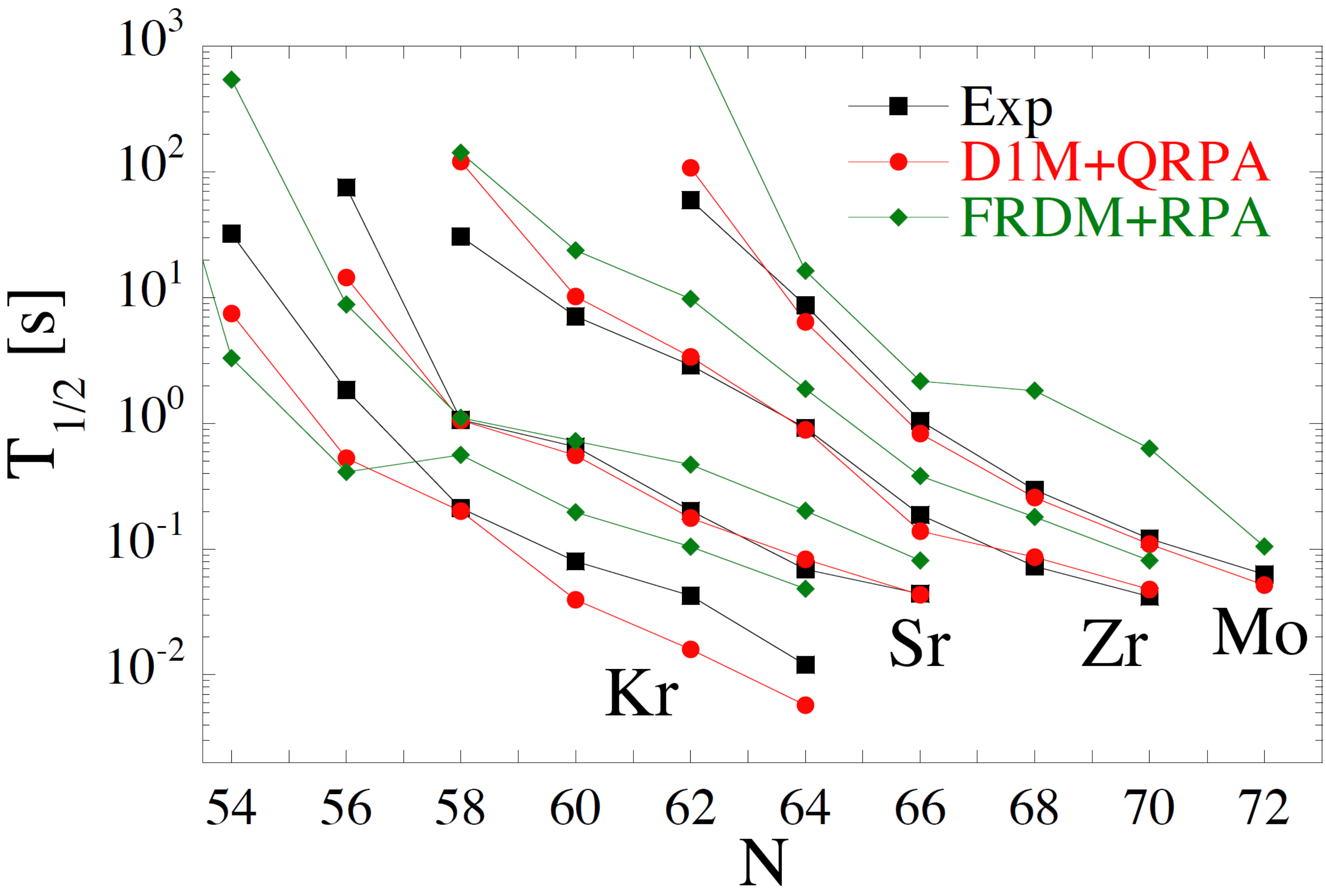
GT2: Tachibana et al.
Prog. Theor. Phys., 84, 641 (1990)

FRDM: Moller et al., ADNDT, 66,131 (1997)

β^- decay half-lives of deformed isotopic chains

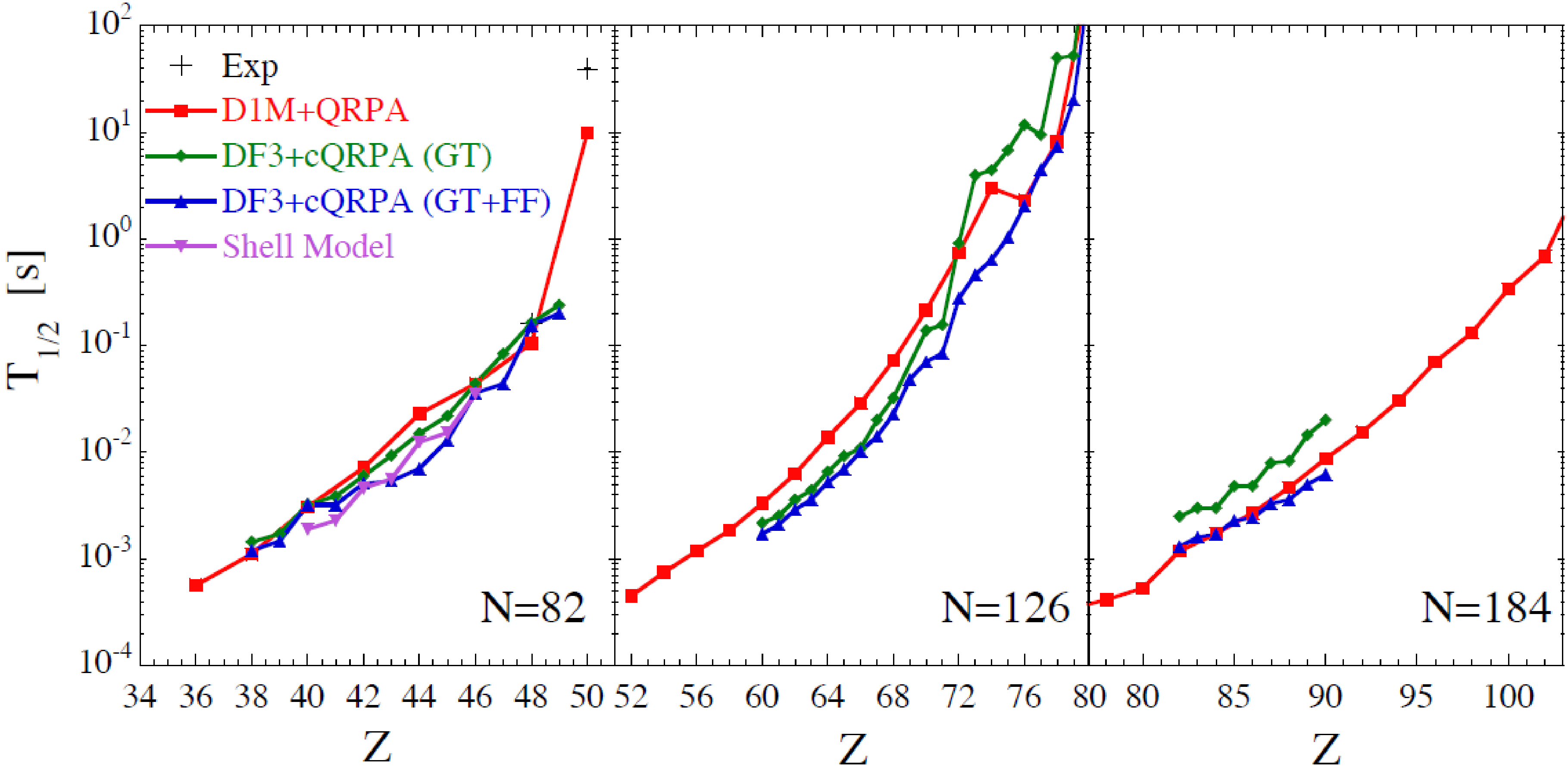


β^- decay half-lives of deformed isotopic chains



β^- decay half-lives of the N=82, 126, 184 isotones

Relevance for the r-process nucleosynthesis

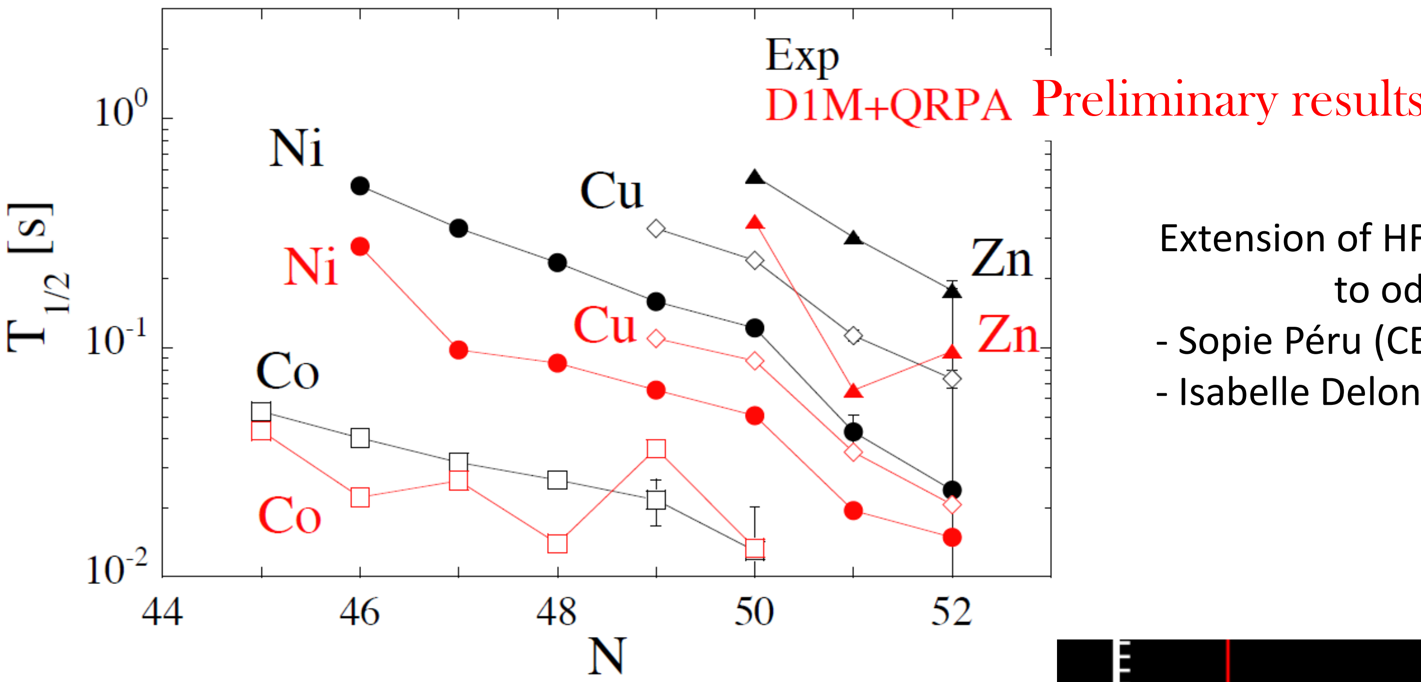


DF3+cQRPA: Borzov et al., PRC 62, 035501 (2000)

Shell Model: Martinez-Pinedo et al., PRL 83, 4502 (1999)

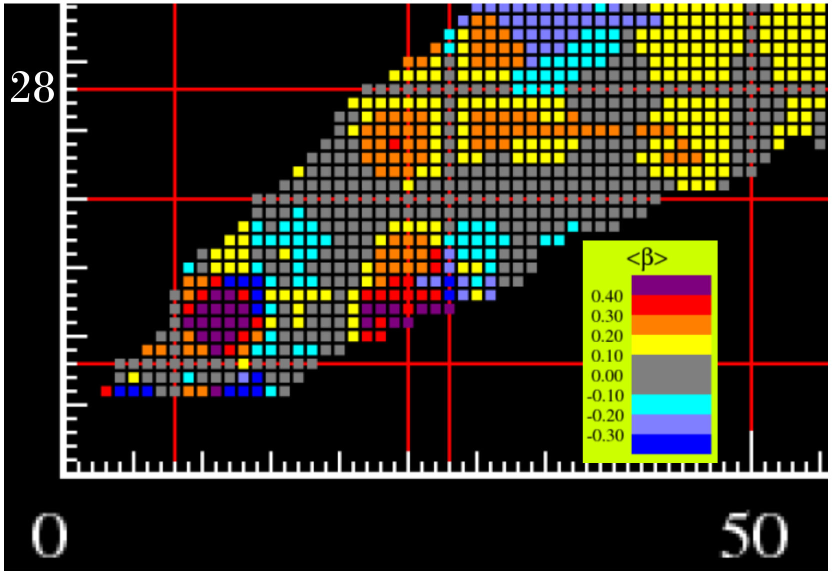
Possible origins of differences:
GT Strengths, estimation of Q_β values, …

Extension to odd systems (Preliminary results)



Extension of HFB+QRPA calculations to odd systems:

- Sopie Péru (CEA/DAM)
- Isabelle Deloncle (CSNSM) Orsay



Recent experimental results

Z.Y. Xu et al, PRL 113, 032505 (2014)

β-decay Half lives of ^{76,77}Co, ^{79,80}Ni and ^{81}Cu :

Experimental indication of a Doubly Magic ^{78}Ni

Summary

Low-energy dipole excitations in Ne isotopes and N=16 isotones

- An example of microscopic analysis with the QRPA approach

Charge exchange excitations and β decay

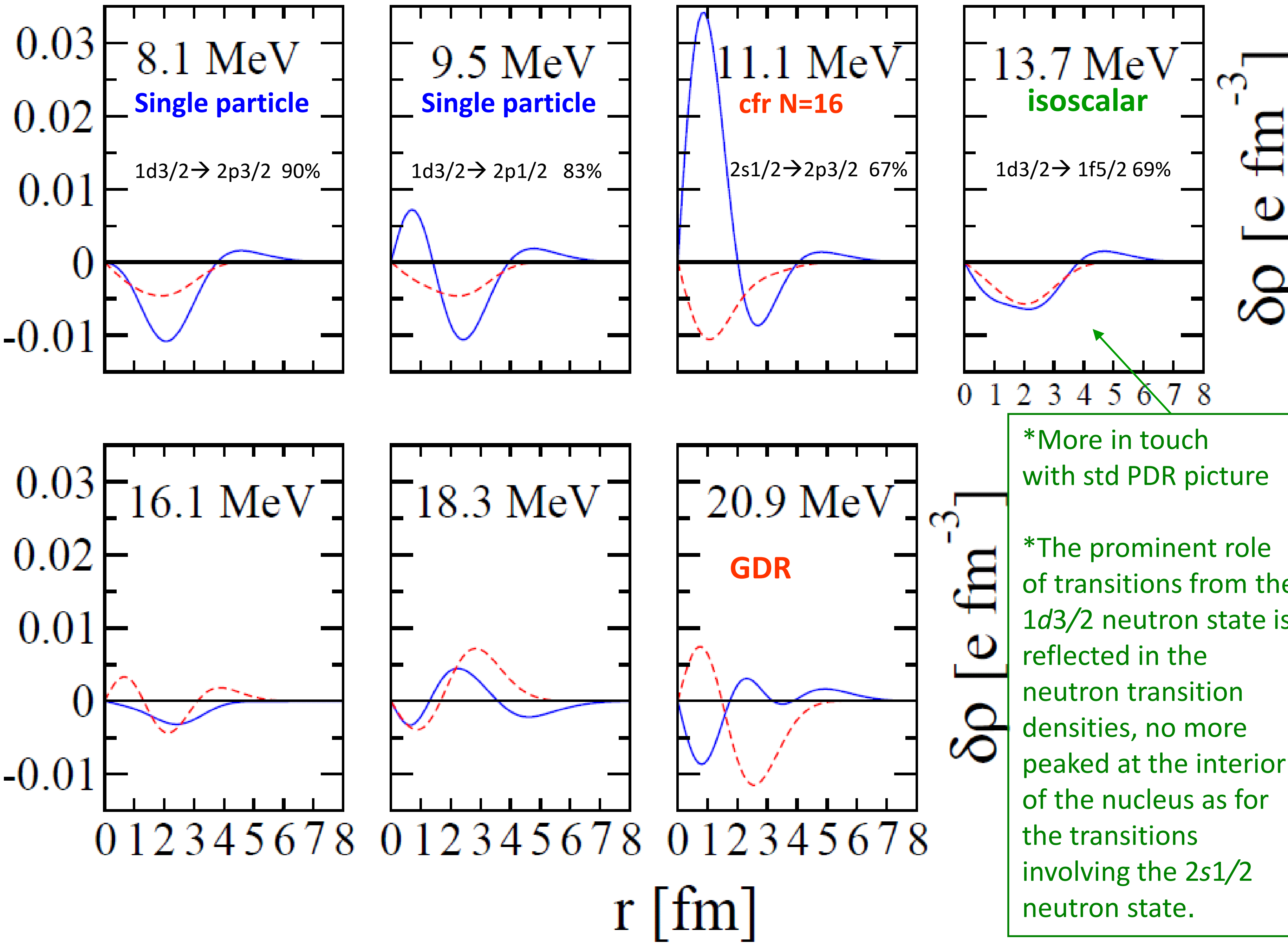
- For spherical nuclei, IAR and GT results in good agreement with data
- The role of the intrinsic deformation has been shown for prolate ^{76}Ge
- Predictions of the β decay half-lives of spherical and deformed nuclei are compatible with experimental data
- The satisfactory agreement with experimental half-lives justifies the additional study on the exotic neutron-rich N = 82, 126 and 184 isotonic chains (r-process)

Perspectives in the charge exchange sector

- Odd systems
- First forbidden transitions
- β delayed processes

Spares

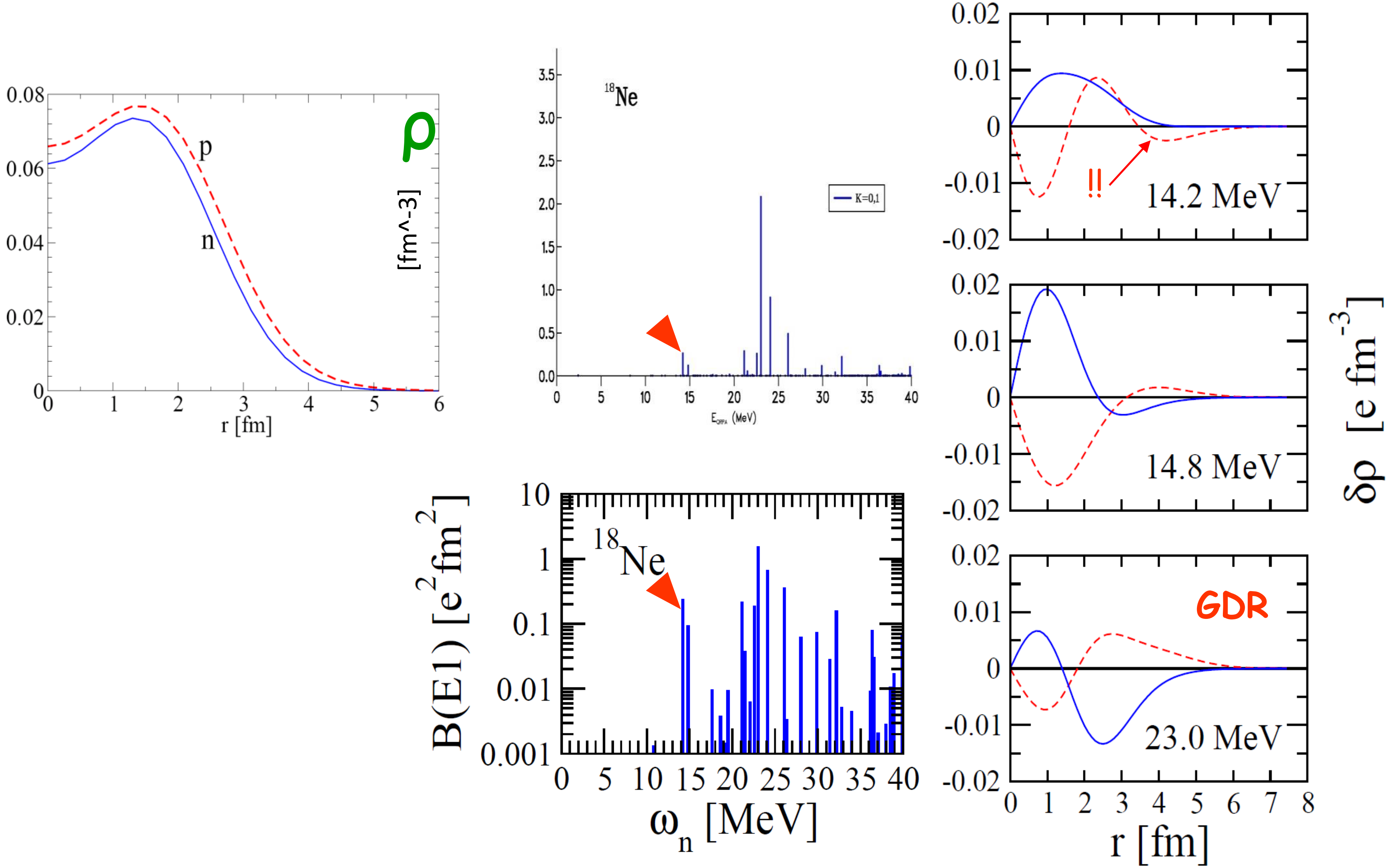
^{28}Ne



*More in touch with std PDR picture

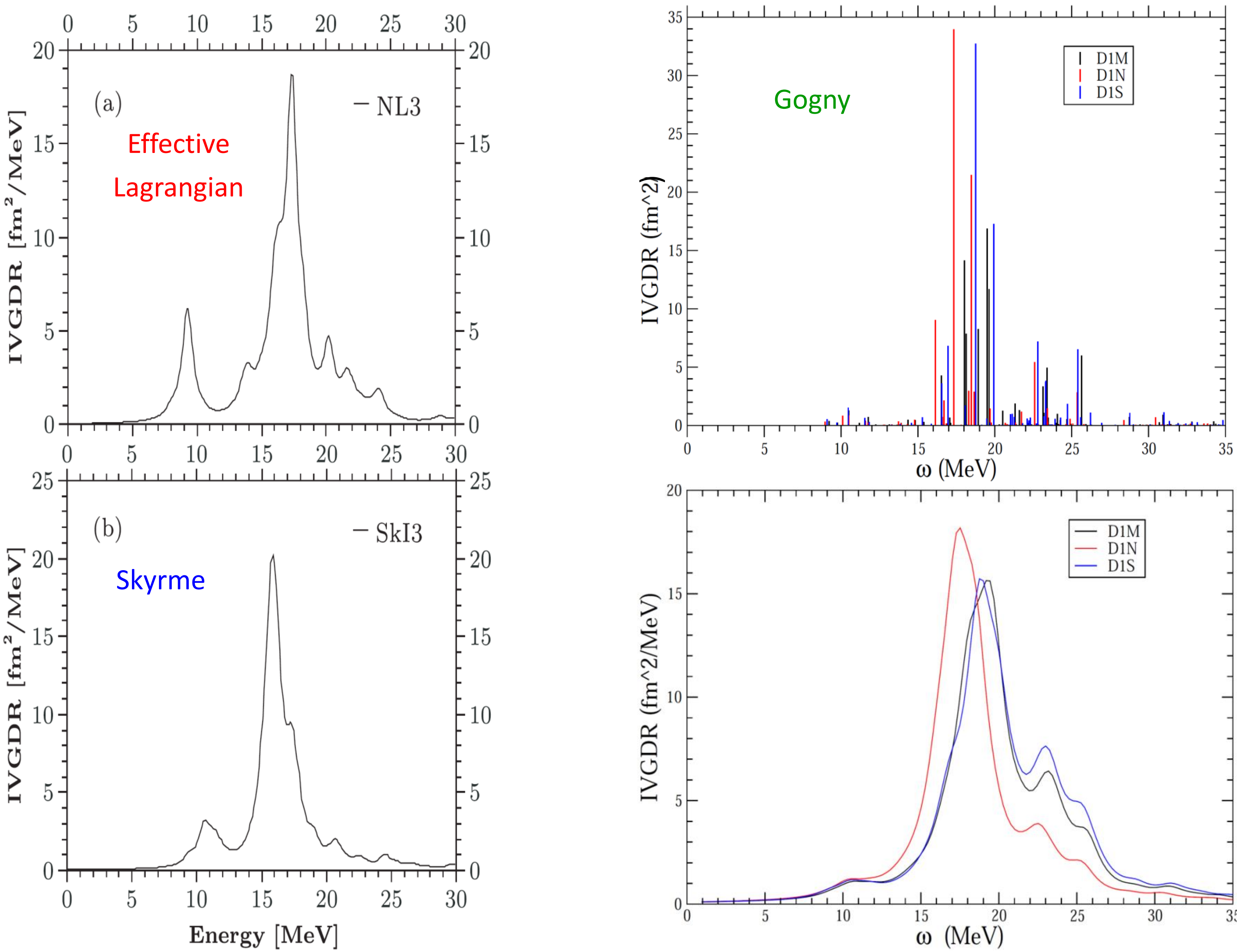
*The prominent role of transitions from the 1*d*3/2 neutron state is reflected in the neutron transition densities, no more peaked at the interior of the nucleus as for the transitions involving the 2*s*1/2 neutron state.

proton PDR: ^{18}Ne



Dipole Resonances in ^{68}Ni

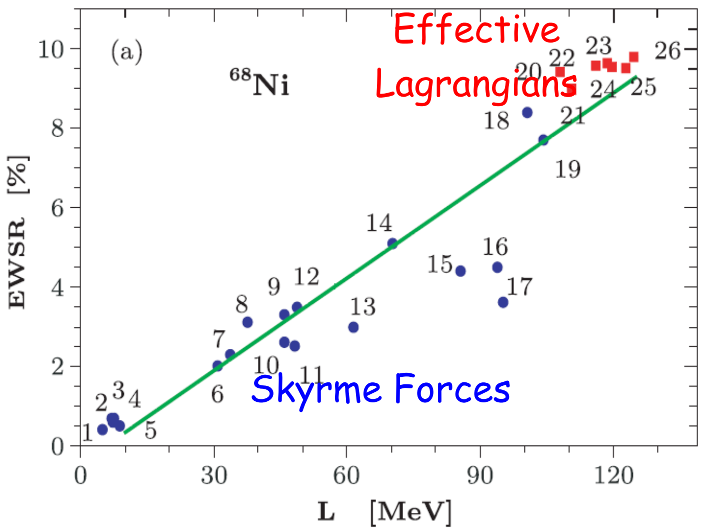
Comparison among models and forces

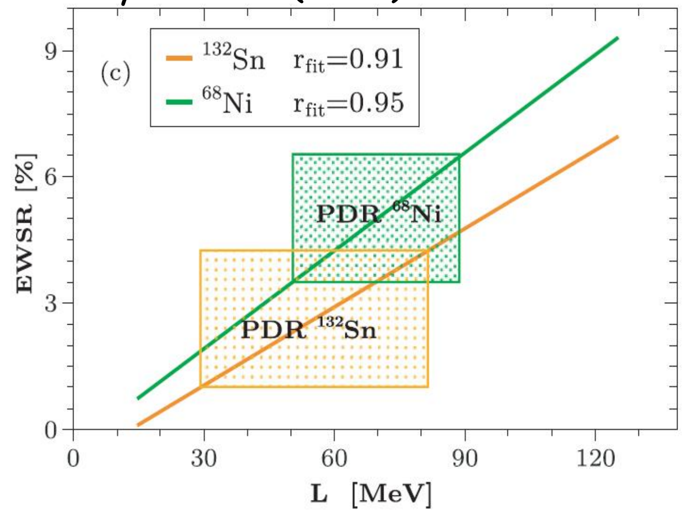


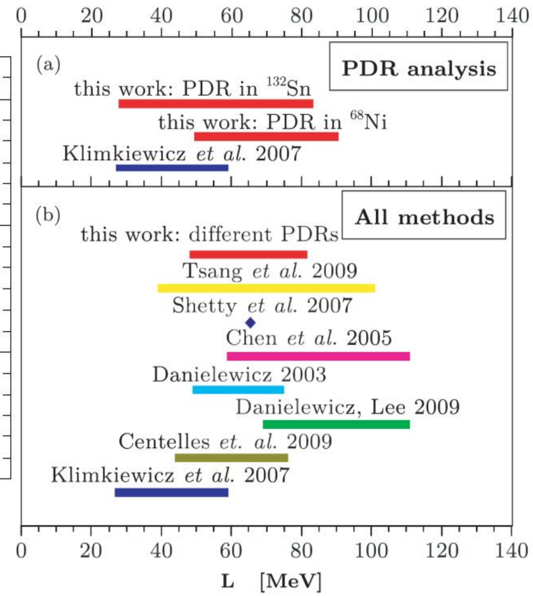
Correlations between PDR and symmetry energy

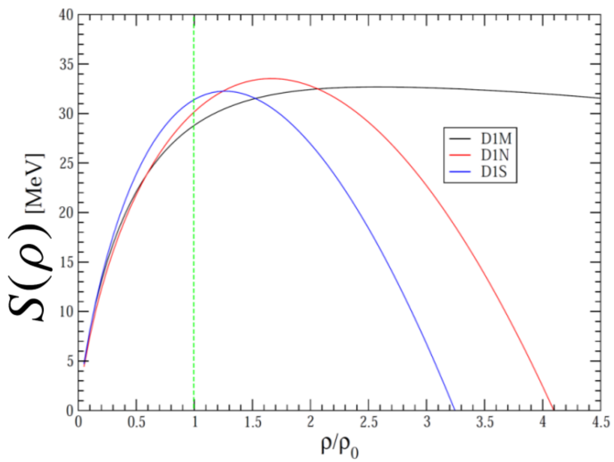
$$\frac{E}{A}(\rho,\delta) = \frac{E}{A}(\rho,\delta=0) + S(\rho)\delta^2 \qquad S'(\rho)|_{\rho=\rho_0} = \frac{L}{3\rho_0}$$

Carbone et al. Phys. Rev. C (2010)





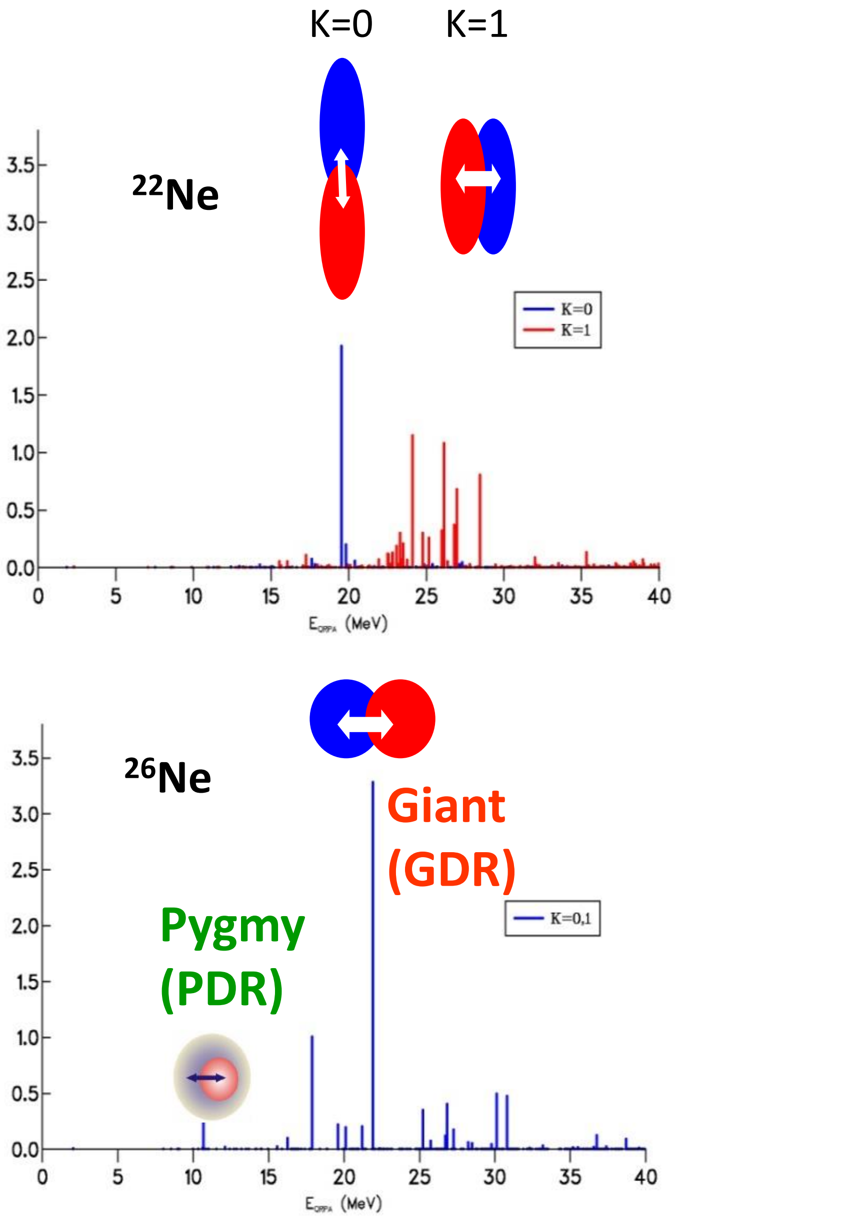




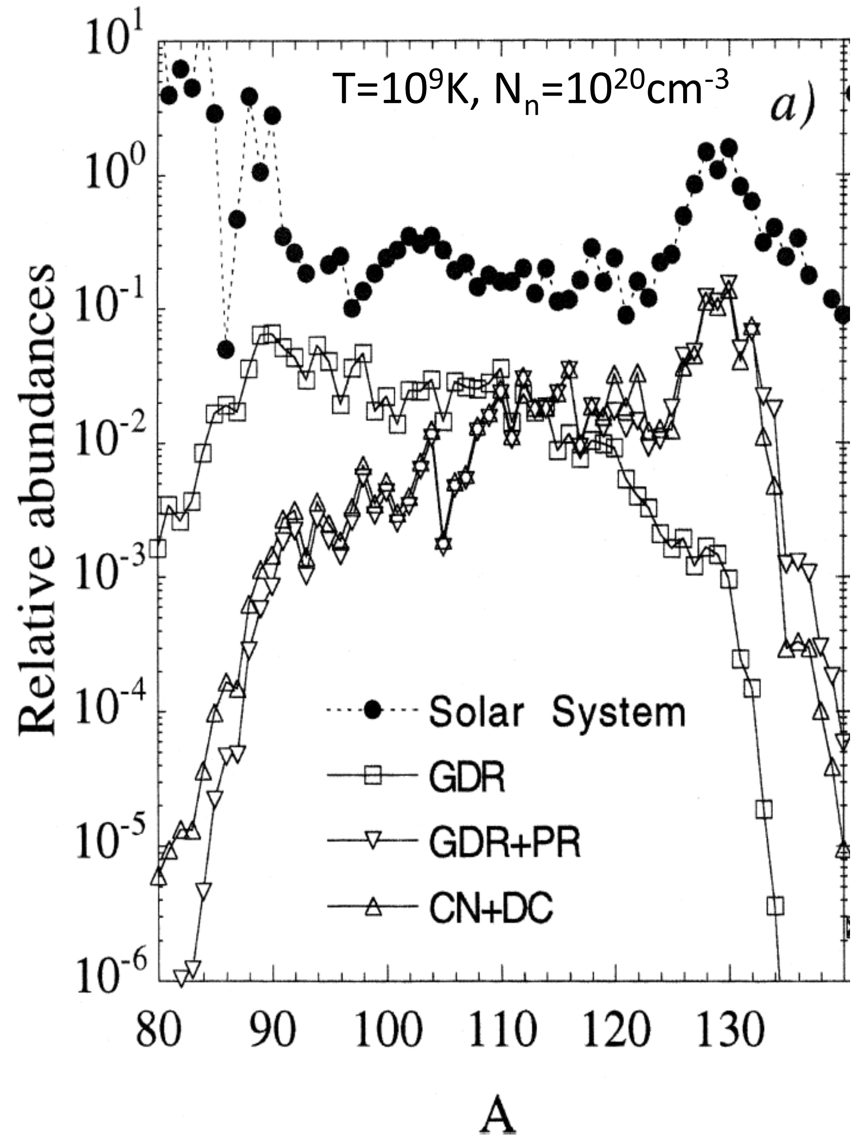
Similar study with Gogny:
(absent in literature)

	EWSR [%]	L [MeV]
D1S	1,2	22,5
D1N	1,5	33,6
D1M	1,3	25,3

Electromagnetic dipole excitations

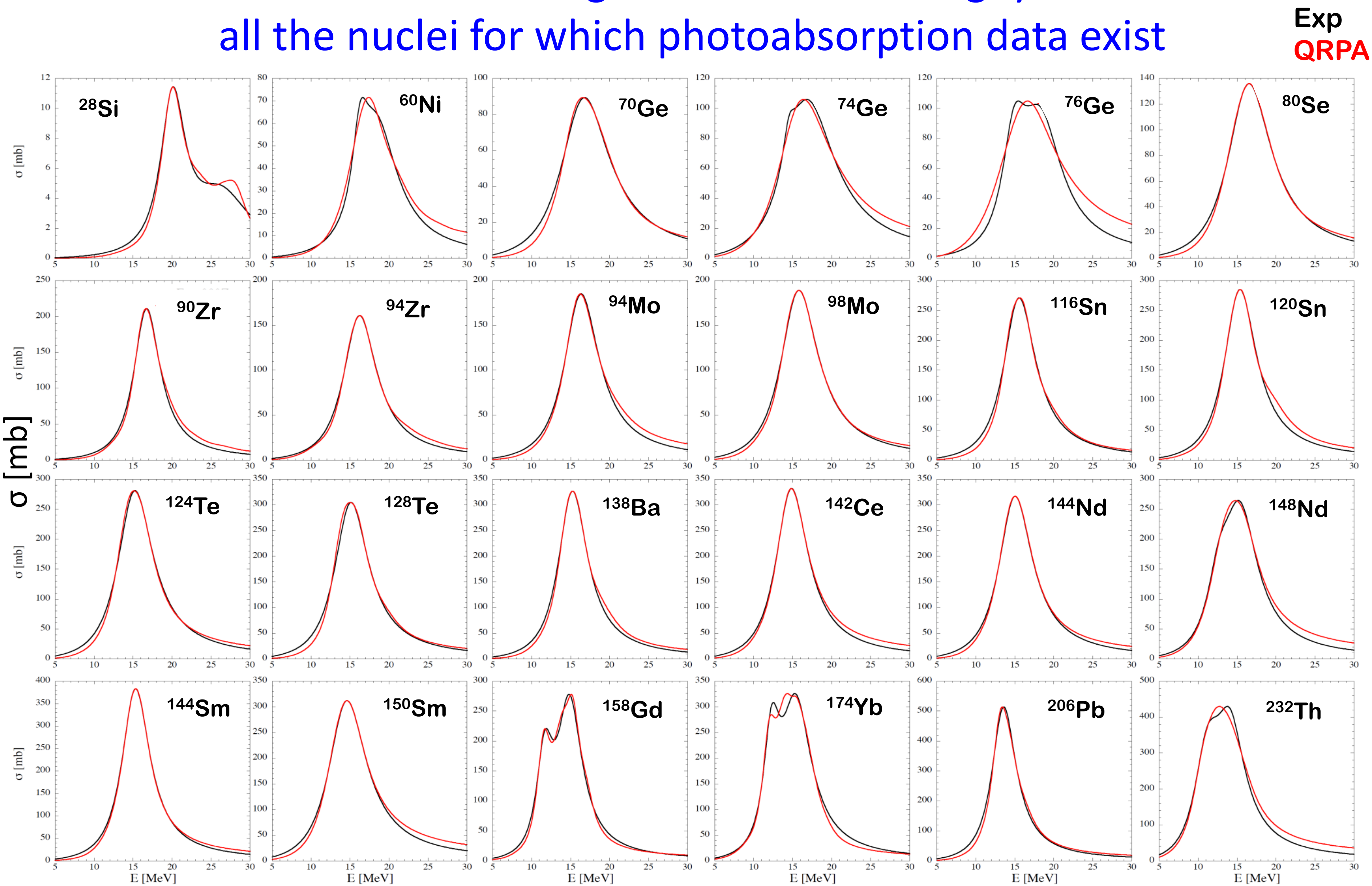


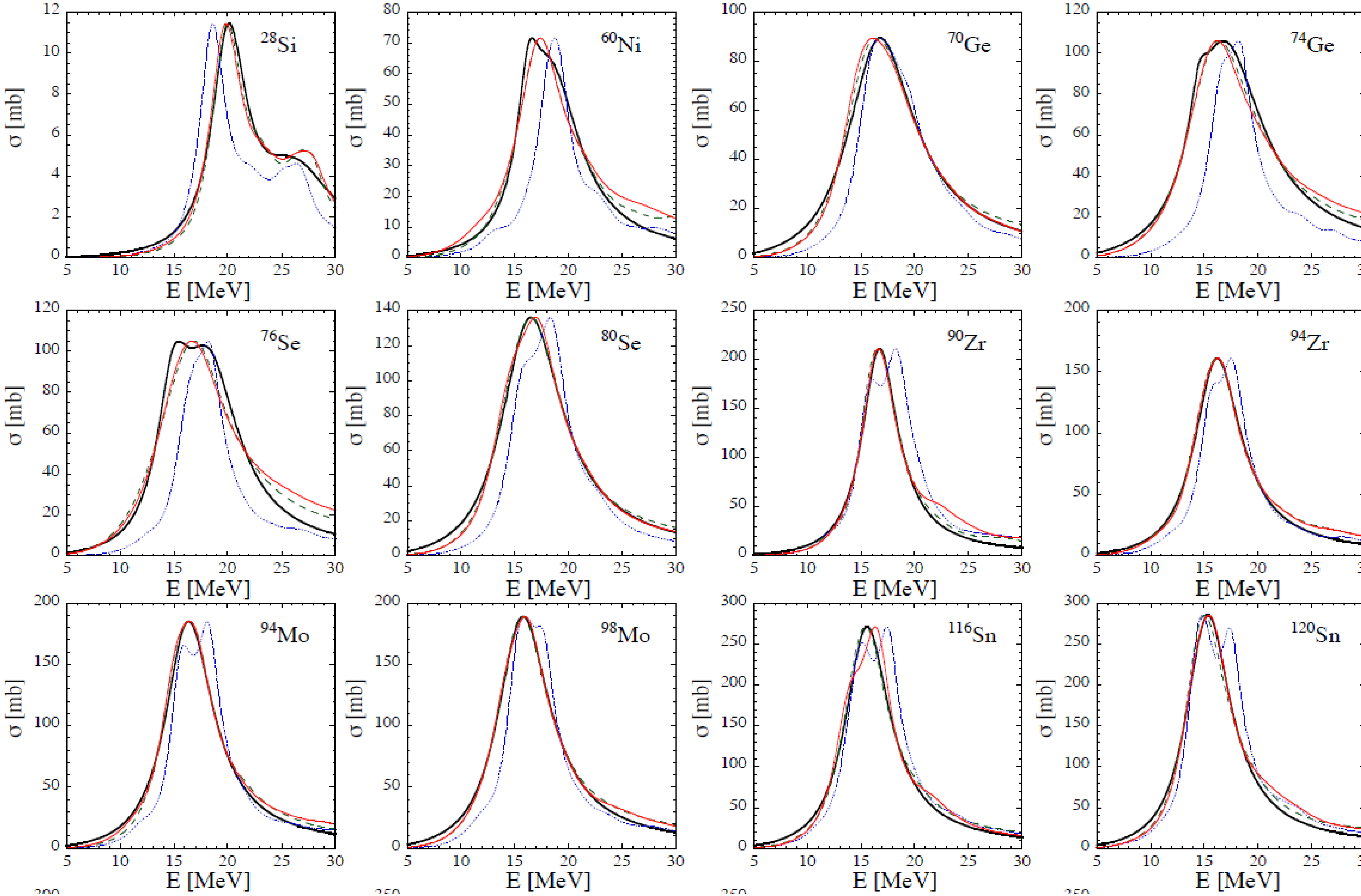
M. Martini, S. Péru and M. Dupuis, Phys. Rev. C 83, 034309 (2011)



S. Goriely, Physics Letters B 436 (1998) 10-18

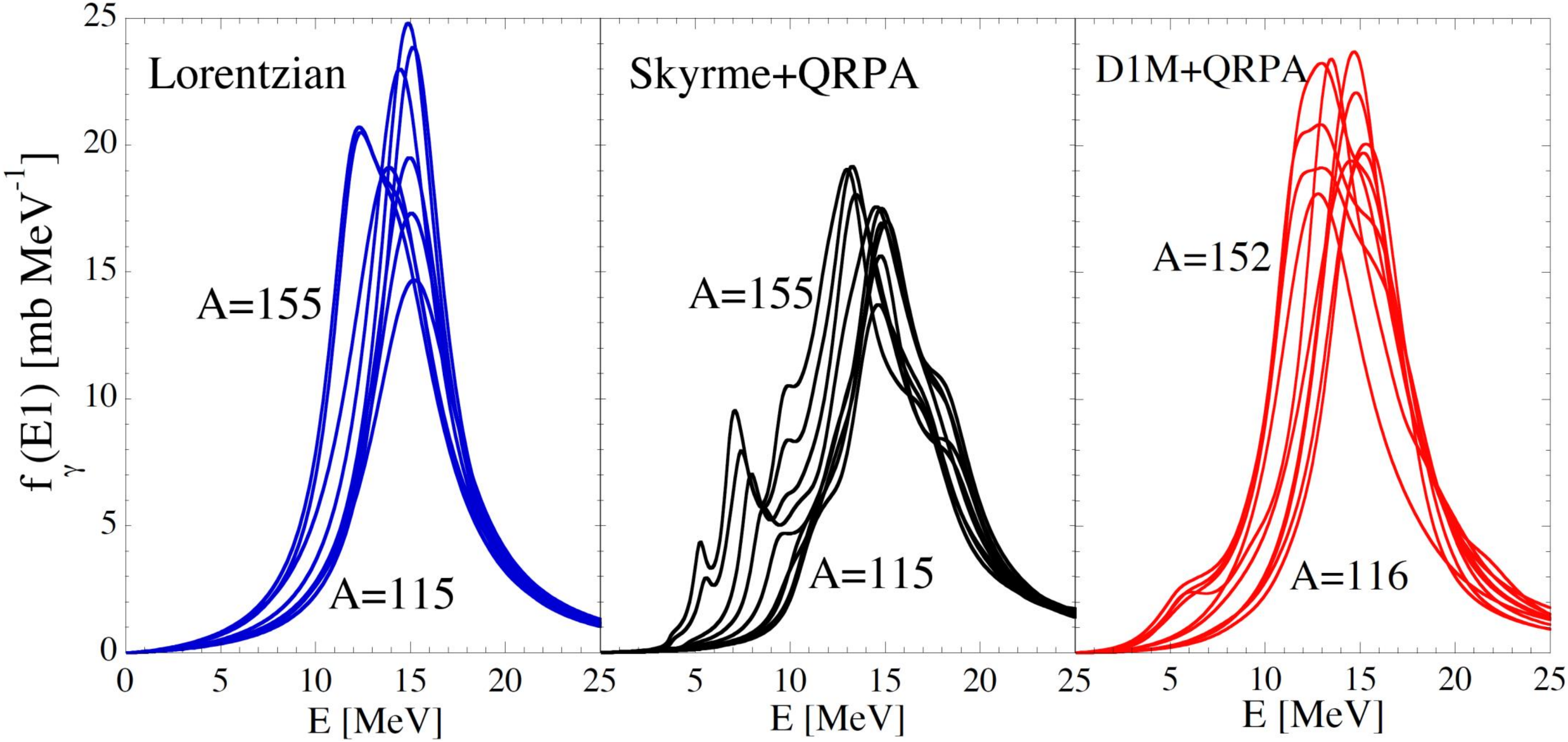
We calculate E1 strength in QRPA with Gogny-D1M for all the nuclei for which photoabsorption data exist





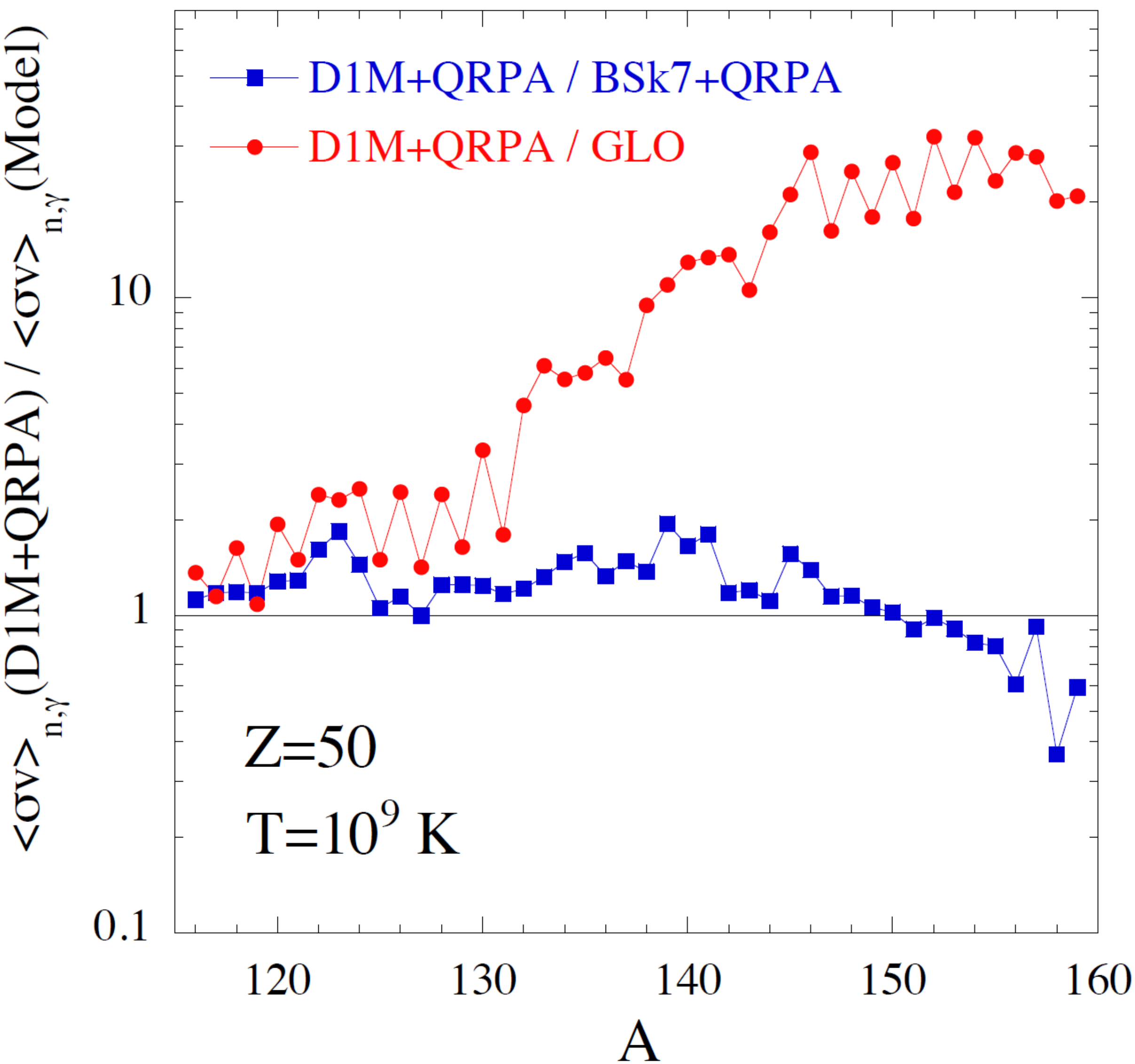
^{28}Si
^{60}Ni
^{70}Ge
^{74}Ge
^{76}Se
^{80}Se
^{90}Zr
^{94}Zr
^{94}Mo
^{98}Mo
^{116}Sn
^{120}Sn
σ [mb]
E [MeV]

Sn isotopes



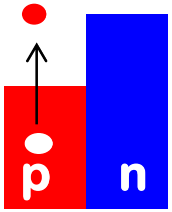
Lorentzian
A=155
A=115
Skyrme+QRPA
A=155
A=115
D1M+QRPA
A=152
A=116
f_γ (E1) [mb MeV^-1]
E [MeV]
E [MeV]
E [MeV]
0
5
10
15
20
25

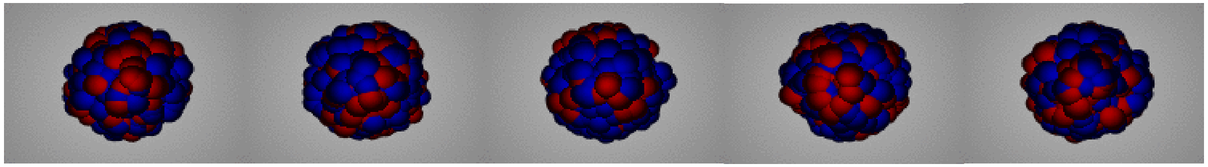
Maxwellian-averaged neutron capture rate for Sn isotopes



M. Martini, S. Hilaire, S. Goriely, A. J. Koning, S. Péru, Nuclear Data Sheets 118 (2014) 273-275

P.S. Other electromagnetic excitations

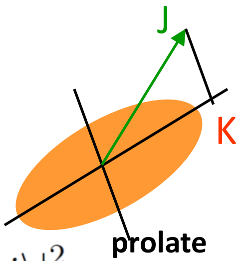


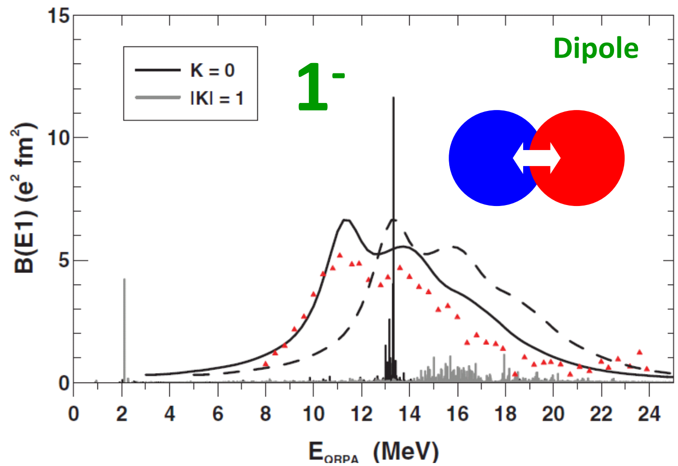


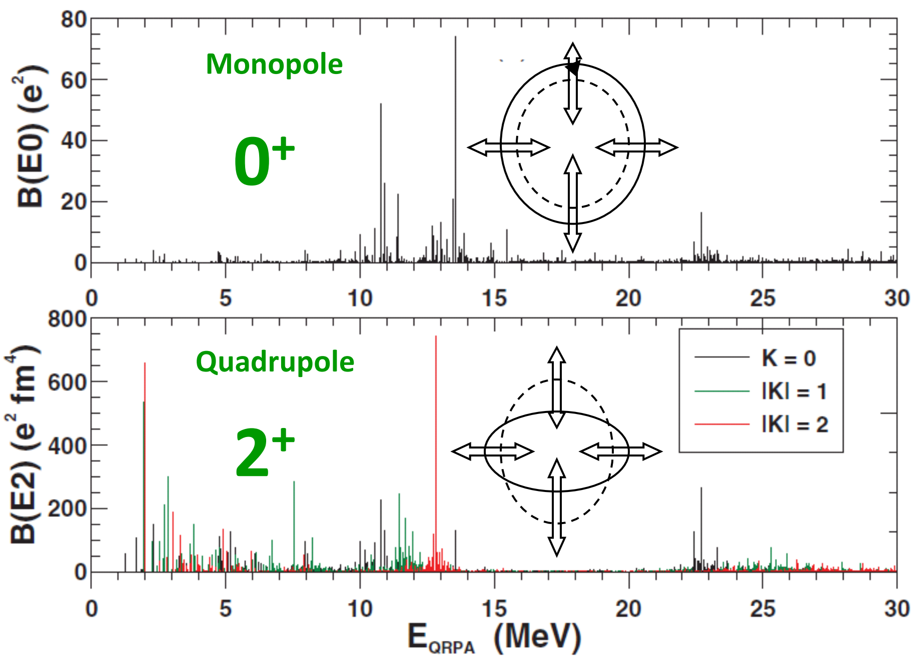
^{238}U Heavy deformed nucleus

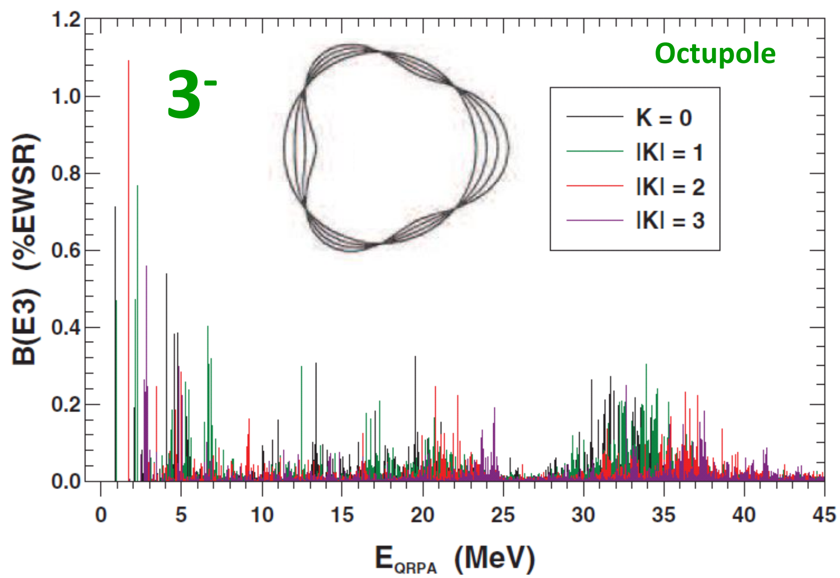
Massively parallel computation

$$B(EJ, J_i \rightarrow J_f) = \frac{1}{2J_i+1} |\langle f || \hat{F}_J || i \rangle|^2$$









S. Péru, G. Gosselin, M. Martini, M. Dupuis, S. Hilaire and J. C. Devaux, Phys. Rev. C 83, 014314 (2011)

Nuclear response to an external perturbation

External operator $\hat{F}$

No charge exchange

- Electromagnetic excitations
- Strong excitations

$$\hat{F}_{JM}(\mathbf{r}) = \sum_{i=1}^{A} r_i^J Y_{JM}(\hat{r}_i)$$

$$\hat{F}_{JM}(\mathbf{r}) = \sum_{i=1}^{A} r_i^J Y_{JM}(\hat{r}_i)\tau_z(i)$$

$$\hat{F}_{JM}(\mathbf{r}) = \sum_{i=1}^{A} r_i^L \left[Y_L(\hat{r}_i) \otimes \vec{\sigma}(i)\right]_{JM}$$

$$\hat{F}_{JM}(\mathbf{r}) = \sum_{i=1}^{A} r_i^L \left[Y_L(\hat{r}_i) \otimes \vec{\sigma}(i)\right]_{JM} \tau_z(i)$$

Charge exchange

- Weak excitations
  (β decay, ν scattering,...)
- Strong excitations

$$\hat{F}_{JM}(\mathbf{r}) = \sum_{i=1}^{A} r_i^J Y_{JM}(\hat{r}_i)\tau_{\pm}(i)$$

$$\hat{F}_{JM}(\mathbf{r}) = \sum_{i=1}^{A} r_i^L \left[Y_L(\hat{r}_i) \otimes \vec{\sigma}(i)\right]_{JM} \tau_{\pm}(i)$$

Sum Rules

Fermi

$$\Sigma_{\pm}(\mathrm{IA}) = \sum_{f} \left| \langle f | \tau_{\pm} | i \rangle \right|^{2}$$

$$\Sigma_{-}(IA) - \Sigma_{+}(IA) = N - Z$$

Ikeda

$$\Sigma_{\pm}(\mathrm{GT}) = \sum_{f,\mu} \left| \left\langle f \left| \sum_{k=1}^{A} \sigma_{\mu k}\, \tau_{\pm k} \right| i \right\rangle \right|^{2}$$

$$\Sigma_{-}(GT) - \Sigma_{+}(GT) = 3(N - Z)$$

114Sn

F: 13.9999993456112

GT/3: 13.9999993456064

132Sn

F: 32.000000000001

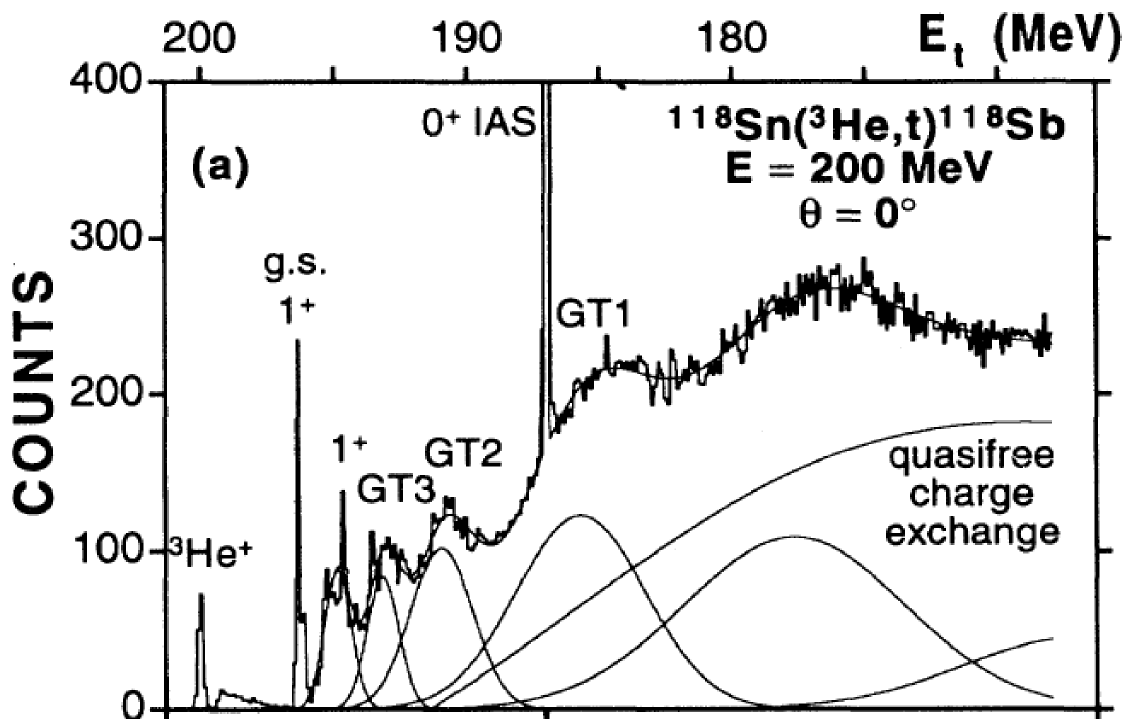
GT/3: 32.000000000001

Folding of GT strength

To take into account more complex configurations as well as coupling with phonons, the deformed GT strength $S_{GT}(\omega)$ can be folded with a Lorentzian function $L(E,\omega)$ of width Γ

$$f_{GT}(E) = \int_{-\infty}^{+\infty} L(E,\omega) S_{GT}(\omega) d\omega \qquad L(E,\omega) = \frac{1}{\pi} \frac{\Gamma/2}{(E-\omega)^2 + \Gamma^2/4}$$

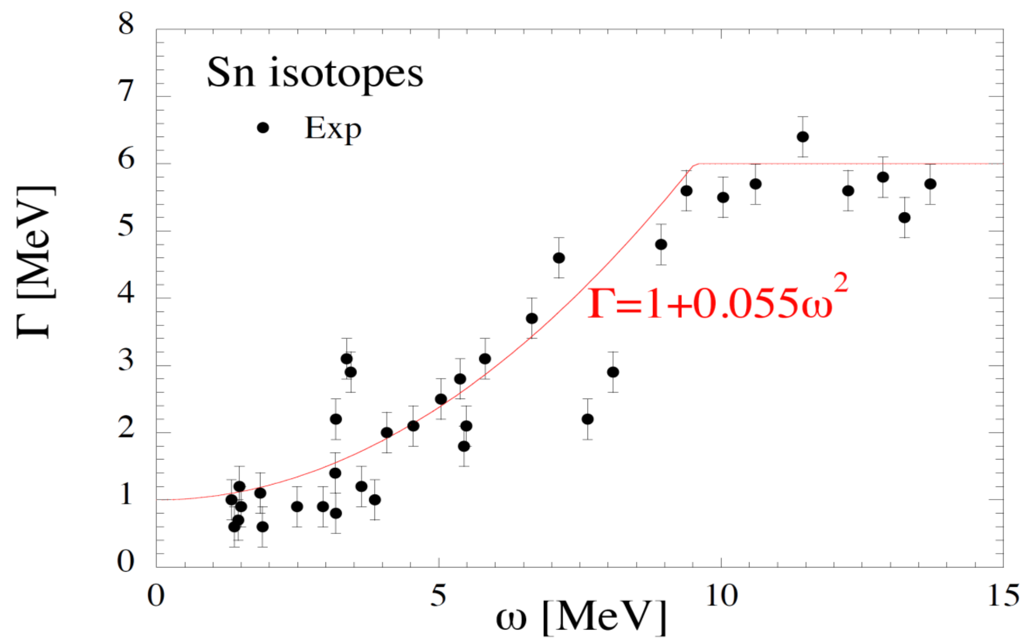
For $\Gamma(\omega)$ we use an analytical expression which reproduces successfully the experimental results on Γ_{GT} of Sn isotopes [**Pham et al. Phys. Rev. C 51, 526 (1995)**].



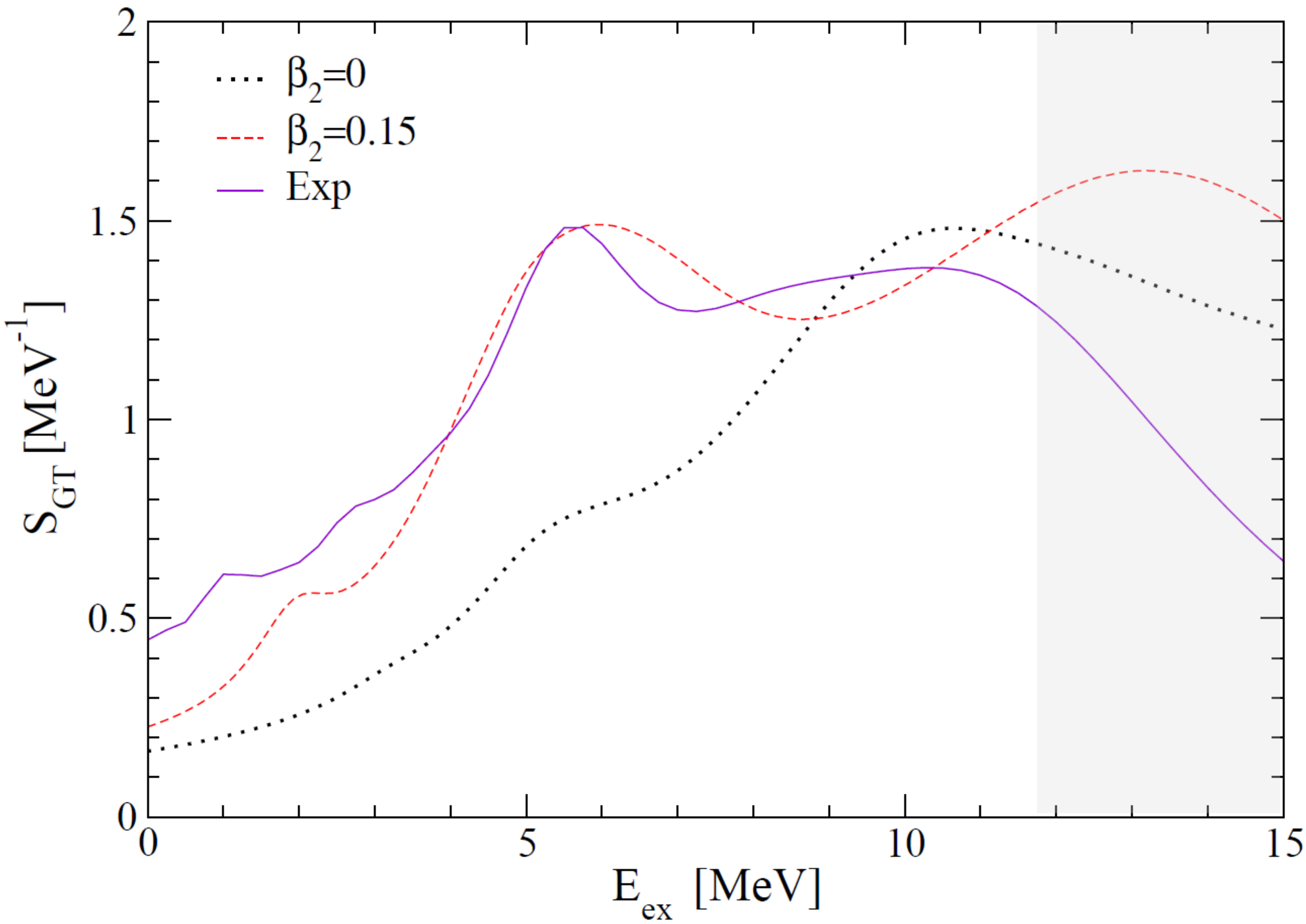
Folding of GT strength (II)

A	GT1 E_x (MeV)	GT1 Γ (MeV)	GT2 E_x (MeV)	GT2 Γ (MeV)	GT3 E_x (MeV)	GT3 Γ (MeV)	GT4 E_x (MeV)	GT4 Γ (MeV)
112	8.94±0.25	4.8±0.3	4.08±0.25	2.0±0.3	2.49±0.20	0.9±0.3	1.33±0.20	1.0±0.3
114	9.39±0.25	5.6±0.3	4.55±0.25	2.1±0.3	2.95±0.20	0.9±0.3	1.88±0.20	0.6±0.3
116	10.04±0.25	5.5±0.3	5.04±0.25	2.5±0.3	3.18±0.20	0.8±0.3	1.84±0.20	1.1±0.3
117	12.87±0.25	5.8±0.3	7.64±0.25	2.2±0.3	5.45±0.20	1.8±0.3	3.87±0.20	1.0±0.3
118	10.61±0.25	5.7±0.3	5.38±0.25	2.8±0.3	3.17±0.20	1.4±0.3	1.47±0.20	1.2±0.3
119	13.71±0.25	5.7±0.3	8.09±0.25	2.9±0.3	5.49±0.20	2.1±0.3	3.63±0.20	1.2±0.3
120	11.45±0.25	6.4±0.3	5.82±0.25	3.1±0.3	3.18±0.20	2.2±0.3	1.38±0.20	0.6±0.3
122	12.25±0.25	5.6±0.3	6.65±0.25	3.7±0.3	3.37±0.20	3.1±0.3	1.45±0.20	0.7±0.3
124	13.25±0.25	5.2±0.3	7.13±0.25	4.6±0.3	3.44±0.20	2.9±0.3	1.50±0.20	0.9±0.3

Pham et al. Phys. Rev. C 51, 526 (1995)



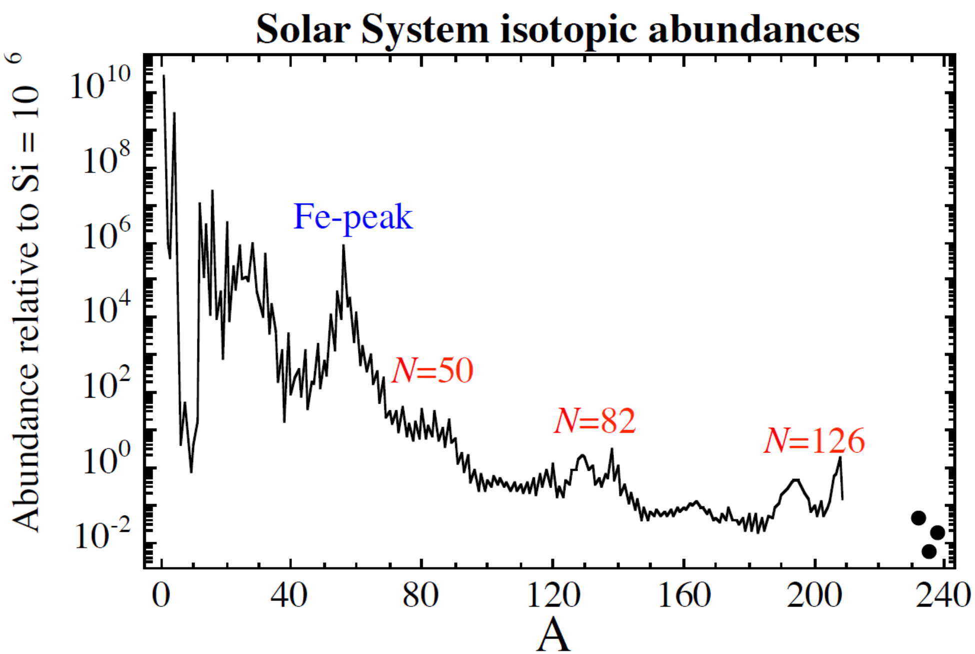
The folded results for the ^{76}Ge

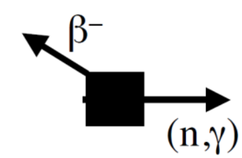


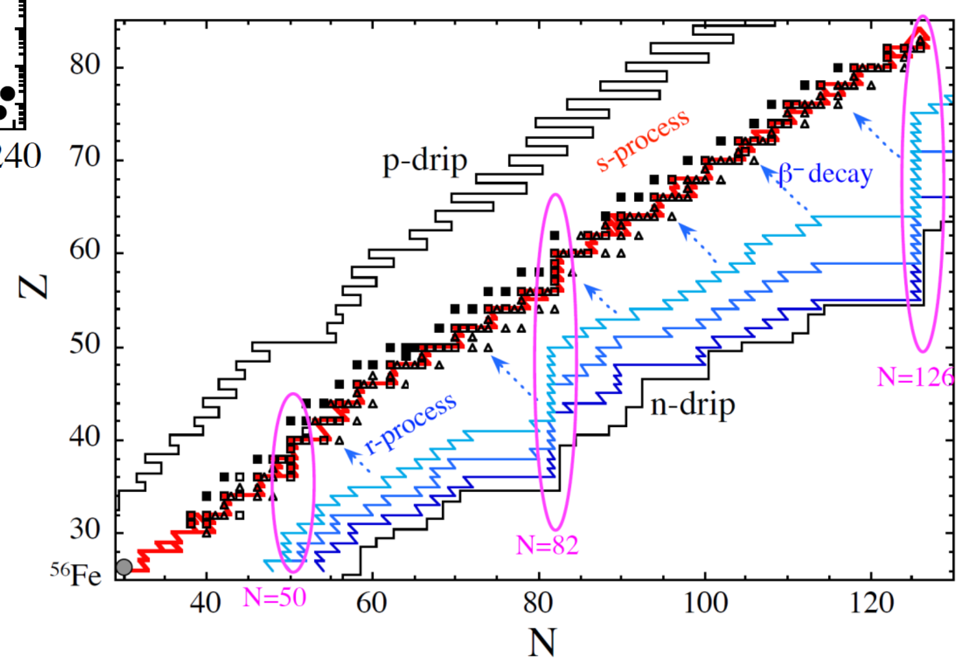
Stellar nucleosynthesis: the production of elements heavier than iron

Slow (s-) and rapid (r-) neutron capture processes

Explanation for the peaks observed in the solar system abundances
at neutron magic numbers N=50, 82 and 126

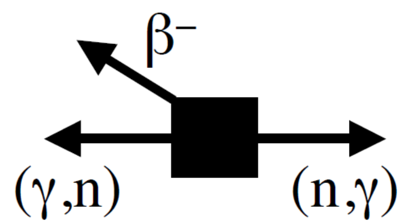






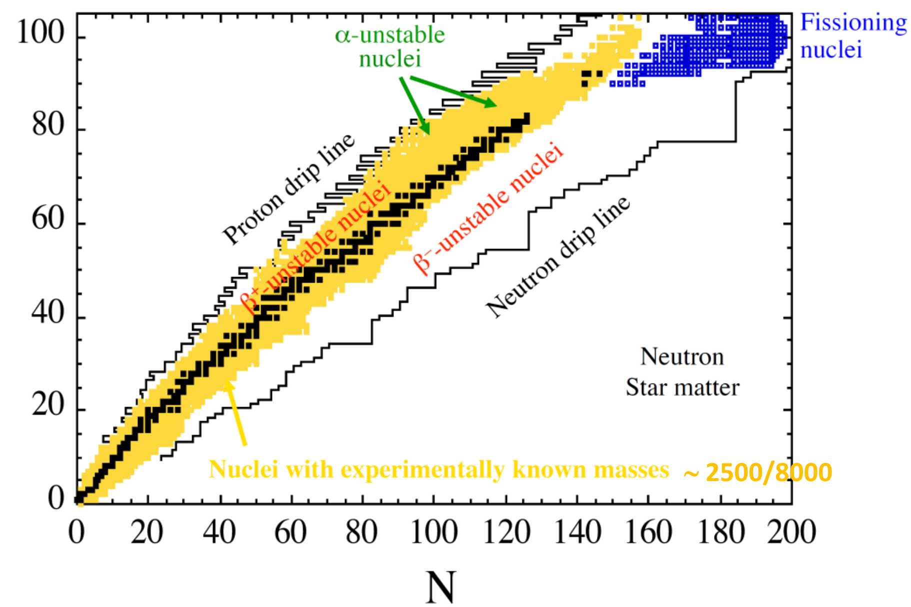
Nuclear Physics associated with the r-process

Competition between
- radiative neutron capture (n,γ)
- β-decay
- photo-neutron emission (γ,n)
- fission for the heaviest species
- ν-nucleus interaction properties (?)



From potentially all nuclei (~ 5000) from the valley of stability to the neutron drip-line

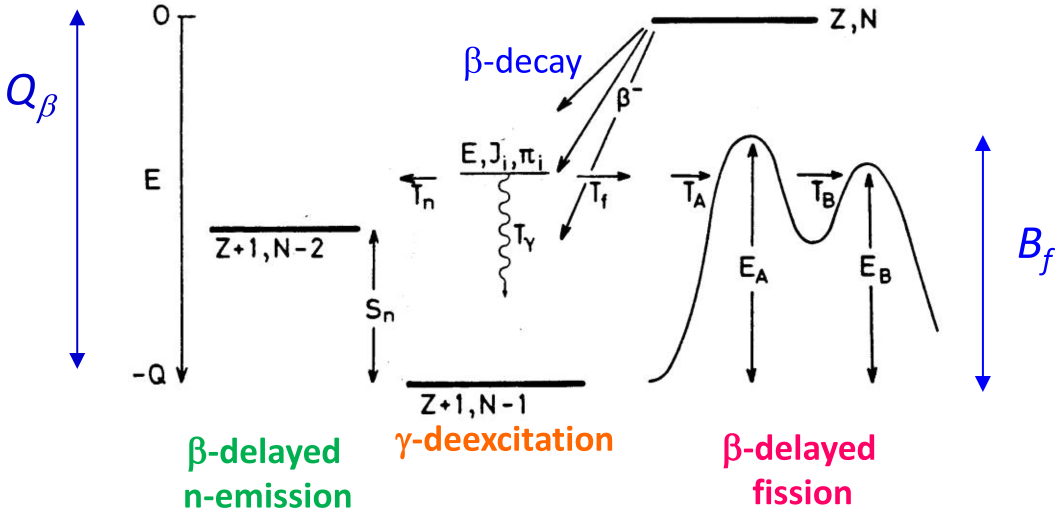
Need of **accurate**, **global** and as **microscopic** as possible **nuclear models**

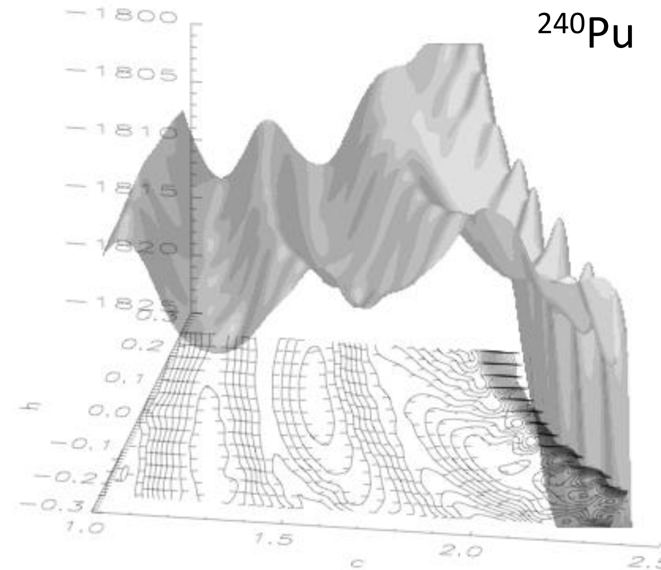


P.S.

The **r-process** puzzle is first of all an **astrophysics problem**: the r-process **site** remains **unknown**

Next step: β delayed processes





$$P_{f,n,\gamma} = \frac{\int_0^{Q_\beta} S_{GT}(E)\, f_0(E)\, \frac{T_{f,n,\gamma}(E)}{T_{tot}(E)}\, dE}{\int_0^{Q_\beta} S_{GT}(E)\, f_0(E)\, dE}$$

Gamow-Teller Strength: pnQRPA with D1M

Transmission coefficients: Talys code

Large scale calculations with Gogny D1M interaction

(S. Goriely-ULB, S. Hilaire, N. Dubray-CEA/DAM/DIF)

Hartree-Fock (-Bogoliubov)

HF

$$|\psi_{HF}\rangle = \left(\prod_{i=1}^{A} a_i^+\right)|0\rangle \qquad H = \sum_{l_1 l_2} t_{l_1 l_2} c_{l_1}^+ c_{l_2} + \frac{1}{4}\sum_{l_1 l_2 l_3 l_4} \bar{v}_{l_1 l_2, l_3 l_4} c_{l_1}^+ c_{l_2}^+ c_{l_4} c_{l_3}$$

$$a_p|\psi_{HF}\rangle = 0 \qquad a_\alpha^+ = \sum U_{i\alpha} C_i^+ \qquad \phi_\alpha = \sum U_{i\alpha}\chi_i \qquad \rho_{ij} = \langle\psi_{HF}|C_j^+ C_i|\psi_{HF}\rangle = \sum_{\alpha=1}^{A} U_{i\alpha} U_{j\alpha}^\star$$

$$a_i^+|\psi_{HF}\rangle = 0 \qquad \mathcal{E}(\psi_{HF}) = \mathcal{E}(\rho) = \mathrm{Tr}(t\rho) + \frac{1}{2}\mathrm{Tr}_1\mathrm{Tr}_2(\rho_1 \bar{v}_{12}\rho_2)$$

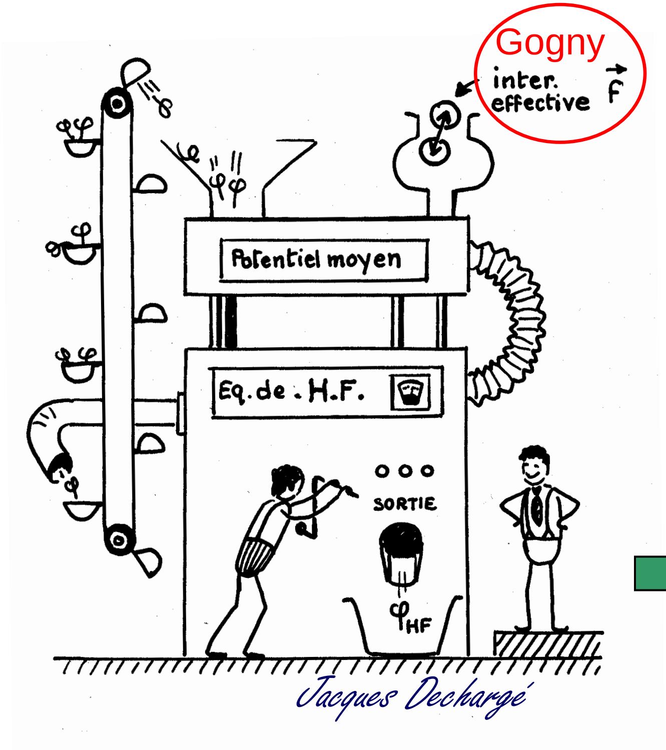
$$[h,\rho] = 0 \qquad h = \frac{\partial\mathcal{E}(\rho)}{\partial\rho} \qquad \sum_{l'} h_{ll'} U_{l'k} = \sum_{l'}\left(t_{ll'} + \sum_{i=1}^{A}\sum_{pp'} \bar{v}_{lp'l'p} U_{pi} U_{p'i}^\star\right) U_{l'k} = \epsilon_k U_{lk}$$

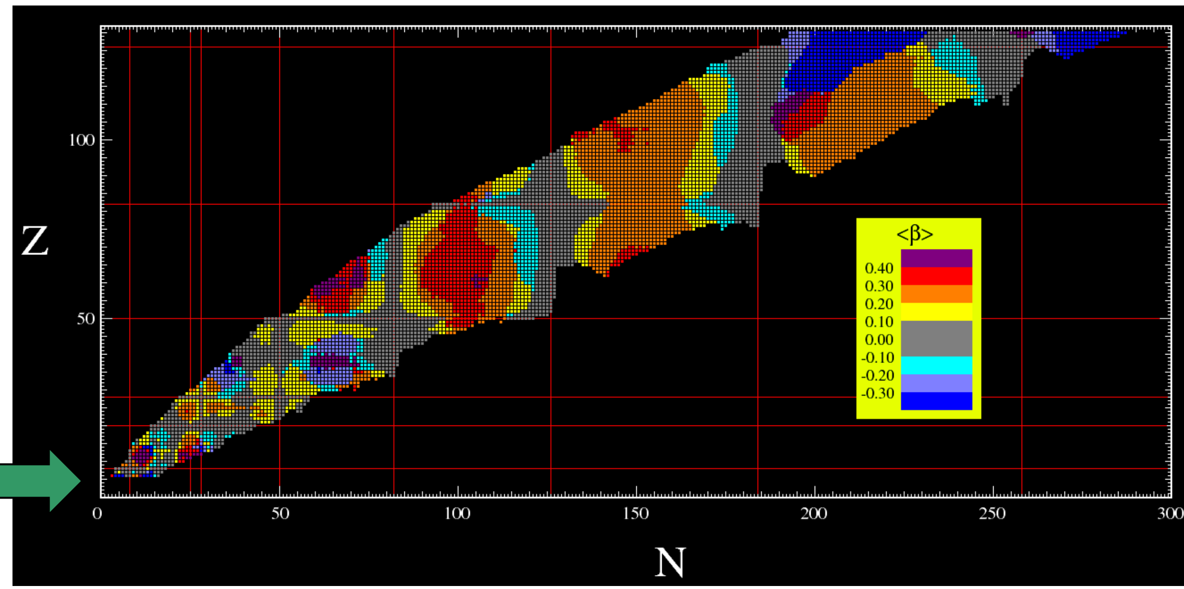
HFB

$$|\psi_{HFB}\rangle = \left(\prod_{\nu} \eta_\nu\right)|0\rangle \qquad \rho_{ij} = \langle\psi_{HFB}|C_j^+ C_i|\psi_{HFB}\rangle \qquad k_{ij} = \langle\psi_{HFB}|C_i C_j|\psi_{HFB}\rangle$$

$$\eta_\nu|\psi_{HFB}\rangle = 0 \qquad \mathcal{E}(\psi_{HFB}) = \mathcal{E}(\rho k) = \mathrm{Tr}(t\rho) + \frac{1}{2}\mathrm{Tr}_1\mathrm{Tr}_2(\rho_1\bar{v}_{12}\rho_2) + \frac{1}{4}\sum_{\alpha\beta\gamma\delta} v_{\alpha\beta\gamma\delta} k_{\beta\alpha}^\star k_{\delta\gamma}$$

$$\begin{pmatrix}\eta \\ \eta^+\end{pmatrix} = B\begin{pmatrix}C \\ C^+\end{pmatrix} = \begin{pmatrix}U & V \\ V^* & U^*\end{pmatrix}\begin{pmatrix}C \\ C^+\end{pmatrix} \qquad H_B = \begin{pmatrix}e & \Delta \\ \tilde{\Delta} & -e\end{pmatrix} \qquad e = \frac{\partial\mathcal{E}(\rho k)}{\partial\rho} \qquad \Delta = \frac{\partial\mathcal{E}(\rho k)}{\partial k} \qquad H_B\tilde{B} = E\tilde{B}$$





Amedee database :
http://www-phynu.cea.fr/HFB-Gogny_eng.htm
*S. Hilaire & M. Girod, EPJ **A33** (2007) 237*

Static mean field (HFB)
for Ground State Properties :

- Masses
- Deformation
- (Single particle levels)

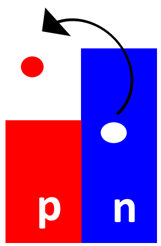
Beyond static mean field approximation (QRPA)
for description of Excited State Properties

- Low-energy collective levels
- Giant Resonances

Spin-isospin nuclear excitations (in particular GT)

- Crucial role in nuclear physics, astrophysics and particle physics
- Experimentally studied via charge exchange reactions , e.g. (p,n) and β decay
- Theoretical models to study the nuclei experimentally inaccessible

Our model: pnQRPA

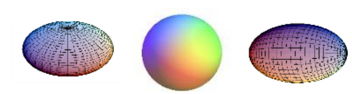


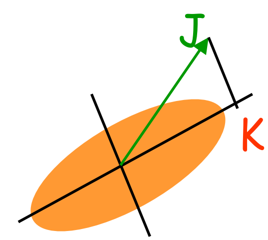
$$\theta^{+}_{\alpha,K} = \sum_{pn} X^{pn}_{\alpha,K}\eta^{+}_{p}\eta^{+}_{n} - (-)^{K} Y^{pn}_{\alpha,K}\eta_{n}\eta_{p} \qquad |\alpha, K\rangle = \theta^{+}_{\alpha,K}|0\rangle$$

$$\begin{pmatrix} A & B \\ B & A \end{pmatrix} \begin{pmatrix} X_{\alpha,K} \\ Y_{\alpha,K} \end{pmatrix} = \omega_{\alpha,K} \begin{pmatrix} X_{\alpha,K} \\ -Y_{\alpha,K} \end{pmatrix}$$

Main features:

- Possibility to treat axially-symmetric **deformed nuclei**





- **Pairing correlations** consistently included

- Use of an **unique nuclear force**: finite range Gogny force
  - same interaction for all the nuclei
  - same interaction for ground state and excited states (self-consistency)

essential features to treat consistently isotopic chains from drip line to drip line

$$|\theta_n, K\rangle = \theta^+_{n,K}|0_{\mathrm{def}}, (K=0)\rangle$$

$$\theta^+_{n,K} = \sum_{i<j} X^{ij}_{n,K}\eta^+_{i,k_i}\eta^+_{j,k_j} - (-)^K Y^{ij}_{n,K}\eta_{j,-k_j}\eta_{i,-k_i}$$

$$|JM(K)_n\rangle = \frac{\sqrt{2J+1}}{4\pi}\int \mathrm{d}\Omega\ \mathcal{D}^J_{MK}(\Omega)R(\Omega)|\theta_n, K\rangle$$
$$+(-)^{J-K}\mathcal{D}^J_{M-K}(\Omega)R(\Omega)|\theta_{\overline{n}}, -K\rangle,$$

$$\langle\tilde{O}_{(J^\pi=0^+)}|\hat{Q}_{\lambda\mu}|JM(K)_n\rangle =$$
$$\sqrt{2J+1}\sum_{\mu'}(-)^{\mu-\mu'}\langle 0_{\mathrm{def}}|r^\lambda\mathcal{Y}_{\lambda\mu'}|\theta_n, K\rangle$$
$$\cdot\begin{pmatrix}0 & \lambda & J\\ 0 & -\mu & M\end{pmatrix}\begin{pmatrix}0 & \lambda & J\\ 0 & -\mu' & K\end{pmatrix}$$
$$+(-)^{J-K+M'-\mu'}\langle 0_{\mathrm{def}}|r^\lambda\mathcal{Y}_{\lambda\mu'}|\theta_{\overline{n}}, -K\rangle$$
$$\cdot\begin{pmatrix}0 & \lambda & J\\ 0 & -\mu & M\end{pmatrix}\begin{pmatrix}0 & \lambda & J\\ 0 & -\mu' & -K\end{pmatrix}.$$

$$\langle\ \tilde{O}_{(J^\pi=0^+)}|\hat{Q}_{10}|JM(K=|1|)\rangle =$$
$$-\frac{1}{\sqrt{3}}\Big(\langle\ 0_{\mathrm{def}}|\hat{Q}_{11}|\theta_n, K=1\rangle$$
$$+\langle\ 0_{\mathrm{def}}|\hat{Q}_{1-1}|\theta_n, K=-1\rangle\Big)$$

$$\langle\ \tilde{O}_{(J^\pi=0^+)}|\hat{Q}_{10}|JM(K=0)\rangle =$$
$$\frac{1}{\sqrt{3}}\langle\ 0_{\mathrm{def}}|\hat{Q}_{10}|\theta_n, K=0\rangle.$$

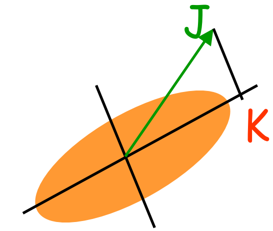
$$\rho^{Tr.}_{n,K}\left(\mathbf{r},\sigma\right)=\sum_{\alpha\beta}\Psi^{*}_{\alpha}\left(\mathbf{r},\sigma\right)\Psi_{\beta}\left(\mathbf{r},\sigma\right)\langle\tilde{0}|c^{+}_{\alpha}c_{\beta}|\theta_{n,K}\rangle$$

$$\rho^{Tr.}_{n,K}\left(\mathbf{r},\sigma\right)=\sum_{\alpha\beta}\Psi^{*}_{\alpha}\left(\mathbf{r},\sigma\right)\Psi_{\beta}\left(\mathbf{r},\sigma\right)$$

$$\sum_{ij}\Big[X^{ij}_{n,K}\left(U^{*}_{\beta i}V_{\alpha j}-V_{\alpha i}U^{*}_{\beta j}\right)$$

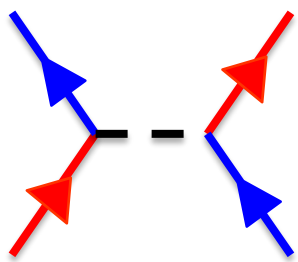
$$+\left(-\right)^{K+1}Y^{ij}_{n,K}\left(U_{\alpha i}V^{*}_{\beta j}-V^{*}_{\beta i}U_{\alpha j}\right)$$

pnQRPA



$$\theta^+_{\alpha,K} = \sum_{pn} X^{pn}_{\alpha,K}\eta^+_p\eta^+_n - (-)^K Y^{pn}_{\alpha,K}\eta_n\eta_p$$

$$\eta^+_p = u_{p\pi}c^+_\pi - v_{p\pi}c_\pi \qquad \begin{pmatrix} A & B \\ B & A \end{pmatrix}\begin{pmatrix} X_{\alpha,K} \\ Y_{\alpha,K} \end{pmatrix} = \omega_{\alpha,K}\begin{pmatrix} X_{\alpha,K} \\ -Y_{\alpha,K} \end{pmatrix}$$



$$\begin{aligned} A_{pn,p'n'} &= (\epsilon_p + \epsilon_n)\delta_{pp'}\delta_{nn'} \\ &+ u_{p\pi}v_{n\nu}u_{p'\pi'}v_{n'\nu'}(\langle\pi\nu'|V|\nu\pi'\rangle - \langle\pi\nu'|V|\pi'\nu\rangle) \\ &+ v_{p\pi}u_{n\nu}v_{p'\pi'}u_{n'\nu'}(\langle\nu\pi'|V|\pi\nu'\rangle - \langle\nu\pi'|V|\nu'\pi\rangle) \\ &+ u_{p\pi}u_{n\nu}u_{p'\pi'}u_{n'\nu'}(\langle\pi\nu|V|\pi'\nu'\rangle - \langle\pi\nu|V|\nu'\pi'\rangle) \\ &+ v_{p\pi}v_{n\nu}v_{p'\pi'}v_{n'\nu'}(\langle\pi'\nu'|V|\pi\nu\rangle - \langle\pi'\nu'|V|\nu\pi\rangle) \end{aligned}$$

Gogny

$$\begin{aligned} V(1,2) &= \sum_{j=1,2} e^{-\frac{(\vec{r}_1-\vec{r}_2)^2}{\mu_j^2}}(W_j + B_jP_\sigma - H_jP_\tau - M_jP_\sigma P_\tau) && \text{central} \\ &&& \textbf{finite range} \\ &+t_0(1+x_0P_\sigma)\delta(\vec{r}_1-\vec{r}_2)\left[\rho\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)\right]^\alpha && \text{density dependent} \\ &+iW_{ls}\overleftarrow{\nabla}_{12}\delta(\vec{r}_1-\vec{r}_2)\times\overrightarrow{\nabla}_{12}.(\overrightarrow{\sigma}_1+\overrightarrow{\sigma}_2) && \text{spin-orbit} \end{aligned}$$

pnQRPA

$$\theta^+_{\alpha,K} = \sum_{pn} X^{pn}_{\alpha,K} \eta^+_p \eta^+_n - (-)^K Y^{pn}_{\alpha,K} \eta_n \eta_p \qquad |\alpha, K\rangle = \theta^+_{\alpha,K} |0\rangle$$

$$\eta^+_p = u_{p\pi} c^+_\pi - v_{p\pi} c_\pi$$

$$\begin{pmatrix} A & B \\ B & A \end{pmatrix} \begin{pmatrix} X_{\alpha,K} \\ Y_{\alpha,K} \end{pmatrix} = \omega_{\alpha,K} \begin{pmatrix} X_{\alpha,K} \\ -Y_{\alpha,K} \end{pmatrix}$$

$$\begin{aligned}
A_{pn,p'n'} &= (\epsilon_p + \epsilon_n)\delta_{pp'}\delta_{nn'} \\
&+ u_{p\pi} v_{n\nu} u_{p'\pi'} v_{n'\nu'} (\langle \pi\nu' | V | \nu\pi' \rangle - \langle \pi\nu' | V | \pi'\nu \rangle) \\
&+ v_{p\pi} u_{n\nu} v_{p'\pi'} u_{n'\nu'} (\langle \nu\pi' | V | \pi\nu' \rangle - \langle \nu\pi' | V | \nu'\pi \rangle) \\
&+ u_{p\pi} u_{n\nu} u_{p'\pi'} u_{n'\nu'} (\langle \pi\nu | V | \pi'\nu' \rangle - \langle \pi\nu | V | \nu'\pi' \rangle) \\
&+ v_{p\pi} v_{n\nu} v_{p'\pi'} v_{n'\nu'} (\langle \pi'\nu' | V | \pi\nu \rangle - \langle \pi'\nu' | V | \nu\pi \rangle)
\end{aligned}$$

$$\begin{aligned}
B_{pn,p'n'} &= u_{p\pi} v_{n\nu} v_{p'\pi'} u_{n'\nu'} (\langle \pi\nu' | V | \nu\pi' \rangle - \langle \pi\nu' | V | \pi'\nu \rangle) \\
&+ v_{p\pi} u_{n\nu} u_{p'\pi'} v_{n'\nu'} (\langle \nu\pi' | V | \pi\nu' \rangle - \langle \nu\pi' | V | \nu'\pi \rangle) \\
&+ u_{p\pi} u_{n\nu} v_{p'\pi'} v_{n'\nu'} (\langle \pi\nu | V | \nu'\pi' \rangle - \langle \pi\nu | V | \pi'\nu' \rangle) \\
&+ v_{p\pi} v_{n\nu} u_{p'\pi'} u_{n'\nu'} (\langle \pi'\nu' | V | \nu\pi \rangle - \langle \pi'\nu' | V | \pi\nu \rangle)
\end{aligned}$$