

Measurement of masses and widths of Σ_c baryons at Belle

Lake Louise Winter Institute, Feb 19, 2015

Soohyung Lee

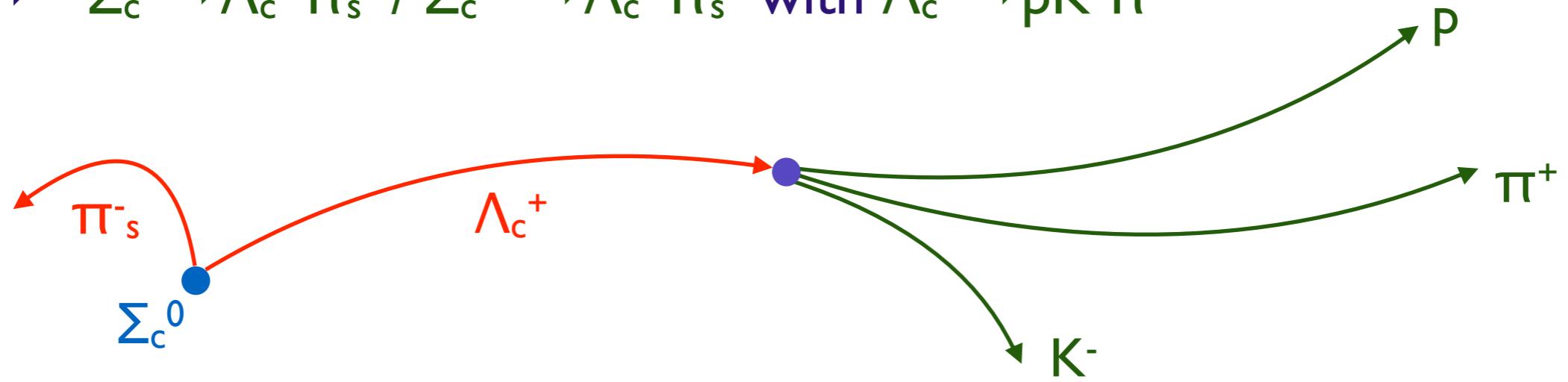
Center for Axion and Precision Physics Research / Institute for Basic Science

On behalf of the Belle Collaboration



Σ_c Baryons

- Σ_c baryons have very short lifetime \rightarrow finite decay widths
 - Decays into $\Lambda_c^+ \pi_s$ ($\sim 100\%$)
 - $\Sigma_c^0 \rightarrow \Lambda_c^+ \pi_s^-$ / $\Sigma_c^{++} \rightarrow \Lambda_c^+ \pi_s^+$ with $\Lambda_c^+ \rightarrow p K^- \pi^+$



- The uncertainties of the current world average values are a bit large
 - e.g. $\sim 10\%$ for widths

World average (MeV/c ²)	$\Sigma_c(2455)^0$	$\Sigma_c(2455)^{++}$	$\Sigma_c(2520)^0$	$\Sigma_c(2520)^{++}$
Mass	2453.74 ± 0.16	2453.98 ± 0.16	2518.8 ± 0.6	2517.9 ± 0.6
Mass Difference	167.27 ± 0.08	167.52 ± 0.08	232.3 ± 0.5	231.4 ± 0.6
Decay Widths	2.16 ± 0.26	2.26 ± 0.25	14.5 ± 1.5	14.9 ± 1.5

* Large uncertainties

Σ_c Baryons



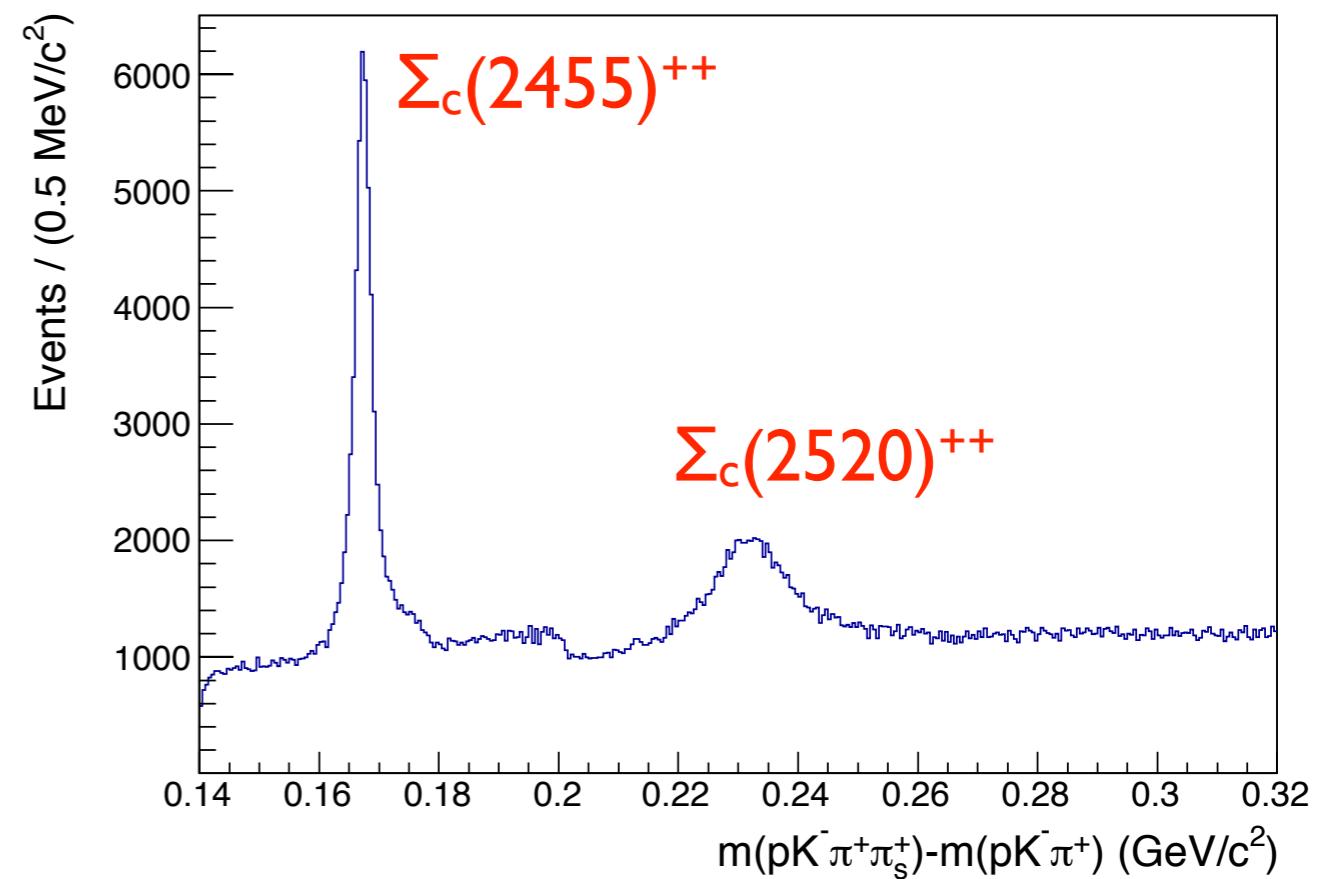
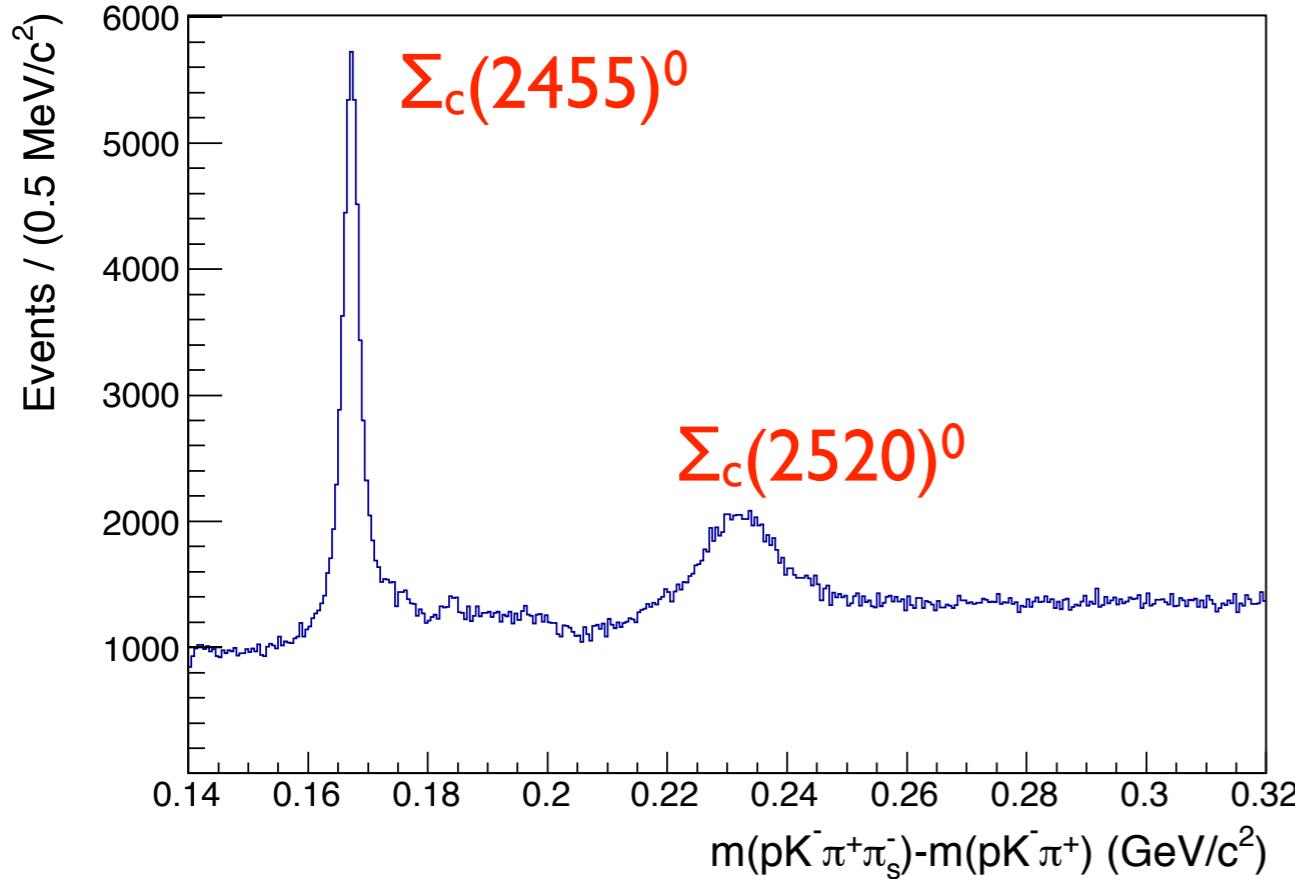
- Isospin mass splittings
 - Since u and d quarks have a bit different masses, their compositions may have different masses, therefore, one may expect that
 - ▶ $m(u) < m(d) \rightarrow m(\Sigma_c^{++}) (uuc) < m(\Sigma_c^0) (ddc)$
 - Experimental measurements show opposite results
 - ▶ $m(\Sigma_c(2455)^{++}) - m(\Sigma_c(2455)^0) = 0.24 \pm 0.09$ (PDG 2014)
- Theoretical models calculate the mass splittings with considering various contributions
 - e.g. electromagnetic potential, hyperfine interaction, ...
 - Precise measurements of the masses of Σ_c baryons will test the models

Theoretical Expectations	$m(\Sigma_c(2455)^{++}) - m(\Sigma_c(2455)^0)$
Chan, <i>Phys. Rev. D</i> 31 , 204 (1985)	0.18
Hwang and Lichtenberg, <i>Phys. Rev. D</i> 35 , 3526 (1987)	3.0
Capstick, <i>Phys. Rev. D</i> 36 , 2800 (1987)	1.4
Verma and Srivastava, <i>Phys. Rev. D</i> 38 , 1623 (1988)	0.5
Cutkosky and Geiger, <i>Phys. Rev. D</i> 48 , 1315 (1993)	0.84
Silvestre-Brac et al., <i>J. Phys. G</i> 29 , 2685 (2003)	0.27 ± 0.11
Guo et al., <i>J. High Energy Phys.</i> 09 , 136 (2008)	0.37

Data Samples and Backgrounds



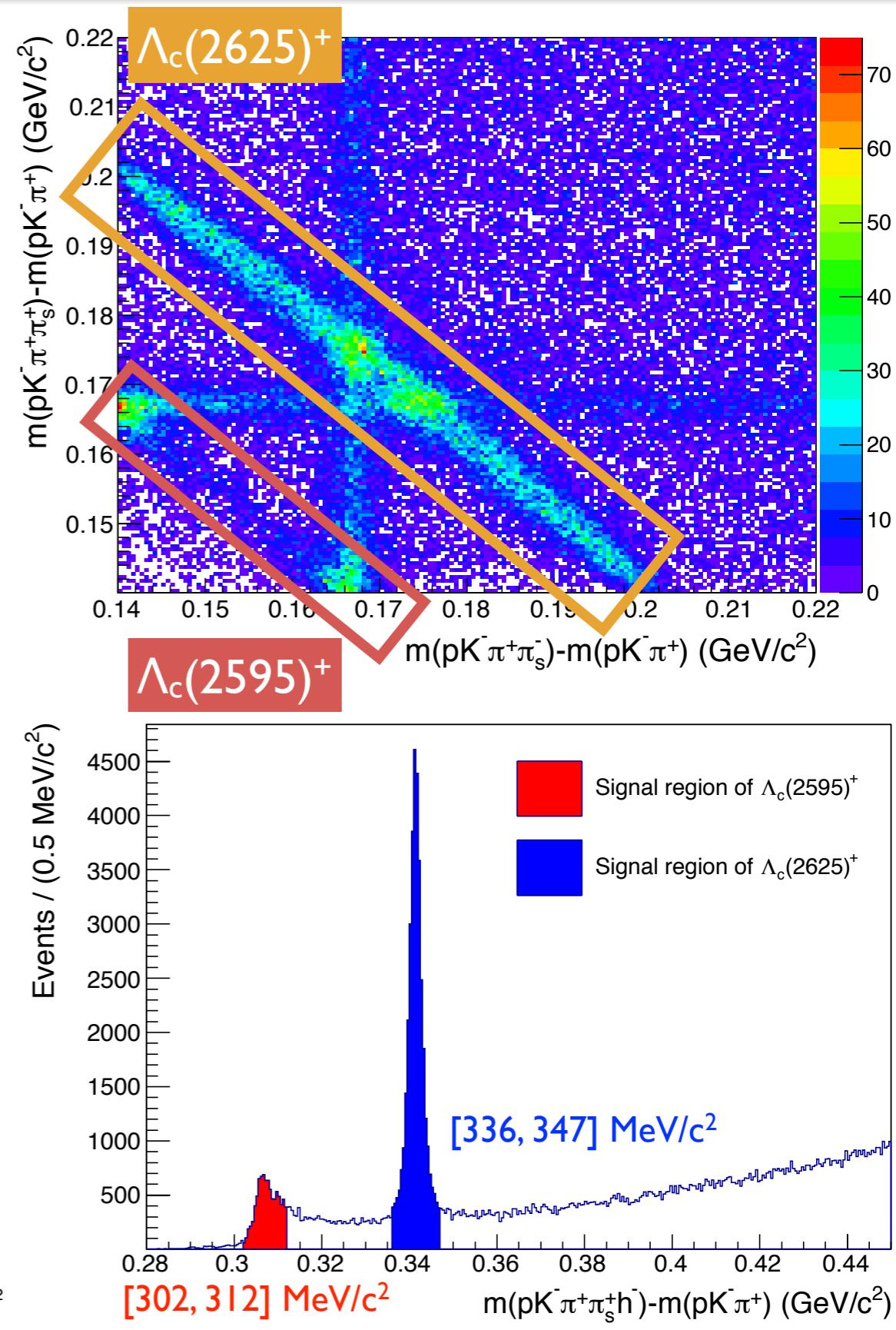
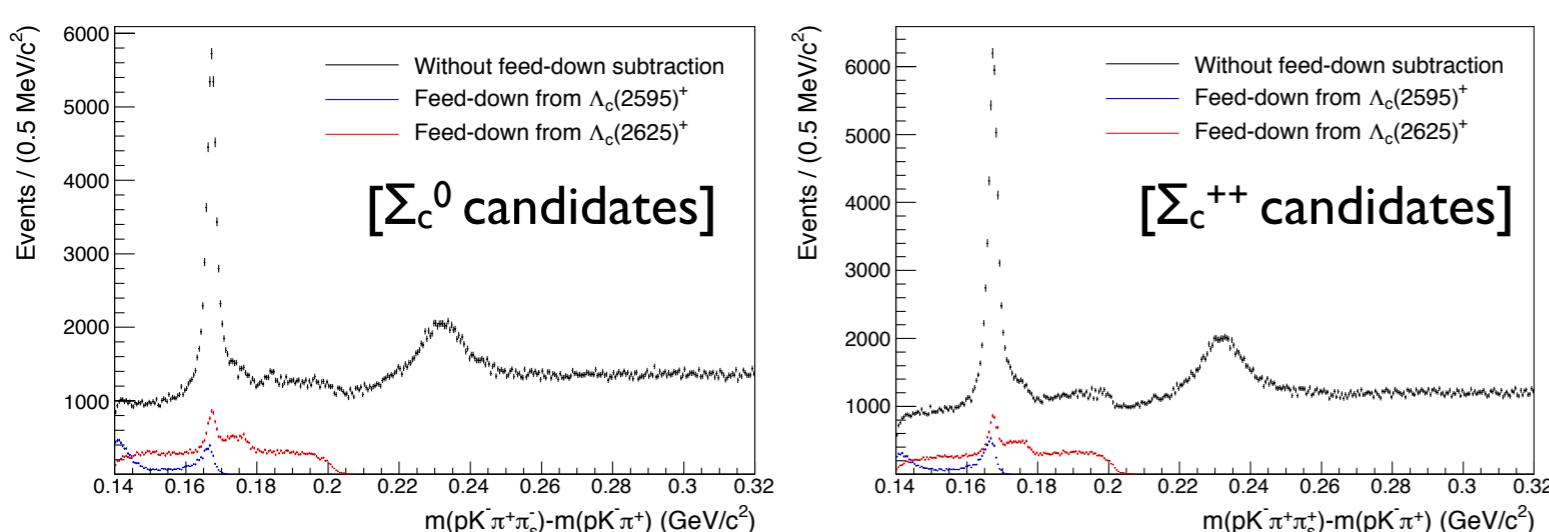
- Belle data of 711 fb^{-1} taken at $\Upsilon(4S)$ used for the study
- Mass differences after event selections



- Backgrounds are classified as:
 - Feed-down backgrounds from excited Λ_c^+ states
 - Reflection backgrounds from D^{*+} decays
 - Contribution from Ξ_c^0 decay
 - Random background due to wrong combinations

Feed-down Backgrounds from Λ_c^{*+} Decays

- Λ_c^{*+} decays can be either signal or background, e.g.:
 - $\Lambda_c^{*+} \rightarrow \Sigma_c^0 \pi^+$ is signal for Σ_c^0 measurement, but background for Σ_c^{++} measurement
- In order to extract the backgrounds, a tagging method is used
 - By appending an additional h^\pm , and test the mass with defined Λ_c^{*+} signal region, the candidates are tagged
- Obtained feed-down backgrounds are corrected with detection efficiency of the additional charged hadrons as a function of their momenta



Reflection Backgrounds from D^{*+} Decays

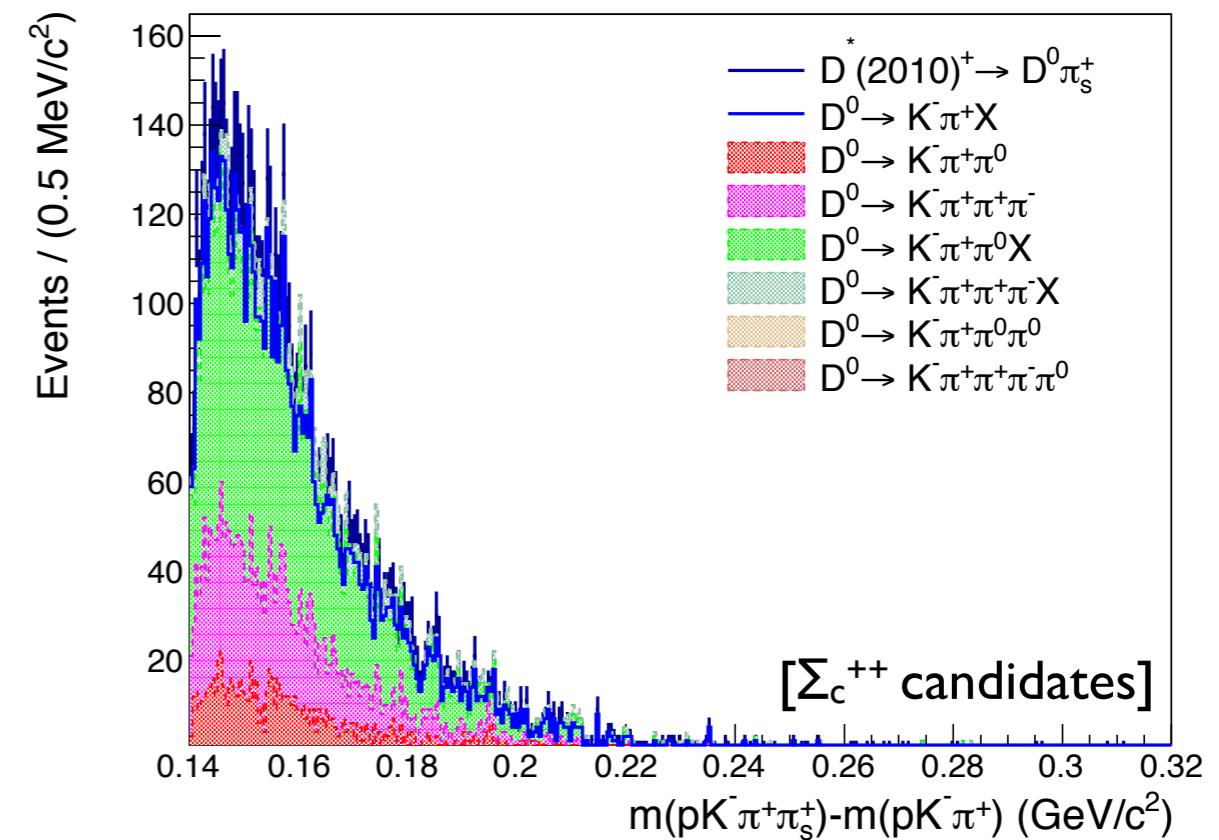
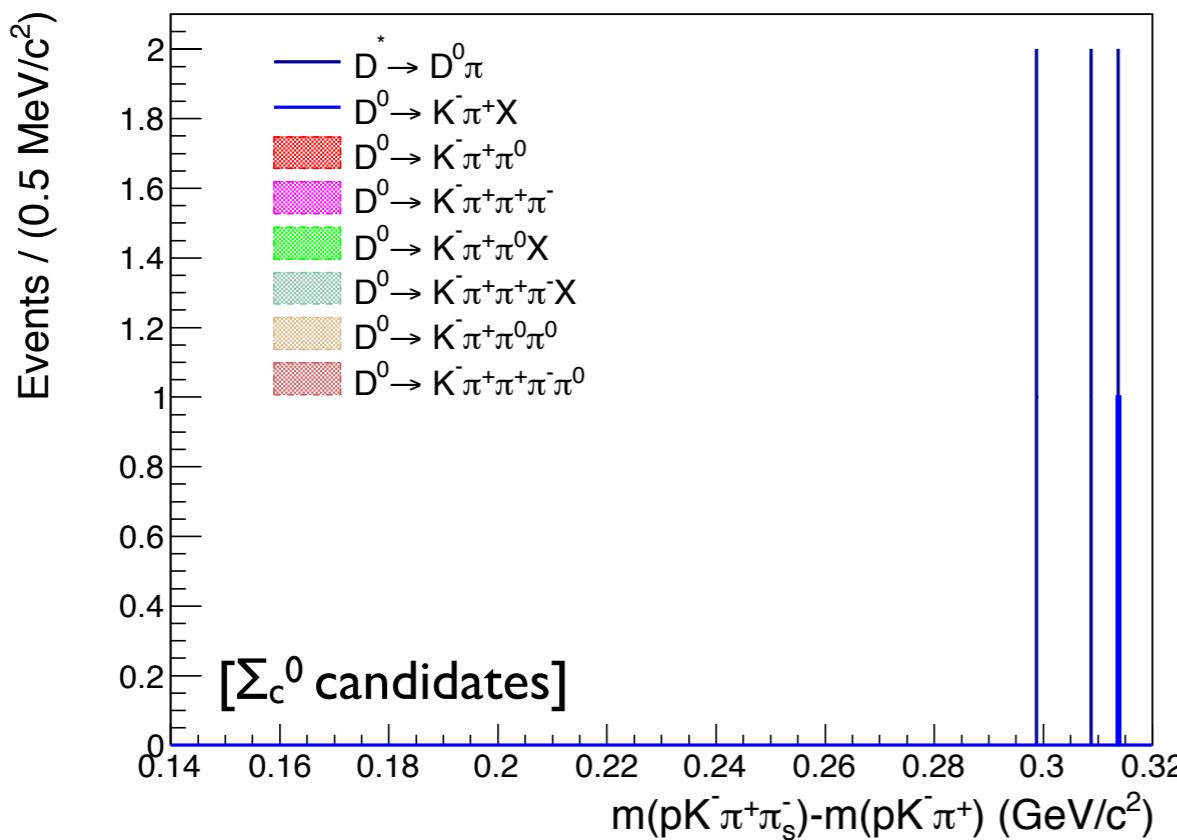


- D^{*+} decays may contribute as a background source

- e.g. $D^{*+} \rightarrow D^0 \pi_s^+$ with $D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$

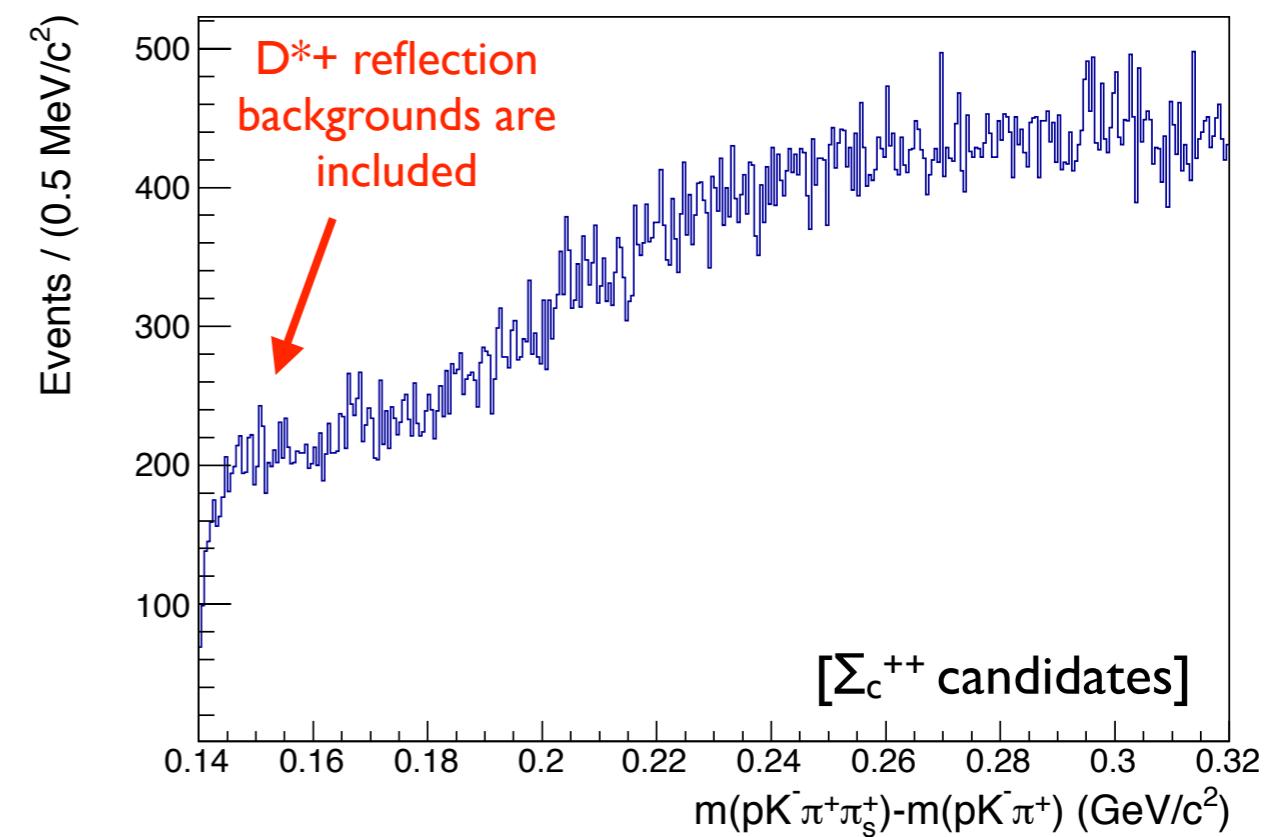
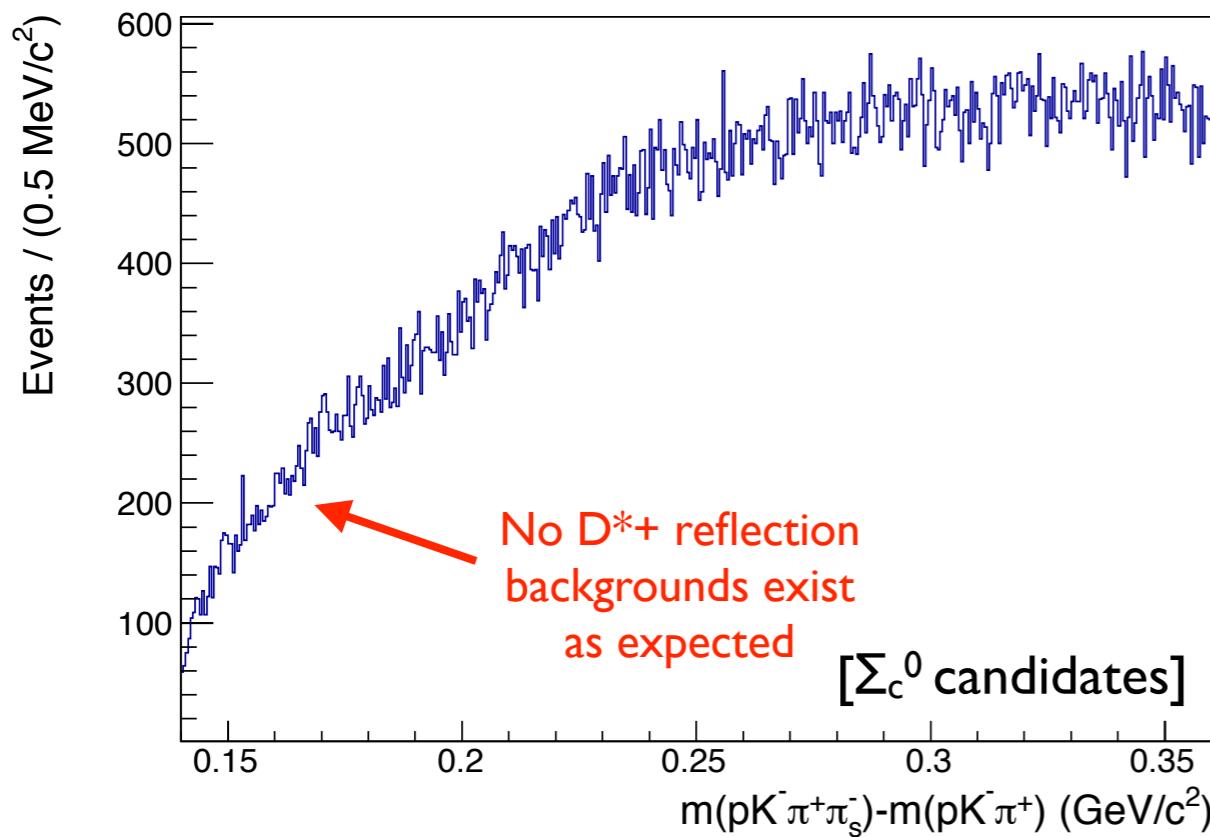
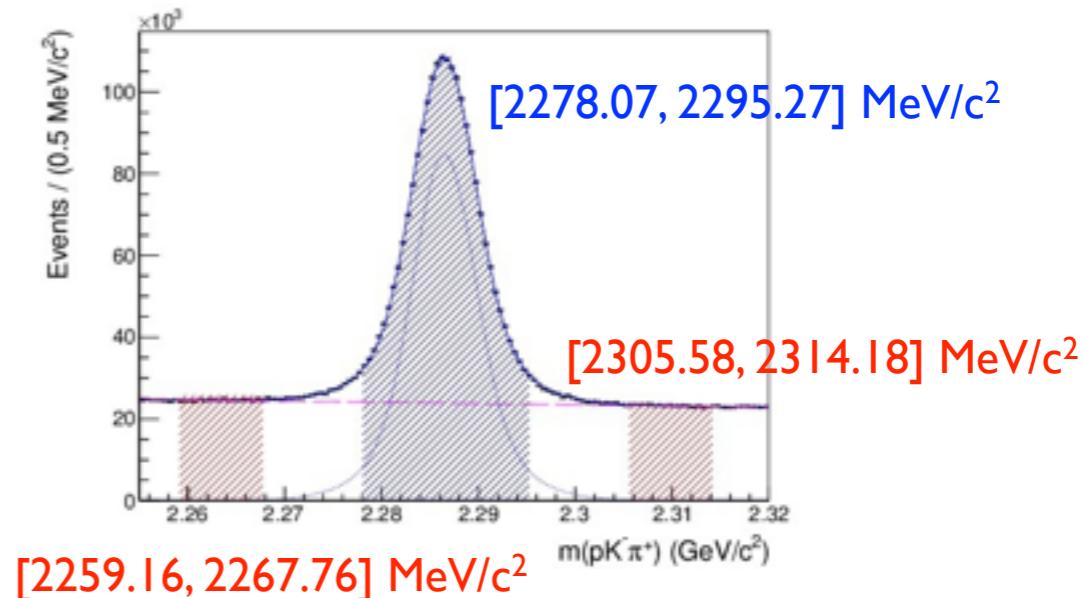
Background contribution of $pK^- \pi^+ \pi^+ \pi^-$ is possible
random p

- The background contribution from D^{*+} decays are confirmed with MC
 - It appears only in $m(pK^- \pi^+ \pi^+) - m(pK^- \pi^+)$



Random Backgrounds

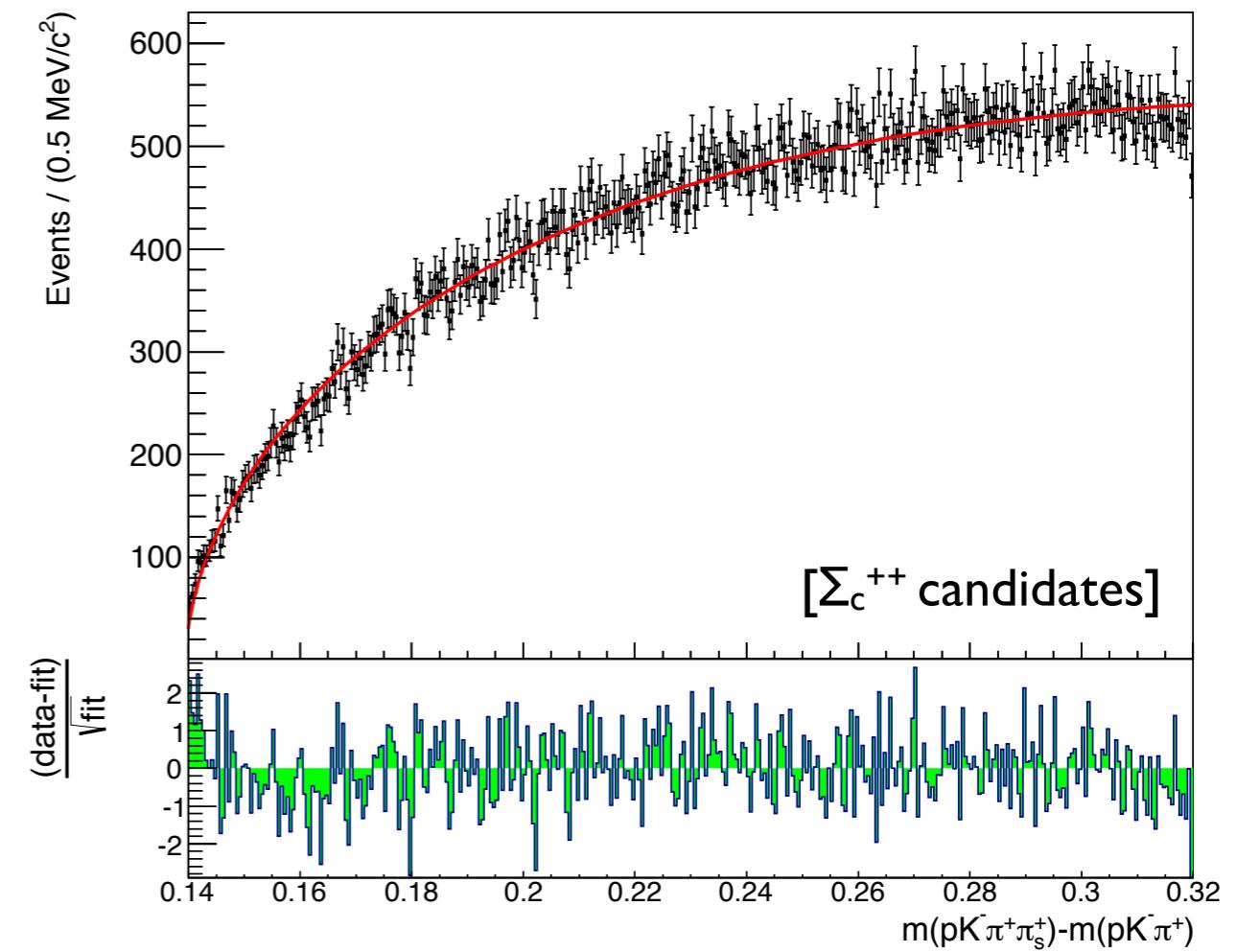
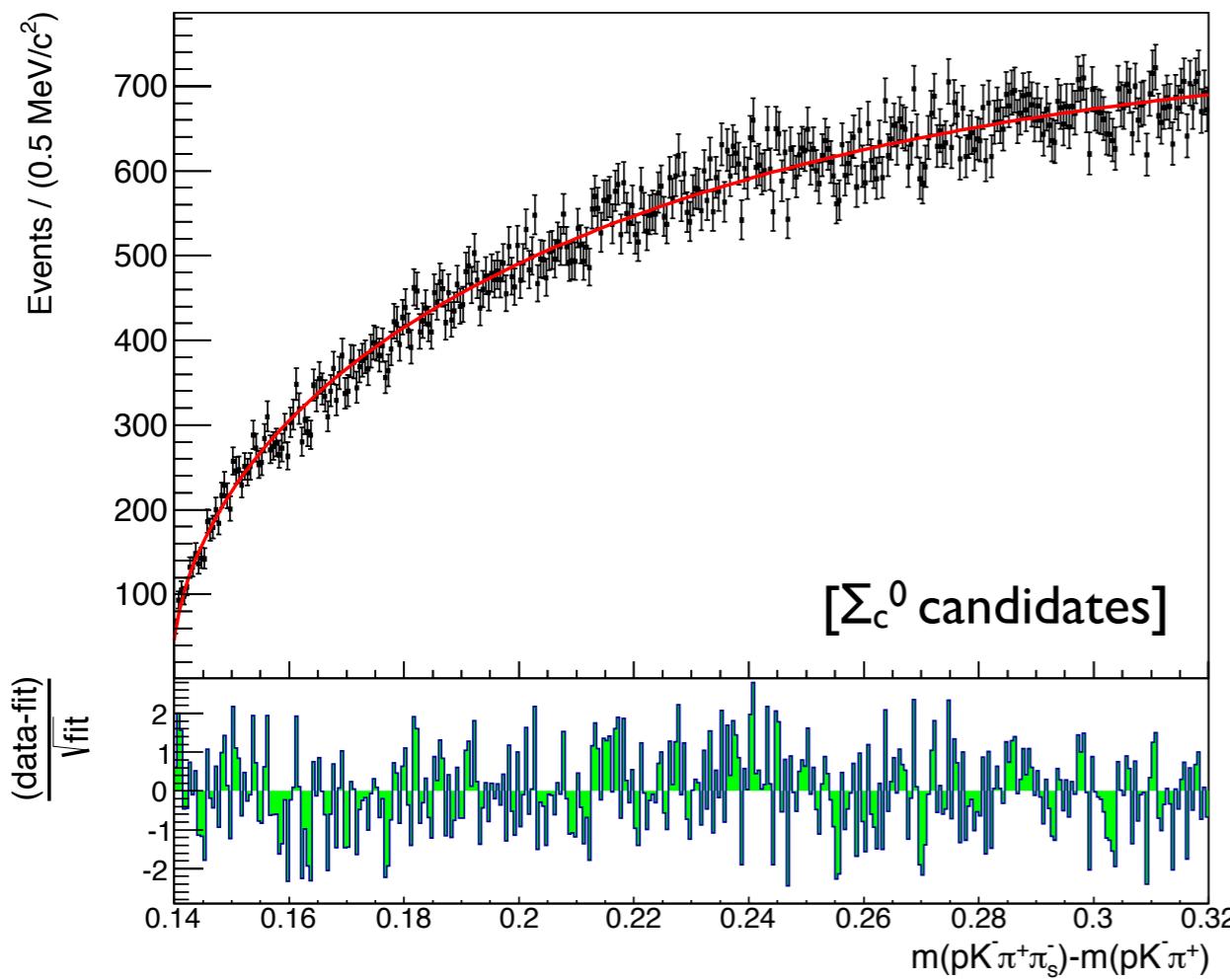
- Random backgrounds are classified by true and fake Λ_c^+
 - Associated with fake Λ_c^+ : Extractable by using Λ_c^+ sideband (sideband subtraction)
 - Since the reflection backgrounds from D^{*+} decays is also a random background associated with fake Λ_c^+ , it appears here as well → no explicit description of the reflection background needed



Random Backgrounds



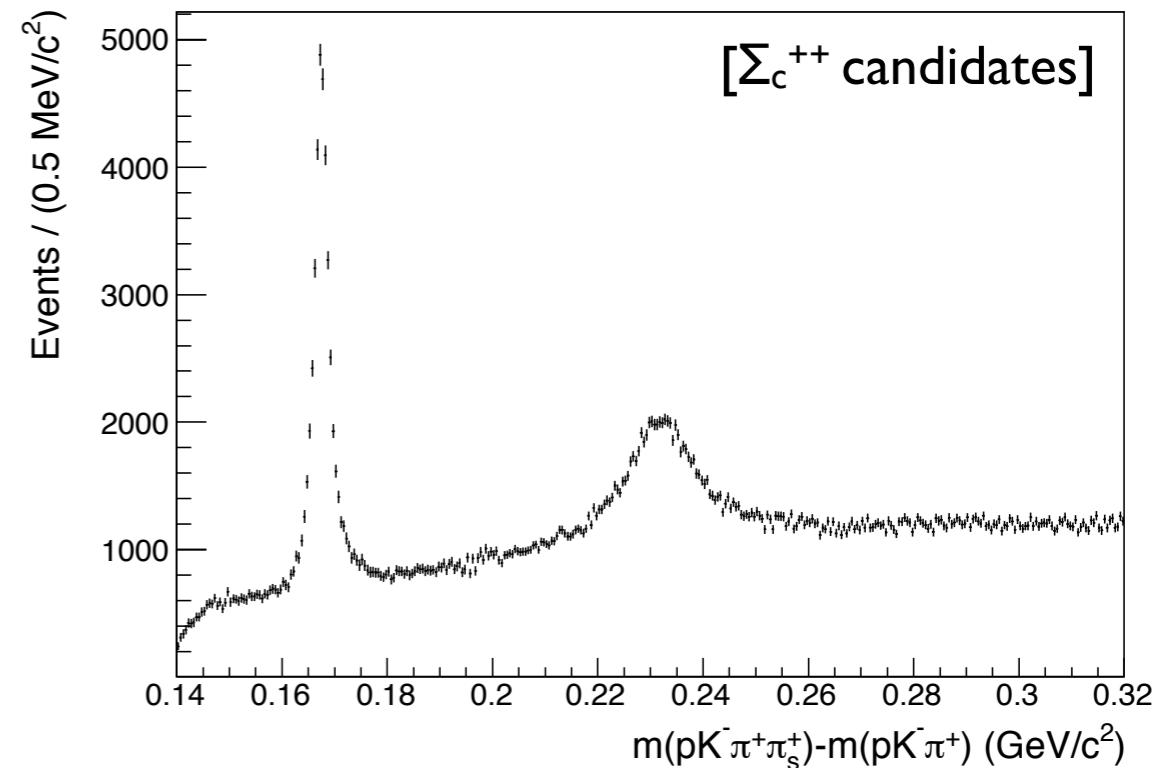
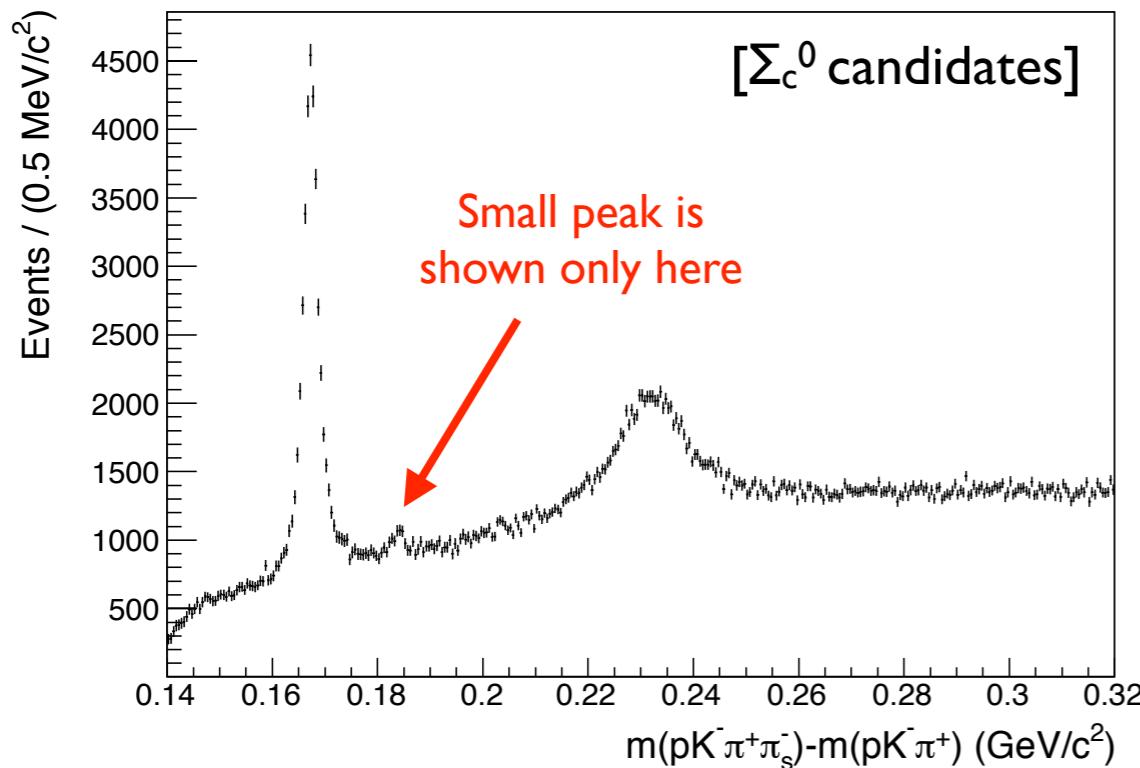
- Random backgrounds are classified by true and fake Λ_c^+
 - Associated with true Λ_c^+ : No way to extract from data
 - Modeled with: $(\Delta m - m_\pi)^{c_0} e^{-c_1(\Delta m - m_\pi)}$
 - The modeling is confirmed by fitting to MC samples



Contribution from Ξ_c^0 Decays



- Small peak is found in $m(p\bar{K}^-\pi^+\pi^-) - m(p\bar{K}^-\pi^+)$

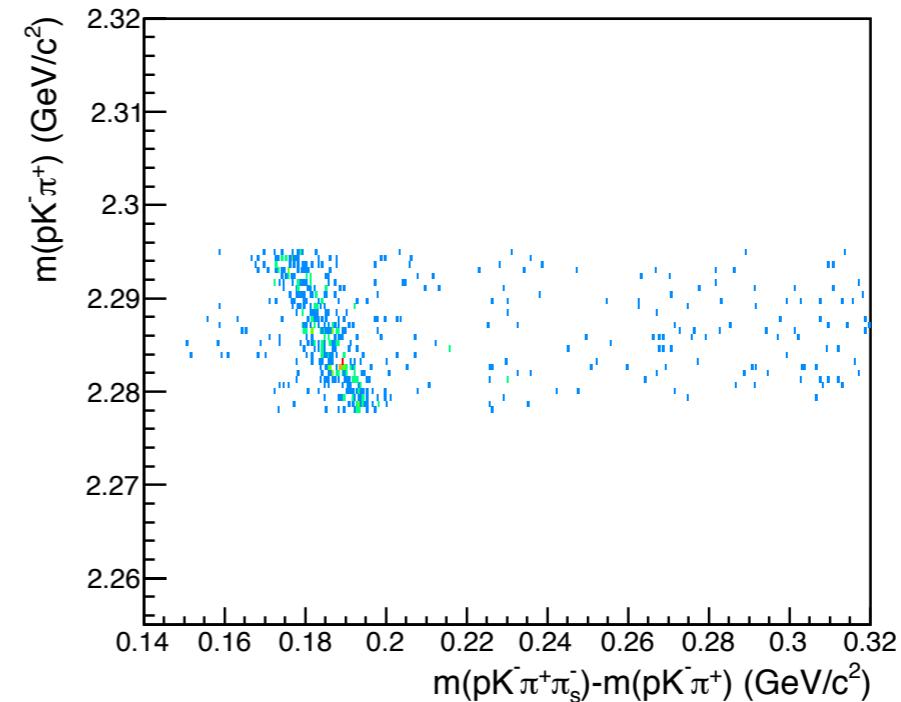
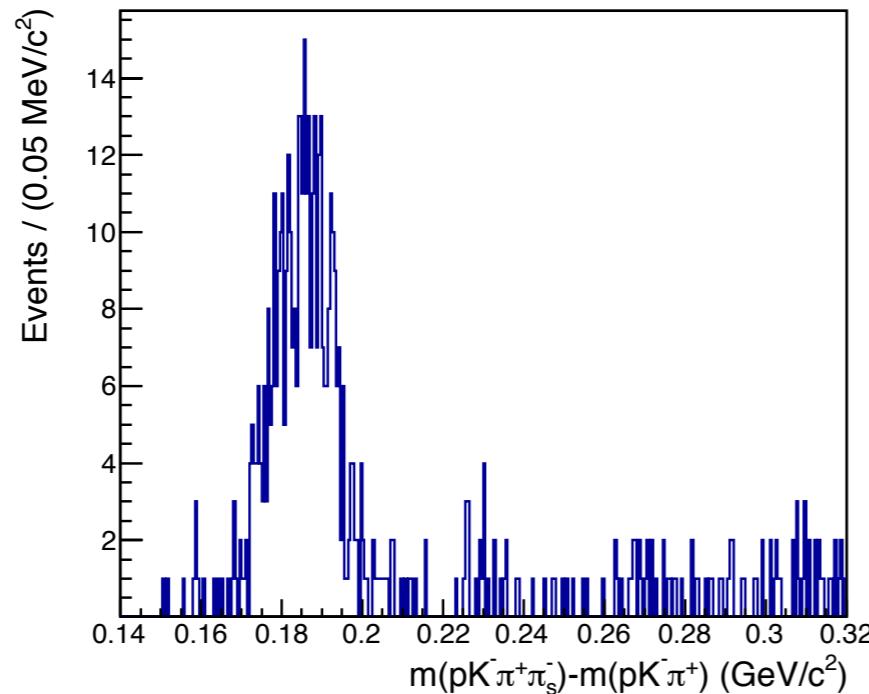


- The peak position is ~ 185 MeV/c²
 - If a nominal mass of Λ_c^+ is added, it is compatible with Ξ_c^0 mass
 - $m(\text{peak}) + m(\Lambda_c^+) = 185 + 2286.46$ (PDG 2013) = 2471.46 MeV/c²
 - $m(\Xi_c^0) = 2470.88$ (PDG 2013)
- Because of the charge conservation, it makes a sense that the peak only appears in the neutral mode (**there is no Ξ_c^{++}**)

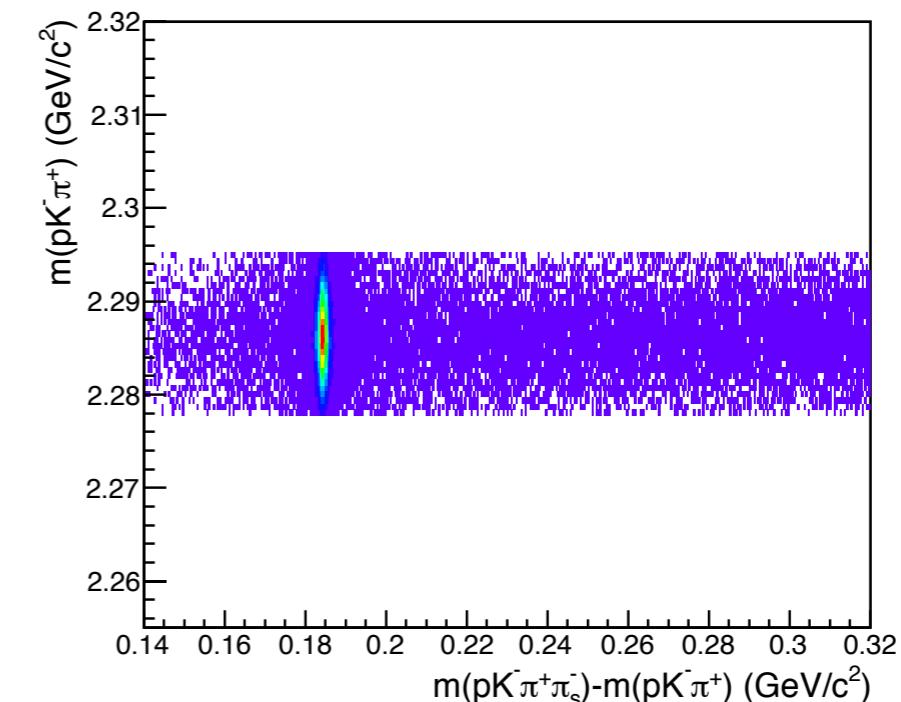
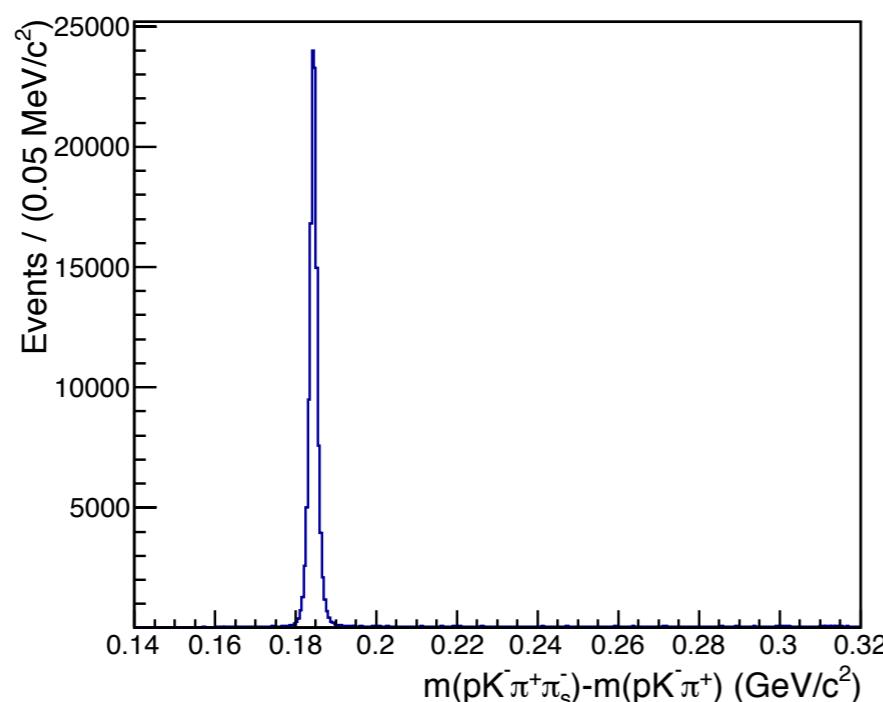
Contribution from Ξ_c^0 Decays



- From MC and data, this background contribution is confirmed
 - $\Xi_c^0 \rightarrow \Lambda_c^+ \pi^-$ is dominant, $\Xi_c^0 \rightarrow p\bar{K} \pi^+ \pi^-$ is negligible



$\Xi_c^0 \rightarrow p\bar{K} \pi^+ \pi^-$ MC simulation
(1M events are generated)



$\Xi_c^0 \rightarrow \Lambda_c^+ \pi^-$ MC simulation
(1M events are generated)

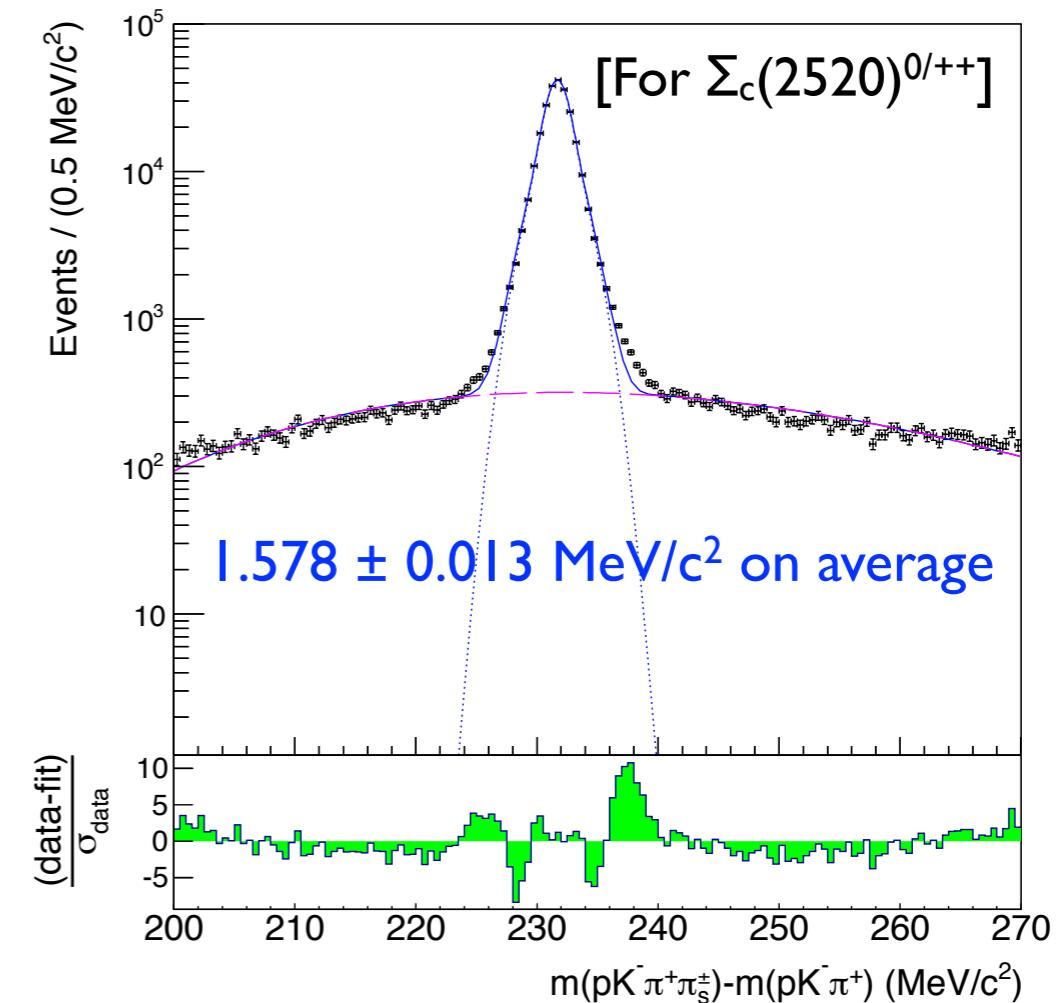
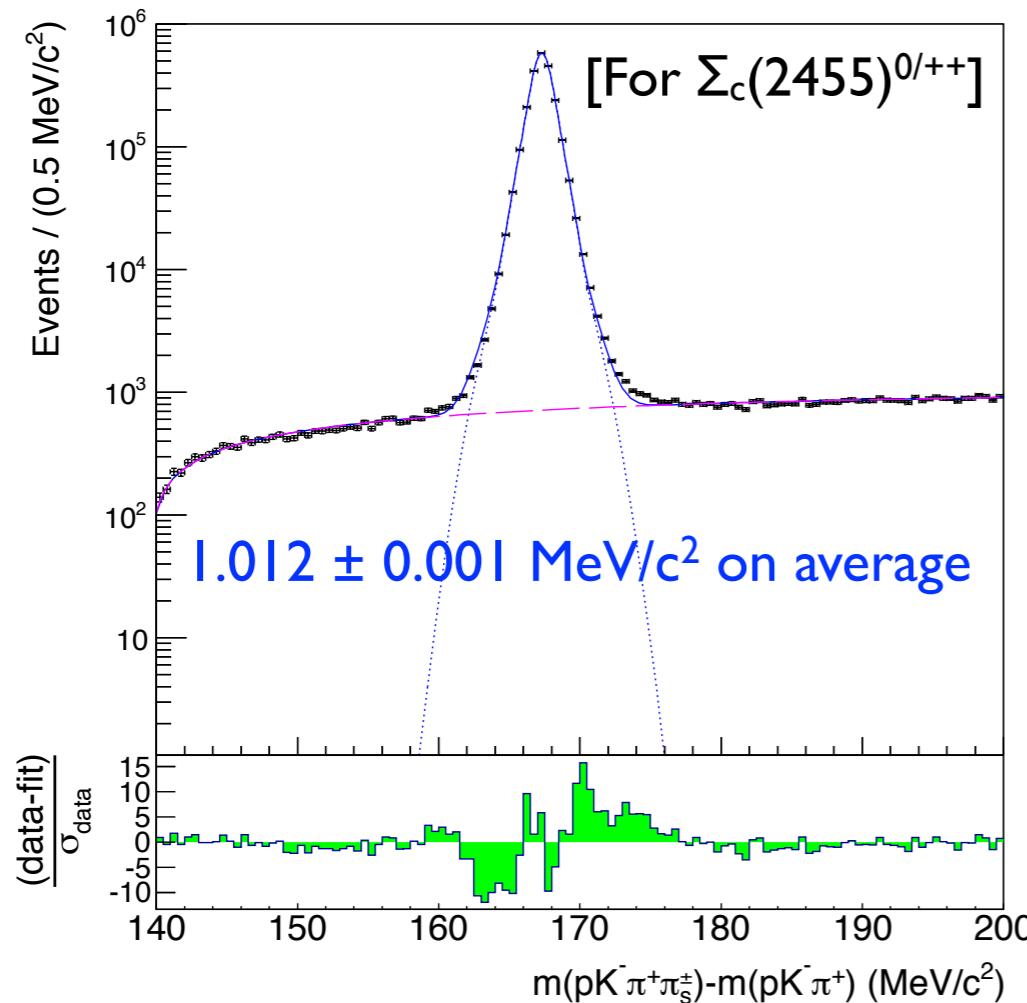
Fit Procedure

- Because of the finite decay width, it should be considered with a detector resolution simultaneously by the convolution theorem

$$M(m; m_0, \Gamma) = T(m; m_0, \Gamma) \otimes R(m)$$

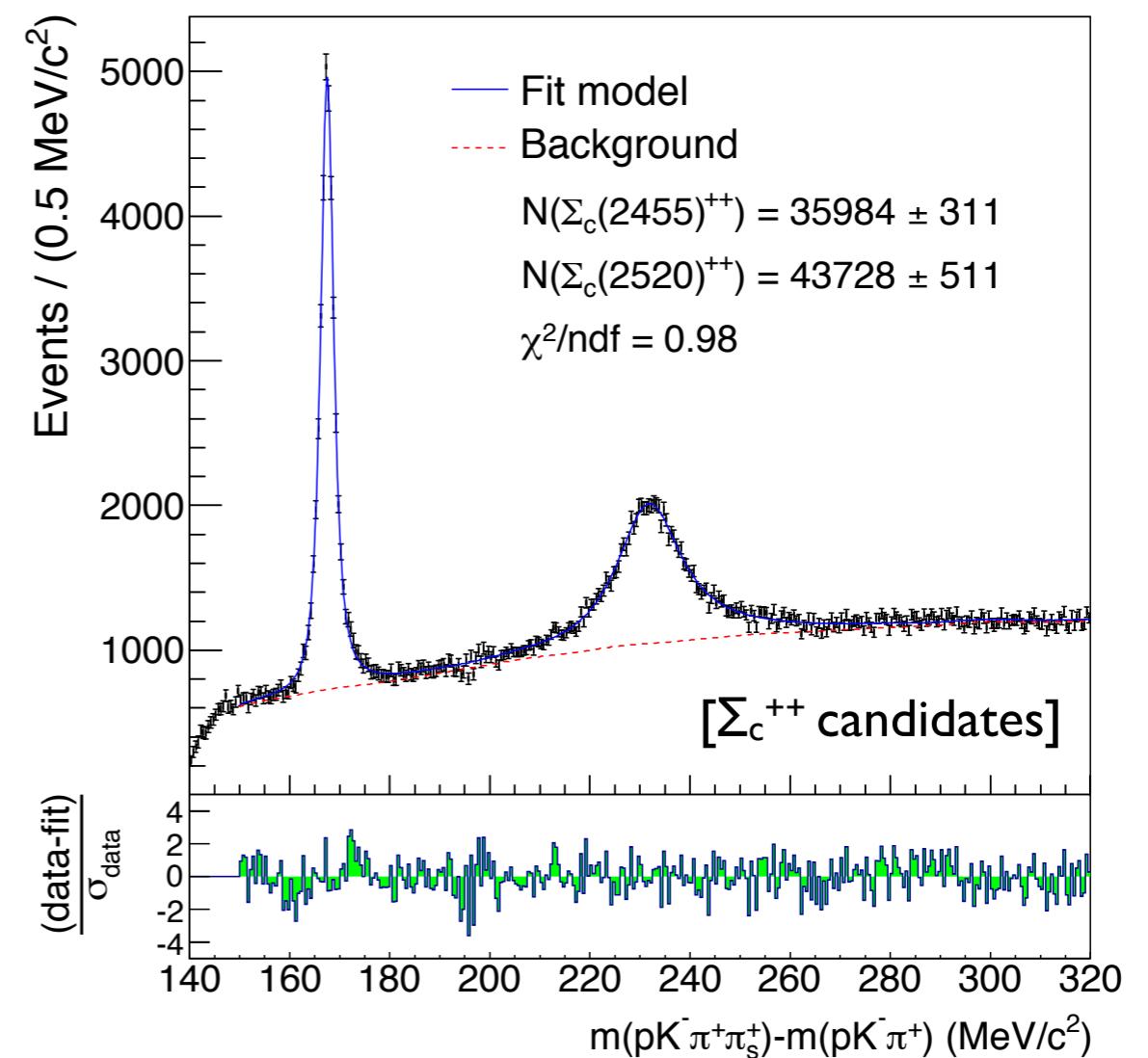
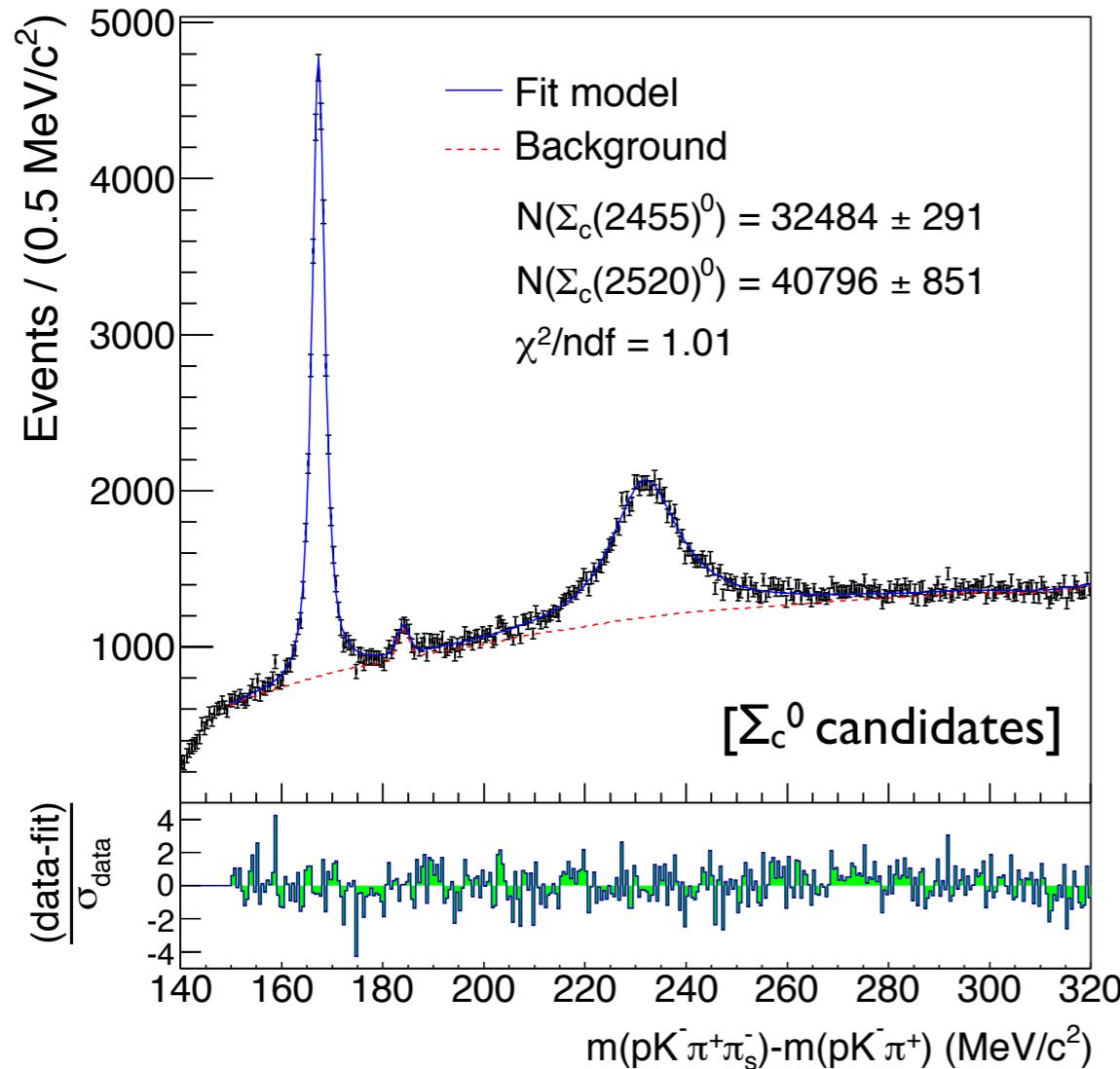
$$= \int_{-\infty}^{+\infty} T(m; m_0, \Gamma) R(m - m') dm'$$

- Detector response functions are obtained from MC with no Σ_c widths



Results

- Binned ML fit done with necessary components
(Ξ_c^0 contribution is modeled by a Gaussian; mean= 184.08 ± 0.15 , width= 1.21 ± 0.18 in MeV/c^2)



(MeV/ c^2)	$\Sigma_c(2455)^0$	$\Sigma_c(2455)^{++}$	$\Sigma_c(2520)^0$	$\Sigma_c(2520)^{++}$
$m(\Sigma_c) - m(\Lambda_c^+)$	167.29 ± 0.01	167.51 ± 0.01	231.98 ± 0.11	231.99 ± 0.10
Decay Widths	1.76 ± 0.04	1.84 ± 0.04	15.41 ± 0.41	14.77 ± 0.25
$m(\Sigma_c)^*$	2453.75 ± 0.01	2453.97 ± 0.01	2518.44 ± 0.11	2518.45 ± 0.10

* Mass is calculated by adding a nominal Λ_c^+ mass to $m(\Sigma_c) - m(\Lambda_c^+)$

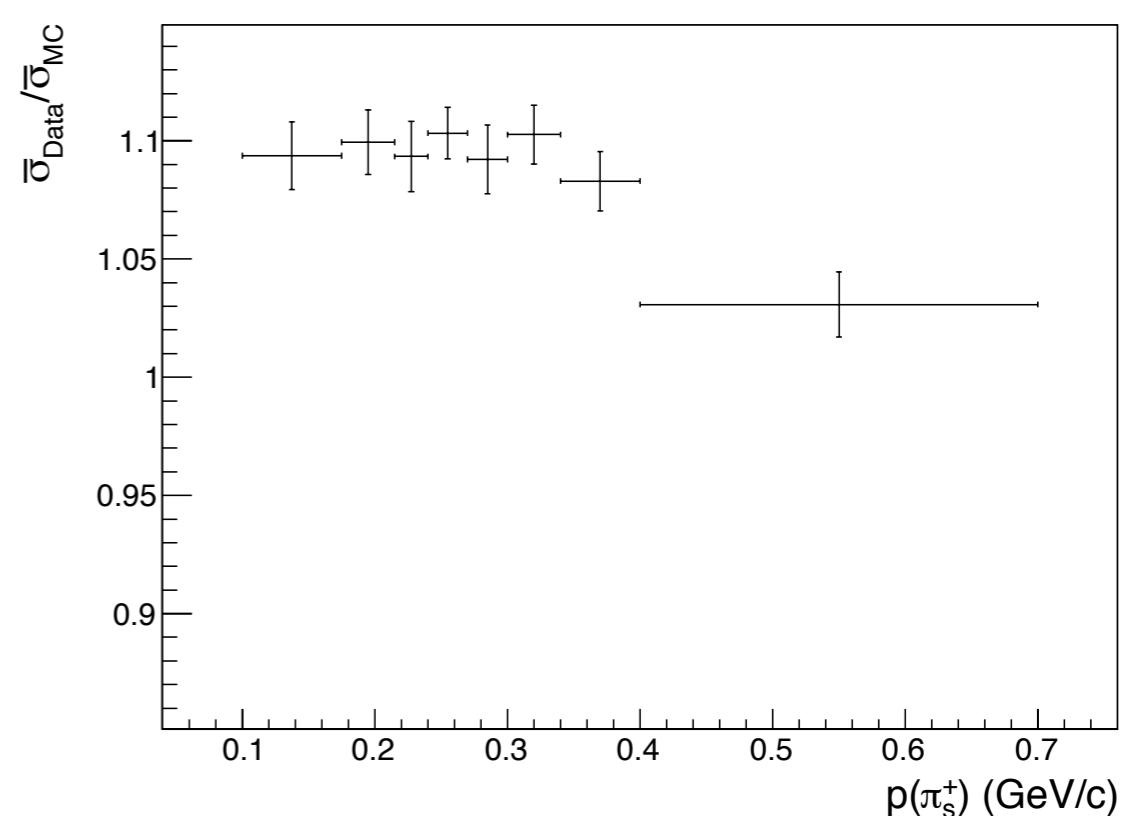
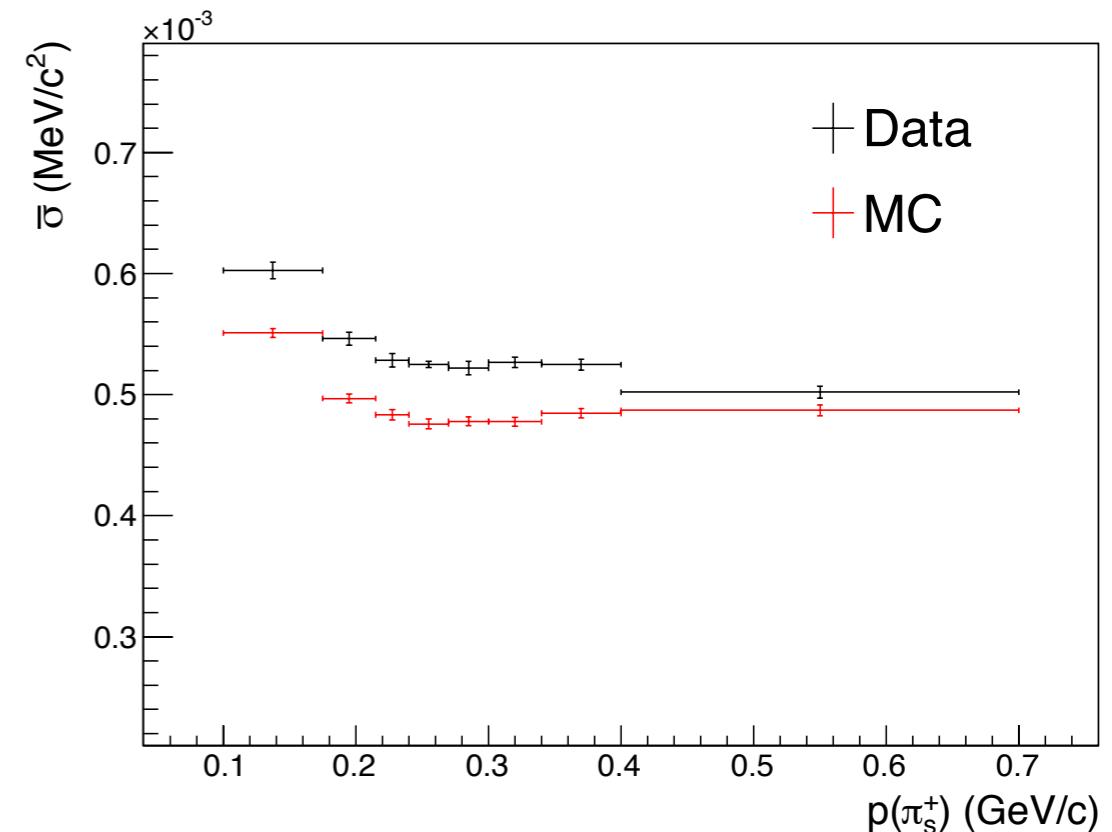
Systematic Uncertainties



- Major contributions of the systematic uncertainties:
 - Resolution model
 - ▶ Discrepancy between data and MC
 - ▶ Statistical fluctuation of the resolution model
 - Momentum scale
 - Fit model
 - ▶ Bias by the fitter
 - ▶ Binning effect
 - ▶ Effect of the fit ranges
 - Background model
 - ▶ Feed-down correction
 - ▶ Statistical fluctuation of the random backgrounds with fake Λ_c^+
 - ▶ Various modeling of the random background with true Λ_c^+

Systematics: Resolution Model

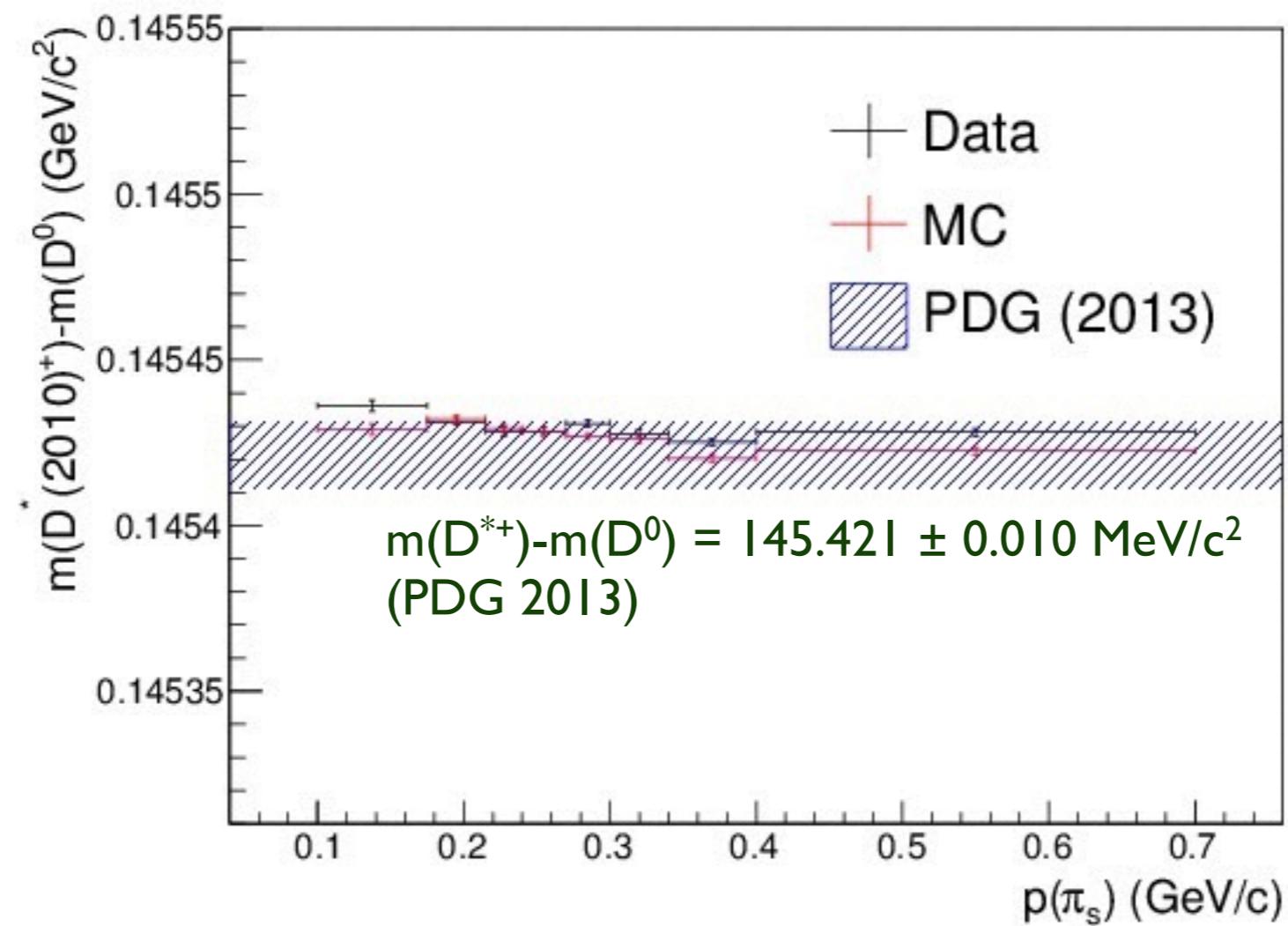
- Discrepancy between data and MC
 - Studied by using a control sample, $D^{*+} \rightarrow D^0 \pi_s^+$
 - By comparing the resolution of $m(D^{*+}) - m(D^0)$ between data and MC, a (single-sided) systematic uncertainties are assigned for the decay widths
 - ▶ **-0.19 MeV/c²** for $\Gamma(\Sigma_c(2455)^{0/++})$
 - ▶ **-0.25 MeV/c²** for $\Gamma(\Sigma_c(2520)^0)$
 - ▶ **-0.24 MeV/c²** for $\Gamma(\Sigma_c(2520)^{++})$
- Statistical fluctuation of the resolutions by $\pm 1\sigma$
 - Small uncertainties are found for the decay widths
 - ▶ **0.01** and **0.04 MeV/c²** for $\Gamma(\Sigma_c(2455)^{0/++})$ and $\Gamma(\Sigma_c(2520)^{0/++})$, respectively



Systematics: Momentum Scale



- Possible bias in the measurements of charged particle momenta
 - Wrong momentum scale of the detector
 - The effect is studied with the control sample of D^{*+} decay
 - By comparing $m(D^{*+}) - m(D^0)$ with the world average, the uncertainty is assigned to be **0.02 MeV/c²** for the mass difference



Systematic Uncertainties

- Major contributions of the systematic uncertainties:

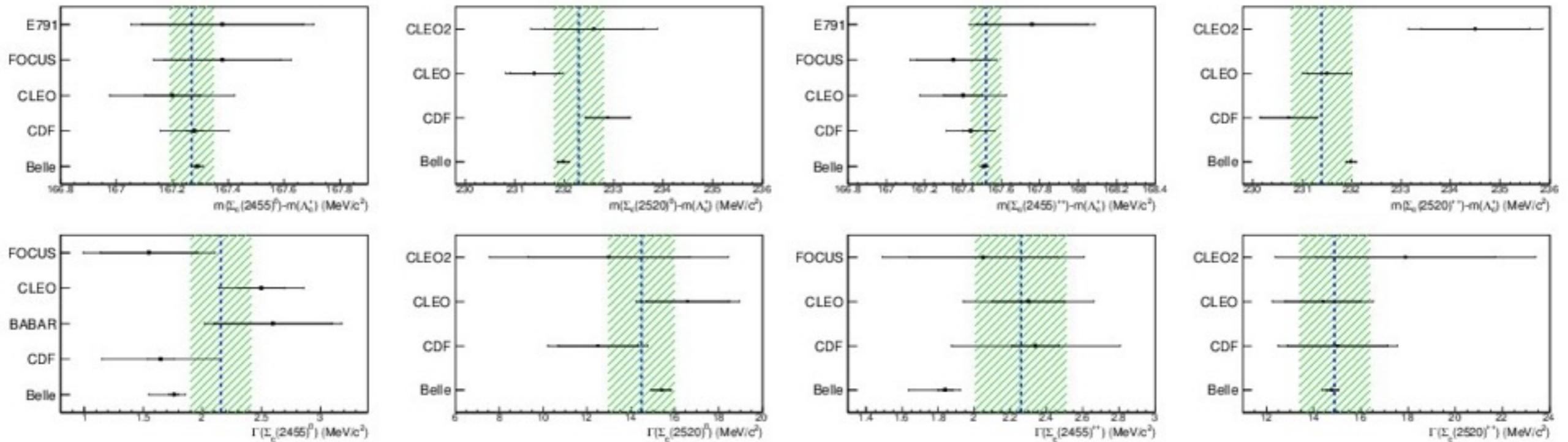
- Resolution model
 - Discrepancy between data and MC
 - Statistical fluctuation of the resolution model
- Momentum scale
- Fit model
 - Bias by the fitter
 - Binning effect
 - Effect of the fit ranges
- ~~Background model (all negligible)~~
 - ~~Feed-down correction~~
 - ~~Statistical fluctuation of the random backgrounds with fake Λ_c^+~~
 - ~~Various modeling of the random background with true Λ_c^+~~

(MeV/c ²)	$\Sigma_c(2455)^0$		$\Sigma_c(2455)^{++}$		$\Sigma_c(2520)^0$		$\Sigma_c(2520)^{++}$	
	Δm	Γ	Δm	Γ	Δm	Γ	Δm	Γ
Resolution model	—	+0.01 -0.19	—	+0.01 -0.19	—	+0.01 -0.19	—	+0.01 -0.19
Momentum scale	± 0.02	—	± 0.02	—	± 0.02	—	± 0.02	—
Fit model	± 0.01	± 0.09	± 0.03	± 0.20	± 0.01	± 0.07	± 0.01	± 0.18
Total	± 0.02	+0.09 -0.21	± 0.04	+0.20 -0.32	± 0.02	+0.07 -0.20	± 0.02	+0.18 -0.30

Results

- The results are the most precise to date
 - $m(\Sigma_c)$'s are calculated by adding a nominal Λ_c^+ mass to the mass difference
 - The most systematic uncertainties cancels (e.g. momentum scale)

(MeV/c ²)	$m(\Sigma_c) - m(\Lambda_c^+)$	Decay widths (Γ)	$m(\Sigma_c)$
$\Sigma_c(2455)^0$	$167.29 \pm 0.01 \pm 0.02$	$1.76 \pm 0.04 {}^{+0.09}_{-0.21}$	$2453.75 \pm 0.01 \pm 0.02 \pm 0.14$
$\Sigma_c(2455)^{++}$	$167.51 \pm 0.01 \pm 0.02$	$1.84 \pm 0.04 {}^{+0.07}_{-0.20}$	$2453.97 \pm 0.01 \pm 0.02 \pm 0.14$
$\Sigma_c(2520)^0$	$231.98 \pm 0.11 \pm 0.04$	$15.41 \pm 0.41 {}^{+0.20}_{-0.32}$	$2518.44 \pm 0.11 \pm 0.04 \pm 0.14$
$\Sigma_c(2520)^{++}$	$231.99 \pm 0.10 \pm 0.02$	$14.77 \pm 0.25 {}^{+0.18}_{-0.30}$	$2518.45 \pm 0.10 \pm 0.02 \pm 0.14$



Summary

- Charmed baryons are an interesting testbed to test QCD models
 - Properties of Σ_c baryons are not well established
 - Isospin mass splitting contradicts a naive expectation
 - Precise measurements will be helpful to develop theoretical models
- Measurements of masses and decay widths of the $\Sigma_c(2455)^{0/++}$ and $\Sigma_c(2520)^{0/++}$ baryons have been performed at Belle
 - Various sources of the backgrounds are analyzed
 - Systematic uncertainties reduces significantly
 - The results are most precise to date

Backup Slides

Data Samples and Selection Criteria



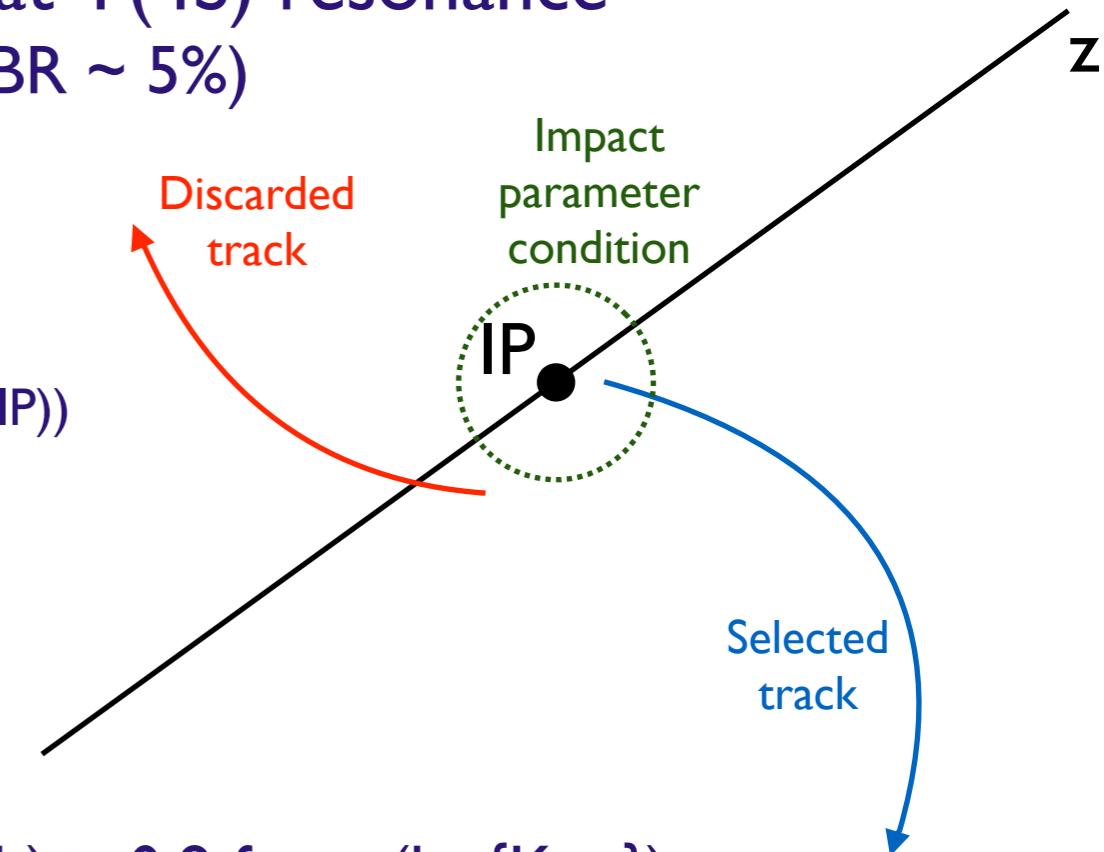
- Integrated luminosity of 711 fb^{-1} collected at $\Upsilon(4S)$ resonance
 - Λ_c^+ decay chain is chosen to be $\Lambda_c^+ \rightarrow p K^- \pi^+$ ($\text{BR} \sim 5\%$)

- Track selection for all charged tracks

- Impact parameters
 - ▶ $|dr| < 1 \text{ cm}$ (radial position from the interaction point (IP))
 - ▶ $|dz| < 3 \text{ cm}$ (z-position from IP)
 - SVD hit requirements
 - ▶ > 1 hits both $r\varphi$ and z direction

- Particle identification

- $P(K:\pi) > 0.6$ for K / $P(\pi:K) > 0.6$ for π / $P(p:h) > 0.9$ for p ($h=\{\text{K},\text{ }\pi\}$)
 - PID efficiencies (ε : efficiency / f : fake rate)
 - ▶ $\varepsilon(K:\pi) = (90.58 \pm 0.53)\%$ $f(K:\pi) = (9.77 \pm 0.33)\%$
 - ▶ $\varepsilon(\pi:K) = (92.63 \pm 0.69)\%$ $f(\pi:K) = (7.84 \pm 0.33)\%$
 - ▶ $\varepsilon(\pi_s:K) = (99.18 \pm 0.89)\%$ $f(\pi_s:K) = (5.93 \pm 0.52)\%$

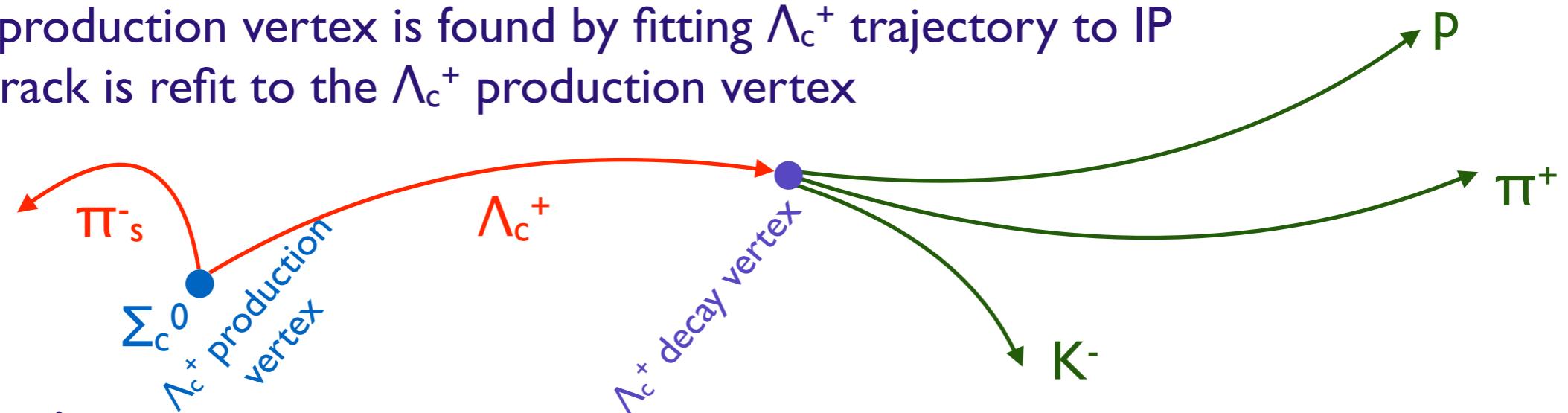


Data Samples and Selection Criteria

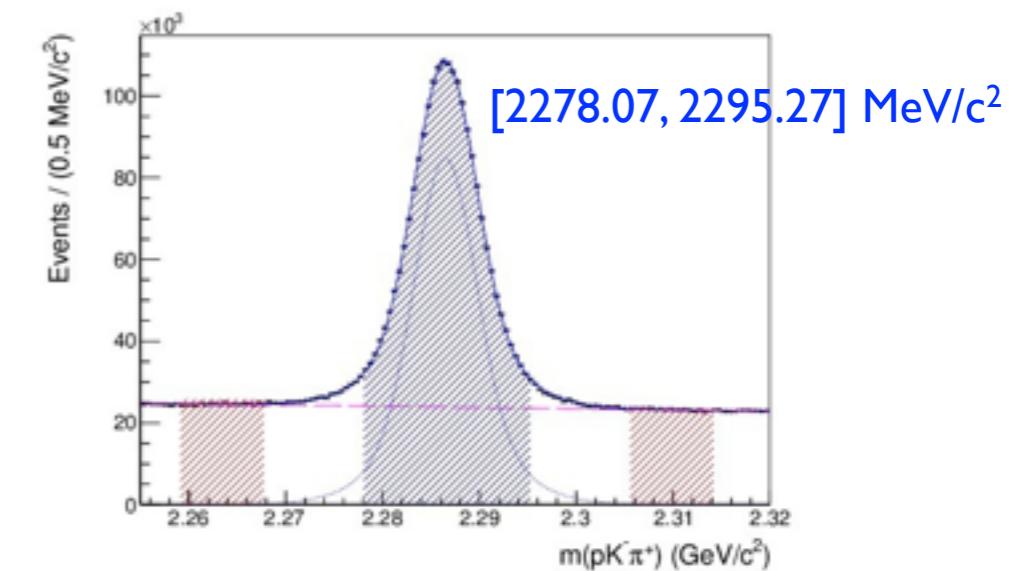


- Λ_c^+ reconstruction
 - Using p, K⁻, π^+ tracks, Λ_c^+ candidates are reconstructed
 - Daughter tracks are refit to a common vertex (Λ_c^+ decay vertex)

- Σ_c reconstruction
 - Attaching π_s^- track to Λ_c^+ candidate, Σ_c candidates are reconstructed
 - Λ_c^+ production vertex is found by fitting Λ_c^+ trajectory to IP
 - π_s^- track is refit to the Λ_c^+ production vertex



- Σ_c selection
 - $m(\Lambda_c^+) \in [2278.07, 2295.79] \text{ MeV}/c^2$
($\pm 2.1\sigma$ around $m(\Lambda_c^+)$)
 - $p(\Sigma_c) > 2.0 \text{ GeV}/c$
 - (Confidence level of π_s vertex fit) $> 0.1\%$



Feed-down Backgrounds from Λ_c^{*+} Decays



- Decays of the excited states of Λ_c^+ baryon can make contributions

	Mass (MeV/c ²)	Decay modes	Branching Fraction
$\Lambda_c(2595)^+$	2592.25 ± 0.28	$\Lambda_c^+\pi^+_s\pi^-_s$	$\sim 67\%$
		$\Sigma_c(2455)^{++}\pi^-_s$	$(24 \pm 7)\%$
		$\Sigma_c(2455)^0\pi^+_s$	$(24 \pm 7)\%$
		$\Lambda_c^+\pi^+_s\pi^-_s$ 3-body	$(18 \pm 10)\%$
$\Lambda_c(2625)^+$	2628.11 ± 0.19	$\Lambda_c^+\pi^+_s\pi^-_s$	$\sim 67\%$
		$\Sigma_c(2455)^{++}\pi^-_s$	$< 5 @ 90\% \text{ C.L.}$
		$\Sigma_c(2455)^0\pi^+_s$	$< 5 @ 90\% \text{ C.L.}$
		$\Lambda_c^+\pi^+_s\pi^-_s$ 3-body	Large
$\Lambda_c(2765)^+$	2766.6 ± 2.4	$\Lambda_c^+\pi^+_s\pi^-_s$ 3-body	seen
$\Lambda_c(2880)^+$	2881.53 ± 0.35	$\Lambda_c^+\pi^+_s\pi^-_s$	seen
		$\Sigma_c(2455)^0/^{++}\pi^{+-}_s$	seen
		$\Sigma_c(2520)^0/^{++}\pi^{+-}_s$	seen

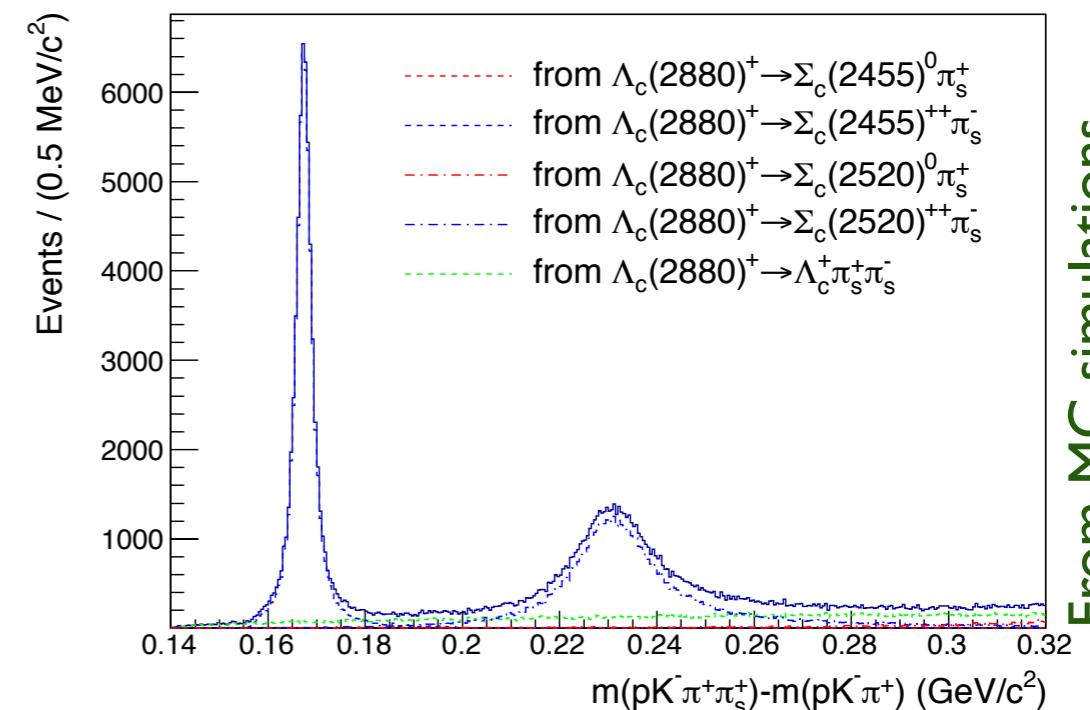
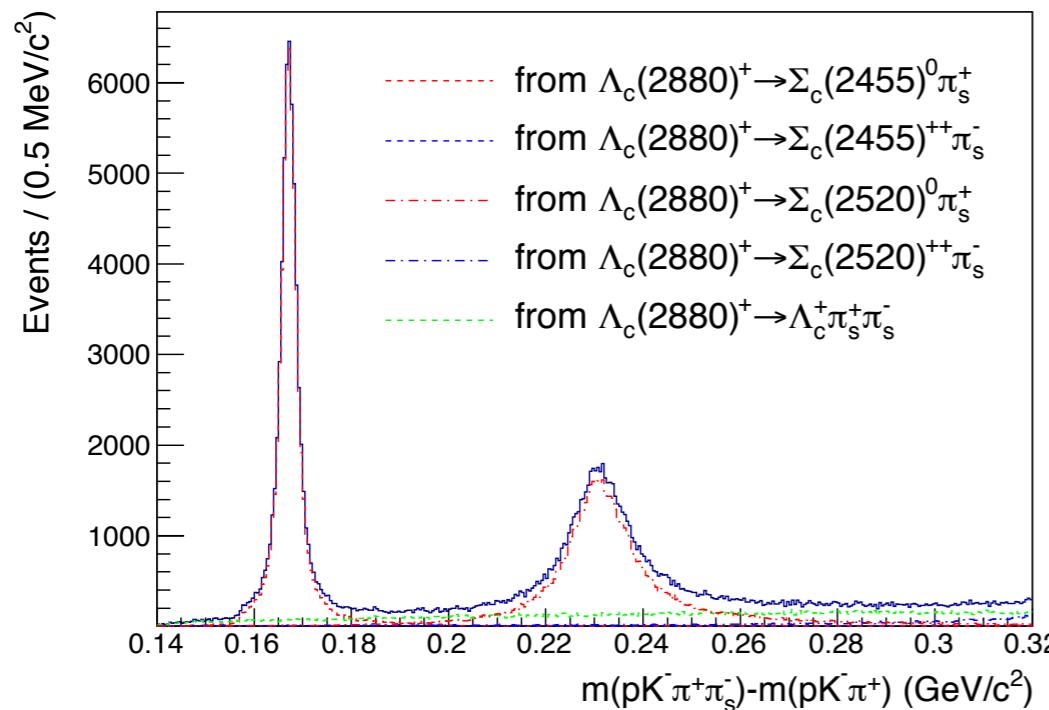
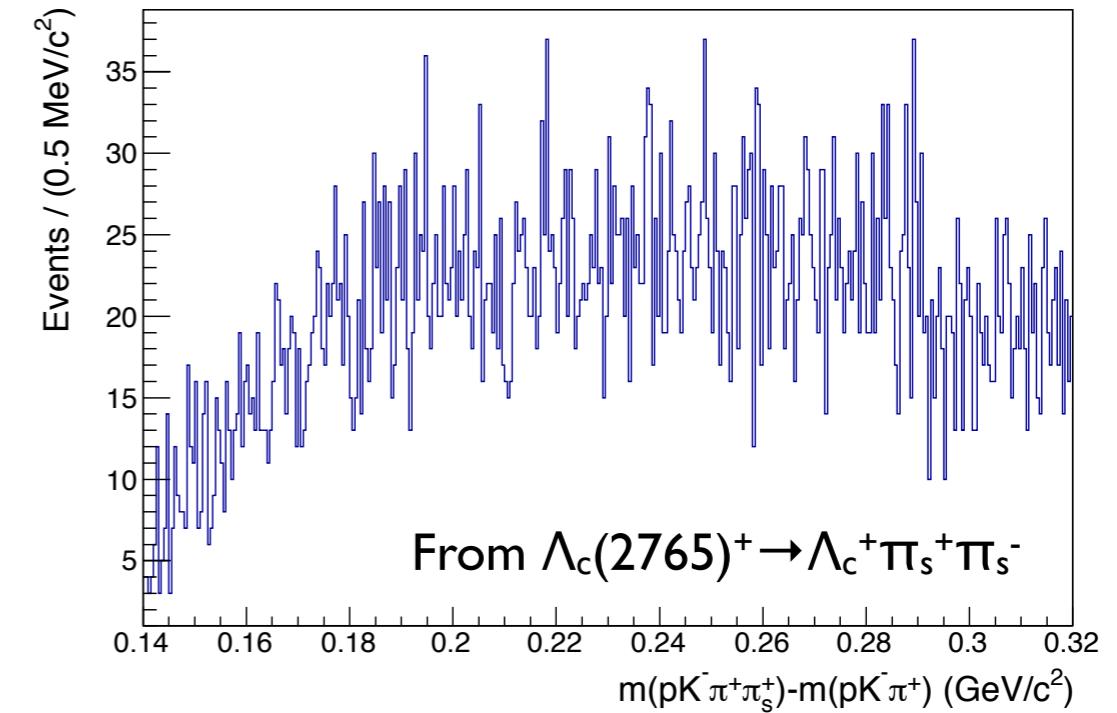
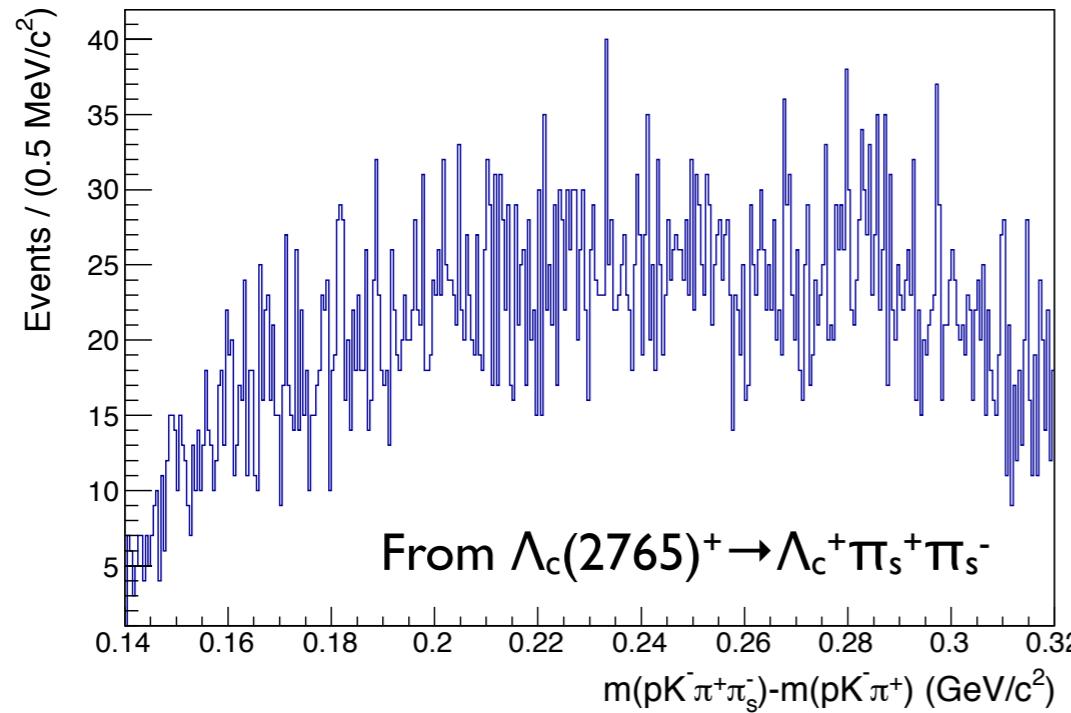
PDG 2013

- Those contributions can be either signal or background events
 - $\Lambda_c(2625)^+ \rightarrow \Sigma_c(2455)^0\pi^+$ (final state: pK⁻ $\pi^+\pi^-\pi^+$): Signal events for $\Sigma_c(2455)^0$ but background events for $\Sigma_c(2455)^{++}$
 - $\Lambda_c(2625)^+ \rightarrow \Sigma_c(2455)^{++}\pi^-$: Background events for $\Sigma_c(2455)^0$ but signal events for $\Sigma_c(2455)^{++}$
 - $\Lambda_c(2625)^+ \rightarrow \Lambda_c^+\pi^+\pi^-$: Background events for both

Feed-down Backgrounds from Λ_c^{*+} Decays



- The contributions from $\Lambda_c(2765)^+$ and $\Lambda_c(2880)^+$ are kinematically negligible

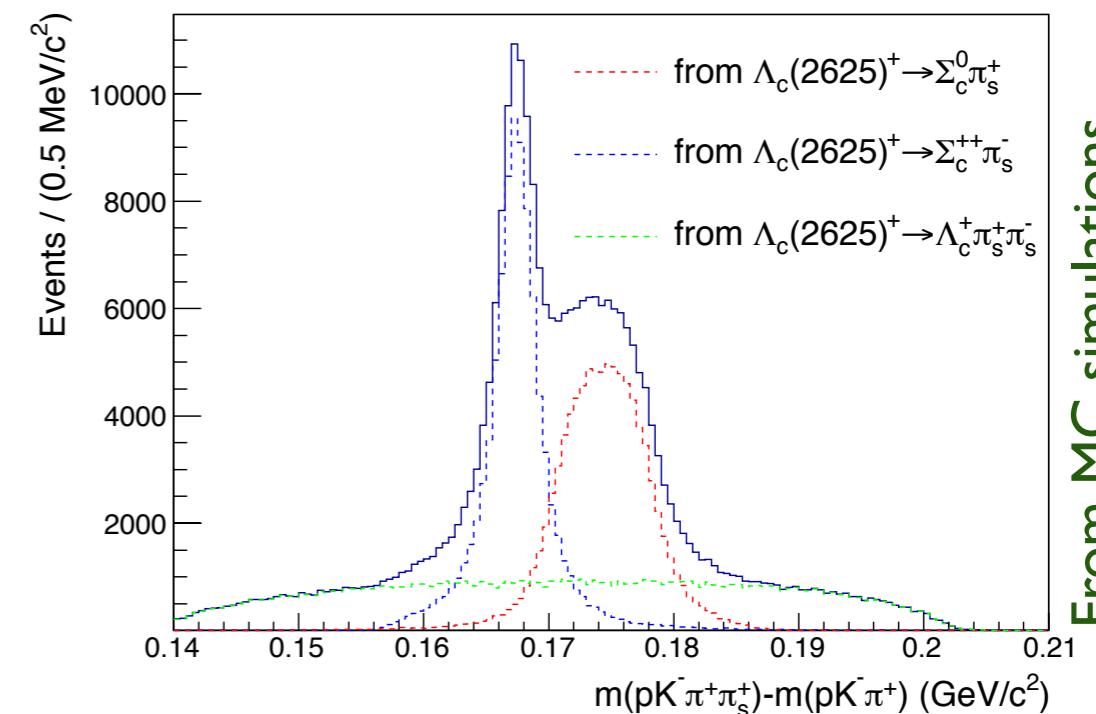
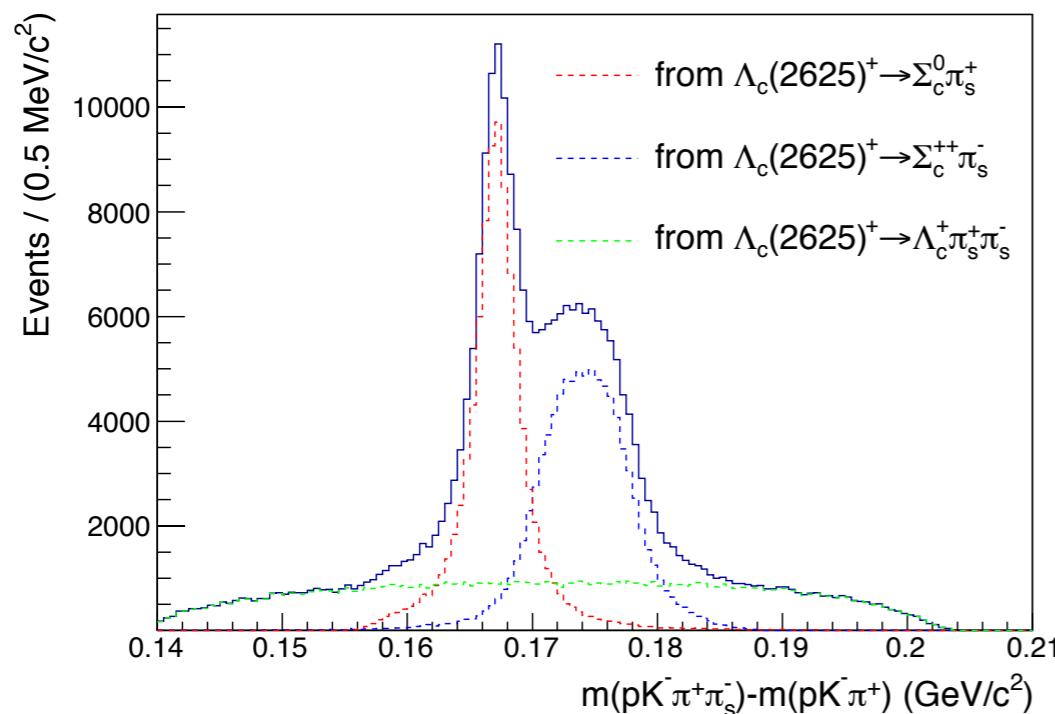
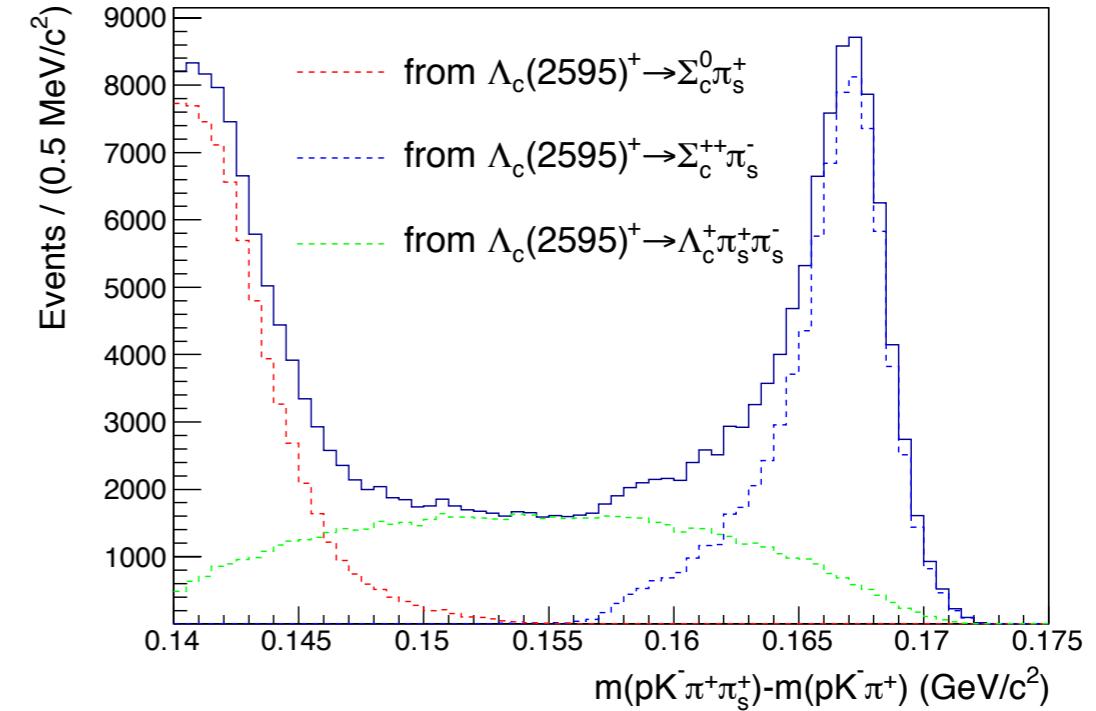
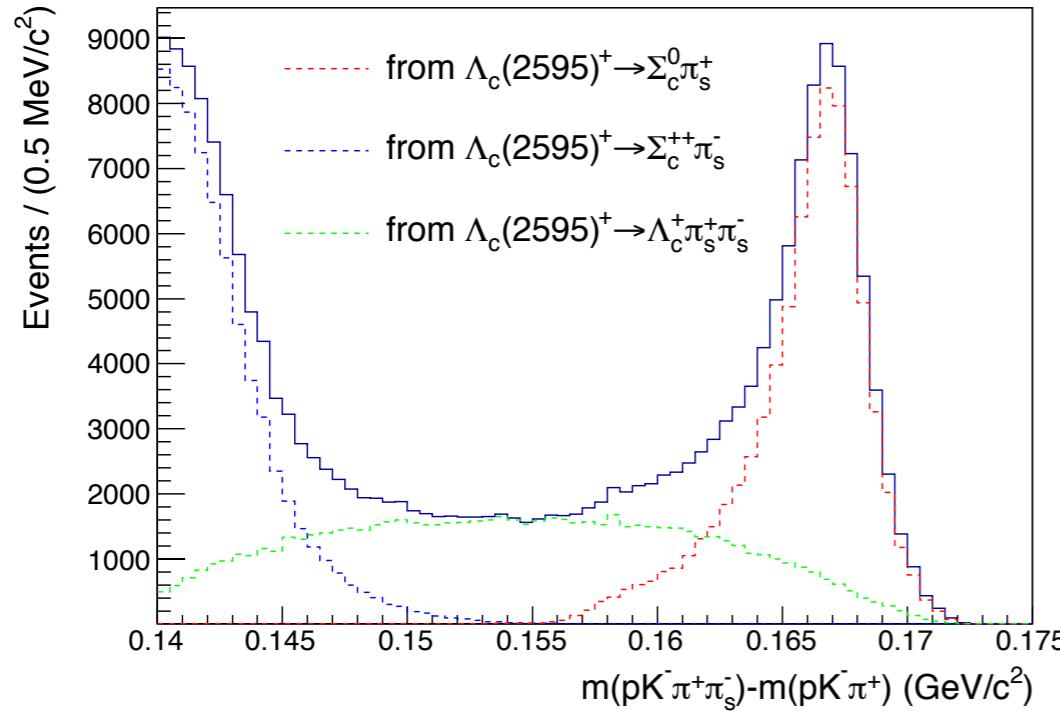


From MC simulations
 (1M events generated for each channels)
 * Branching fractions are not accounted

Feed-down Backgrounds from Λ_c^{*+} Decays



- The contributions from $\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$ are found to be dominant



From MC simulations
(1M events generated for each channels)
* Branching fractions are not accounted

Feed-down Backgrounds from Λ_c^{*+} Decays



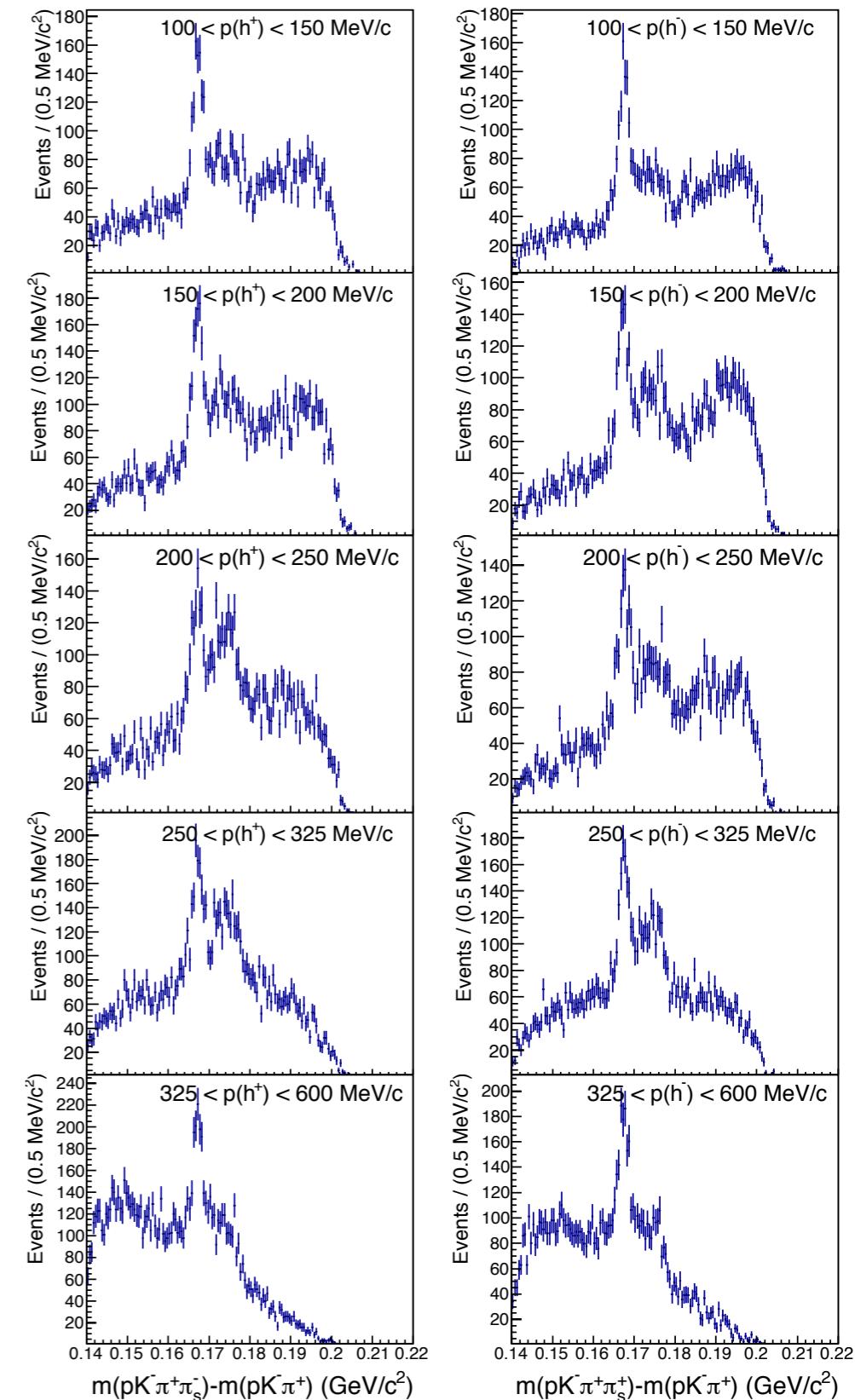
- Feed-down corrections

- Partial reconstruction of $pK^-\pi^+\pi^+\pi^-$ events
→ no efficiency involves
- Tagging by attaching an additional charged track
→ efficiency involves
- Since the feed-down obtained by the tagging method reflects the detector efficiency, it should be corrected to get real feed-down normalization

$$\mathcal{N}_{\text{corrected}}(\text{Feed-down}) = \frac{\mathcal{N}_{\text{uncorrected}}(\text{Feed-down})}{\epsilon_{\text{tracking}}(h^\pm) \cdot \epsilon_{\text{acceptance}}(h^\pm)}$$

~74% on average

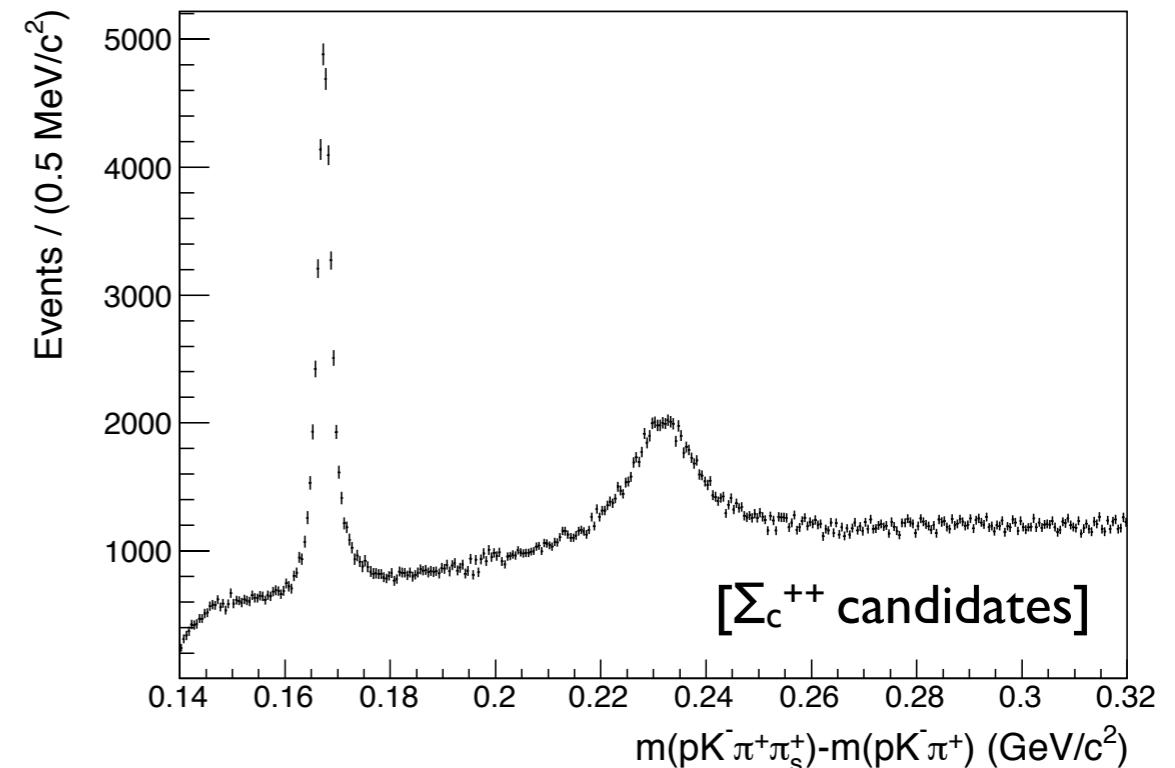
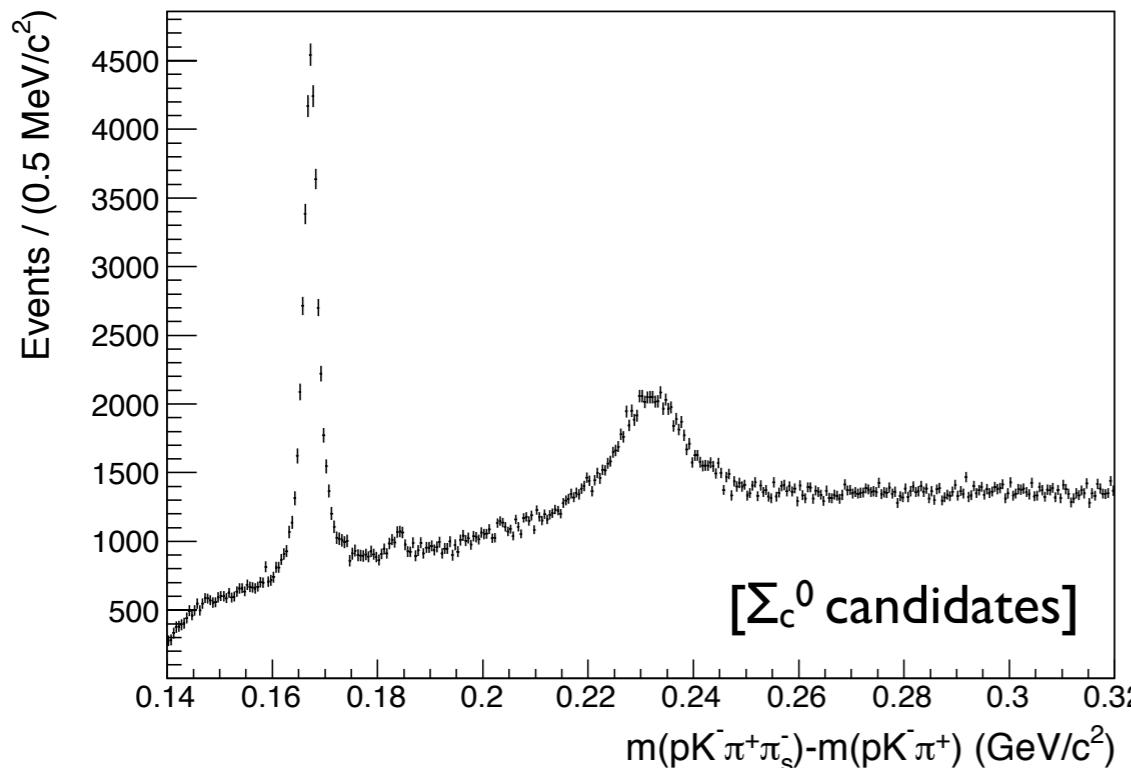
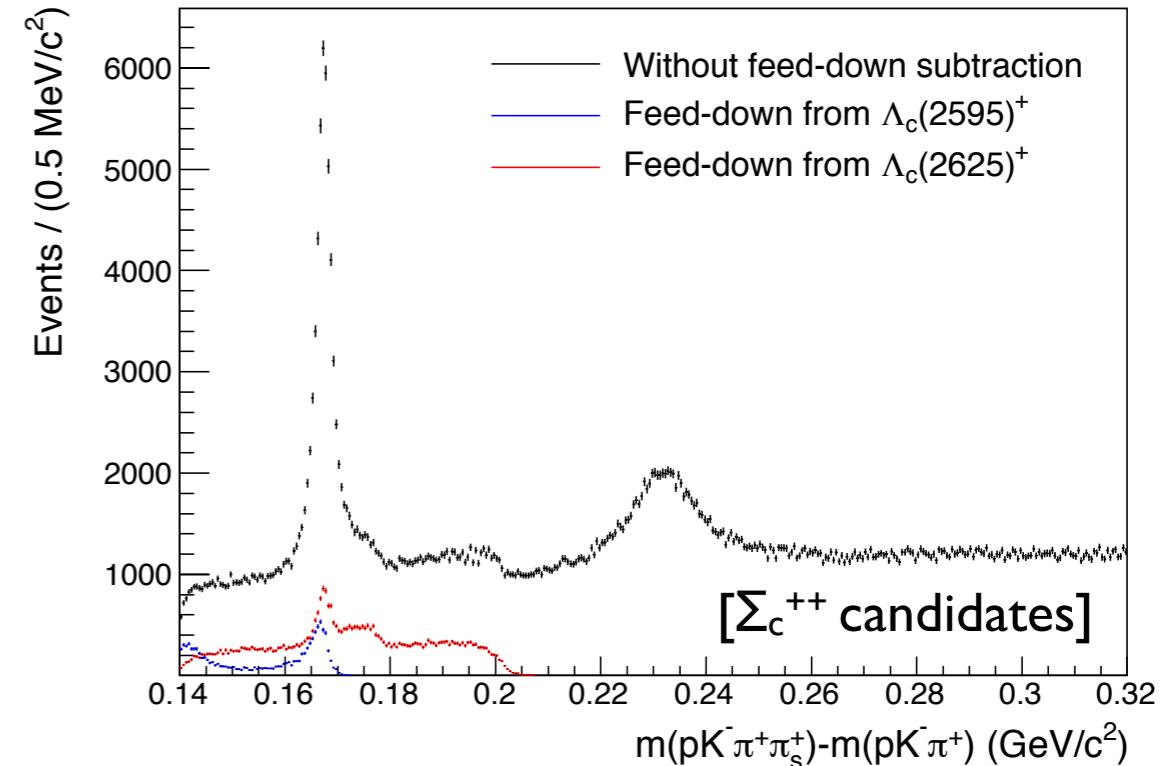
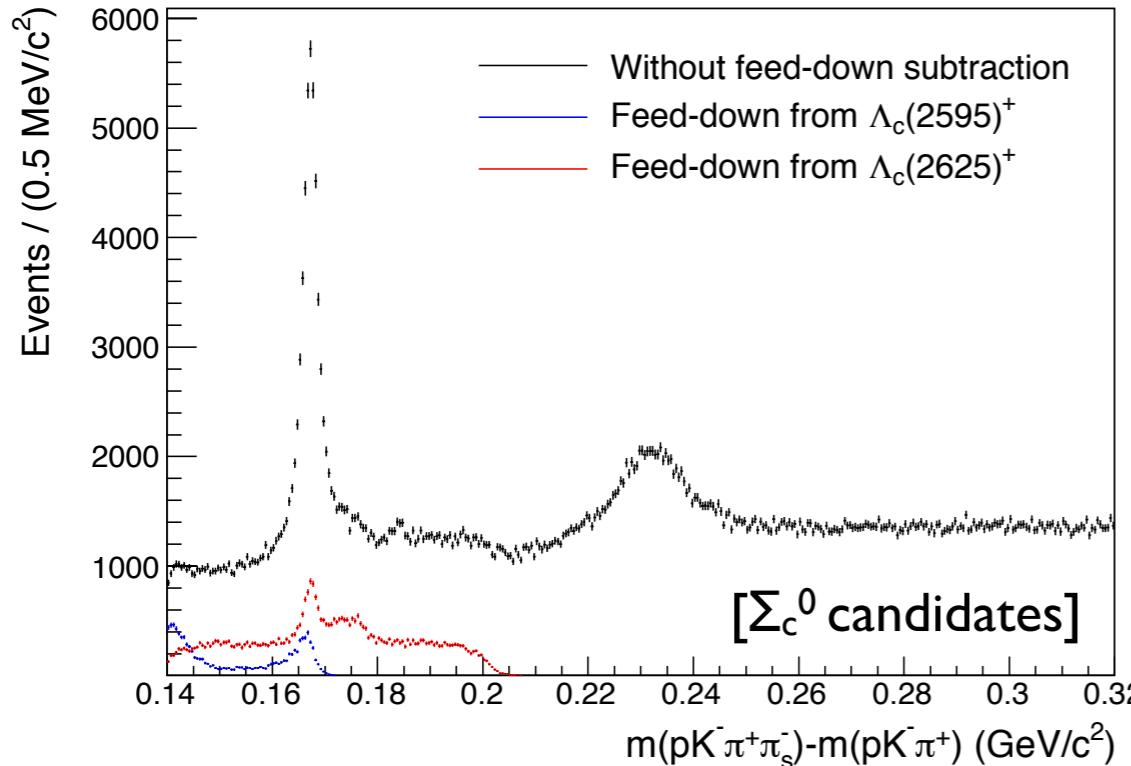
- The feed-down shape depends on charged track momentum
 - The efficiencies are obtained as a function of charged track momentum
 - The correction is applied as a function of charged track momentum



Feed-down Backgrounds from Λ_c^{*+} Decays



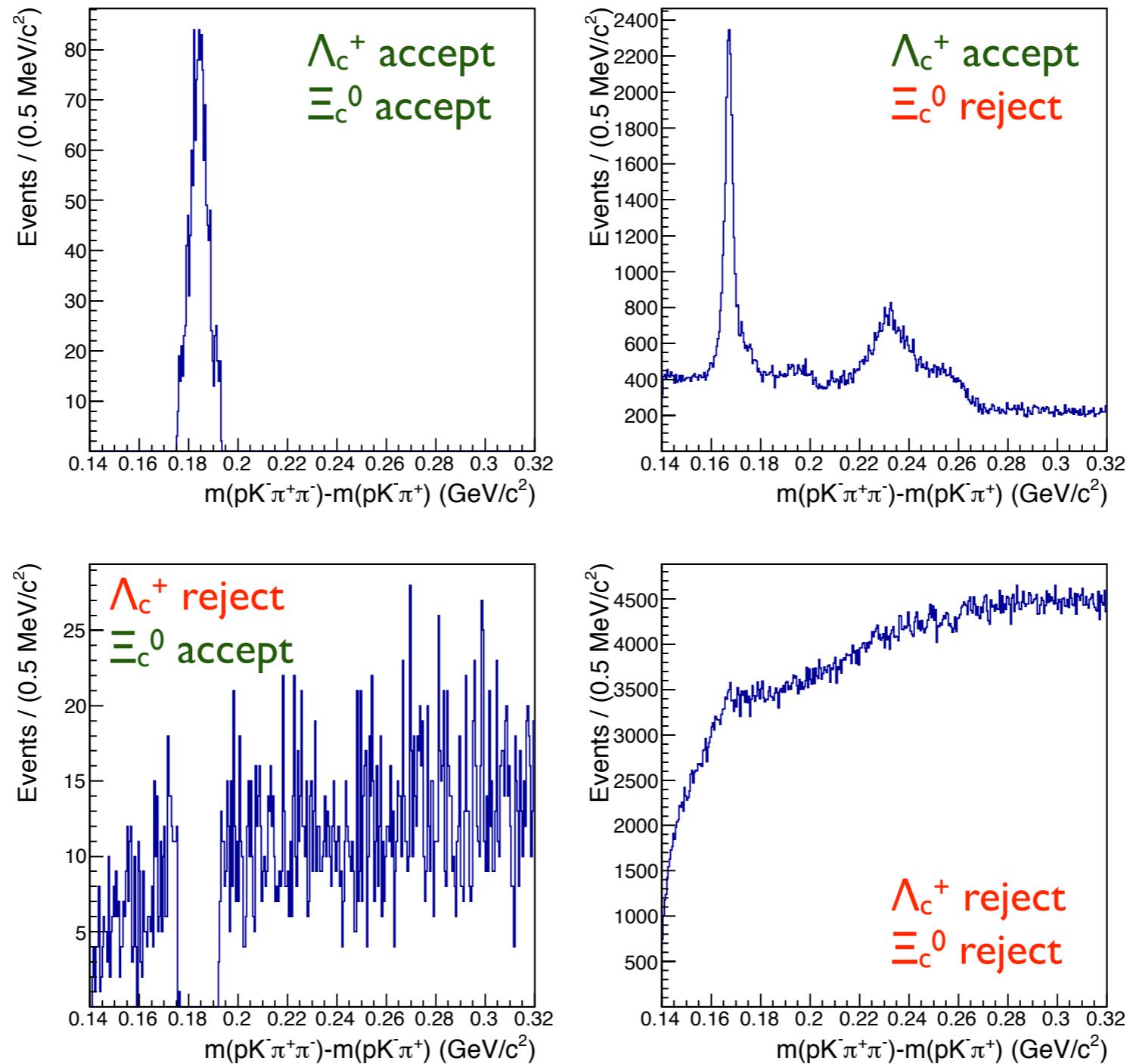
- Before and after the feed-down subtraction



Backgrounds from Ξ_c^0 Decays



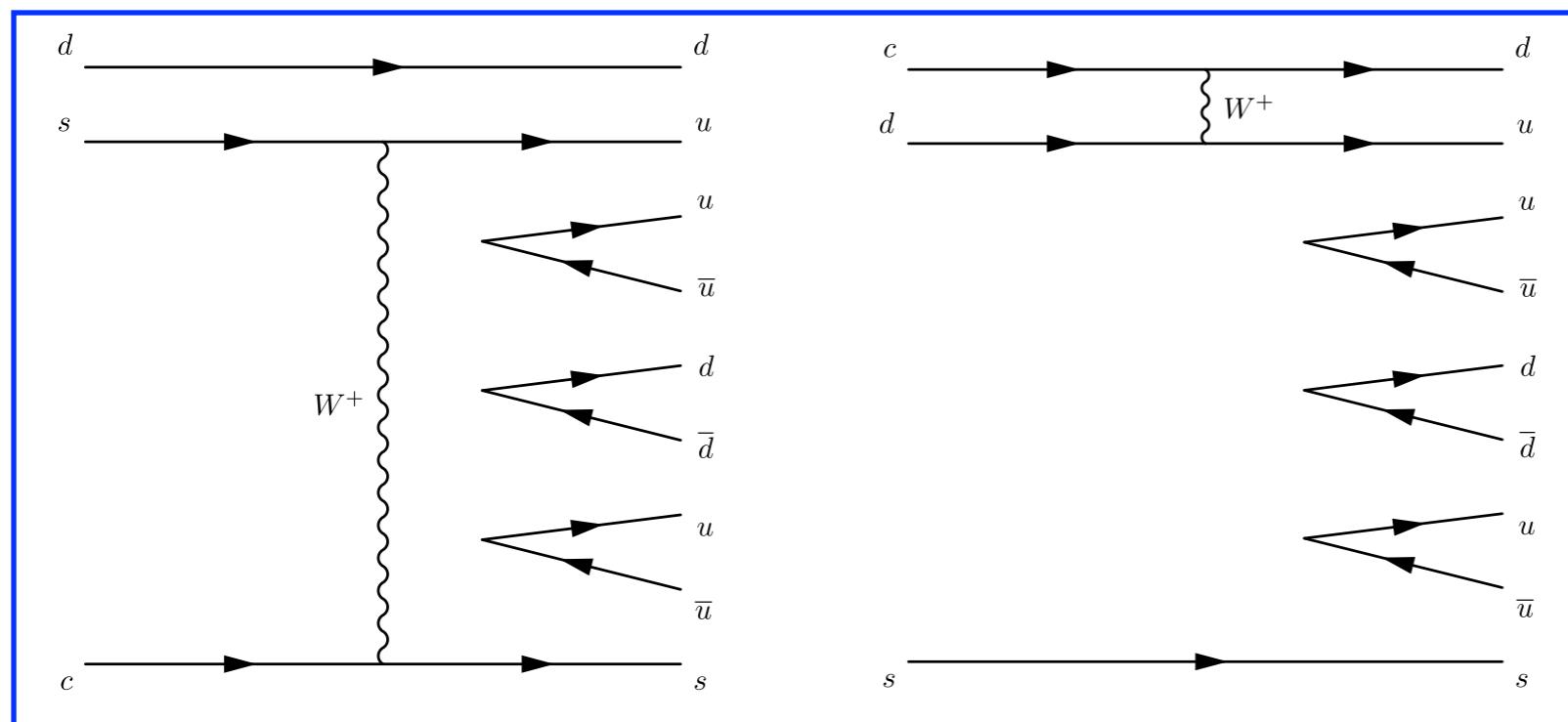
- From MC and data, this background contribution is confirmed
 - Data also shows that the peak is related to Λ_c^+ and Ξ_c^0 simultaneously



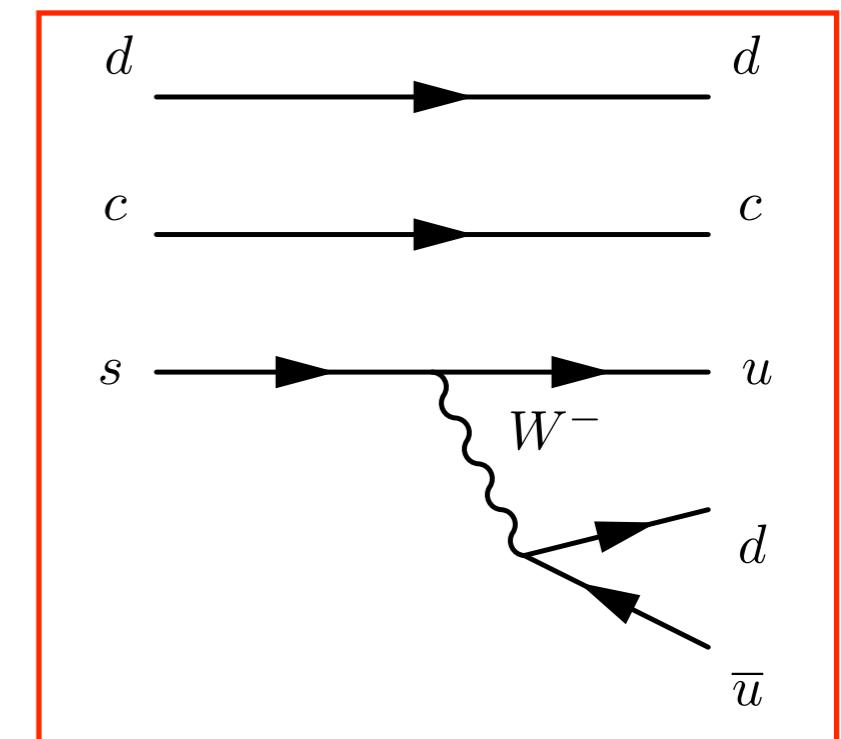
Backgrounds from Ξ_c^0 Decays



- However, there is no known decays of $\Xi_c^0 \rightarrow \Lambda_c^+ \pi^-$ or $\Xi_c^0 \rightarrow p K^- \pi^+ \pi^-$
 - Theoretically possible (Cabibbo-suppressed) but no experimental observation to date
 - No such an observation of which charmed baryon decays into charmed baryon through weak decay
 - The observation of this will be addressed in a separate study



$$\Xi_c^0 \rightarrow p K^- \pi^+ \pi^-$$



$$\Xi_c^0 \rightarrow \Lambda_c^+ \pi^-$$

Fit Procedure

- Since the Σ_c baryons have finite decay widths, the decay widths and detector resolutions should be considered simultaneously in the modeling

$$M(\Delta m; \Delta m_0, \Gamma) = T(\Delta m; \Delta m_0, \Gamma) \otimes R(\Delta m)$$

$$= \int_{-\infty}^{+\infty} T(\Delta m; \Delta m_0, \Gamma) R(\Delta m - \Delta m') d(\Delta m')$$

- Relativistic Breit-Wigner function is employed for the theoretical model

$$\frac{dN}{dm} \propto \frac{m \cdot \Gamma(m)}{(m_0^2 - m^2)^2 + m_0^2 \cdot \Gamma^2(m)} \quad \text{where} \quad \Gamma(m) = \Gamma_0 \frac{m_0}{m} \left(\frac{q}{q_0} \right)^{2L+1} \frac{F(Rq)}{F(Rq_0)}$$

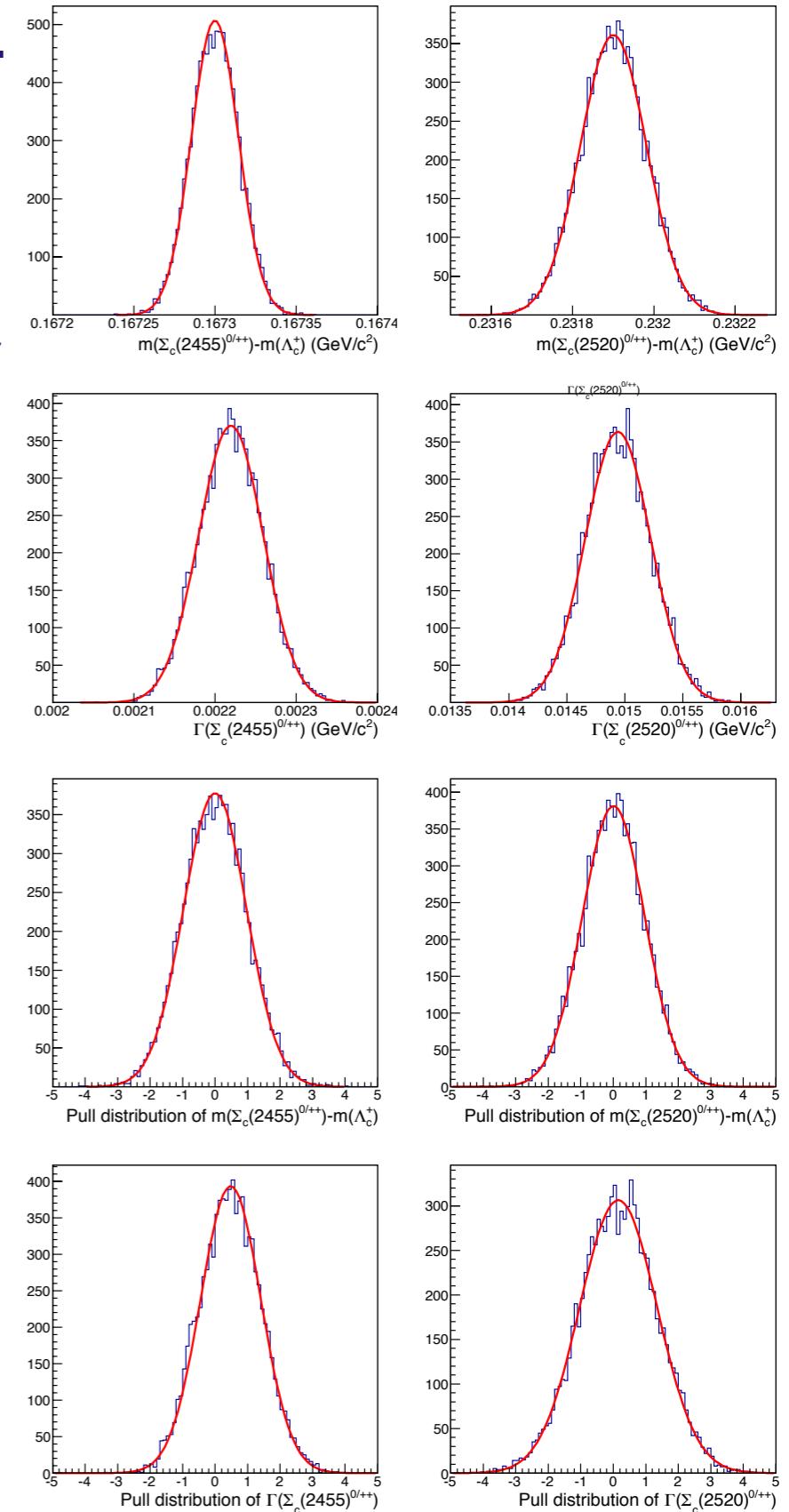
$$q = \frac{\sqrt{(m^2 - (m_{\Lambda_c^+} + m_\pi)^2)(m^2 - (m_{\Lambda_c^+} - m_\pi)^2)}}{2m}$$

- Since $L=0$ for the Σ_c baryons in SM, that is, the width reduces $\Gamma(m) = \Gamma_0 \frac{m_0}{m} \frac{q}{q_0}$ and Blatt-Weisskopf form factor $F(Rq)=1$, therefore,

$$\frac{dN}{dm} \propto \frac{m \cdot \left(\Gamma_0 \frac{m_0}{m} \frac{q}{q_0} \right)}{(m_0^2 - m^2)^2 + m_0^2 \cdot \left(\Gamma_0 \frac{m_0}{m} \frac{q}{q_0} \right)^2}$$

Systematics: Fit Model

- Fit bias from the fitter is estimated with pseudo-experiments
 - Each pseudo-experiment has the same input values obtained from data
 - Only small uncertainties of **0.02** and **0.04 MeV/c²** for $\Sigma_c(2455)^{0/++}$ and $\Sigma_c(2520)^{0/++}$ widths
- Binning effect
 - Fits are tested with various bin size from 0.1 to 1.0 MeV/c²
 - Uncertainties:
 - ▶ For $m(\Sigma_c)-m(\Lambda_c^+)$: negligible
 - ▶ For $\Gamma(\Sigma_c)$: **0.09, 0.06, 0.04, and 0.05 MeV/c²** for $\Sigma_c(2455)^0$, $\Sigma_c(2455)^{++}$, $\Sigma_c(2520)^0$, $\Sigma_c(2520)^{++}$, respectively
- Fit ranges
 - Fits are tested with various fit ranges
 - Uncertainties:
 - ▶ For $m(\Sigma_c)-m(\Lambda_c^+)$: **0.03** and **0.01 MeV/c²** for $\Sigma_c(2520)^0$ and $\Sigma_c(2520)^{++}$
 - ▶ For $\Gamma(\Sigma_c)$: **0.19** and **0.17 MeV/c²** for $\Sigma_c(2520)^0$ and $\Sigma_c(2520)^{++}$



10,000 pseudo-experiments are performed

Systematics: Background Model



- Feed-down correction

$$\frac{\delta \mathcal{N}_{\text{corrected}}(\text{Feed-down})}{\mathcal{N}_{\text{corrected}}(\text{Feed-down})} = \left[\left(\frac{\delta \mathcal{N}_{\text{uncorrected}}(\text{Feed-down})}{\mathcal{N}_{\text{uncorrected}}(\text{Feed-down})} \right)^2 + \left(\frac{\delta \epsilon_{\text{tracking}}(h^\pm)}{\epsilon_{\text{tracking}}(h^\pm)} \right)^2 + \left(\frac{\delta \epsilon_{\text{acceptance}}(h^\pm)}{\epsilon_{\text{acceptance}}(h^\pm)} \right)^2 \right]^{1/2}$$

0.46 %
1.80%
0.17%

- Total uncertainty is estimated to be 1.87% - negligible
- Statistical fluctuation of the random background with fake Λ_c^+
 - Normalizations of fake Λ_c^+ backgrounds by $\pm 1\sigma$
 - No uncertainties are found
- Various modeling of the random background with true Λ_c^+
 - e.g. $c_0(\Delta m - m_\pi)^{1/2} + c_1(\Delta m - m_\pi)^{3/2}$
 $c_0(\Delta m - m_\pi)^{1/2} + c_1(\Delta m - m_\pi)^{3/2} + c_2(\Delta m - m_\pi)^{5/2}$
 - No uncertainties are found