NLO Event Simulation for Chargino Production at the ILC

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Tania Robens
in collaboration with W. Kilian, J. Reuter

RWTH Aachen University

Tools 08, MPI Munich

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Chargino and Neutralino sector: Reconstruction of SUSY parameters

- Charginos $\tilde{\chi}_i^\pm$ and Neutralinos $\tilde{\chi}_i^0$:
  superpositions of gauge and Higgs boson superpartners
- Chargino/ Neutralino sector:

  $$\tan\beta,\ \mu\ \text{(Higgs sector)},\ M_1,\ M_2\text{(soft breaking terms)}$$

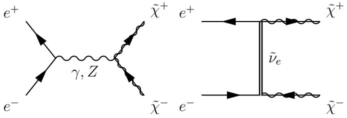
  can be reconstructed from

  $$\text{masses of } \tilde{\chi}_1^\pm,\ \tilde{\chi}_2^\pm,\ \tilde{\chi}_1^0,\ 2\ \sigma \text{ in the } \tilde{\chi}^\pm \text{ sector}$$

  (Choi ea 98, 00, 01)
- low-scale parameters + evolution to high scales (RGEs):
  $\Rightarrow$ hint at SUSY breaking mechanism (Blair ea, 02)
- requires high precision in ew-scale parameter determination

Chargino production at the ILC: experimental accuracy, theoretical NLO contributions

- **ILC**: future e^+e^- collider, $\sqrt{s} = 500\,\mathrm{GeV}\,(1\,\mathrm{TeV})$
- LO chargino production at the ILC:



- decays: typically long decay chains
- combined LHC + ILC accuracy ‰
  (cf. eg. LHC/ ILC study, Weiglein ea)
- Full NLO SUSY corrections for $\sigma(ee \rightarrow \widetilde{\chi}\,\widetilde{\chi})$ at ILC: in the % regime (Fritzsche ea 04, Öller ea 04, 05)

  $\Rightarrow$ include complete NLO contributions in analyses$\Leftarrow$

MC event generators: How do they work ??

- $m \rightarrow n$ particle process: in phase space Γ_n, $3\,n-4$ independent variables x_i define "event"
- determine random values for x_i (within allowed regions), calculate $d\sigma(x_i)$ ("weight" of event)
- for event simulation, calculate relative probability $p_{\rm evt} = \frac{d\sigma(x_i)}{d\sigma_{\rm max}}$ ("unweigthing")
- hit-and-miss technique: take $r \,\epsilon\, [0,1]$, accept event if $p \geq r$
- in practice: more sophisticated adaption methods for most MC generators

"unweighting" requires $p \geq 0$!

(not always fulfilled for NLO calculations, $\mathcal{O}(\alpha)$ corrections $\propto \ln(k_0)$ can be solved by resumming all orders)

NLO cross section contributions

σ_{tot} contributions and dependencies:

- σ_{born}
- virtual $\mathcal{O}(\alpha)$ corrections: $\sigma_{\text{virt}}(\lambda)$
- emission of soft/ hard collinear/ hard non-collinear photons:

  $\sigma_{\text{soft}}(\Delta E_\gamma, \lambda) + \sigma_{\text{hc}}(\Delta E_\gamma,\ \Delta\theta_\gamma) + \sigma_{2\to 3}(\Delta E_\gamma, \Delta\theta_\gamma)$
- higher order initial state radiation: $\sigma_{\text{ISR}} - \sigma_{\text{ISR}}^{\mathcal{O}(\alpha)}(Q)$

  λ: photon mass , ΔE_γ: soft cut , $\Delta\theta_\gamma$: collinear angle

Including FormCalc $\mathcal{O}(\alpha)$ results in WHIZARD (1)

- inclusion in WHIZARD : split photon phase space for real photon into soft/ hard-collinear/ hard non-collinear region:

$$\sigma_{\text{Born}+\gamma} = \sigma_{\text{soft}} + \sigma_{\text{hard, coll}} + \sigma_{\text{hard,noncoll}}$$

- soft photons ($E_\gamma \leq \Delta E_\gamma$): use soft photon approximation, add to virtual contribution ($\Rightarrow$ cancellation of IR divergencies): $\Rightarrow$ integrate over effective matrix element in Γ_2:

$$\sigma_{\text{Born}} + \sigma_{\text{virt}}(\lambda) + \sigma_{\text{soft}}(\Delta\, E_\gamma,\, \lambda) = \int d\Gamma_2\, |\mathcal{M}_{\text{eff}}|^2(\Delta\, E_\gamma)$$

$$|\mathcal{M}_{\text{eff}}|^2(\Delta E_\gamma) = (1 + f_s(\Delta E_\gamma,\, \lambda))\, |\mathcal{M}_{\text{born}}|^2 + 2\, Re(\mathcal{M}_{\text{born}}\, \mathcal{M}^*_{\text{virt}}(\lambda))$$

$\Delta\, E_\gamma$: soft photon cut, λ: photon mass

- in practice: create library from FormCalc code, link this to WHIZARD

Including FormCalc $\mathcal{O}(\alpha)$ results in WHIZARD (2)

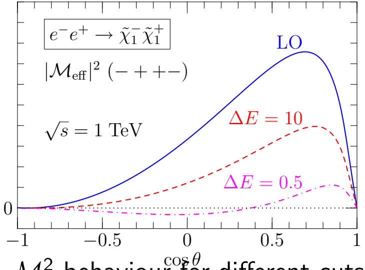
- hard collinear photons: $E_\gamma > \Delta E_\gamma,\ \theta_\gamma \leq \Delta \theta_\gamma$
  use hard collinear approximation (Dittmaier ea, 1993):

$$\sigma_{\text{hard, coll}} = \int_{\text{hard, coll}} d\Gamma_3 |\mathcal{M}_{2\to 3}|^2$$
$$\longrightarrow \int d\Gamma_2 \int_0^{x_0} dx_i\, f_\pm(x_i)\, |\mathcal{M}^{(\pm)}_{\text{Born}}|^2(x_i, s),$$

  x_i: energy fraction of incoming fermion after photon radiation
  integrate in Γ_2
- hard, non-collinear photons: calculated exactly using $\mathcal{M}_{(2\to 3)}$
  generated by separate WHIZARD run using Γ_3

Fixed order method: Result and Drawback

- corresponds to analytic results (Fritzsche ea/ Öller ea)
- Drawback: $|\mathcal{M}_{\text{eff}}|^2 < 0$ for small values of $\frac{\Delta E_\gamma}{\sqrt{s}}$
- well-known problem at LEP
- ad hoc solution: set $|\mathcal{M}_{\text{eff}}|^2 = 0$ for these cases
- too low energy cuts: $\mathcal{O}(\alpha)$ not sufficient, leads to "wrong" σ_{tot}



$\mathcal{M}^2$ behaviour for different cuts

θ: angle between e^- and $\tilde{\chi}^-$

remark: **event generator specific problem** ($\sigma_{\text{tot}} \geq 0$)

Resumming leading logs to all orders

solution to fixed order drawback:
$\Rightarrow$ resumm respective contributions to all orders $\Leftarrow$

- in practice: subtract $\mathcal{O}(\alpha)$ soft + virtual collinear contributions in $\mathcal{M}_{\text{eff}}$:

$$\begin{aligned} |\widetilde{\mathcal{M}}_{\text{eff}}|^2 &= (1 + f_s(\Delta E_\gamma))\,|\mathcal{M}_{\text{born}}|^2 + 2\,Re(\mathcal{M}_{\text{born}}\,\mathcal{M}^*_{\text{virt}}) \\ &- 2\,f_s^{ISR,\mathcal{O}(\alpha)}(\Delta E_\gamma)\,|\mathcal{M}_{\text{born}}|^2 \end{aligned}$$

- add the resummed contribution by folding with ISR structure function:

$$\int d\Gamma \int_0^1 dx_1 \int_0^1 dx_2\, f^{\text{ISR}}(x_1)\, f^{\text{ISR}}(x_2)\, |\widetilde{\mathcal{M}}_{\text{eff}}|^2(s, x_i)$$

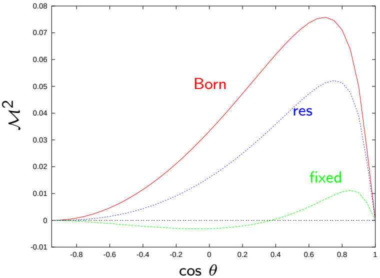
- $f^{\text{ISR}}(x)$: Initial state radiation (Jadach, Skrzypek, Z.Phys. 1991), describes collinear (real + virtual) photons in leading log accuracy
- $f_s^{\text{ISR},\mathcal{O}(\alpha)}$: soft integrated $\mathcal{O}(\alpha)$ contribution

Resumming: What do we get ??

- $\mathcal{O}(\alpha)$: equivalent to fixed order method

$\Rightarrow$ got rid of $|\mathcal{M}|^2 < 0$ effects !!

no negative weights



(-++-),

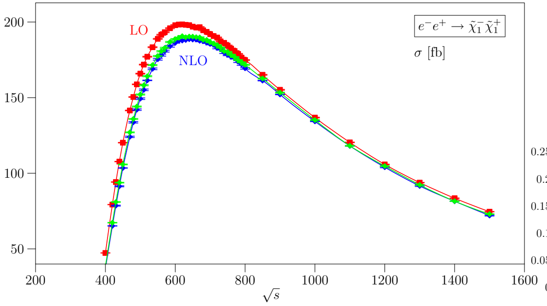
$\Delta E_\gamma = 0.5$ GeV

- higher orders:
  higher order ISR for $|\mathcal{M}_{\text{born}}|^2$ as well as Re $(\mathcal{M}_{\text{born}}\,\mathcal{M}^*_{\text{virt}})$!!!

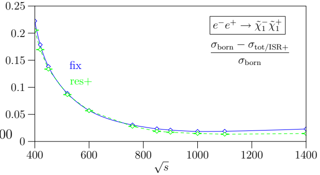
$\Rightarrow$ new higher order effects $\Leftarrow$

- additional possibility: also fold $2 \rightarrow 3$ process with ISR ("res+")

Results: cross sections



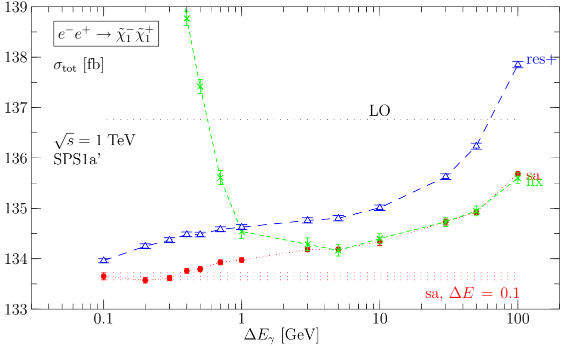
$\sigma(\sqrt{s})$



relative corrections

agrees with results in the literature (Fritzsche ea, Öller ea)

A closer look: ΔE_γ dependence of σ_{tot}



$\sigma_{tot}(\Delta E_\gamma)$:

semianalytic tests soft photon approx shift: 2 - 5 ‰ ($\Delta E_\gamma \leq 10\,\mathrm{GeV}$)

fixed order $|\mathcal{M}|^2 \leq 0$ effects for $\Delta E_\gamma \leq 3\,\mathrm{GeV}$

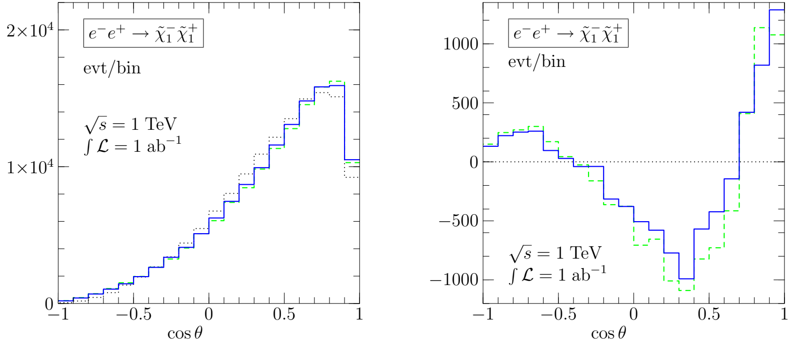
resummation includes higher order effects, 5‰ difference to 'sa' for $\Delta E_\gamma \leq 10\,\mathrm{GeV}$

In summary:

shift in ΔE_γ leads to ‰ effects, match ILC accuracy

$\Rightarrow$ careful choice of ΔE_γ, method important

simulation results: angular distributions



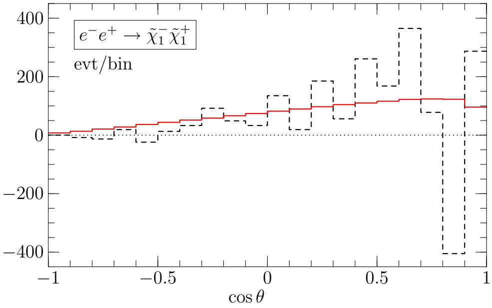
N_{tot} $\quad$ $N_i - N_{\text{Born}}$

Born, fixed order, resummation

!! more than 1 σ deviation !! $\sqrt{n_{\text{max}}} \approx \mathcal{O}(10^2)$; nbins = 20

...also $\mathcal{O}(\alpha^2)$ and higher orders are important...

Angular distributions: higher orders



$N_{\mathrm{res},+} - N_{\mathrm{fix}}$
red: 1 standard dev from Born result

N^+_{res}: resummation, additionaly $2 \to 3$ folded w ISR; most complete

also higher order contributions statistically significant

Treatment of narrow width particles (LO)

(Work in progress)

Treatment of narrow width particles (1)

- consider process with
  - many particles in final state,
  - very narrow width particle in intermediate states
- popular example: processes involving $\tau\tau$ production and decay

$$e^+e^- \longrightarrow X \longrightarrow e^-\mu^+\bar{\nu}_e\nu_\mu\nu_\tau\bar{\nu}_\tau\ldots$$

- hard to find resonances in phase space generation

$\Rightarrow$ implies use of specialized routines, as eg Tauola

Treatment of narrow width particles (2)

- for WHIZARD : "offline" decay program exists
- handles hepevt standard input files, lets particles decay
- output: also hepevt standard files
- used so far for τ decays, weak c decays (parton level)
- in principle useable for any narrow width particle (eg SUSY scenarios with very narow $\widetilde{\chi}$s)
- at the moment: personalized version, but ready to be handed over for external use
- so if you are interested let me know
- of course: last words not said, full integration into WHIZARD in process

Very short Summary

- WHIZARD can (in principle) handle NLO pairproduction for e^+e^- colliders at event generation level
- interface to FormCalc generated matrix elements
- soft photons resummed to all orders
- done this for $\widetilde{\chi}_1^+\widetilde{\chi}_1^-$ production
- routine for treatment of narrow width particles exists, bound to be included

! Thanks for listening !

Long Summary and Outlook

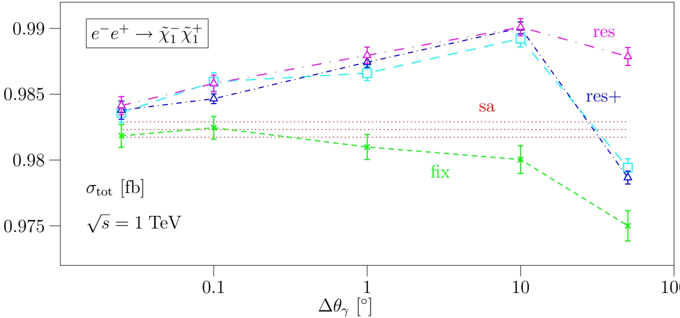
- Chargino/ neutralino sector of MSSM: high precision in SUSY paramater analysis at EW scale (‰ at ILC)
- same size/ larger NLO corrections

$\Rightarrow$ include NLO results in Monte Carlo Event generators

- resummation method for photons allows lower soft cuts/ inclusion of higher order contributions
- NLO as well as higher order contributions significant !!
- next steps: include NLO corrections to $\tilde{\chi}$ decays, non-factorizing contributions (start with photonic corrections in the double-pole approximation)
- general interface to FormCalc generated matrix elements: extendable to other processes...

cut dependencies: $\Delta\theta_\gamma$

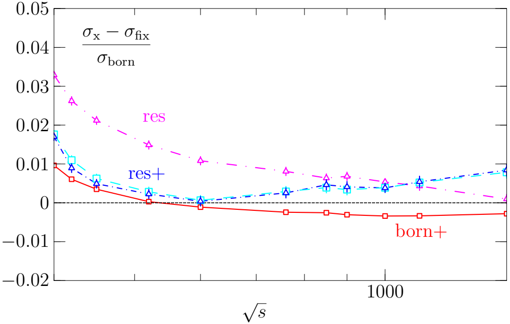
tests: collinear photon approximation



$\frac{\sigma_i}{\sigma \text{ Born}}$

σ_{tot} again larger for resummation method
for higher angles: second order ISR effects between 0.05^o and 0.1^o ($\mathcal{O}(‰)$)

$\sqrt{s}$ dependence of different higher order contributions



relative difference:

$$\frac{\sigma_X - \sigma_{fix}}{\sigma_{Born}}$$

Born+: only Born folded w ISR (standard way in the literature),
fully resummed result: subtraction, also fold $2 \to 3$ part with ISR
difference between Born+ and fully resummed result: multiple photon emission from interaction term

Interlude:
Treatment of higher order contributions

Higher order photonic contributions (1)

Methods so far

- σ_{fix}: no higher order contributions
- σ_{res}: fold both σ_{born} and σ_{virt} with ISR structure functions

consider $\mathcal{O}(\alpha^2)$ contributions:

* two photon emission from the Born ($|\mathcal{M}_{\text{born}}|^2$) contribution
* additional collinear photon emission from interference term ($2\,Re(\mathcal{M}_{\text{born}}\,\mathcal{M}^*_{\text{virt}})$)
* naive guess: first photon given by ISR-description
* more detailed investigation: exact up to $\mathcal{O}\left((f_{\text{ISR}} - f_{\text{soft}})^2\right)$ for emission of real photons
* similar for real-virtual and virtual-virtual combinations

Higher order photonic contributions (2)

Additional possibilities (1)

- $\sigma_{\text{born}+}$: fold σ_{born} w ISR structure functions:

$$\begin{aligned}\sigma_{\text{born}+}(s) &= \int dx_1 \int dx_2\, f^{\text{ISR}}(x_1)\, f^{\text{ISR}}(x_2)\sigma_{\text{born}}(x_i, s) \\ &+ \sigma_{\text{virt}}(s) + \sigma_{\text{real}} - \sigma_{\text{ISR},\mathcal{O}(\alpha)}\end{aligned}$$

* up to $\mathcal{O}(\alpha)$: same as fixed order result
* emission of additional photons: described according to ISR

  $\Rightarrow$ considers only dominant contributions $\propto (\alpha \log(\theta))^n$
* no additional photon emission from interference term
* standard method in the literature

Higher order photonic contributions (3)

Additional possibilities (2)

- $\sigma_{\text{res},+}$: σ_{res}, also fold real $\sigma_{2\to3}$ w ISR:

$$\sigma_{2\to3}(s) \to \int dx_1 \int dx_2\, f^{\text{ISR}}(x_1)\, f^{\text{ISR}}(x_2) \sigma_{2\to3}(x_i, s)$$

* again: Born and interference combined with higher order radiation given by ISR ("quasi"-exact)
* now: also combine second and higher order collinear photon(s) with a hard, non-collinear last photon
* ⇒ all tree-level and first order processes combined with higher order photon emission

⇒ most complete description ⇐

η, f_s, hard collinear approximation, $ISR^{\mathcal{O}(\alpha)}$

- $\eta = \frac{2\alpha}{\pi}\left(\log\left(\frac{Q^2}{m_e^2}\right) - 1\right)$ (Q = scale of process)
- $$f_s = -\frac{\alpha}{2\pi} \sum_{i,j=e^\pm} \int_{|\mathbf{k}| \leq \mathbf{\Delta E}} \frac{d^3k}{2\omega_k} \frac{(\pm)\, p_i\, p_j\, Q_i\, Q_j}{p_i\, k\, p_j\, k},$$

  (Denner 1992)

  $\omega_k = \sqrt{\mathbf{k^2} + \lambda^2}$, p_i initial/ final state momenta, k: γ momentum
- hard collinear factor ($\pm$ helicity conserving/ flipping):

  $$f^+(x) = \frac{\alpha}{2\,\pi} \frac{1+x^2}{(1-x)} \left(\ln\left(\frac{s\,(\Delta\theta)^2}{4\,m^2}\right) - 1\right), f^-(x) = \frac{\alpha}{2\,\pi} x.$$

  (Dittmaier 1993)
- $$f_s^{ISR,\,\mathcal{O}(\alpha)} = \left[\int_{x_0}^1 f_{ISR}(x)\, dx\right]_{\mathcal{O}(\alpha)} = \frac{\eta}{4}\left(2\ln(1-x_0) + x_0 + \frac{1}{2}x_0^2\right)$$

Resummation: general idea

k_0 problem for MC Generators at LEP

- Event generators: split photon phase space into soft ($E_\gamma \leq k_0$) and hard ($E_\gamma > k_0$) regions (with different kinematics)
- dominant behaviour in soft region:

$$\sigma_{\text{Born}} + \sigma_{\text{virt}} + \sigma_{\text{soft}}(k_0) \sim (1 + \beta \ln(k_0))\, \sigma_{\text{Born}}$$

problem for $k_0 \to 0$

- way out: include dominant multiphoton emission to all orders

$$(1 + \beta \ln(k_0)) \to \exp[\beta \ln(k_0)] = (k_0)^\beta > 0$$

- $\beta \propto \alpha \ln\left(\frac{s}{m_e^2}\right)$

ISR in its full beauty (Skrzypek ea, 91)

$$\Gamma_{ee}^{LL}(x, Q^2) = \frac{\exp(-\frac{1}{2}\eta\gamma_E + \frac{3}{8}\eta)}{\Gamma(1+\frac{\eta}{2})} \frac{\eta}{2} (1-x)^{(\frac{\eta}{2}-1)}$$

$$- \frac{\eta}{4}(1+x) + \frac{\eta^2}{16}\Bigg(-2(1-x)\log(1-x) - \frac{2\log x}{1-x} + \frac{3}{2}(1+x)\log x - \frac{x}{2}$$

$$- \frac{5}{2}\Bigg) + \left(\frac{\eta}{2}\right)^3 \Bigg[-\frac{1}{2}(1+x)\Bigg(\frac{9}{32} - \frac{\pi^2}{12} + \frac{3}{4}\log(1-x) + \frac{1}{2}\log^2(1-x)$$

$$- \frac{1}{4}\log x \log(1-x) + \frac{1}{16}\log^2 x - \frac{1}{4}\mathrm{Li}_2(1-x)\Bigg)$$

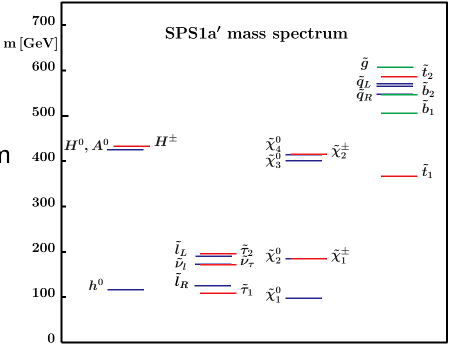
$$+ \frac{1}{2}\frac{1+x^2}{1-x}\left(-\frac{3}{8}\log x + \frac{1}{12}\log^2 x - \frac{1}{2}\log x \log(1-x)\right)$$

$$- \frac{1}{4}(1-x)\left(\log(1-x) + \frac{1}{4}\right) + \frac{1}{32}(5-3x)\log x\Bigg] \; ; \; \eta = \frac{2\,\alpha}{\pi}\left(\log\left(\frac{Q^2}{m_e^2}\right) - 1\right)$$

Point SPS1a'

- mSUGRA scenario
- according to Snowmass Points (Allanach ea, 02), in agreement with cosmology data/ WMAP ($\tilde{\chi}_1^0$ as DM candidate)

spectrum $\Longrightarrow$



light sleptons
heavy squarks
some light $\tilde{\chi}$s
all masses < 1 TeV