5th International Workshop on Mechanisms of Vacuum Arcs

Modern theory of field electron emission

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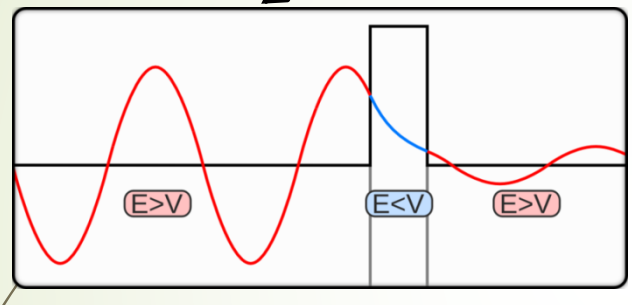
Outline

1) JWKB approximation for 1-D problems
2) Classical theory of field emission (FE) (Fowler-Nordheim, planar emitters)
3) Modern FE theory (curved-nanometric emitters): Generalization of the F-N equation
4) 3-D JWKB approximation for the analysis of the beam distribution
5) Conclusions

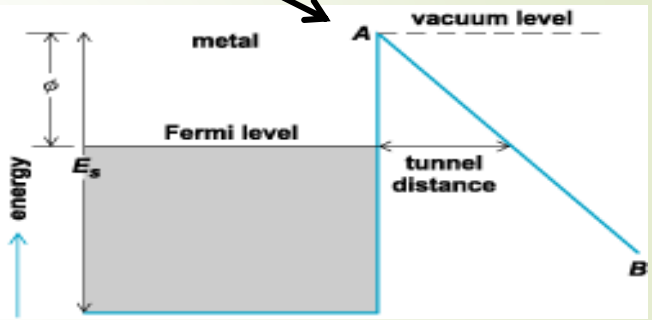
1-D tunneling and JWKB (1926)

Tunneling problems solved exactly-algebraically



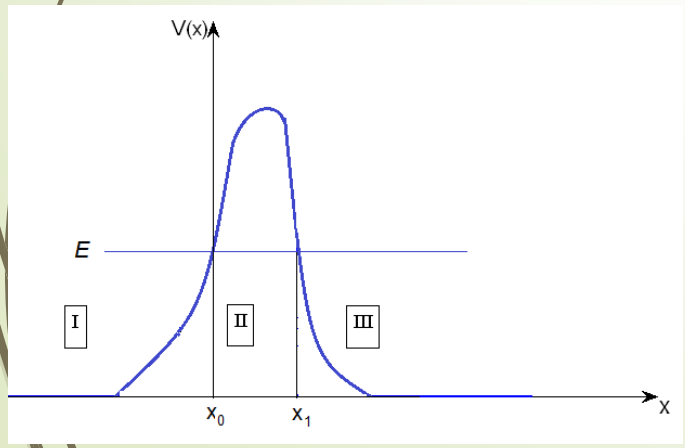
Rectangular Barrier

- Plane waves
- $T \sim \exp(-g\sqrt{V-E}L)$



Triangular barrier

- Airy functions
- $T \sim \exp(-g\frac{2}{3}\varphi^{\frac{3}{2}}/F)$



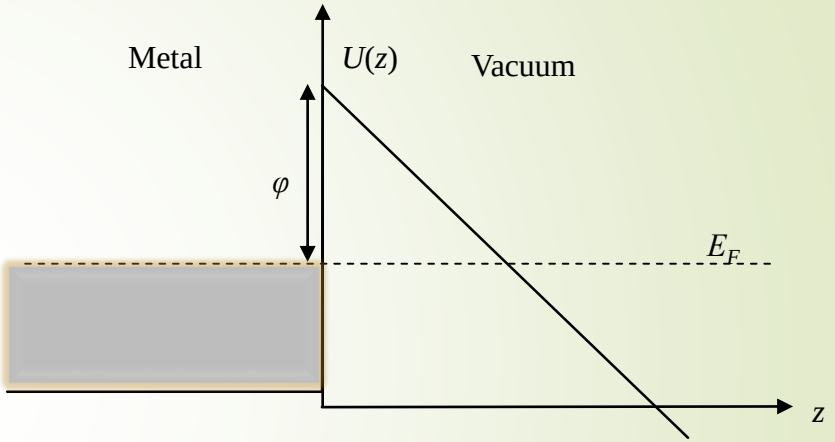
The general barrier cannot be solved exactly. But the JWKB approximation gives a very good representation of the transmission coefficient T:

$$T \sim exp\left(-g\int_{x_0}^{x_1}\sqrt{V-E}\,dx\right)$$

First explanation of Field emission (F-N 1928)

- Exact triangular barrier tunneling:

$$V(z) = \varphi - Fz$$

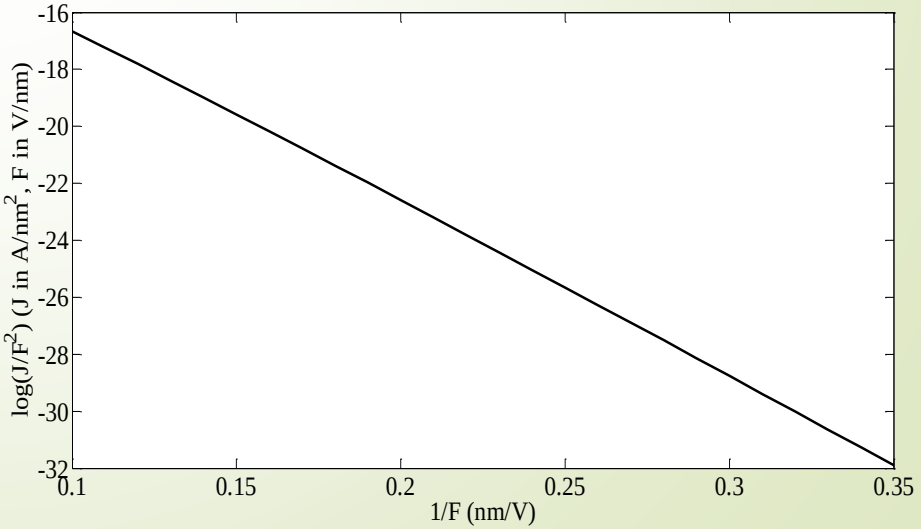


- Free-electron model

$$J = \frac{emk_B T_{th}}{2\pi^2 \hbar^3} \int_{-\infty}^{+\infty} T(E) \log\left[1 + \exp\left(\frac{E_F - E}{k_B T_{th}}\right)\right] dE$$

- First Fowler-Nordheim equation:

$$J \propto F^2 \exp\left(-\frac{4\sqrt{2m}}{3\hbar}\frac{\varphi^{3/2}}{eF}\right) \longrightarrow$$



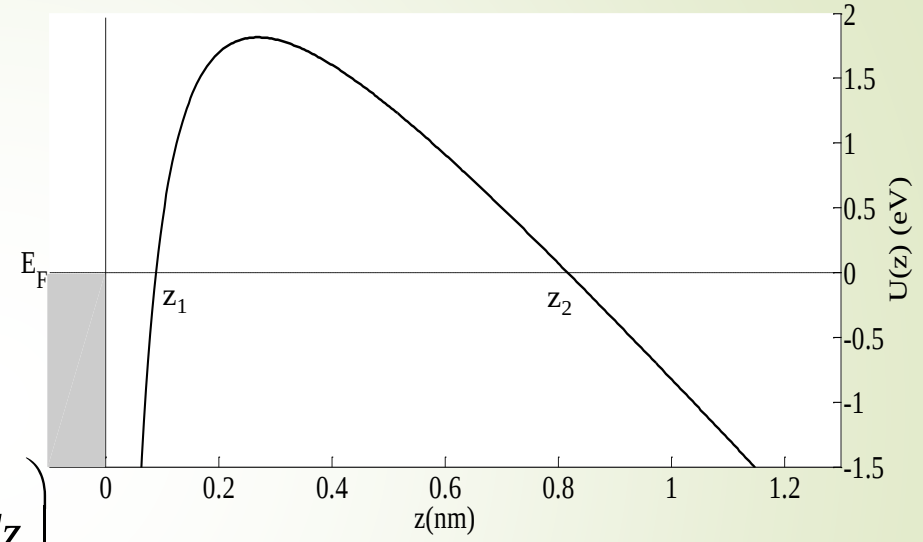
$\log\left(I/V^2\right) - 1/V$ Plot : straight line

Image correction (Nordheim 1928)

- Emitted current density
  - Image potential correction

$$V(z) = \varphi - Fz - b/z$$

  - Transmission coefficient can be found by the JWKB approximation:



$$T(E) = \exp\left(-\frac{2}{\hbar}\int_{z_1}^{z_2}\sqrt{2m(V(z)-E)}\,dz\right)$$

$$T(E) = \exp\left[-\frac{4\sqrt{2m}}{3\hbar}\frac{(\varphi - E)^{3/2}}{eF}v(y)\right] \qquad y = 2\sqrt{bF}/(\varphi - E)$$

$$v(y) = \frac{3}{2}\int_{\zeta_1}^{\zeta_2}\sqrt{1-\zeta-y^2/4\zeta}\,d\zeta = \sqrt{1+y}\left[E\left(\frac{1-y}{1+y}\right) - yK\left(\frac{1-y}{1+y}\right)\right]$$

Standard form of F-N equation

$$J\left(F,\varphi,T_{th}\right)=\sigma\Theta\frac{e^3}{8\pi\hbar}\frac{F^2}{t^2(y)\varphi}\exp\left(-\frac{4}{3}\frac{\sqrt{2m}}{\hbar}\frac{v(y)\varphi^{3/2}}{eF}\right)$$

$$t(y)=v(y)-\frac{2}{3}yv'(y)$$

- Correction when T_{th}>0 (Murphy & Good 1956):

$$\Theta=1+\frac{1}{6}\left(\pi d_0 k_{\mathrm{B}}T_{th}\right)^2+O\left(\left(\pi d_0 k_{\mathrm{B}}T_{th}\right)^4\right) \qquad d_0=\frac{2\sqrt{2m}}{\hbar}\frac{\varphi^{1/2}}{eF}t(y)$$

- Correction due to band structure different to free electron metal (Modinos 2001): Multiply by an unknown correction pre-factor σ varying between 0.1 and 100 in standard FE cases.
- Sharp emitters, enhancement factor correction:

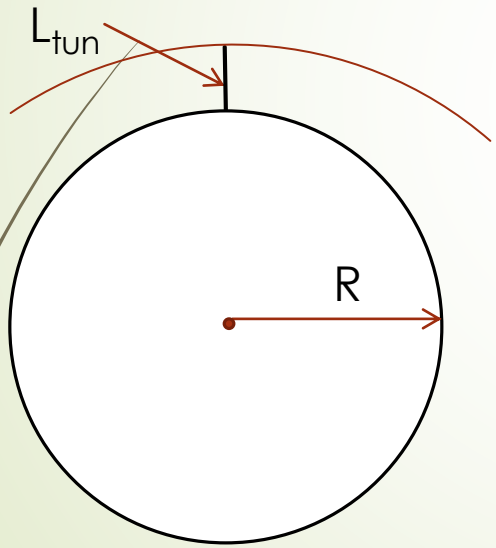
$$F=\gamma F_M, F=\beta V$$

Theoretical problems of the F-N theory

$$J = at^{-2}(y)\frac{F^2}{\varphi}\exp\left[-\frac{2}{3}g\frac{\varphi^{3/2}}{eF}v(y)\right]$$

inadequate

- Assumption:
  Planar emitter
- When can it be considered planar?

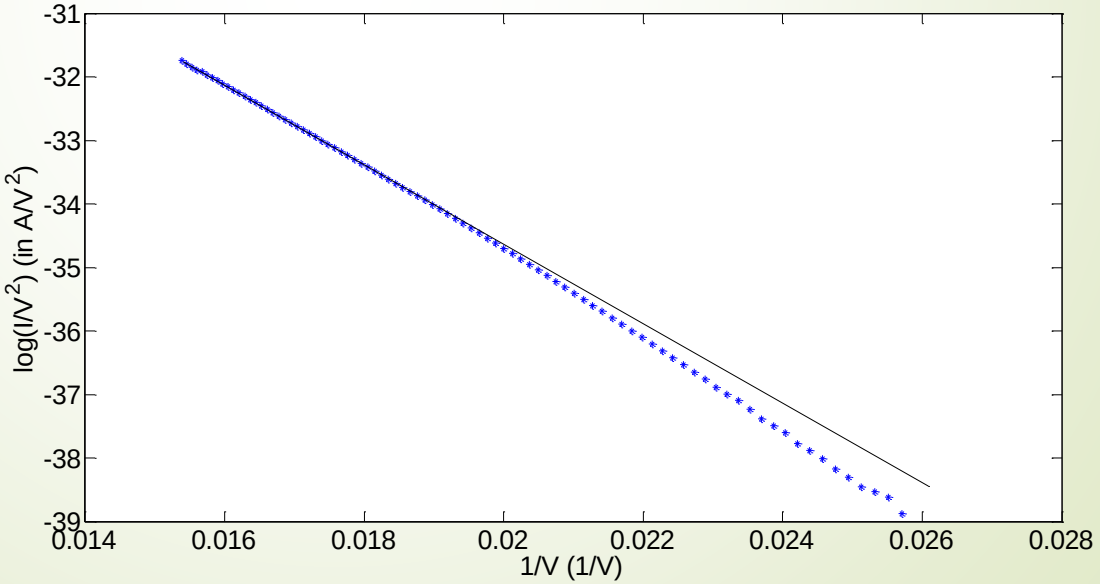


- Sphere~Plane ὁταν $L_{tun} << R$
- Does this hold nowadays?
- $L_{tun} \approx 0.5\text{-}2nm$
- R curvature nowadays ~1-25nm

Non-planar!!

FN equation inadequacy evidence

- Theoretically known since the 90's:
  - P.H Cutler et al, *Progress in Surface Science* 42 p169, (1993)
  - G.N. Fursey, D.V. Galzanov, JVSTB 16(2), p910 (1998)
  - C.J. Edgcombe PRB (2007)
  - A. Mousa, M.S. Fischer, R.G. Forbes, JBSTB (2013)
- Experimental evidence: curved F-N plot

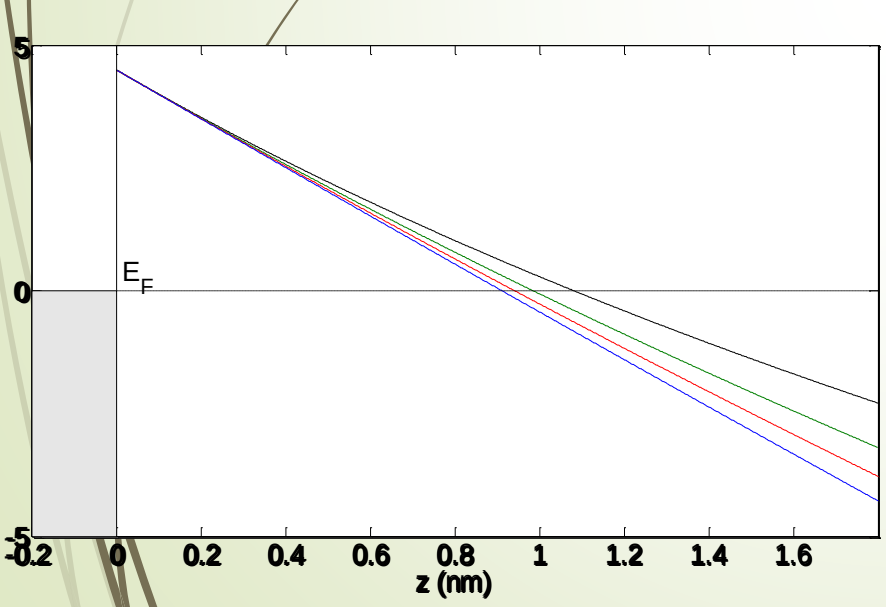


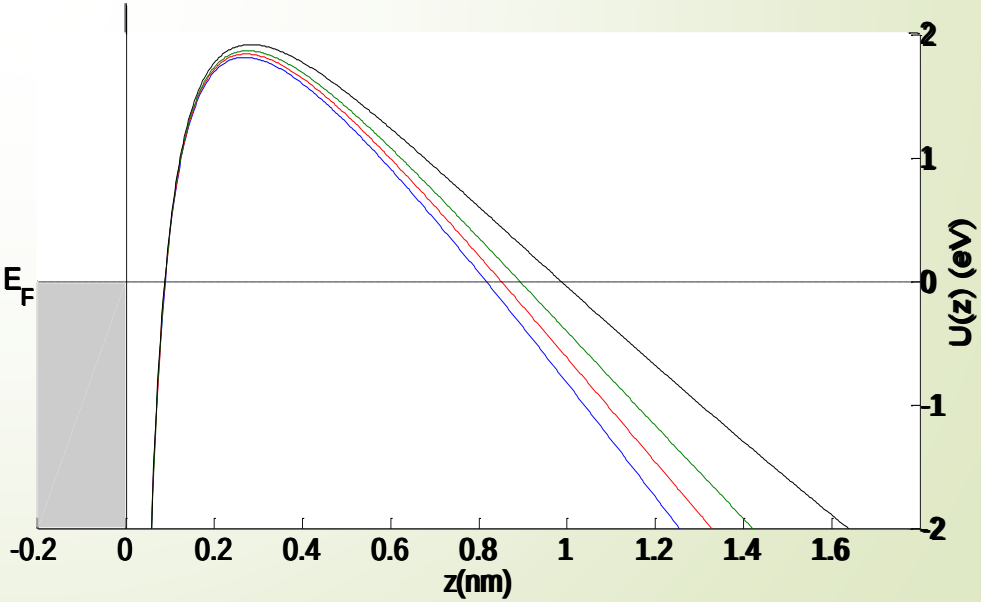
- ✓ H. Cabrera et. al. PRB 2013: If the F-N equation is used to extract the work function φ, an erroneous value 5.5eV is found for Tungsten.

Numerical results: Non-linearity of the potential

- Basic problem when the emitter is curved:
  Electrostatic potential is not linear: U(z)≠φ-Fz-Image
- ✓ Not equivalent to planar-plate capacitor

R=100nm (FN) R=20nm R=10nm R=5nm

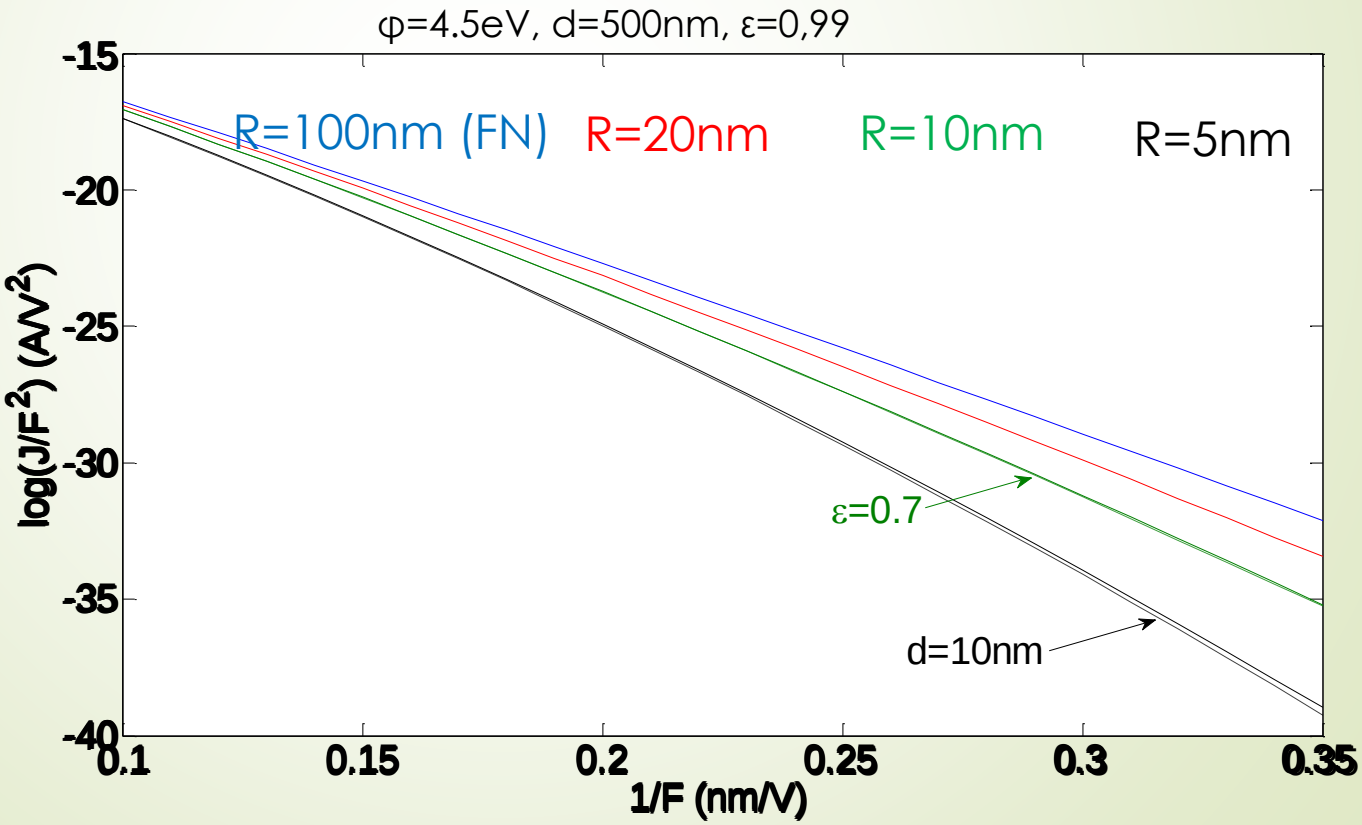




F=5V/nm, φ=4.5eV, d=500nm, ε=0,99

Numerical results: Current density and current

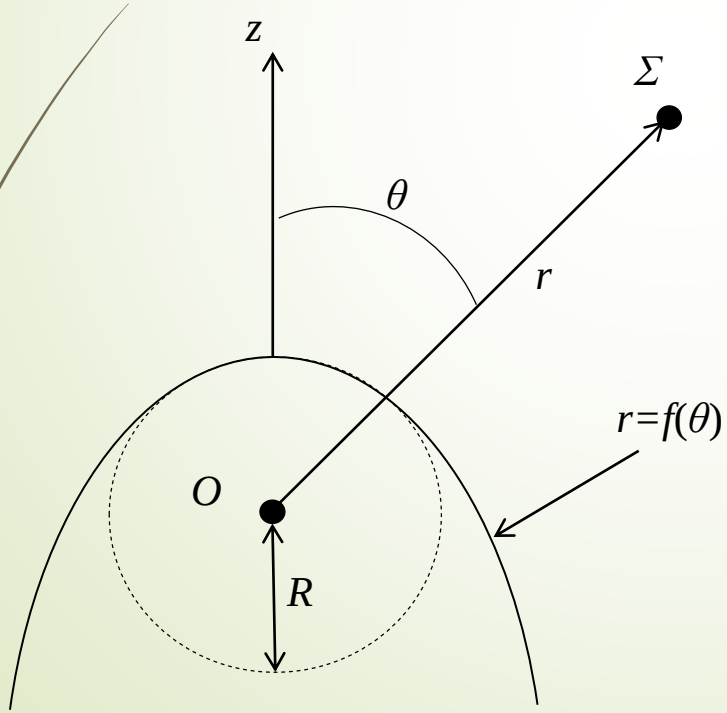
- J-F plot: Very strongly dependent on R
- Almost independent on other geometry parameters



Algebraic approximation and extension of the F-N equation

$$\Phi(z) = \Phi(0) + Fz + \frac{1}{2}\frac{\partial^2\Phi}{\partial r^2}\bigg|_{(R,0)} z^2 + \frac{1}{6}\frac{\partial^3\Phi}{\partial r^3}\bigg|_{(R,0)} z^3 + \ldots = Fz\left(1 - \frac{z}{R} + O\left(\frac{z}{R}\right)^2\right)$$

We keep up to the 2nd term

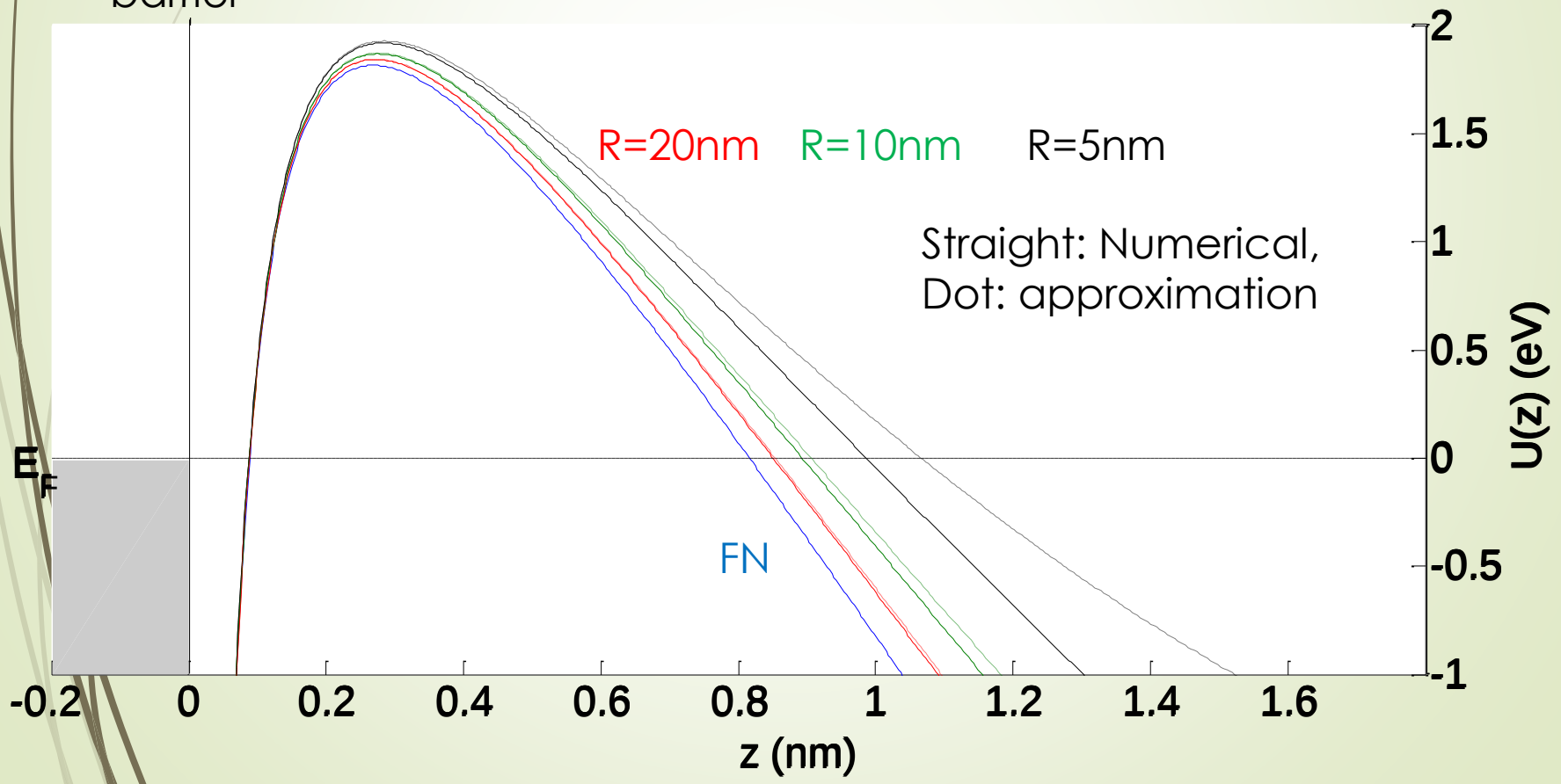


- ✓ Tunneling region length $L_{tun} \sim \varphi/eF$

Inserting the 2nd order term

Approximation for the tunneling barrier

$$\longrightarrow U(z) = W - eFz - \frac{e^2}{16\pi\varepsilon_0}\frac{1}{z(1+z/2R)} + \frac{eF}{R}z^2$$

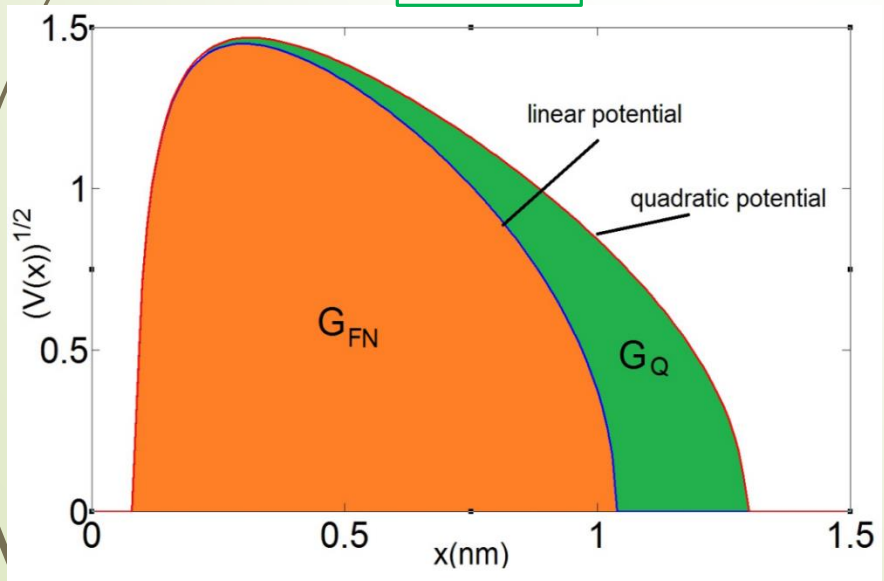


The JWKB integral

$$G = g\frac{\varphi^{3/2}}{eF}\int_{z_1}^{z_2}\sqrt{\varphi - eFz - \frac{b}{z(1+z/2R)} + \frac{eF}{R}z^2}\,dz$$

$$G = g\frac{\varphi^{3/2}}{eF}\int_{\zeta_1}^{\zeta_2}\sqrt{1-\zeta - \frac{y^2/4}{\zeta + x\zeta^2/2} + x\zeta^2}\,d\zeta \qquad \zeta = \frac{eF}{\varphi}z,\ x = \frac{\varphi}{eFR},\ y = 1.2e\sqrt{Vnm}\frac{\sqrt{F}}{\varphi}$$

$$\Xi(x,y) = \Xi(0,y) + x\left.\frac{\partial\Xi}{\partial x}\right|_{x=0} + O(x^2) \longrightarrow G = \frac{2}{3}g\left(\frac{\varphi^{3/2}}{eF}v(y) + \frac{1}{R}\frac{\varphi^{5/2}}{(eF)^2}\omega(y)\right)$$



$$v(y) = \frac{3}{2}\int_{\zeta_1}^{\zeta_2}\sqrt{1-\zeta - y^2/4\zeta}\,d\zeta$$

$$\omega(y) = \frac{3}{4}\int_{\zeta_1}^{\zeta_2}\frac{\zeta^2 + y^2/8}{\sqrt{1-\zeta-y^2/4\zeta}}d\zeta$$

The current density J

- Given the Gamow exponent we may obtain J

$$J = Z_S \left[\frac{\partial G}{\partial \varphi}\right]^{-2} \exp(-G)$$

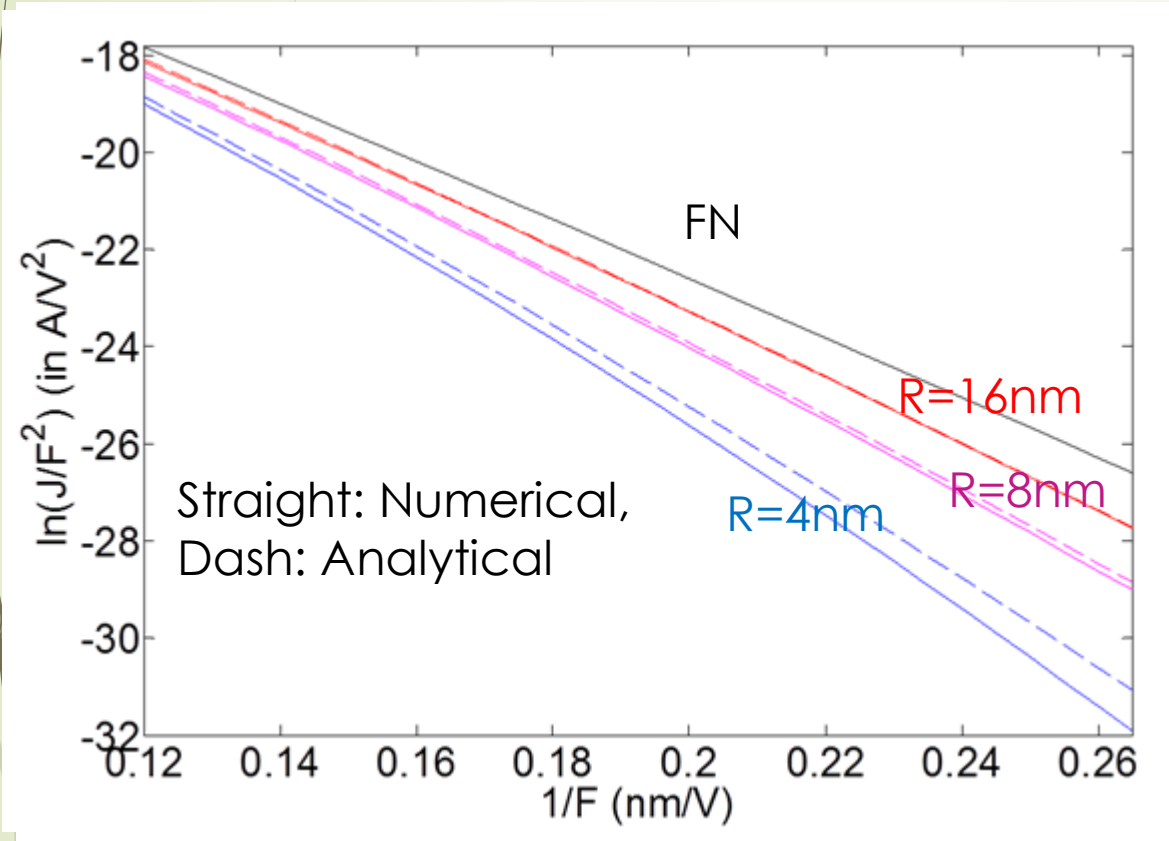
$$J = \frac{Z_S}{g^2}\frac{F^2}{\varphi}\left(t(y) + \frac{\varphi}{eFR}\psi(y)\right)^{-2} \exp\left[-\frac{2}{3}g\left(\frac{\varphi^{3/2}}{eF}v(y) + \frac{1}{R}\frac{\varphi^{5/2}}{(eF)^2}\omega(y)\right)\right]$$

$$v(y) \approx 1 - y^2 - (y^2 \ln y)/3$$
$$t(y) \approx 1 + y^2/9 - (y^2 \ln y)/11$$

$$\omega(y) \approx 4/5 - 7y^2/40 - (y^2 \ln y)/100$$
$$\psi(y) \approx 4/3 - y^2/500 - (y^2 \ln y)/15$$

- When φ/eFR→0 all green parts→0…
  Reduces to the standard FN equation (expected)

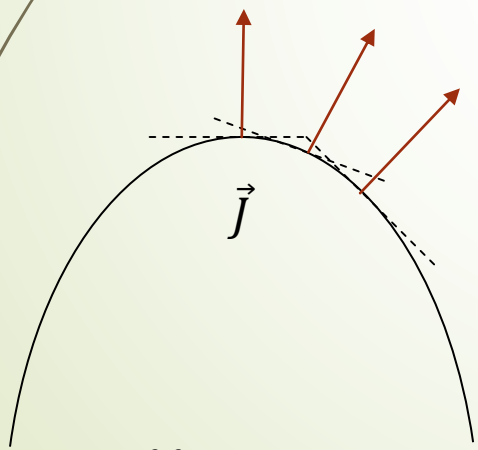
Algebraic Results: Current density



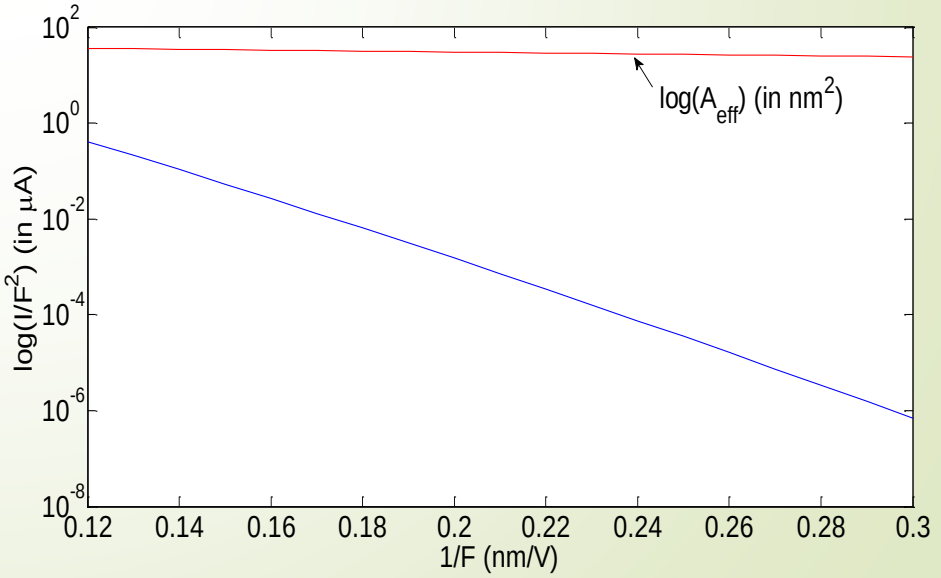
- Very good results for R>4-5nm
- Orders of magnitude better than F-N
- The F-N plot is significantly curved
- The curvature of the F-N plot depends on R
- **We may extract R from I-V measurements!**

3-D extension: the effective area A_{eff}

- The classical theory is planar. It does not give information about the beam distribution
- 3-D extension: Assume adjacent planes
- In every plane we introduce the field F(θ) in the F-N equation and obtain J(θ).
- Sufficient approximation: A_{eff}=const

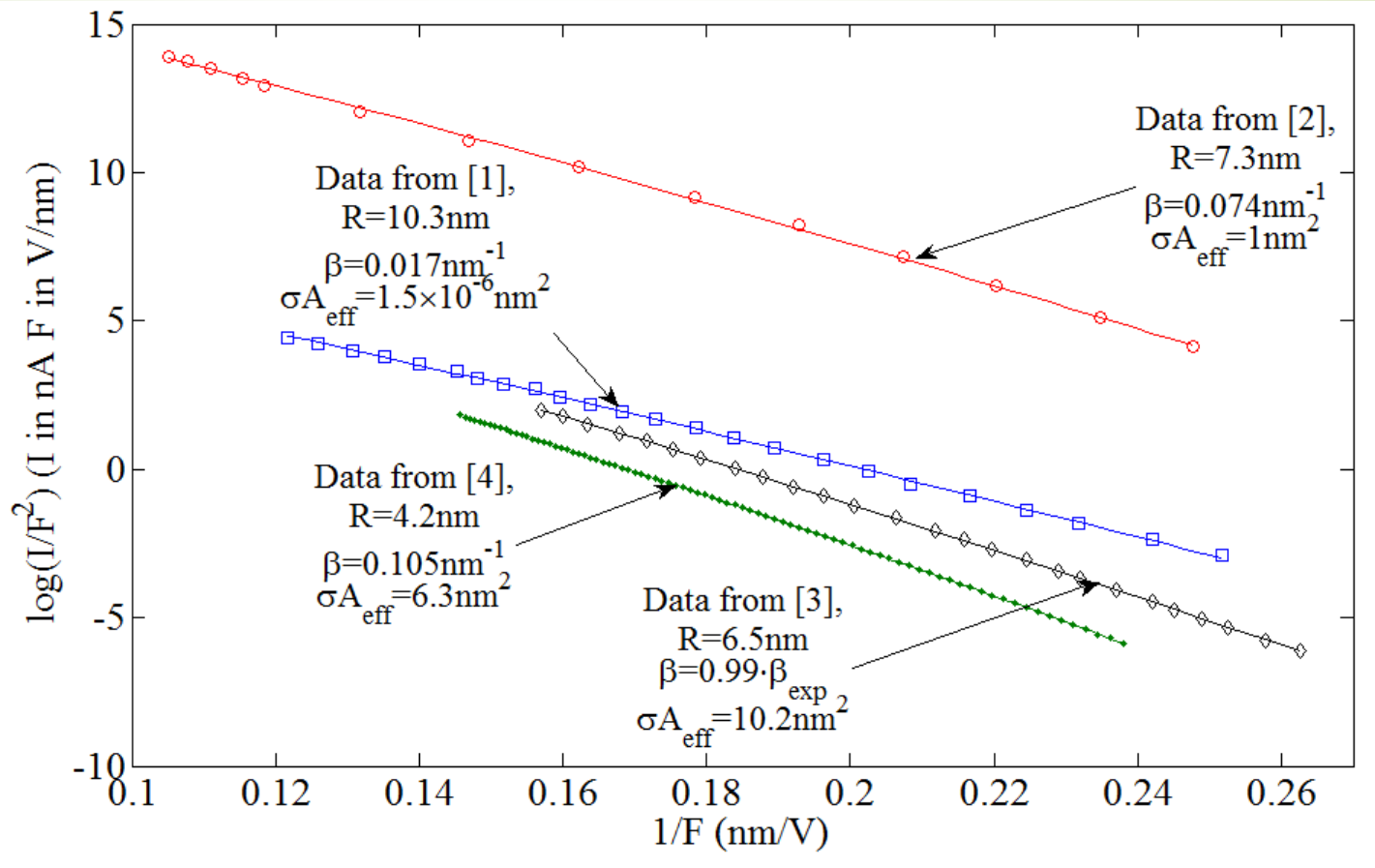


$$I = \iint_{\substack{emitting \\ surface}} \vec{J} \cdot d\vec{S} = JA_{eff}$$



Extraction of R from I-V data

- From experimental to theoretical variables: $I_{exp}=\sigma A_{eff}\cdot J$, $F=\beta V$
- We can fit the I-V measurements to our equation and extract adjustable parameters σA_{eff}, β and the radius R
- We have developed a computer program to do it in sec.



The Weaknesses of the 1-D theory

- When we are interested in the current density distribution, 1-D JWKB is not enough.

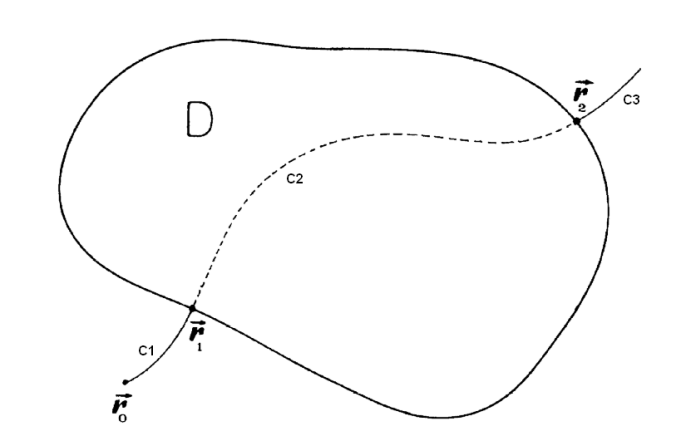
  Radial current – straight line paths-> The 3-D analysis of the tunneling problem requires more complicated calculations when we are interested on the current distribution

3-D JWKB Approximation (Kapur-Peierls 1936, Das-Mahanty 1987)

- Extension of the standard JWKB semiclassical approximation to give answer to 3-D problems
- The WKB integration is performed along the "electron paths"
- The "electron paths" are found by solving the Newtonean mechanics problem in the inaccessible region, with the inverted barrier (V-> –V, E-> –E)

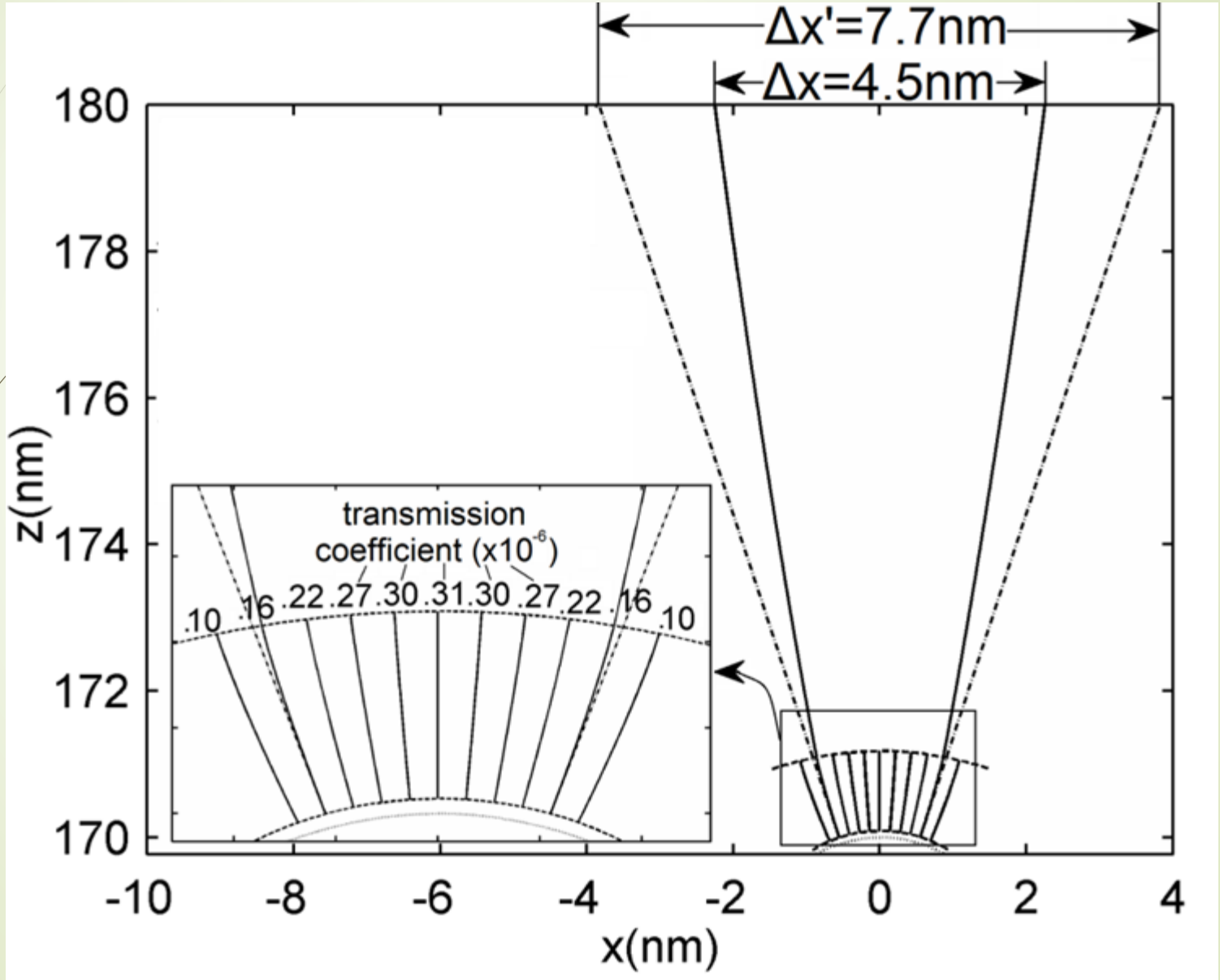
$$T = \exp(-G)$$

$$G = \frac{2}{\hbar} \int_{\vec{r}_1(C_2)}^{\vec{r}_2} \sqrt{2m(V(\vec{r}) - E)}\, d\vec{r}$$

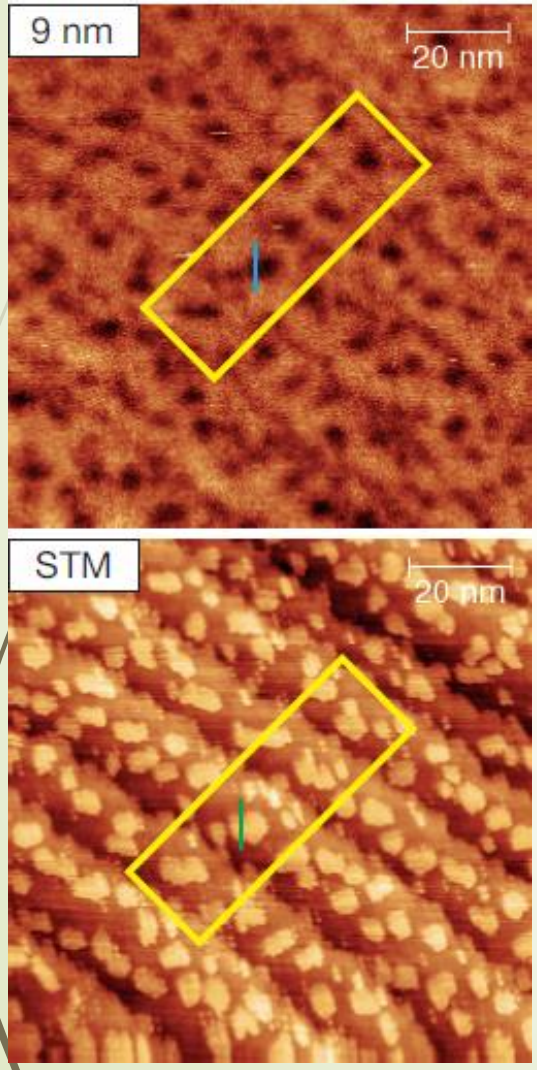


$$C_2 : \left(\frac{d\vec{r}}{d\tau}\right)^2 = \frac{2}{m}\left(V(\vec{r}) - E\right)$$

Beam width calculation



The problem of the lateral resolution of the NFESEM



- The LR of the NFESEM for d=9nm is LR~3nm.
- The 1-D FE theory predicts a beam width of about BW~17nm

Incompatible

- 3-D JWKB calculations with curved line paths predict BW~4.5 nm in much better agreement

Conclusions

1. Original F-N theory corrected many times since 1928
2. Standard FE theory needs extension to include nanometric emitters' curved electrostatic potential
3. Both numerical solution and algebraic generalization of the standard F-N equation
4. Extraction of R from experimental I-V data
5. Use of 3-D JWKB approximation with curved-line electron paths when emitted current distribution has to be calculated

Full reference papers

- **Generalized F-N equation:**

A. Kyritsakis and J.P. Xanthakis, "*Derivation of a Fowler-Nordheim type equation for nanoscopic field-emitters*", Proc. R. Soc. A 471: 2176 http://dx.doi.org/10.1098/rspa.2014.0811

- **3-D JWKB analysis of FE current distribution:**

- A. Kyritsakis, G.C. Kokkorakis, J.P. Xanthakis, T.L. Kirk, and D. Pescia. "*Self focusing of field emitted electrons at an ellipsoidal tip*", Appl. Phys. Lett. 97: 023104 (2010) http://dx.doi.org/10.1063/1.3462934
- A. Kyritsakis and J.P. Xanthakis. "Beam *spot diameter of the near-field scanning electron microscopy*", Ultramicroscopy 125: p.24-28. (2013) http://dx.doi.org/10.1016/j.ultramic.2012.10.015

Thank you very much!!

- Looking for a post-doc position in the field of theory-simulation in applied physics
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