Direct detection module in micrOMEGAs_2:

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Status micrOMEGAs.

micrOMEGAs is a package for calculation of properties of Dark Matter particles in a generic model.

Generic Model

- written in CalcHEP format.
- has Z_2 symmetry, names of odd particles start with ~
- cross sections are of EW order.

What we calculate?

- Relic density of WIMP which in our case is the lightest odd particle.
- γ, ν indirect detection signals as well as e^-, p^+ yield without propagation.
- Direct detection signal.
- Cross sections and width for any particles involved in the model.

User languages: C, C++, Fortran.

Unix dialects: Linux, OSF1, Darwin,CYGWIN.

Models implemented: MSSM (SuSpect, SoftSusy, SPheno, Isajet), NMSSN(NMSSMTools), CPVMSSM(CPsuperH), LittleHiggs, RHNM(KK model).

Method of calculation

- microOMEGAs uses CalcHEP for automatic generation of matrix elements.
- All matrix elements are loaded dynamically in runtime. See

```
man dlopen;  man  dlsym
```

When micrOMEGAs needs matrix element it

a) tries to find it among downloaded ones;

if not b) tries to find the corresponding shared library on the disk and downloads it.

if not c) sends request to CalcHEP to generate the needed library.

micrOMEGAs user as well has a possibility to add and check codes for any matrix element:

```
numout* cc=newProcess("e,E->2*x", "eE_2x");
cs= cs22(cc,n_channel,Pcm,cosmin,cosmax,&err);
```

Advantages of our approach

- very simple model implementation.

MSSM, NMSSM, CPVMSSM,Little Higgs, RHNM are implemented.

- flexibility to correct/upgrade a model

- no need to compile codes for numerous co-annihilation channels in advance

- short code 14Mb all together

  150Kb/5000lines - micrOMEGAs kernel.
  1.5Mb spectra for indirect detection.
  4Mb - CalcHEP,
  4Mb - SuSpect, NMSSMTools, CPsuperH
  1+1.5Mb -model files

- possibility to check cross sections and widths.

Direct detection.

Direct detection module calculates reaction of WIMP $v \approx 220 km/s$ with neutrons and nuclei.

Typical transfer momenta $\Delta p \approx 100 MeV \approx 1/(2fm)$

Actually one can work in $\Delta p = 0$ limit where there are two independent amplitudes: spin independent-scalar and spin dependent-vector.

Path of calculation.

- Lagrangian
- WIMP-quark interaction via operator expansion
- WIMP-nucleon interaction - quark condensates in nucleons.
- WINP-nucleus interaction - Nuclei form factors
- integration over WIMP velocity distribution.

From Lagrangian to operator expansion

We have complete Lagrangian $\mathcal{L}$. But for WIMP-nucleon scattering we need WIMP-quark amplitude expanded over local operators.

$$\mathcal{L}_{eff} = \lambda\bar{\psi}\psi\bar{q}q + \xi(\bar{\psi}\gamma_\mu\gamma_5\psi)(\bar{q}\gamma_\mu\gamma_5 q) + ...$$

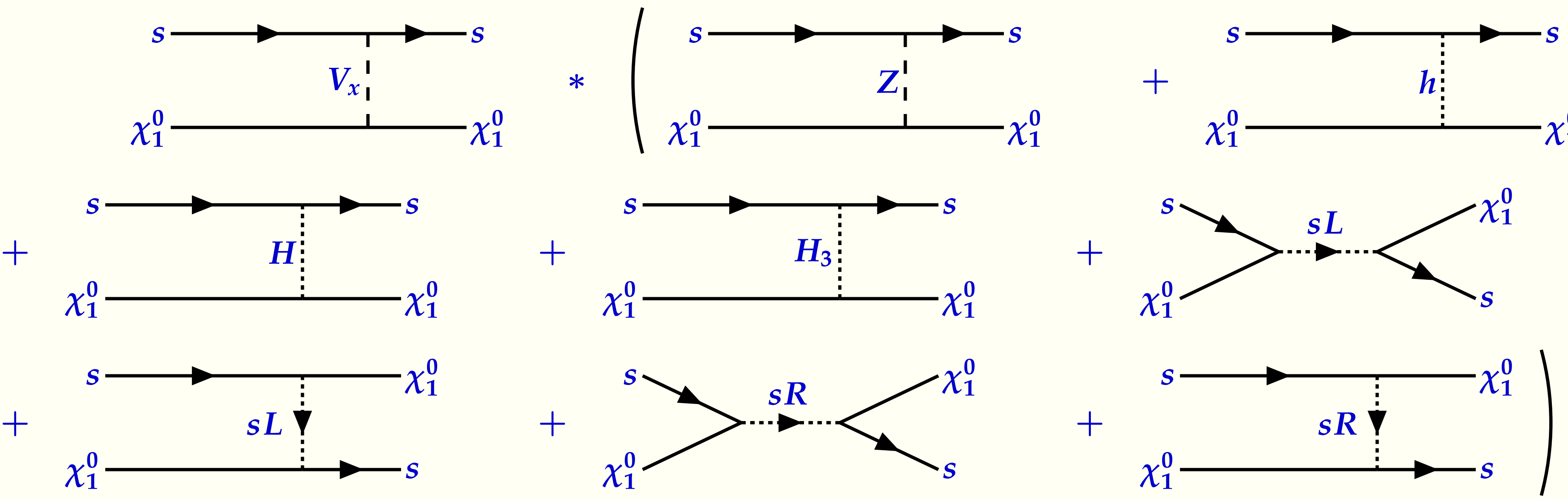
CalcHEP calculates squared diagrams ...
MicrOMEGAs automatically creates a new model with $\mathcal{L} + \mathcal{L}_{eff}$ interaction and calculates matrix elements

$$< \bar{q}(p_1), \psi(p_2)|\mathcal{L}_{eff} \cdot \mathcal{L}|\bar{q}(p_1), \psi(p_2) >$$

for collision at rest.

Operator expansion at diagram level ....

Indeed CalcHEP calculates part of squared diagrams which contains in one side the auxiliary vertex, and in another side diagrams of physical matrix element.



Local operators in generic model

We don't restrict ourself by Majorana DM. Spin of DM particle could be 0, 1/2, 1, and it can be described as well as by neutral or complex field. In case of complex field DM-quark interactions can be separated on even and odd ones respect to $DM - \overline{DM}$ swapping.

	J_{DM}	Even operators	Odd operators
	0	$\phi_\chi \phi^*_\chi \overline{\psi}_q \psi_q$	$-i(\partial_\mu \phi_\chi \phi_\chi \chi^* - \phi_\chi \partial_\mu \phi^*_\chi)\overline{\psi}_q \gamma^\mu \psi_q$
SI	1/2	$\overline{\psi}_\chi \psi_\chi \overline{\psi}_q \psi_q$	$\overline{\psi}_\chi \gamma_\mu \psi_\chi \overline{\psi}_q \gamma^\mu \psi_q$
	1	$M_\chi A_{\chi,\mu} A^\mu_\chi \overline{\psi}_q \psi_q$	$+i(A^{*\alpha}_\chi \partial_\mu A_{\chi,\alpha} - A_\chi{}^\alpha \partial_\mu A^*_{\chi\alpha})\overline{\psi}_q \gamma_\mu \psi_q$
SD	1/2	$\overline{\psi}_\chi \gamma_\mu \gamma_5 \psi_\chi \overline{\psi}_q \gamma_\mu \gamma_5 \psi_q$	$\overline{\psi}_\chi \sigma_{\mu\nu} \psi_\chi \overline{\psi}_q \sigma^{\mu\nu} \psi_q$
	1	$\frac{\sqrt{3}}{2}(\partial_\alpha A_{\chi,\beta} A_{\chi\nu} - A_{\chi\beta}\partial_\alpha A_{\chi\nu})\epsilon^{\alpha\beta\nu\mu}\overline{\psi}_q \gamma_5 \gamma_\mu \psi_q$	$i\frac{\sqrt{3}}{2}(A_{\chi\mu} A^*_{\chi\nu} - A^*_{\chi\mu} A_{\chi\nu})\overline{\psi}_q \sigma^{\mu\nu} \psi_q$

Next term of operator expansion is presented by twist-2 operators.

$$\mathcal{L}_{eff} += \lambda_2(\bar{\psi}\gamma_\mu \partial_\nu \psi)\bar{q}(\gamma_\mu \overrightarrow{\partial}_\nu - \gamma_\mu \overleftarrow{\partial}_\nu + \gamma_\nu \overrightarrow{\partial}_\mu - \gamma_\nu \overleftarrow{\partial}_\mu - i m_q g^{\mu\nu})q$$

To extract coefficients for twist-2 operator we test matrix elements of collision close to rest.

Nucleon form factors of light quarks

Nucleon form factors work in the following manner:

$$< N|m_q\bar{q}q| >= f_q^s M_N < N|\bar{\Psi}_N\Psi_N|N >$$

$$< N|\bar{q}\gamma_\mu\gamma_5 q| >= f_q^v < N|\bar{\Psi}_N\gamma_\mu\gamma_5\Psi_N|N >$$

They allow to transform WIMP-quark operator to WIMP-nucleon one.

Default values we use are(proton):

Even sector form factors

	d	u	s
scalar	0.033	0.023	0.26
vector	-0.427	0.842	-0.085

Odd sector (not neutral WIMP) form fact

	d	u	s
scalar γ_μ	1	2	0
vector $\sigma_{\mu\nu}$	-0.231	0.839	-0.046

In even sector main uncertainty comes from s-quark scalar FF.

Form factors for $\sigma_{\mu\nu}$ operators are not known precisely

To change form factors we have a procedure

setProtonFF(scalar,vector,sigma)

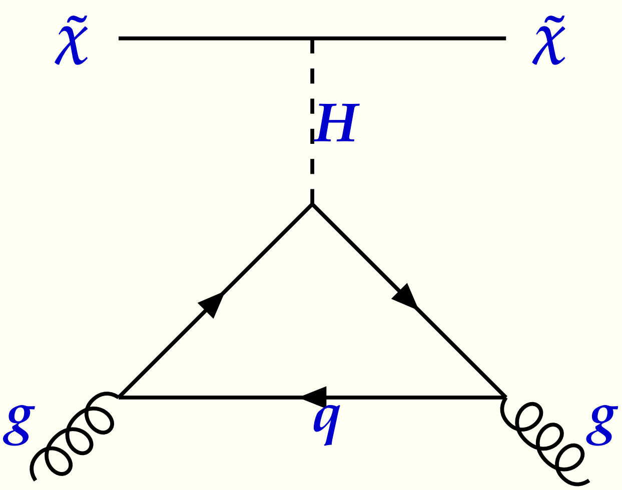
Heavy quark form factors

Heavy quarks form factors can be obtained from energy-momentum tensor trace anomaly:

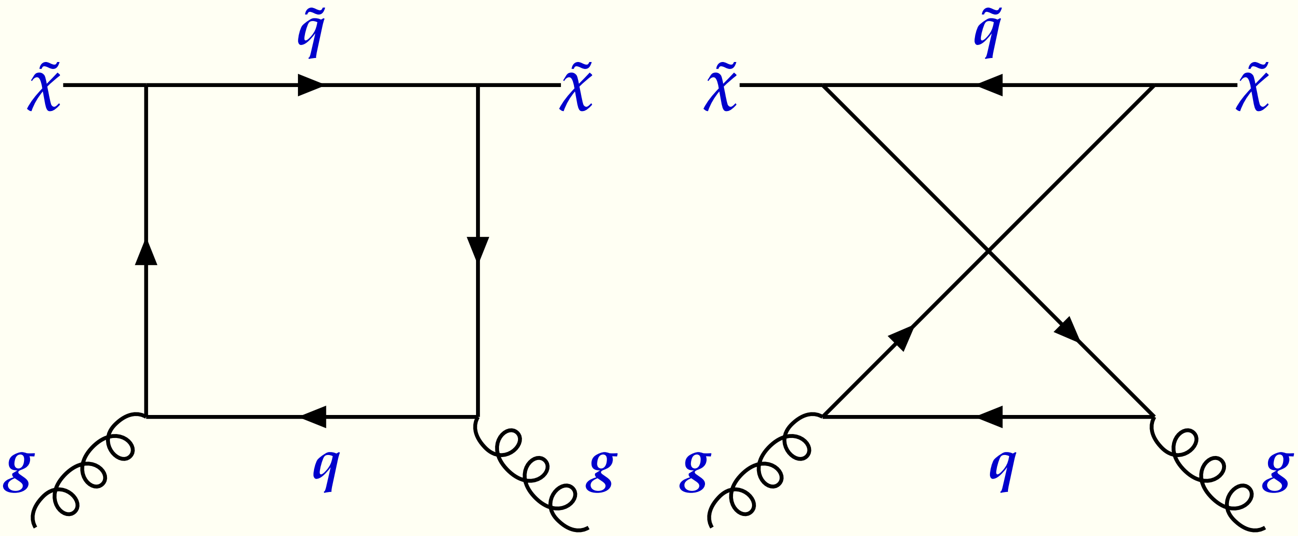
$$M_N < N|N> = < N| \sum_{q \le n_f} M_q \bar{\psi}_q \psi_q (1+\gamma) + (\frac{\beta^{n_f}}{2\alpha^2}) \alpha G_{\mu\nu} G^{\mu\nu} |N>$$

This leads to LO result plus some QCD corrections.

For Higgs diagrams this approach works perfect.



Box diagrams



These diagrams were calculated for MSSM by

M. Drees and M. Nojiri, Phys. Rev. D48 *(1993) 3483–3501*

We note that their result can be presented as substitution of some effective propagators

$$\frac{1}{M_{\tilde{q}}^2-(M_\chi \pm m_q)^2} \rightarrow FeScLoop(\pm 1, m_q, M_{\tilde{q}}, M_\chi)$$

FeScLoop is universal function for all models with fermion WIMP and scalar squark*. The order of correction is* $0.5 M_q/(M_{\tilde{q}} - M_\chi)$

Twist-2 form factors

twist-2 form factors are just momenta of parton distributions. We use CTEQ6l set of parton functions.

WIMP-nucleon amplitudes and cross sections

Evaluated by

nucleonAmplitude(LoopKfactor,SProton,SNeutron,VProton,Vneutron);

`Sproton[0]` *- for WIMP*

`Sproton[1]` *- for anti-WIMP*

Cross section:

$$\sigma_{Proton} = \frac{4}{\pi}\left(\frac{M_N M_{wimp}}{M_N + M_{wimp}}\right)^2 (S_{Proton}^2 + 3V_{proton}^2)$$

Effect of higher-order corrections on σ_p^{SI} in sample SUSY models.

	BP	KP	IP	NUH	MSSM1
tb	10	40	35	30	10
μ	+	-	+	+	+
Ωh^2	0.101	0.111	0.111	0.104	0.100
$\sigma_p^{SI} \times 10^9 pb$ tree-level	7.84	7.87	24.5	8.36	18.1
QCD	8.08	8.33	25.1	8.60	18.8
Δm_b	7.16	7.78	18.8	7.11	17.5
QCD+Δm_b	7.39	8.24	19.3	7.32	18.1
QCD+Δm_b +box	7.41	8.24	19.3	7.32	18.1

Uncertainty caused by nucleon form factors.

There is a noticeable uncertainty of SI amplitude caused by form factor of strange quark 0.26 ± 0.11.

`getScalarFF(`$m_u/m_d, m_s/m_d, \sigma_{\pi N}, \sigma_0$`,FF_proton,FF_neutron )` calculates nucleon SI form factors using πN scattering data:

$$\sigma_{\pi N} = 55 - 73 \text{ MeV} \quad \text{and} \quad \sigma_0 = 35 \pm 5 \text{ MeV}$$

	AP	BP	CP	DP	IP	KP	MP	NUG	NUH
m_0	130	70	90	120	180	2500	1100	1620	250
$M_{1/2}$	600	250	400	500	350	550	1100	300	530
A_0	0	-300	0	-400	0	-80	0	0	0
tb	5	10	10	10	35	40	50	10	30
μ	+	+	+	-	+	-	+	+	+
$\sigma_p^{SI} \times 10^9$ pb									
$\sigma_{\pi N} = 55, \sigma_0 = 35$	0.96	7.41	2.45	0.019	19.3	8.24	0.503	46.4	7.32
$\sigma_{\pi N} = 70, \sigma_0 = 35$	1.77	15.7	4.92	0.055	43.3	13.1	1.05	84.9	16.0
$\sigma_{\pi N} = 55, \sigma_0 = 40$	0.75	5.43	1.85	0.014	13.6	7.00	0.371	36.8	5.26

Uncertainty caused by nucleon form factors.

Uncertainty of SD amplitude is not so large but also is noticeable. Below we compare two sets

Set A: $\Delta_u^p = 0.842 \pm 0.012$, $\Delta_d^p = -0.427 \pm 0.013$, $\Delta_s^p = -0.085 \pm 0.018$

Set B: $\Delta_u^p = 0.780 \pm 0.020$, $\Delta_d^p = -0.48 \pm 0.02$, $\Delta_s^p = -0.15 \pm 0.02$

	AP	BP	CP	DP	IP	KP	MP	NUG	NUH
$\sigma_p^{SD} \times 10^6$pb									
set A	0.294	4.94	1.95	0.355	3.71	128.	0.0952	504.	2.13
set B	0.224	3.65	1.54	0.257	3.00	118.	0.0806	464.	1.81
$\sigma_n^{SD} \times 10^6$pb									
set A	0.245	4.29	1.53	0.318	2.84	79.8	0.068	313.	1.51
set B	0.326	5.83	1.99	0.438	3.63	90.9	0.084	357.	1.86

Comparison with DarkSuSy and Isajet.

DarkSuSy and Isajet have not implemented QCD andSUSY-QCD corrections.

Ignoring these correction and substituting the same nucleon form factors we have a good agreement with latest Isajet_7.76 while we had disagreement with Isajet_7.75.

Comparison with DarkSusy looks like:

$$\begin{aligned} A^{SD}_{micrO} &= A^{SD}(Z) + A^{SD}(\tilde{q}) \\ A^{SD}_{DarkSusy} &= A^{SD}(Z) + 0.5A^{SD}(\tilde{q}) && (1) \\ A^{SI}_{micrO}(q_{light}) &= A^{SI}_{DarkSusy}(q_{light}) && (2) \\ (Mb_{run}/Mb_{pole})A^{SI}_{micrO}(q_b) &= A^{SI}_{DarkSusy}(q_b) && (3) \end{aligned}$$

Table of comparison for SUGRA points.

	AP	BP	CP	DP	IP	MP
m_0	130	70	90	120	180	1100
$M_{1/2}$	600	250	400	500	350	1100
A_0	0	-300	0	-400	0	0
$\tan\beta$	5	10	10	10	35	50
μ	+	+	+	-	+	+
$\sigma_p^{SI} \times 10^9$ pb						
micrOMEGAs	0.466	3.65	1.17	0.0067	9.57	0.16
DarkSUSY_4.1	0.357	2.89	0.895	0.0054	7.54	0.118
$\sigma_p^{SD} \times 10^6$ pb						
micrOMEGAs	0.248	4.44	1.66	0.306	3.19	0.068
DarkSUSY_4.1	0.370	6.79	2.29	0.506	4.21	0.082
$\sigma_n^{SD} \times 10^6$ pb						
micrOMEGAs	0.203	3.75	1.29	0.267	2.41	0.0489
DarkSUSY_4.1	0.254	4.71	1.54	0.353	2.80	0.054

Spin independent nuclei form factors

$$\frac{d\sigma^{SI}}{dE} = \frac{4\mu_\chi^2}{\pi E_{max}}\left(\left(\lambda_p Z + \lambda_n (A-Z)\right)\right)^2 Fermi(R_A, a, q)^2$$

where

$$\mu_\chi = \frac{M_{wimp} M_A}{M_{wimp} + M_A}, \quad E_{max}(v) = 2\left(\frac{v^2\mu_\chi^2}{M_A}\right), \quad R_A = C \cdot A^{1/3} + B$$

Default value for C, B and thickness a corresponds to

J. D. Lewin and P. F. Smith, Astropart. Phys. *6 (1996) 87–112.*

and can be changed by

```
SetFermi(C,B,a)
```

We have disagreement with DarkSUSY about q. In spite of correct comment

```
dsddffsd.f:c   q: real*8  : momentum transfer in GeV (q=sqrt(M*E/2/mu^2))
```

DarkSUSY uses

```
dsddsigmaff.f:          q = sqrt(mni*e*1.d-6)
```

Spin dependent nuclei form factors

Implemented in 2 forms, approximate **and** precise **identical at $q = 0$.**

$$\frac{d\sigma^{SD}_{approx}}{dE} = \frac{\mu_\chi^2}{\pi E_{max}} \frac{J_A+1}{J_A} \left(\xi_p S_p^A + \xi_n S_n^A\right)^2 Fermi(R_A^{improved}, a, q)^2$$

$$\frac{d\sigma^{SD}_{precise}}{dE} = \frac{16\mu_\chi^2}{(2J_A+1)E_{max}} \left(S_{00}^A(q)a_0^2 + S_{01}^A(q)a_0a_1 + S_{11}^A(q)a_1^2\right)$$

where $a_0 = \xi_p + \xi_n$, $a_1 = \xi_p - \xi_n$.

We have implemented all spin dependent form factors and S_p^A, S_n^A coefficients collected in review

V. A. Bednyakov and F. Simkovic, Phys. Part. Nucl. 37 *(2006) S106–S128*

For nuclei ^{19}F, ^{23}Na, ^{27}Al, ^{29}Si, ^{39}K, ^{73}Ge, ^{93}Nb, ^{125}Te, ^{127}I, ^{129}Xe, ^{131}Xe, ^{207}Pb **we have presented two sets of form factors/coefficients with names**

`S00I127, S01I127, S11I127` ***(Bonn-A potential)***

`S00I127A, S01I127A, S11I127A` ***(Nijmegen II potential)***

DM velocity distribution

Two distributions are implemented.

Maxwell distribution motivated by isothermal model

$$d\rho/d^3V \sim exp(-|\vec{V} - \vec{V}_{Earth}|^2/V_{rot}^2)\Theta(v_{max} - |\vec{V}|)$$

Default parameters $V_{rot} = 220km/s$, $V_{Earth} = 225.2km/s$, $V_{vmax} = 700km/s$ can be overwritten by $SetfMaxwell(V_{rot}, V_{Earth}, V_{max})$

δ-function distribution corresponds to constant velocity DM stream with velocity which which can be specified by $SetfDelta(V_{DM})$.

The main function 'nucleusRecoil'

nEvents = nucleusRecoil(ρ_{DM},$dFdv$,A, Z,J,S_{00},S_{01},S_{11},box,$dNdE$)

ρ_{DM} density of DM near the Earth $\sim 0.3 GeV/cm^3$

dFdv velocity distribution normalized $\int_0^\infty v \cdot dFdv(v)dv = 1$

`fDvMaxwell, fDvDelta` or any user function can be substituted.

For Z and J there are predefined constants like `Z_Ge, J_Ge73`.

The main output value is $dNdE[200]$ which gives recoil energy distribution with step 1KeV normalized to $kg \cdot day$ of exposure.

$dNdE[200]$ can be integrated **cutRecoilResult(dNdE, E1,E2)**
and presented as a plot **displayRecoilPlot(dNdE, text, E1, E2)**

If spin depended form factor are not known one can use

nEvents = nucleusRecoil0(ρ_{DM},$dFdv$,A, Z,J,Sp_A,Sn_A,box,$dNdE$)

For Sp_A, Sn_A there are predefined constants **Sp_He3, Sn_He3**

Example: DAMA signal in E1 - E2 recoil energy interval

```
SetfMaxwell(vRotation,vSun+15,vEsc);  /* Summer  signal */
nucleusRecoil(0.3,fDvMaxwell,23,Z_Na,J_Na23,S00Na23,S01Na23,S11Na23,FeSc
                                                              dNdE_Na);
nucleusRecoil(0.3,fDvMaxwell,127,Z_I,J_I127,S00I127,S01I127,S11I127,FeSc
                                                              dNdE_I);
N_summer=  (23*cutRecoilResult(dNdE_Na,E1/q_na,E2/q_na)
         +127*cutRecoilResult(dNdE_I,E1/q_i,E2/q_i))/(23+127);

SetfMaxwell(vRotation,vSun-15,vEsc);  /* Winter  signal */
nucleusRecoil(0.3,fDvMaxwell,23,Z_Na,J_Na23,S00Na23,S01Na23,S11Na23,FeSc
nucleusRecoil(0.3,fDvMaxwell,127,Z_I,J_I127,S00I127,S01I127,S11I127,FeSc
N_winter=  (23*cutRecoilResult(dNdE_Na,E1/q_na,E2/q_na)
         +127*cutRecoilResult(dNdE_I,E1/q_i,E2/q_i))/(23+127);

Modulation=(N_summer - N_winter)/(E1-E2);   /*  day kg KeV */
```

One byproduct!

nEvents = nucleusRecoilAux(ρ_{DM}, $dFdv$, A, Z, J, S_{00}, S_{01}, S_{11}, M_{wimp}, CS_P^{SI}, $\pm CS_N^{SI}$, CS_P^{SD}, $\pm CS_N^{SD}$, $dNdE$**)**

Assuming that we know WIMP mass and cross sections one can predict a DM signals on different detectors.

Can set limits on cross sections when no signal is seen in a detector

By this way we reproduce experimental plots for limits on WIMP-nucleon cross sections for Xenon10, CDMS, and DAMA experiments.

The last slide.

For details see

Dark matter direct detection rate in a generic model with micrOMEGAs_2.1.
G. Belanger, F. Boudjema, A. Pukhov, A. Semenov.
e-Print: arXiv:0803.2360 (hep-ph)

Current version is micrOMEGAs_2.2. Public code can be found
http://lappweb.in2p3.fr/lapth/micromegas/

There is WEB online version (in progress) prepared by Rashid Lemrani
http://pisrv0.pit.physik.uni-tuebingen.de/darkmatter/micromegas
See his talk.