FeynHiggs
The Swiss Army Knife for MSSM Higgs Physics*

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based on collaboration with
T. Hahn, W. Hollik, H. Rzehak, G. Weiglein

1. Introduction
2. FeynHiggs features
3. Recent changes and additions
4. How to run FeynHiggs
5. On-line demonstration
6. Conclusions

* thanks to Pietro Slavich

1. Introduction

The LHC is coming . . .
first collisions by the end of this year?

The ILC is still coming . . .
. . . a bit later than anticipated

⇒ New Physics is certainly around the corner

⇒ Time to get ready

The big question:

Which Lagrangian describes the world?

My guess:

It is a supersymmetric one

$\Rightarrow$ concentrate on the MSSM from now on

(other people $\Rightarrow$ other guesses $\Rightarrow$ other priorities . . .)

In any case:

$\Rightarrow$ we have to measure as many observables as possible

- masses
- branching ratios
- angular distributions
- cross sections
- . . .

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In any case:

$\Rightarrow$ we have to measure as many observables as possible

– masses
– branching ratios
– angular distributions
– cross sections
– . . .

$\Rightarrow$ compare with theory calculations at the same level of accuracy

The Minimal Supersymmetric Standard Model (MSSM)

Superpartners for Standard Model particles

$$\left[u, d, c, s, t, b\right]_{L,R} \quad \left[e, \mu, \tau\right]_{L,R} \quad \left[\nu_{e,\mu,\tau}\right]_{L} \qquad \text{Spin } \tfrac{1}{2}$$

$$\left[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}\right]_{L,R} \quad \left[\tilde{e}, \tilde{\mu}, \tilde{\tau}\right]_{L,R} \quad \left[\tilde{\nu}_{e,\mu,\tau}\right]_{L} \qquad \text{Spin } 0$$

$$g \quad \underbrace{W^{\pm}, H^{\pm}} \quad \underbrace{\gamma, Z, H_1^0, H_2^0} \qquad \text{Spin 1 / Spin 0}$$

$$\tilde{g} \quad \tilde{\chi}^{\pm}_{1,2} \quad \tilde{\chi}^{0}_{1,2,3,4} \qquad \text{Spin } \frac{1}{2}$$

Enlarged Higgs sector: Two Higgs doublets $\Leftarrow$ focus here!

Problem in the MSSM: many scales

Problem in the MSSM: complex phases

Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) + \underbrace{\frac{g'^2 + g^2}{8}}(H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM

physical states: $h^0, H^0, A^0, H^\pm$

Goldstone bosons: $G^0, G^\pm$

Input parameters: (to be determined experimentally)

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2 (\tan\beta + \cot\beta)$$

Enlarged Higgs sector: Two Higgs doublets with $\mathcal{CP}$ violation

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix} e^{i\xi}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}}(H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM

physical states: $h^0, H^0, A^0, H^\pm$

2 $\mathcal{CP}$-violating phases: ξ, arg(m_{12}) $\Rightarrow$ can be set/rotated to zero

Input parameters: (to be determined experimentally)

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_{H^\pm}^2$$

Effects of complex parameters in the Higgs sector:

Complex parameters enter via loop corrections:

– μ : Higgsino mass parameter

– $A_{t,b,\tau}$: trilinear couplings $\Rightarrow$ $X_{t,b,\tau} = A_{t,b} - \mu^* \{\cot\beta, \tan\beta\}$ complex

– $M_{1,2}$: gaugino mass parameter (one phase can be eliminated)

– $m_{\tilde{g}}$: gluino mass

$\Rightarrow$ can induce $\mathcal{CP}$-violating effects

Result:

$$(A, H, h) \rightarrow (h_3, h_2, h_1)$$

with

$$M_{h_3} > M_{h_2} > M_{h_1}$$

Propagator/Mass matrix at tree-level:

$$\begin{pmatrix} q^2 - m_A^2 & 0 & 0 \\ 0 & q^2 - m_H^2 & 0 \\ 0 & 0 & q^2 - m_h^2 \end{pmatrix}$$

Propagator / mass matrix with higher-order corrections
($\rightarrow$ Feynman-diagrammatic approach):

$$M_{hHA}^2(q^2) = \begin{pmatrix} q^2 - m_A^2 + \hat{\Sigma}_{AA}(q^2) & \hat{\Sigma}_{AH}(q^2) & \hat{\Sigma}_{Ah}(q^2) \\ \hat{\Sigma}_{HA}(q^2) & q^2 - m_H^2 + \hat{\Sigma}_{HH}(q^2) & \hat{\Sigma}_{Hh}(q^2) \\ \hat{\Sigma}_{hA}(q^2) & \hat{\Sigma}_{hH}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_{hh}(q^2) \end{pmatrix}$$

$\hat{\Sigma}_{ij}(q^2)$ $(i, j = h, H, A)$: renormalized Higgs self-energies

$\hat{\Sigma}_{Ah}, \hat{\Sigma}_{AH} \neq 0 \;\Rightarrow\; \mathcal{CP}$V, $\mathcal{CP}$-even and $\mathcal{CP}$-odd fields can mix

$\Rightarrow$ complex roots of $\det(M_{hHA}^2(q^2))$: $\mathcal{M}_{h_i}^2 (i = 1, 2, 3)$: $\mathcal{M}^2 = M^2 - iM\Gamma$

2. FeynHiggs features

Latest version: FeynHiggs 2.6.4 (07/08)

version FeynHiggs 2.6.5 to be released next month . . . ?

FeynHiggs compiles on all modern platforms

$\longrightarrow$ start compilation

FeynHiggs provides:

- Higgs boson masses
- Higgs boson couplings
- Higgs boson decay widths
- Higgs boson production cross sections ($4\,\pi$)
- evaluation of other observables to test the validity of parameters
- all this for the neutral and charged Higgs bosons
- all this in the MSSM with real or complex parameters

Included in FeynHiggs 2.6 (I):

Evaluation of all Higgs boson masses and mixing angles

- $M_{h_1}, M_{h_2}, M_{h_3}, M_{H^\pm}$, $\alpha_{\rm eff}$, $\mathbf{Z}_{ij}$, $\mathbf{U}_{ij}$, $\mathbf{R}_{ij}$, ...

Evaluation of all neutral Higgs boson decay channels $\Leftarrow$ with $\mathbf{Z}$

- total decay width $\Gamma_{\rm tot}$
- $\mathrm{BR}(h_i \to f\bar{f})$: decay to SM fermions
- $\mathrm{BR}(h_i \to \gamma\gamma, ZZ^{(*)}, WW^{(*)}, gg)$: decay to SM gauge bosons
- $\mathrm{BR}(h_i \to h_j Z^{(*)}, h_j h_k)$: decay to gauge and Higgs bosons
- $\mathrm{BR}(h_i \to \tilde{f}_i \tilde{f}_j)$: decay to sfermions
- $\mathrm{BR}(h_i \to \tilde{\chi}^\pm_i \tilde{\chi}^\pm_j, \tilde{\chi}^0_i \tilde{\chi}^0_j)$: decay to charginos, neutralinos

Evaluation for the SM Higgs (same masses as the three MSSM Higgses)

- total decay width $\Gamma^{\rm SM}_{\rm tot}$
- $\mathrm{BR}(h_i^{\rm SM} \to f\bar{f})$: decay to SM fermions
- $\mathrm{BR}(h_i^{\rm SM} \to \gamma\gamma, ZZ^{(*)}, WW^{(*)}, gg)$: decay to SM gauge bosons

Included in FeynHiggs 2.6 (II):

Evaluation of all neutral Higgs boson production cross sections at Tevatron/LHC $\Leftarrow$ with **Z**

SM: (more or less) up-to-date, MSSM: additional effective couplings

- $gg \to h_i$: gluon fusion
- $WW \to h_i$, $ZZ \to h_i$: gauge boson fusion
- $W \to Wh_i$, $Z \to Zh_i$: Higgs strahlung
- $b\bar{b} \to b\bar{b}h_i$: bottom Yukawa process
- $b\bar{b} \to b\bar{b}h_i$, $h_i \to b\bar{b}$, one b tagged
- $t\bar{t} \to t\bar{t}h_i$: top Yukawa process
- $\tilde{t}\bar{\tilde{t}} \to \tilde{t}\bar{\tilde{t}}h_i$: stop Yukawa process

Evaluation for the SM Higgs (same masses as the three MSSM Higgses)

- all SM channels as above

Included in FeynHiggs 2.6 (III):

Evaluations for the charged Higgs boson (rMSSM/cMSSM)

- total decay width $\Gamma_{\rm tot}$
- $\mathrm{BR}(H^+ \to f^{(*)} \bar{f}')$: decay to SM fermions
- $\mathrm{BR}(H^+ \to h_i W^{+(*)})$: decay to gauge and Higgs bosons
- $\mathrm{BR}(H^+ \to \tilde{f}_i \tilde{f}'_j)$: decay to sfermions
- $\mathrm{BR}(H^+ \to \tilde{\chi}^0_i \tilde{\chi}^+_j)$: decay to charginos and neutralinos
- H^+ production cross sections at the LHC
- $\mathrm{BR}(t \to H^+ \bar{b})$ for $M_{H^\pm} \le m_t$ ($H^\pm$ production)

Evaluation of additional couplings: $\Leftarrow$ with $\mathbf{U}$ or $\mathbf{R}$

- $g(V \to V h_i, h_i h_j)$: coupling of gauge and Higgs bosons
- $g(h_i h_j h_k)$: all Higgs self couplings (including charged Higgs)

Included in FeynHiggs 2.6 (IV):

Evaluation of theory error on masses and mixing

$\rightarrow$ estimate of uncertainty in $M_{h_i}, \mathbf{U}_{ij}, \mathbf{Z}_{ij}$ from unknown higher-order corr.

Evaluation of masses, mixing and decay in the NMFV MSSM

NMFV: Non Minimal Flavor Violation *[Hahn, S.H., Hollik, Merz, Peñaranda '04-'08]*

$\Rightarrow$ Connection to Flavor physics

Evaluation of additional constraints (rMSSM/cMSSM)

- ρ-parameter: $\Delta\rho^{\text{SUSY}}$ at $\mathcal{O}(\alpha)$, $\mathcal{O}(\alpha\alpha_s)$, ..., including NMFV effects
  $\Rightarrow M_W$, $\sin^2\theta_{\text{eff}}$ via SM formula + $\Delta\rho^{\text{SUSY}}$, including NMFV effects
- anomalous magnetic moment of the μ: $(g-2)_\mu$
- BR($b \to s\gamma$) , including NMFV effects *[T. Hahn, W. Hollik, J. Illana, S. Peñaranda '06]*
- LEP Higgs constraints (preliminary) $\rightarrow$ link to HiggsBounds
- EDMs of electron, neutron, Hg, . . .

Included in FeynHiggs 2.6 (V):

Predefined scenarios:

- the "normal" Higgs benchmark scenarios:
  m_h^{max}, no-mixing, small α_{eff}, gluophobic Higgs

- the "normal" SUSY benchmarks: SPS 1–9

- the new CDM benchmarks:
  M_A–$\tan\beta$ planes that are in agreement with Cold Dark Matter
  (and to a large extend also with all other EWPO and BPO)
  [*J. Ellis, T. Hahn, S.H., K. Olive, A.M. Weber, G. Weiglein '07*]

- user-defined parameter planes

New M_A–$\tan\beta$ planes:

Data accessed within FeynHiggs in terms of tables
with a grid for M_A and $\tan\beta$

MT	MSUSY	MA0	TB	AT	MUE	...
171.4	500	200	5	1000	761	...
171.4	500	210	5	1000	753	...
⋮	⋮	⋮	⋮	⋮	⋮	⋮
171.4	500	200	6	1000	742	...
171.4	500	210	6	1000	735	...
⋮	⋮	⋮	⋮	⋮	⋮	⋮

FeynHiggs interpolates between the four NWSE points in M_A and $\tan\beta$
FeynHiggs gives an error if $\{M_A, \tan\beta\}$ combination is not allowed

4 M_A–$\tan\beta$ planes can be downloaded from `www.feynhiggs.de`
Definition of new planes by the user is possible (respect table format)

Planned to be included in FeynHiggs:

- ILC production cross sections
- γC production cross sections
- full one-loop corrections to all (remaining) Higgs decays
- flavor violating Higgs decays
- decay of sfermions to Higgs bosons (full one-loop)
- . . .

If you need something, just let us know!

3. Recent changes and additions

- inclusion of the $\mathcal{O}(\alpha_t \alpha_s)$ corrections in the cMSSM
- equivalently: evaluation of $\mathcal{O}(\alpha_t \alpha_s)$ corrections to $M_{H^\pm}$
- Higgs boson mass determination:
  complex roots of $\det(M^2_{hHA}(q^2))$: $\mathcal{M}^2_{h_i} (i = 1, 2, 3)$: $\mathcal{M}^2 = M^2 - iM\Gamma$
  $\text{Im}\,\hat{\Sigma}$ included consistently in mass and coupling evaluation
- $\Rightarrow$ complex 3×3 mixing matrix Z $\Rightarrow$ external (on-shell) Higgs bosons

  unitary 3×3 mixing matrix U or R $\Rightarrow$ Higgs bosons in loops

  $\Rightarrow$ included in all Higgs production and decay
- inclusion of full one-loop NMFV effects (also for BR($b \to s\gamma$))
- Implementation of new M_A–$\tan\beta$ planes in agreement with CDM
- EDMs of electron, neutron, Hg, . . .

Comparison with other codes/calculations:

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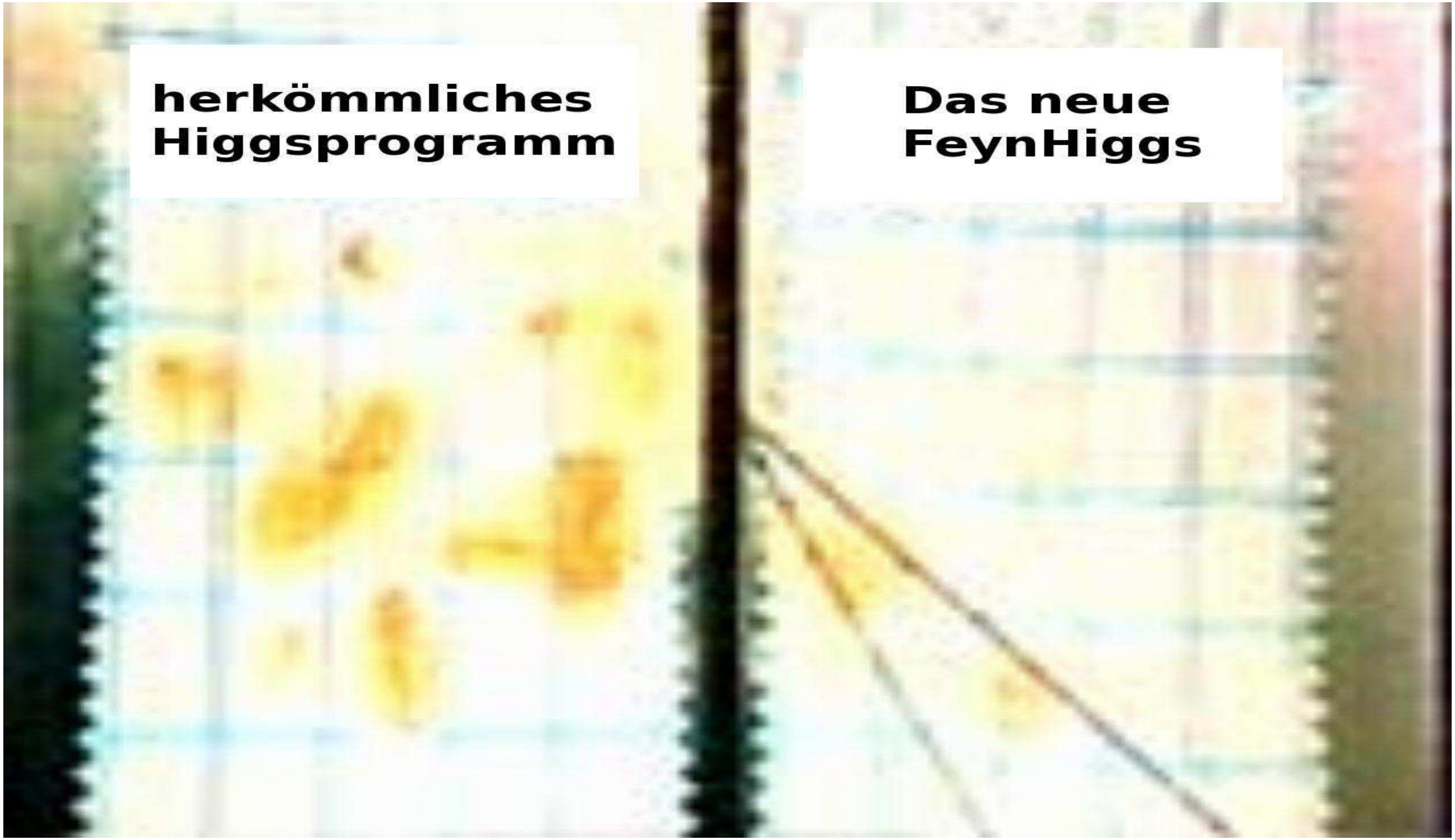


Comparison with other codes/calculations:

FeynHiggs is the only code that has

- estimate of missing higher-order corrections
- evaluation of $\Gamma(h_i \to \ldots)$ with external Higgs, bosons on-shell i.e. evaluated with $\mathbf{Z}$
- evaluation of $\mathrm{BR}(h_i \to \ldots)$ with external Higgs, bosons on-shell i.e. evaluated with $\mathbf{Z}$
- evaluation of $\sigma_{\text{Tev,LHC}}(\ldots \to h_i + X)$ with external Higgs bosons on-shell, i.e. evaluated with $\mathbf{Z}$
- evaluation of effective couplings with $\mathbf{U}$ or $\mathbf{R}$
- $\mathrm{Im}\,\hat{\Sigma}$ included consistently in mass and coupling evaluation

Other codes/calculations:

- rely on evaluation of Γ, BR with $\mathbf{R}$ (possibly with $\mathbf{U}$)
- effective potential approach corresponds to $\mathbf{R}$

$\Rightarrow$ see numerical examples (in the back-up slides) for size of effects

4. How to run FeynHiggs

1. Go to www.feynhiggs.de
2. Download the latest version
3. type `./configure`, `make`, `make install`
   $\Rightarrow$ library `libFH.a` is created
4. 4 possible ways to use *FeynHiggs*:

   A) Command-line mode (allows also running on the GRID)

   B) called from a Fortran/C++ code

   C) called within Mathematica

   D) WWW mode

   processing of Les Houches Accord data possible
5. Detailed instructions and help are provided in the man pages

A) Command-line mode

Input File

```
MT           172.7
MB           4.7
MW           80.4
MZ           91.1
MSusy        975
MA0          200
Abs(M_2)     332
Abs(MUE)     980
TB           50
Abs(At)      -300
Abs(Ab)      1500
Abs(M_3)     975
```

Command

`FeynHiggs` *file* [*flags*]

Screen Output

```
------------------- HIGGS MASSES -------------------
| Mh0     =   116.022817
| MHH     =   199.943497
| MA0     =   200.000000
| MHp     =   216.973920
| SAeff   =  -0.02685112
| UHiggs  =   0.99999346   -0.00361740    0.00000000 \
|             0.00361740    0.99999346    0.00000000 \
|             0.00000000    0.00000000    1.00000000

-------------- ESTIMATED UNCERTAINTIES -------------
| DeltaMh0   =  1.591957
| DeltaMHH   =  0.004428
| DeltaMA0   =  0.000000
| DeltaMHp   =  0.152519
 ...
```

- Loops over parameter values possible (parameter scans).
- Mask off details with `FeynHiggs` *file [flags]* `| grep -v %`
- `table` utility converts to machine-readable format, e.g.
  `FeynHiggs` *file [flags]* `| table TB Mh0 > outfile`

Example for new M_A–$\tan\beta$ planes:

Input File ("normal")

```
MT          172.7
MB          4.7
MW          80.4
MZ          91.1
MSusy       975
MA0         200
Abs(M_2)    332
Abs(MUE)    980
TB          50
Abs(At)     -300
Abs(Ab)     1500
Abs(M_3)    975
```

Input File ("new")

```
MA0         227
TB          23
table ehoww.A2.dat MA0 TB
```

- Loops over parameter values possible (parameter scans).
  ```
  MA0         200 500 10
  TB          5 50 *2
  ```
- Mask off details with `FeynHiggs` *file [flags]* `| grep -v %`
- `table` utility converts to machine-readable format, e.g.
  `FeynHiggs` *file [flags]* `| table TB Mh0 > outfile`

SUSY Les Houches Accord(2) Format

Input File

```
BLOCK MODSEL
    1    1
BLOCK MINPAR
    1   0.10000E+03   # m0
    2   0.25000E+03   # m12
    3   0.10000E+02   # tanb
    4   0.10000E+01   # sgn mu
    5  -0.10000E+03   # A
BLOCK SMINPUTS
    4   0.91187E+02   # MZ
    5   0.42500E+01   # mb(mb)
    6   0.17500E+03   # t
  ...
```

Command

`FeynHiggs` *file* [*flags*] →

file.fh

```
BLOCK MASS
   25   1.12697840E+02   # Mh0
   35   4.00145460E+02   # MHH
   36   3.99769788E+02   # MA0
   37   4.08050556E+02   # MHp
  ...
BLOCK ALPHA
       -1.10658125E-01   # Alpha
  ...
```

- { Uses / was developed into } the SLHA(2) I/O Library. [*T. Hahn '04, '06*]
- SLHA(2) can also be used in Library Mode with `FHSetSLHA`.
- *FeynHiggs* tries to read each file in SLHA(2) format first.
  If that fails, fallback to native format.

B) Called from a Fortran/C++ code

Link *FeynHiggs* as a subroutine ⇒ link `libFH.a`

`call FHSetFlags(...)` :
→ specification of accuracy etc.

`call FHSetPara(...)` :
→ specify input parameters

`call FHGetPara(...)` :
→ obtain derived parameters

`call FHHiggsCorr(...)` :
→ obtain Higgs boson masses and mixings

`call FHUncertainties(...)` :
→ obtain theory error on Higgs boson masses and mixings from unknown higher-order corrections

`call FHCouplings(...), FHHiggsProds(...), ...` :
→ obtain decay widths, BRs, XSs, etc.

C) Called within Mathematica

- install the math link to *MFeynHiggs* , e.g.:

  `Install[''MFeynHiggs'']`

- `FHSetFlags[...]` :
  → specification of accuracy etc.

  `FHSetPara[...]` :
  → specify input parameters

  `FHGetPara[]` :
  → obtain derived parameters

  `FHHiggsCorr[]` :
  → obtain Higgs boson masses and mixings

  `FHUncertainties[]` :
  → obtain theory error on Higgs boson masses and mixings from unknown higher-order corrections

  `FHCouplings[], FHHiggsProds[], ...` :
  → obtain decay widths, BRs etc.

D) WWW mode

1. The FeynHiggs User Control Center is available at
   www.feynhiggs.de/fhucc

2. Enter you parameters on-line in the web page

3. Obtain your results with a mouse click

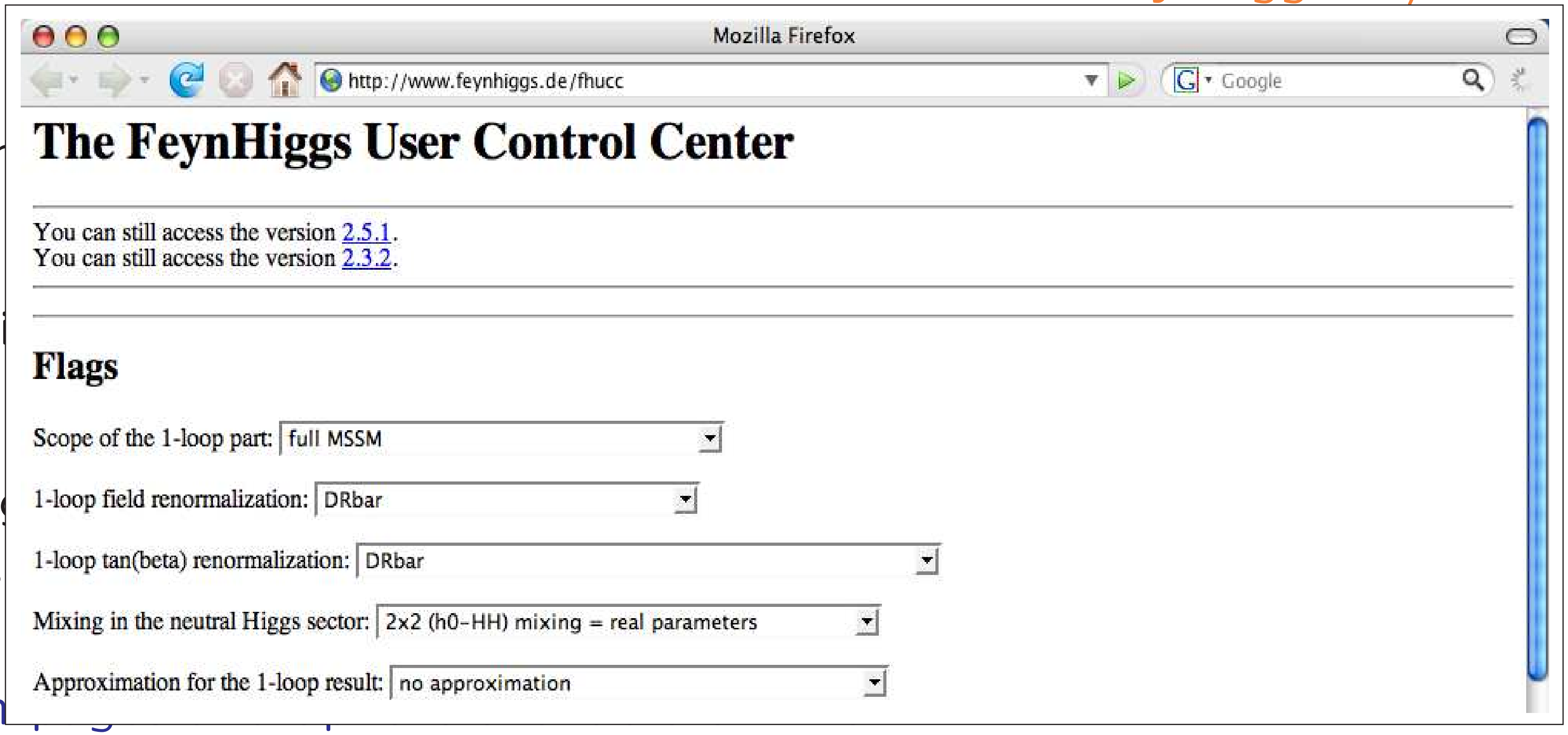
⇒ for single points and checks of your downloaded version of FeynHiggs
⇒ always the latest version

Also man pages and api are available on-line

D) WWW mode

1. The FeynHiggs User Control Center is available at

www.feynhiggs.de/fhucc



2. Enter

3. Obtai

$\Rightarrow$ for sin

$\Rightarrow$ always

Also man

5. On-line demonstration

6. Conclusinos

- Precise MSSM Higgs sector evaluation necessary to
  - do phenomenological analyses at the Tevatron and the LHC
  - exploit anticipated ILC precision, be sensitive to small deviations

- FeynHiggs 2.6 provides Higgs boson masses, mixing angles, couplings, branching ratios, Tev/LHC XS, etc.
  in the MSSM with/without complex parameters (and for NMFV)

- New features:
  - Correction of $\mathcal{O}(\alpha_t\alpha_s)$ in the cMSSM included
  - rotation matrices for the correct treatment of
    external (on-shell) Higgs bosons
    internal Higgs bosons
  - (CDM) parameter planes ($\to$ "FeynHiggs Record" structure)

- How to run FeynHiggs:
  - download the code from `www.feynhiggs.de`
  - Command-line mode (allows also running on the GRID)
  - called from a Fortran/C++ code
  - called within Mathematica
  - WWW mode (`www.feynhiggs.de/fhucc`)

6. Conclusinos

- Precise MSSM Higgs sector evaluation necessary to
  - do phenom ... and the LHC
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www.feynhiggs.de

- How to run F
  - download t
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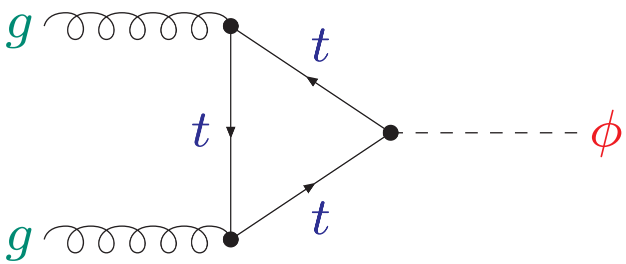
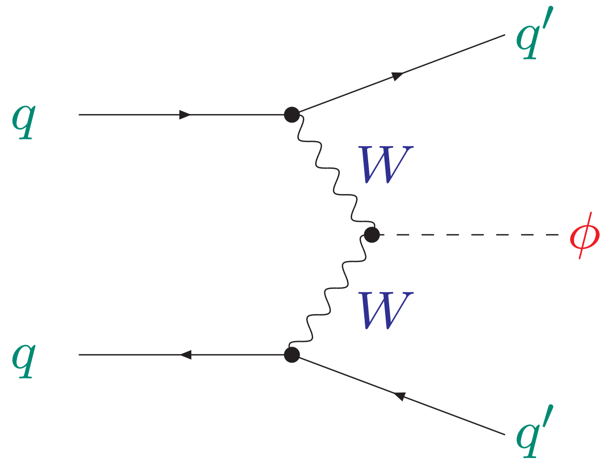
Back-up

External (on-shell) Higgs bosons

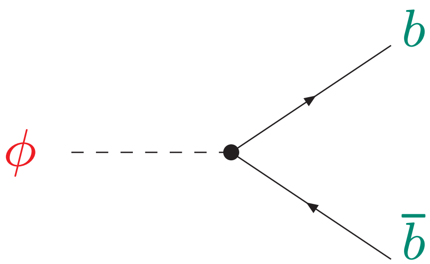
Examples for external (on-shell) Higgs bosons ($\phi = h_1, h_2, h_3$):

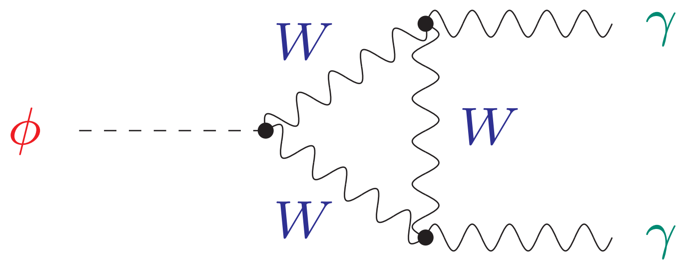
Higgs production:





Higgs decays:





⇒ important to ensure on-shell properties of external Higgs boson

The $\mathbf{Z}$ matrix:

Amplitude with external Higgs h_i:

$$A(h_i) = \sqrt{Z_i}\left(\Gamma_{h_i} + Z_{ij}\Gamma_{h_j} + Z_{ik}\Gamma_{h_k}\right)$$

$\sqrt{Z_i}$: ensures that the residuum of the external Higgs boson is set to 1

Z_{ij} : describes the transition from $i \to j$

$$Z_i = \left[1 + \left(\hat{\Sigma}^{\text{eff}}_{ii}\right)'(\mathcal{M}_i^2)\right]^{-1}$$

$$\hat{\Sigma}^{\text{eff}}_{ii}(p^2) = \hat{\Sigma}_{ii}(p^2) - i\frac{2\hat{\Gamma}_{ij}(p^2)\hat{\Gamma}_{jk}(p^2)\hat{\Gamma}_{ki}(p^2) - \hat{\Gamma}^2_{ki}(p^2)\hat{\Gamma}_{jj}(p^2) - \hat{\Gamma}^2_{ij}(p^2)\hat{\Gamma}_{kk}(p^2)}{\hat{\Gamma}_{jj}(p^2)\hat{\Gamma}_{kk}(p^2) - \hat{\Gamma}^2_{jk}(p^2)}$$

$$Z_{ij} = \frac{\Delta_{ij}(p^2)}{\Delta_{ii}(p^2)}\bigg|_{p^2=\mathcal{M}_i^2}$$

$$\hat{\Gamma}(p^2) = iM^2_{hHA}(p^2) \qquad \Delta(p^2) = \left(-\Gamma(p^2)\right)^{-1}$$

m_i: tree-level masses $\qquad$ M_i: higher-order corrected masses

Written more compact with the $\mathbf{Z}$ matrix : $\mathbf{Z}_{ij} = \sqrt{Z_i}\,Z_{ij}$

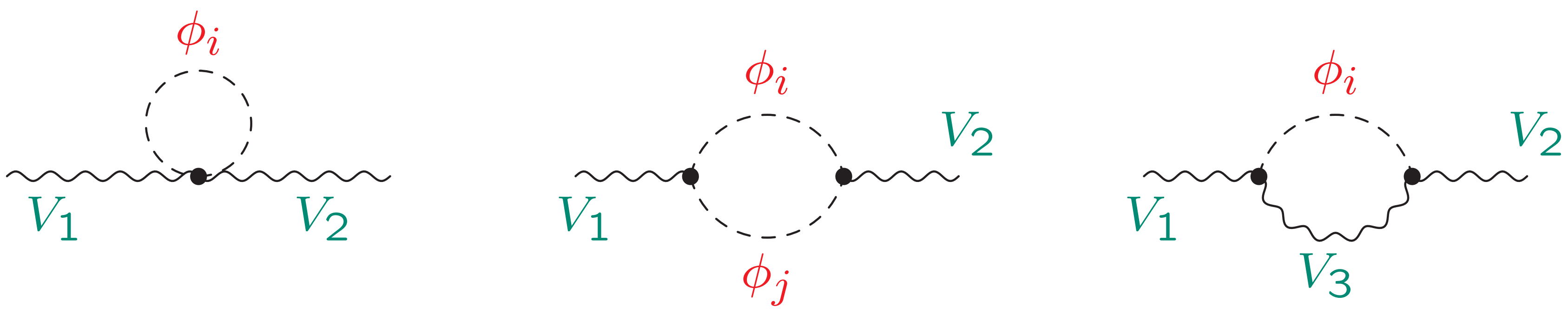
Internal Higgs bosons

Examples for Higgs bosons entering loop corrections:

Vector boson self-energies:

e.g. in μ decay, precision observables, . . .

$(V_{1,2,3} = Z, W^{\pm})$



$\phi_{i,j} = h, H, A$ (tree-level states): $\Rightarrow$ ok

But what if $\phi_{i,j} = h_1, h_2, h_3$?

$\Rightarrow$ How to include higher-order corrections to the Higgs bosons properly?

$\Rightarrow$ How to define "effective couplings"?

Two possibilities:

1.) "p^2 on-shell" : $\mathbf{U}$

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}_{p^2 \text{ on-shell}} = \mathbf{U} \cdot \begin{pmatrix} h \\ H \\ A \end{pmatrix}, \quad p^2 \text{ on-shell} : \begin{array}{lcl} \hat{\Sigma}_{ii}(p^2) & \to & \hat{\Sigma}_{ii}(m_i^2) \\ \hat{\Sigma}_{ij}(p^2) & \to & \hat{\Sigma}_{ij}((m_i^2 + m_j^2)/2) \end{array}$$

$$\mathbf{U}\, \mathrm{Re}\left(\mathbf{M}_{hHA}(p^2 \text{ on-shell})\right) \mathbf{U}^\dagger = \begin{pmatrix} M^2_{h_1,p^2\text{os}} & 0 & 0 \\ 0 & M^2_{h_2,p^2\text{os}} & 0 \\ 0 & 0 & M^2_{h_3,p^2\text{os}} \end{pmatrix}$$

2.) "$p^2 = 0$" : $\mathbf{R}$ ($\mathcal{CP}$C case, 2×2 mixing $\Rightarrow \alpha_{\text{eff}}$)

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}_{p^2=0} = \mathbf{R} \cdot \begin{pmatrix} h \\ H \\ A \end{pmatrix}, \quad \mathbf{R}\, \mathbf{M}_{hHA}(0)\, \mathbf{R}^\dagger = \begin{pmatrix} M^2_{h_1,p^2=0} & 0 & 0 \\ 0 & M^2_{h_2,p^2=0} & 0 \\ 0 & 0 & M^2_{h_3,p^2=0} \end{pmatrix}$$

Limit $p^2 \to 0$:

$$\mathbf{Z} \;\to\; \mathbf{R} \;: \qquad \mathbf{R} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix}$$

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}_{p^2=0} = \mathbf{R} \cdot \begin{pmatrix} h \\ H \\ A \end{pmatrix}, \; \mathbf{R}\,\mathbf{M}_{hHA}(0)\,\mathbf{R}^\dagger = \begin{pmatrix} M^2_{h_1,p^2=0} & 0 & 0 \\ 0 & M^2_{h_2,p^2=0} & 0 \\ 0 & 0 & M^2_{h_3,p^2=0} \end{pmatrix}$$

– $\mathbf{R}$ in the 2×2 case is exactly α_{eff}

– $\mathbf{R}$ corresponds to the effective potential approach

What is better?

1.) "p^2 on-shell" : $\mathbf{U}$

2.) "$p^2 = 0$" : $\mathbf{R}$

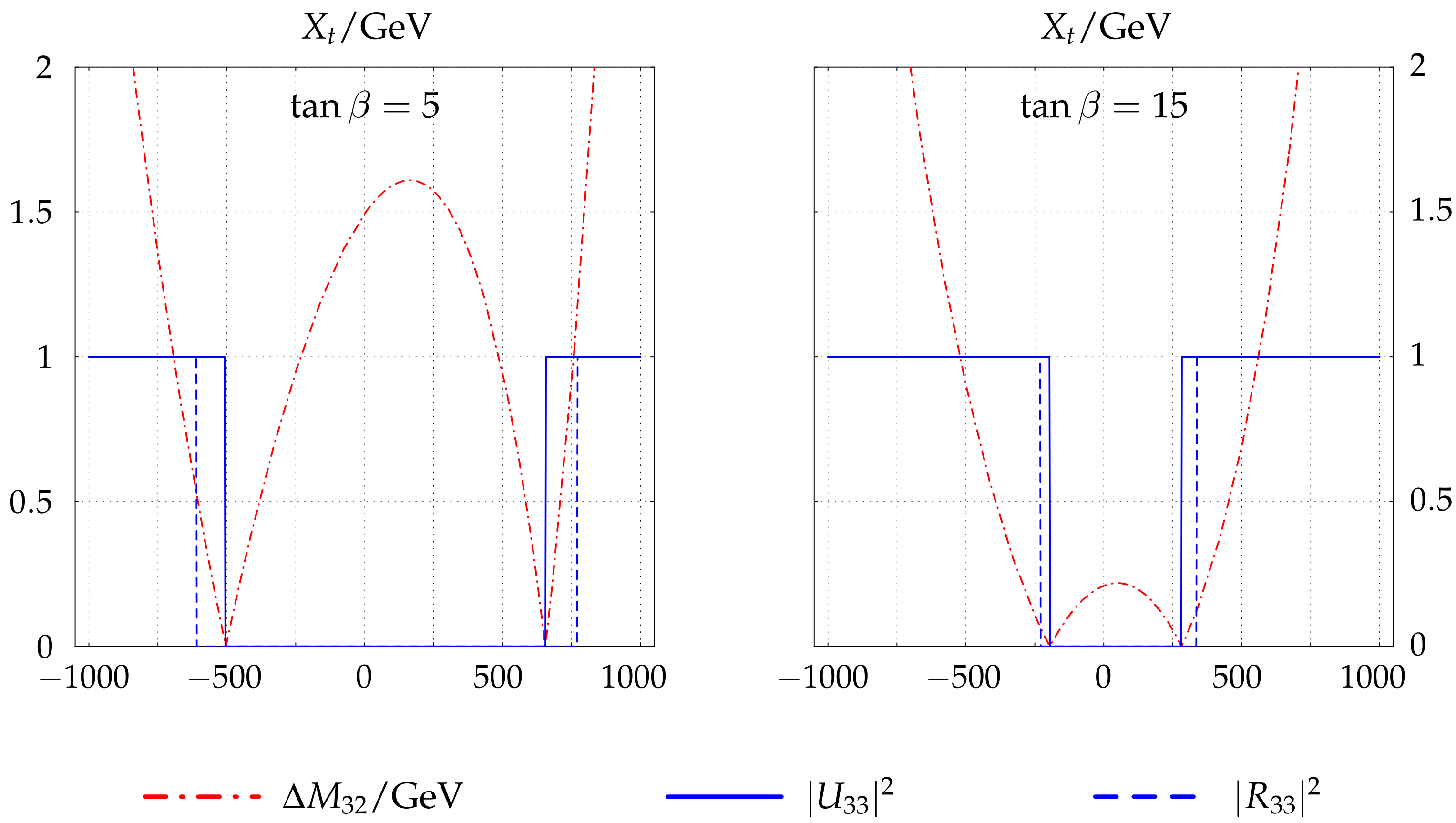
Two possible tests:

1. Compare full decay width, evaluated with $\mathbf{Z}$, with approximations, evaluated with $\mathbf{U}$ or $\mathbf{R}$
   $\rightarrow$ see later in "Numerical examples"

2. $\mathbf{U}_{33}^2$ and $\mathbf{R}_{33}^2$ correspond to the $\mathcal{CP}$-odd part of h_3
   In the rMSSM: $\mathbf{U}_{33}^2$, $\mathbf{R}_{33}^2 = 0$ or 1 (depending on mass ordering)
   Switch-over from 0 to 1 should happen for $\Delta M_{32} := M_{h_3} - M_{h_2} = 0$
   $\rightarrow$ compare switch-over with ΔM_{32}

$\rightarrow$ Compare switch-over with ΔM_{32}:

$M_{\text{SUSY}} = m_{\tilde{g}} = M_2 = 500$ GeV, $\mu = 1000$ GeV, $M_{H^\pm} = 150$ GeV



$\Rightarrow$ **U** gives the better results

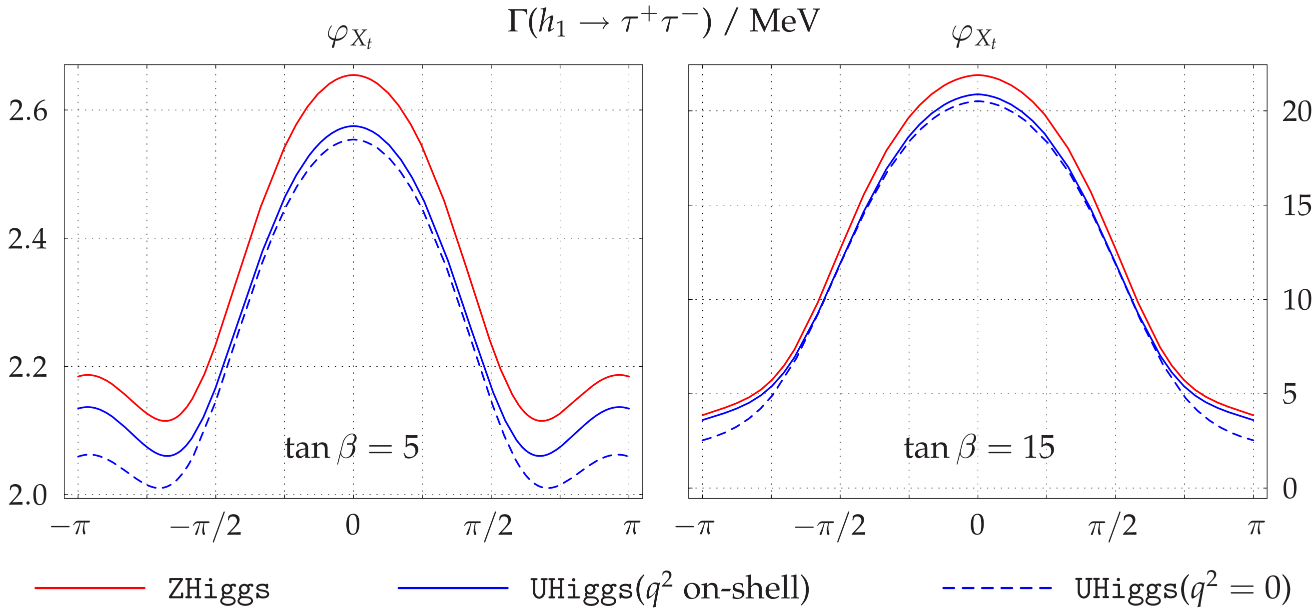
$\Rightarrow$ use **U** for effective couplings

Numerical example for external Higgs bosons:

[*T. Hahn, S.H., W. Hollik, H. Rzehak, G. Weiglein '07*]

$M_{\text{SUSY}} = m_{\tilde{g}} = M_2 = 500$ GeV, $A_t = 1000$ GeV, $\mu = 1000$ GeV, $M_{H^\pm} = 150$ GeV

$\Gamma(h_1 \to \tau^+\tau^-)$ as a function of ϕ_{X_t}



red solid: **Z** , blue solid: **U** , blue dashed: **R**

⇒ **U** gives results closer to full result than **R**

⇒ deviations at the 5-10% level

Numerical example for external Higgs bosons:

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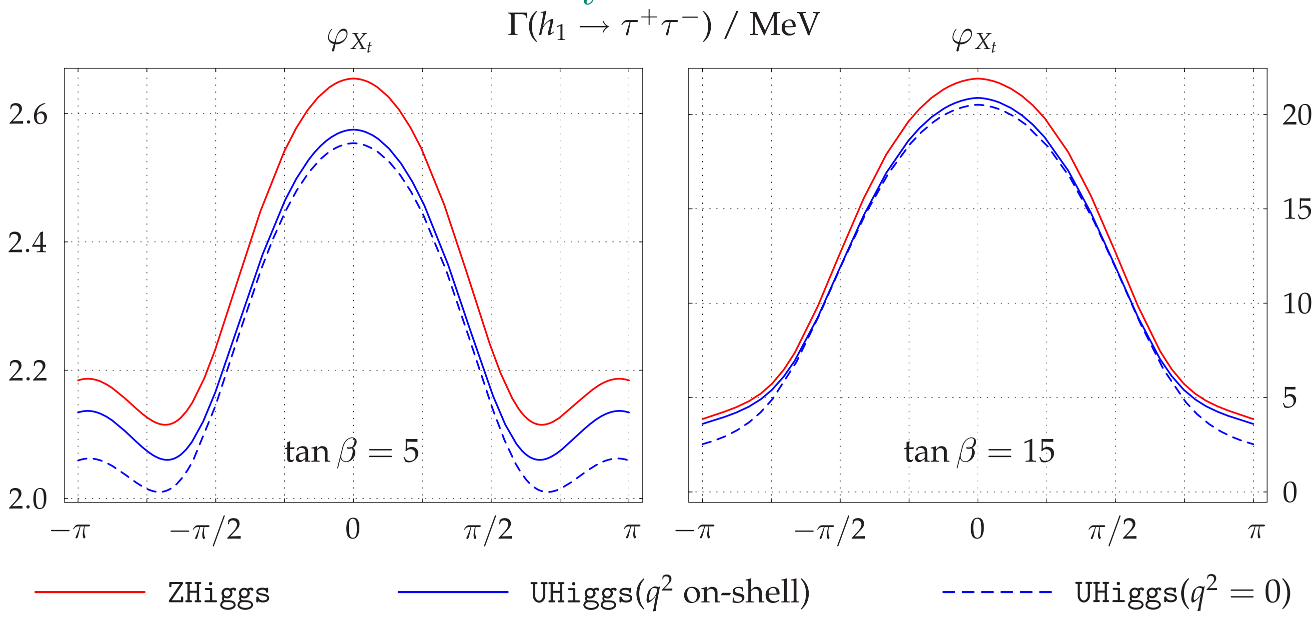


red solid: **Z** , blue solid: **U** , blue dashed: **R**

Included in FeynHiggs: **Z, U, R**

Summary: treatment of "higher-order" corrected Higgs bosons:

1. external/on-shell Higgs bosons

   amplitude with on-shell Higgs boson i:

$$A_{h_i xy} \sim \sqrt{Z_i}\left(Z_{ih} C_{hxy} + Z_{iH} C_{Hxy} + Z_{iA} C_{Axy}\right)$$

   Z_i, Z_{ij}: finite wave function renormalizations

   Written more compact with the $\mathbf{Z}$ matrix:

$$\mathbf{Z}_{ij} = \sqrt{Z_i}\, Z_{ij}$$

   resulting in

$$A_{h_i xy} \sim \mathbf{Z}_{ih}\, C_{hxy} + \mathbf{Z}_{iH}\, C_{Hxy} + \mathbf{Z}_{iA}\, C_{Axy}$$

2. Higgs bosons in loop corrections

   rotate tree-level couplings with $\mathbf{U}$ or $\mathbf{R}$:

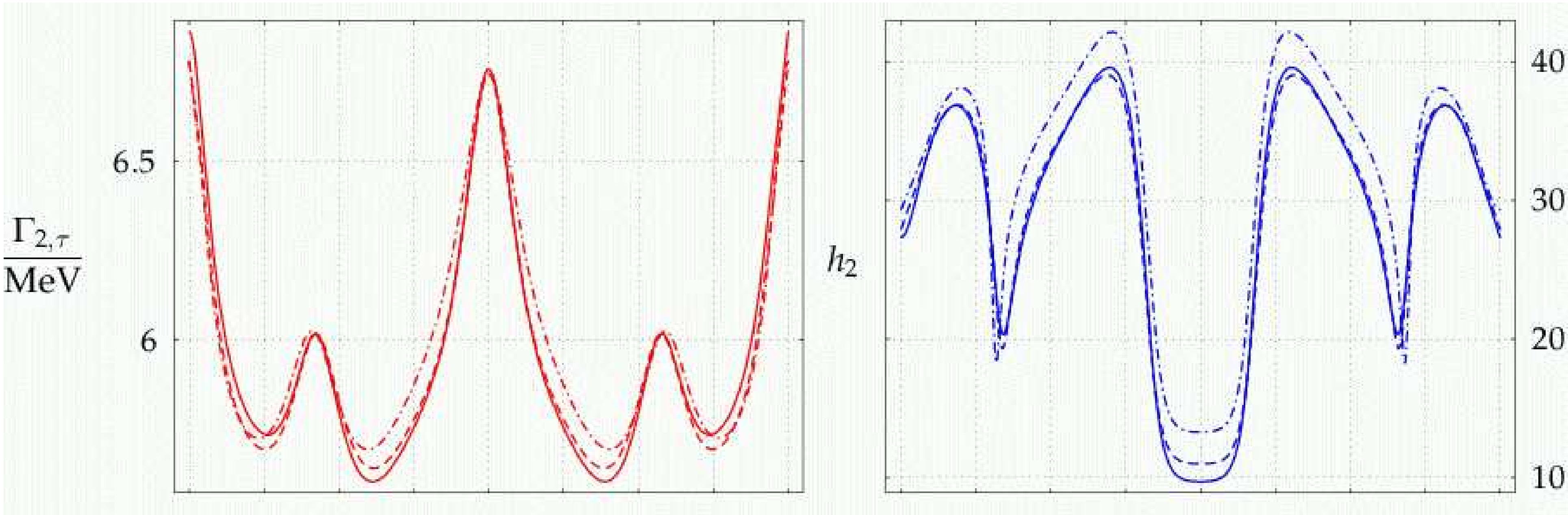
$$C_{h_i xy} = \mathbf{U}_{ih}\, C_{hxy} + \mathbf{U}_{iH}\, C_{Hxy} + \mathbf{U}_{iA}\, C_{Axy}$$
$$C_{h_i xy} = \mathbf{R}_{ih}\, C_{hxy} + \mathbf{R}_{iH}\, C_{Hxy} + \mathbf{R}_{iA}\, C_{Axy}$$

Numerical results (II):

[*M. Frank, T. Hahn, S.H., W. Hollik, H. Rzehak, G. Weiglein '06*]

$M_{\text{SUSY}} = m_{\tilde{g}} = M_2 = 500$ GeV, $A_t = 1000$ GeV, $\mu = 1000$ GeV, $M_{H^\pm} = 150$ GeV

$\Gamma(h_1 \to \tau^+\tau^-)$ as a function of ϕ_{X_t}



solid: **Z** , dashed: **U** , dot-dashed: **R**

⇒ **U** gives results closer to full result than **R**

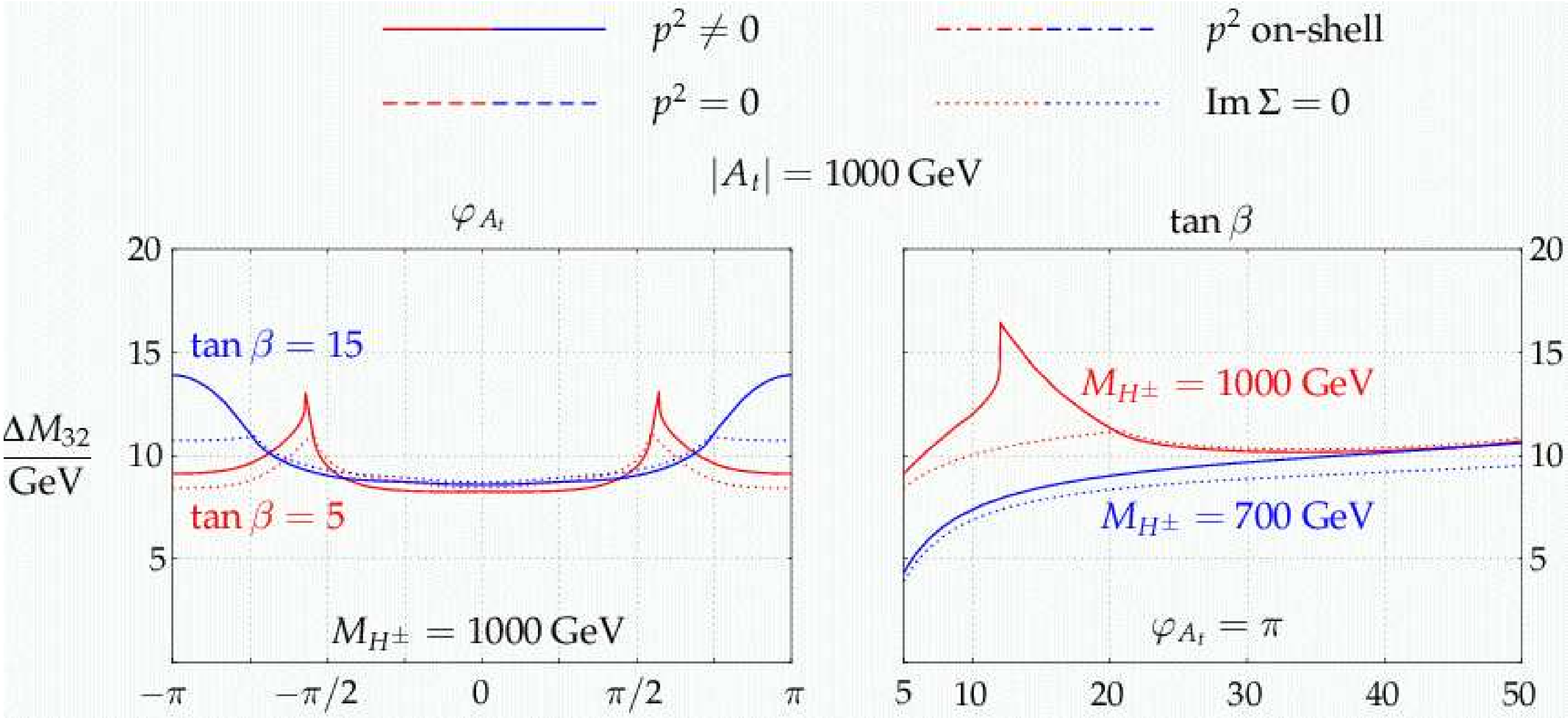
⇒ deviations at the 5-10% level

Numerical results (III):

[*M. Frank, T. Hahn, S.H., W. Hollik, H. Rzehak, G. Weiglein '06*]

$M_{\text{SUSY}} = m_{\tilde{g}} = M_2 = 500$ GeV, $|A_t| = 1000$ GeV, $\mu = 1000$ GeV, $M_{H^\pm} = 1000$ GeV

Effects of $\text{Im}\,\hat{\Sigma}$ on $\Delta M_{32} := M_{h_3} - M_{h_2}$

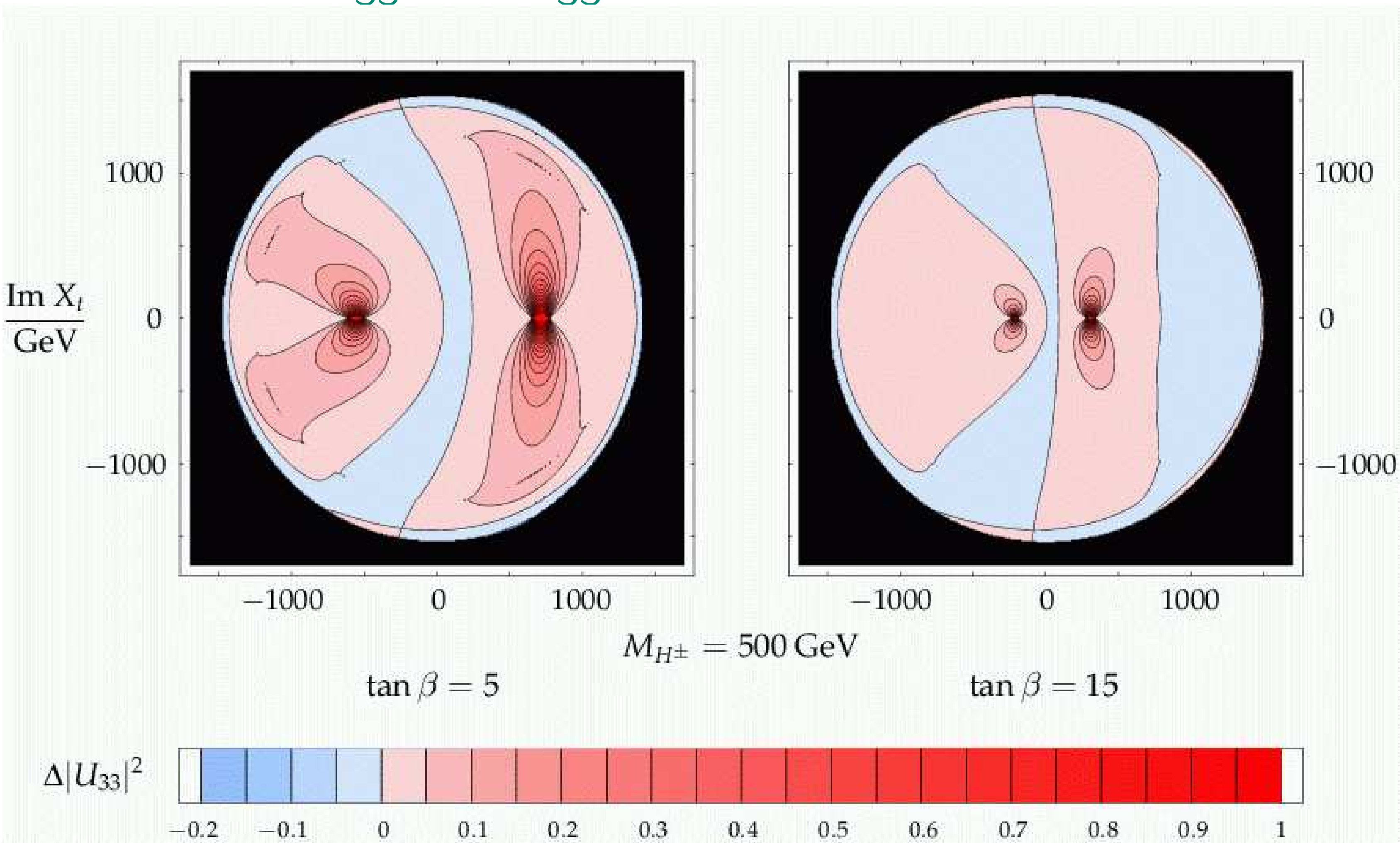


$\Rightarrow$ differences of up to 5 GeV

Numerical results (IV): [M. Frank, T. Hahn, S.H., W. Hollik, H. Rzehak, G. Weiglein '06]

$M_{\rm SUSY} = m_{\tilde g} = M_2 = 500$ GeV, $\mu = 1000$ GeV, $M_{H^\pm} = 150$ GeV

Difference between $\mathbf{U}_{33}^2$ and $\mathbf{R}_{33}^2$:

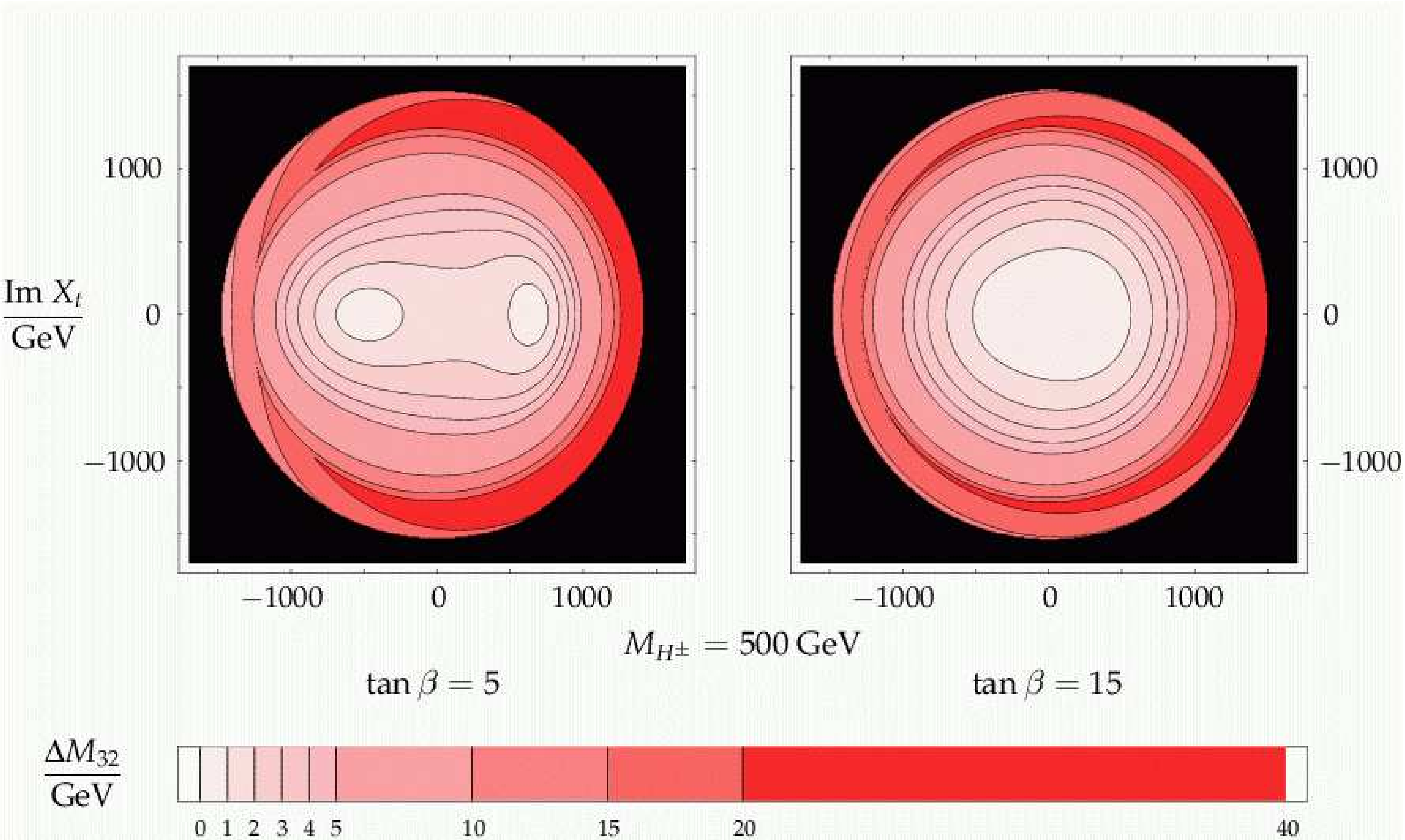


⇒ large deviations where ΔM_{32} is small

Numerical results (IV): [M. Frank, T. Hahn, S.H., W. Hollik, H. Rzehak, G. Weiglein '06]

$M_{\text{SUSY}} = m_{\tilde{g}} = M_2 = 500$ GeV, $\mu = 1000$ GeV, $M_{H^\pm} = 150$ GeV

Difference between $\mathbf{U}^2_{33}$ and $\mathbf{R}^2_{33}$:



$\Rightarrow$ large deviations where ΔM_{32} is small

Parameter planes

Search for the MSSM Higgs bosons:

$\longrightarrow$ investigate benchmark scenarios:

> $\longrightarrow$ Vary only M_A and $\tan\beta$
> $\longrightarrow$ Keep all other SUSY parameters fixed

1. m_h^{max} scenario:
   $\longrightarrow$ obtain conservative $\tan\beta$ exclusion bounds ($X_t = 2\,M_{\text{SUSY}}$)
2. no-mixing scenario
   $\longrightarrow$ no mixing in the scalar top sector ($X_t = 0$)
3. small α_{eff} scenario
   $\longrightarrow$ $hb\bar{b}$ coupling $\sim \sin\alpha_{\text{eff}}/\cos\beta$ can be zero: $\alpha_{\text{eff}} \to 0$:
   main decay mode vanishes, important search channel vanishes
4. gluophobic Higgs scenario
   $\longrightarrow$ hgg coupling is small: main LHC production mode vanishes

[*M. Carena, S.H., C. Wagner, G. Weiglein '02*]

$\longrightarrow$ included in FeynHiggs for a long time

Possible external constraints:

– cold dark matter (CDM)
– BR($b \to s\gamma$)
– anomalous magnetic moment of the μ
(reason for change from $\mu = -200$ GeV $\to$ $\mu = +200$ GeV)

$\Rightarrow$ so far ignored (for (good) reasons)

Wanted: M_A–$\tan\beta$ planes in agreement with CDM

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$\Rightarrow$ so far ignored (for (good) reasons)

Wanted: M_A–$\tan\beta$ planes in agreement with CDM

Possible models:

1.) CMSSM (or mSUGRA):

$\Rightarrow$ Scenario characterized by

$$m_0,\ m_{1/2},\ A_0,\ \tan\beta,\ \text{sign}\,\mu$$

$\Rightarrow$ too restricted

Possible external constraints:

- cold dark matter (CDM)
- BR($b \to s\gamma$)
- anomalous magnetic moment of the μ
  (reason for change from $\mu = -200$ GeV $\to$ $\mu = +200$ GeV)

$\Rightarrow$ so far ignored (for (good) reasons)

Wanted: M_A–$\tan\beta$ planes in agreement with CDM

2.) NUHM: (Non-universal Higgs mass model)

Assumption: no unification of scalar fermion and scalar Higgs parameters at the GUT scale

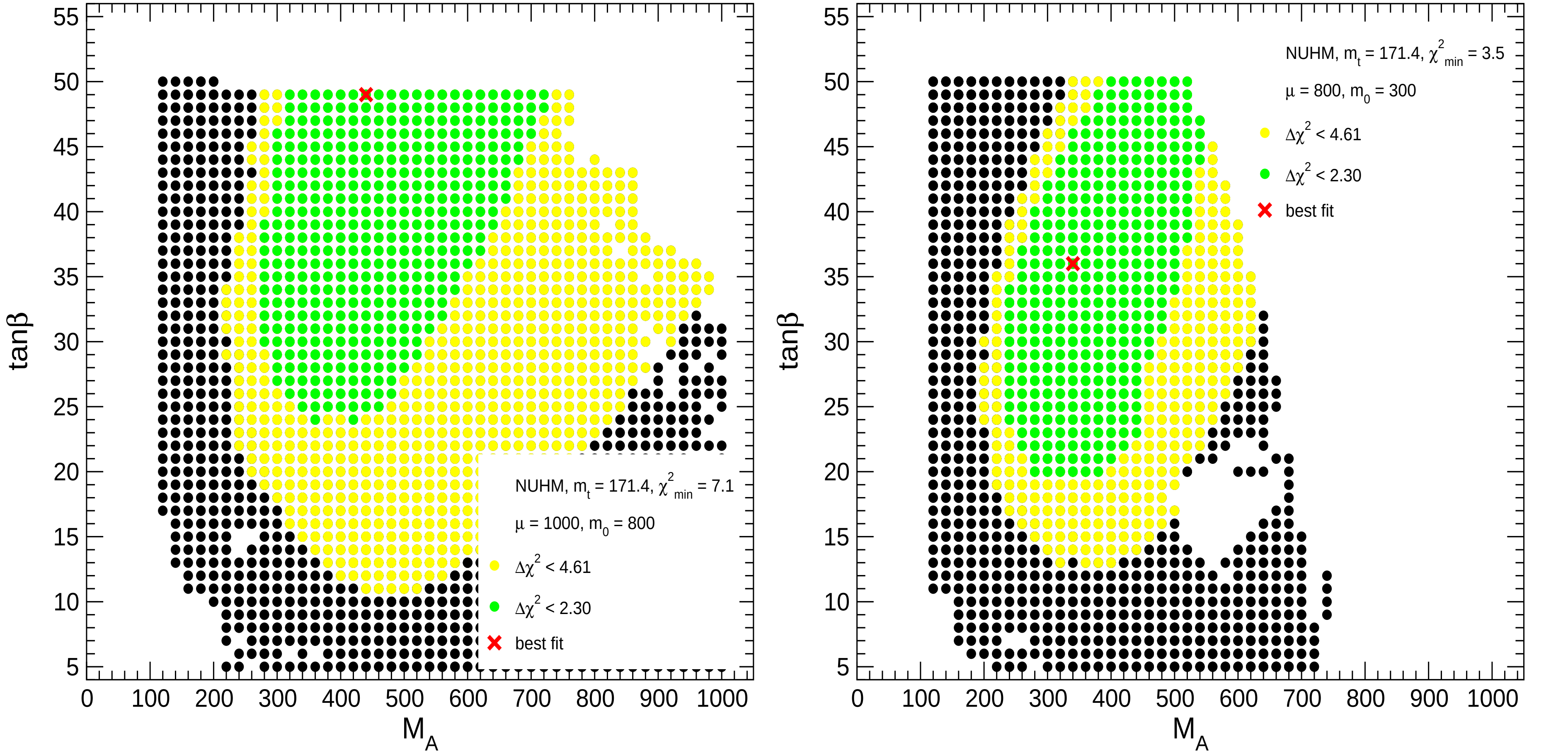
$\Rightarrow$ effectively M_A and μ free parameters at the EW scale

$\Rightarrow$ besides the CMSSM parameters

M_A and μ

Results: NUHM: planes 1,2

[*J. Ellis, T. Hahn, S.H., K. Olive, G. Weiglein '07*]

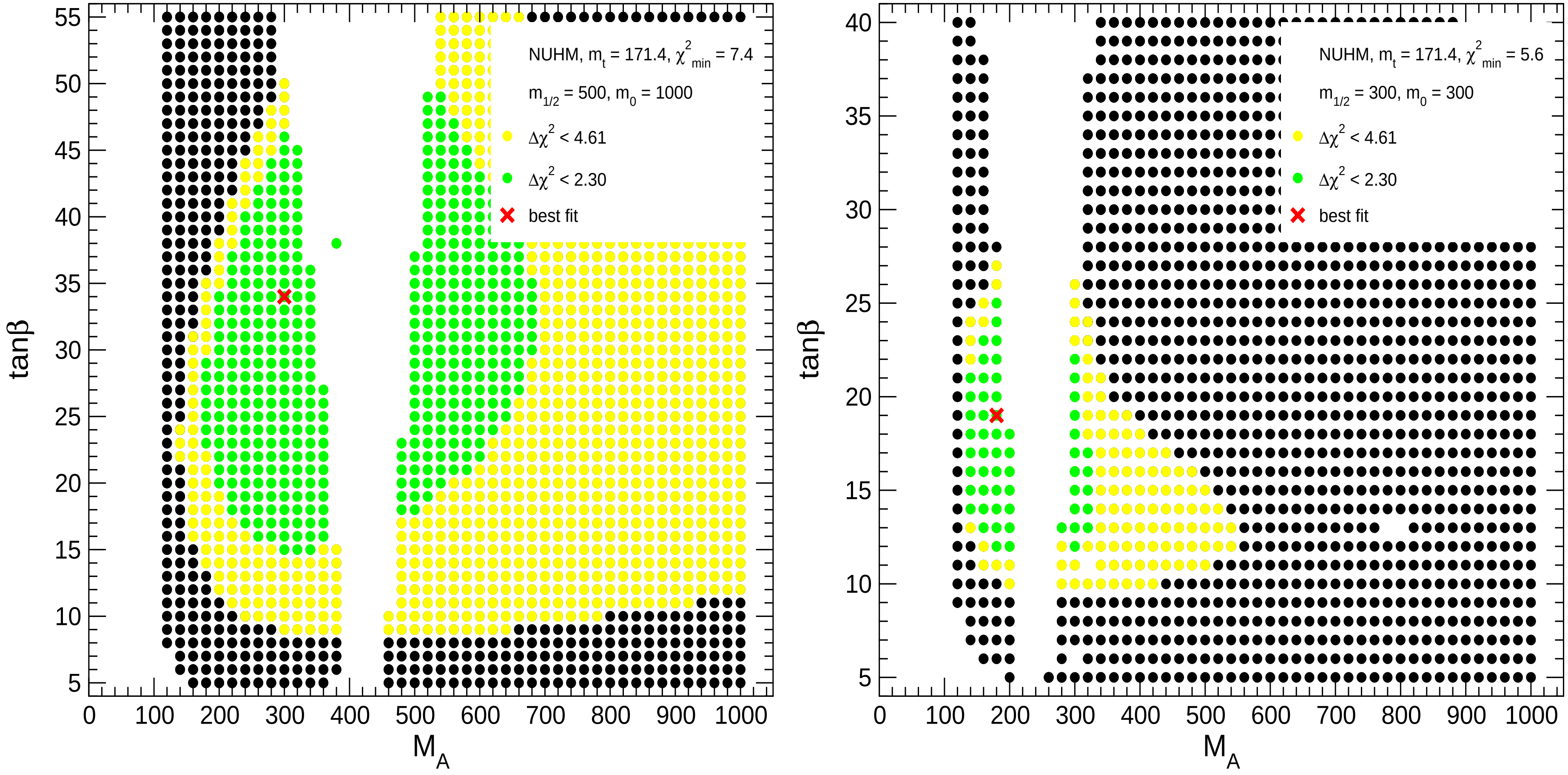


$\Rightarrow$ good χ^2 (M_W, $\sin^2\theta_{\rm eff}$, Γ_Z, M_h, $(g-2)_\mu$, BR($b \to s\gamma$) and other BPO)

$\Rightarrow$ larger regions o.k.

Results: NUHM: planes 3,4

[*J. Ellis, T. Hahn, S.H., K. Olive, G. Weiglein '07*]



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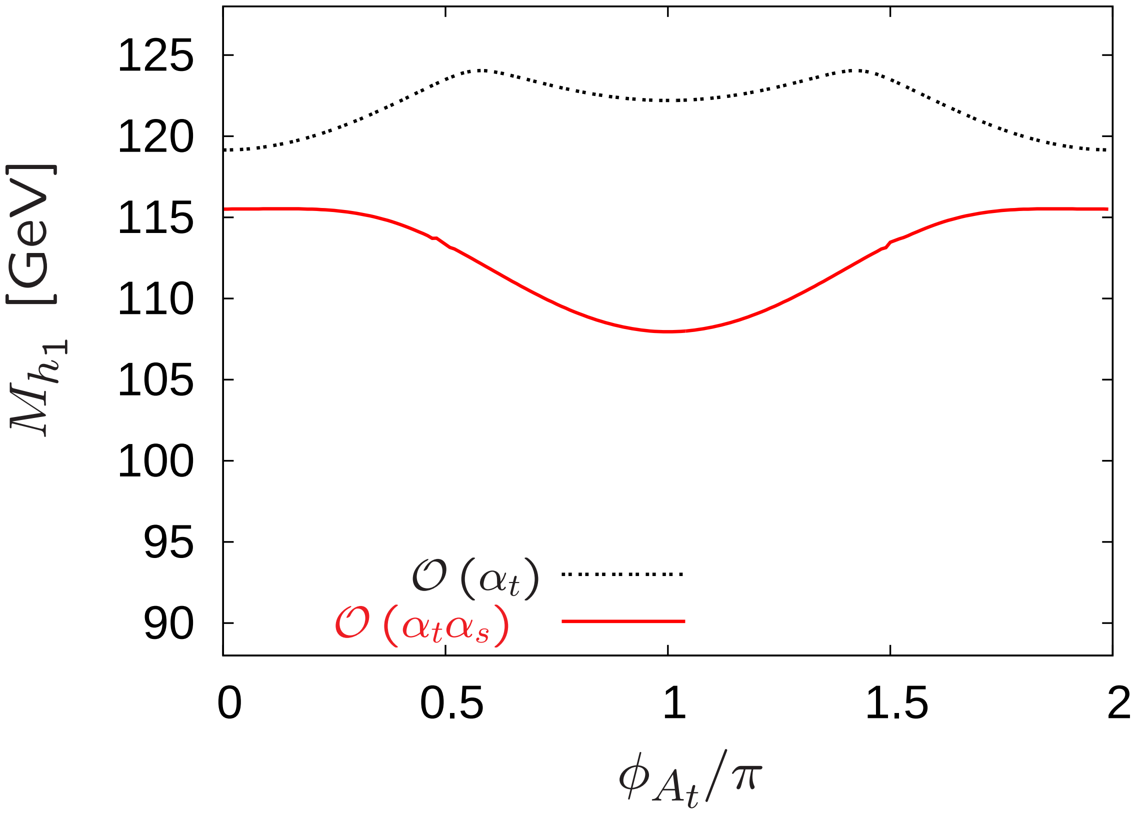
M_{h_1} as a function of ϕ_{A_t}:

[S.H., W. Hollik, H. Rzehak, G. Weiglein '07]



$M_{\mathrm{SUSY}} = 1000$ GeV

$|A_t| = 2000$ GeV

$\tan\beta = 10$

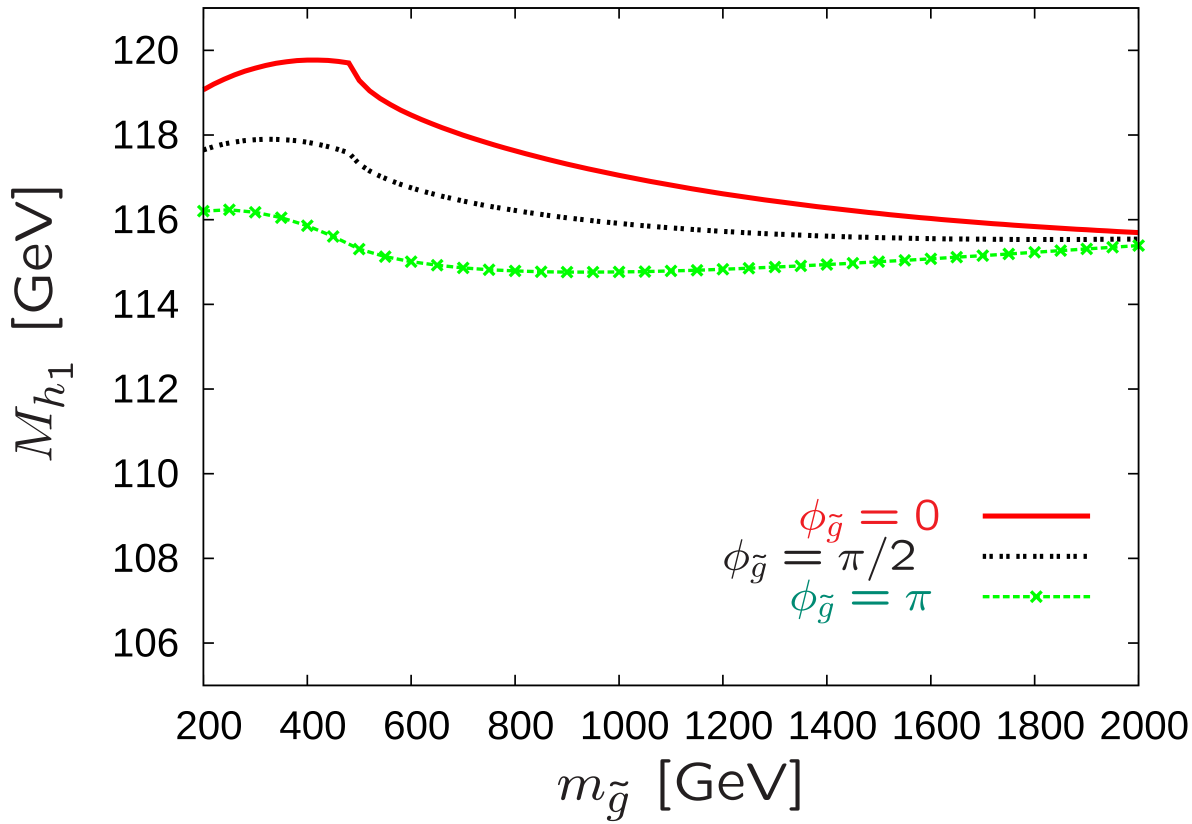
$M_{H^\pm} = 150$ GeV

OS renormalization

$\Rightarrow$ modified dependence on ϕ_{A_t} at the 2-loop level

M_{h_1} as a function of $\phi_{\tilde{g}}$:

[S.H., W. Hollik, H. Rzehak, G. Weiglein '07]



$M_{\text{SUSY}} = 500$ GeV
$A_t = 1000$ GeV
$\tan\beta = 10$
$M_{H^\pm} = 500$ GeV

OS renormalization

$\Rightarrow$ threshold at $m_{\tilde{g}} = m_{\tilde{t}} + m_t$

$\Rightarrow$ large effects around threshold

$\Rightarrow$ phase dependence has to be taken into account