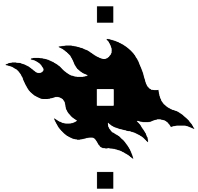
Implementing the general thermal-field emission equation to the high electric field nanoprotrusion model

Kristjan Eimre [1], Stefan Parviainen [2], Alvo Aabloo [1], Flyura Djurabekova [2], Vahur Zadin [1]



[1] University of Tartu [2] University of Helsinki

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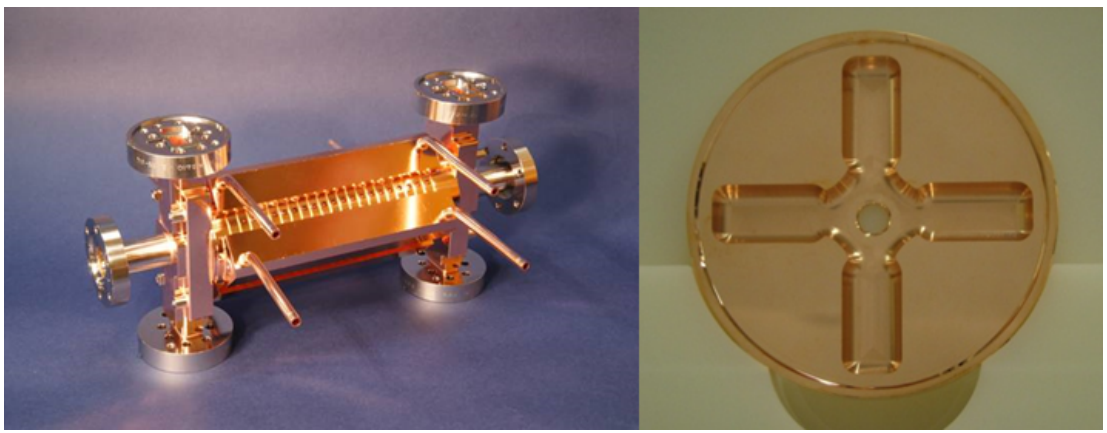
Overview of the problem

- high electric fields cause vacuum breakdowns
- CLIC (up to $E \approx 250\,\mathrm{MV/m}$)
- electrodes have nanoscale protrusions, where the applied field is amplified (β) significantly
- emission currents flow out of the protrusions
- emission currents heat the protrusion until melting

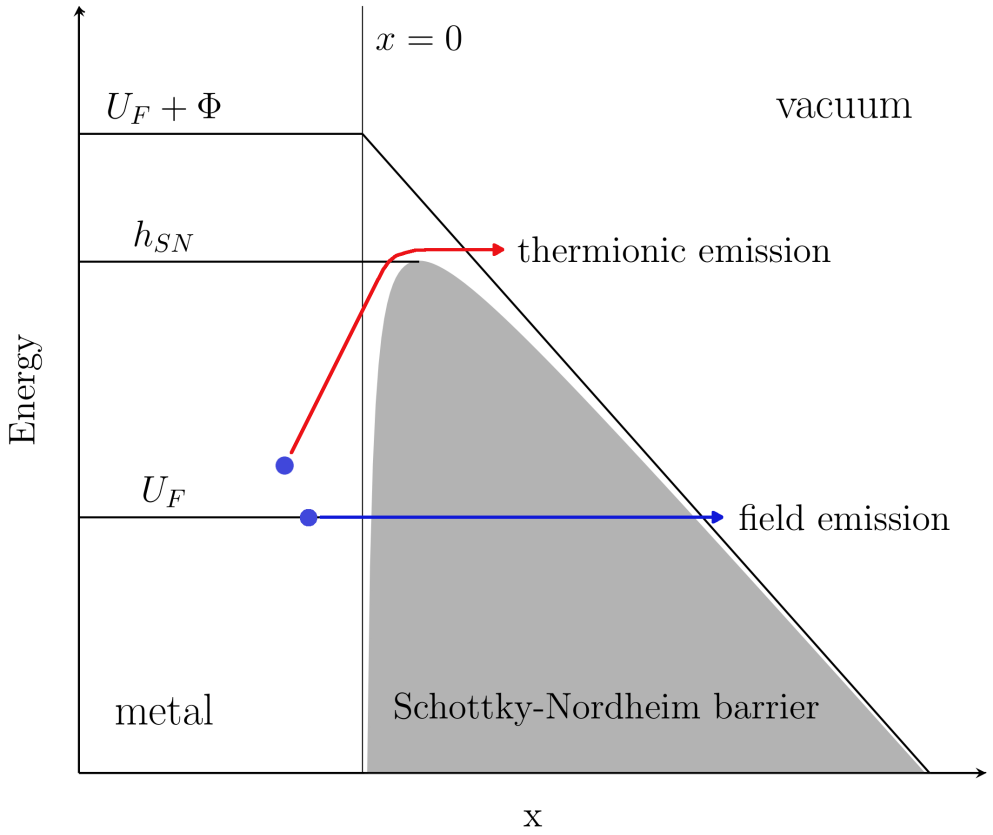


Aim of the work

- improve the accuracy of the field emitter simulations
  - implement GTF equation instead of FN
  - implement Nottingham effect
- study nanoprotrusion using FEM in high electric fields
- compare the GTF and FN equations in context of the nanoprotrusion model
- investigate possible thermal effects to the Fowler-Nordheim plot
- evaluate the influence of the Nottingham effect

Electron emission mechanisms

- thermionic emission: thermal excitations
- field emission: quantum tunnelling
- Nottingham effect: emitting electrons above U_F causes cooling, under U_F causes heating.



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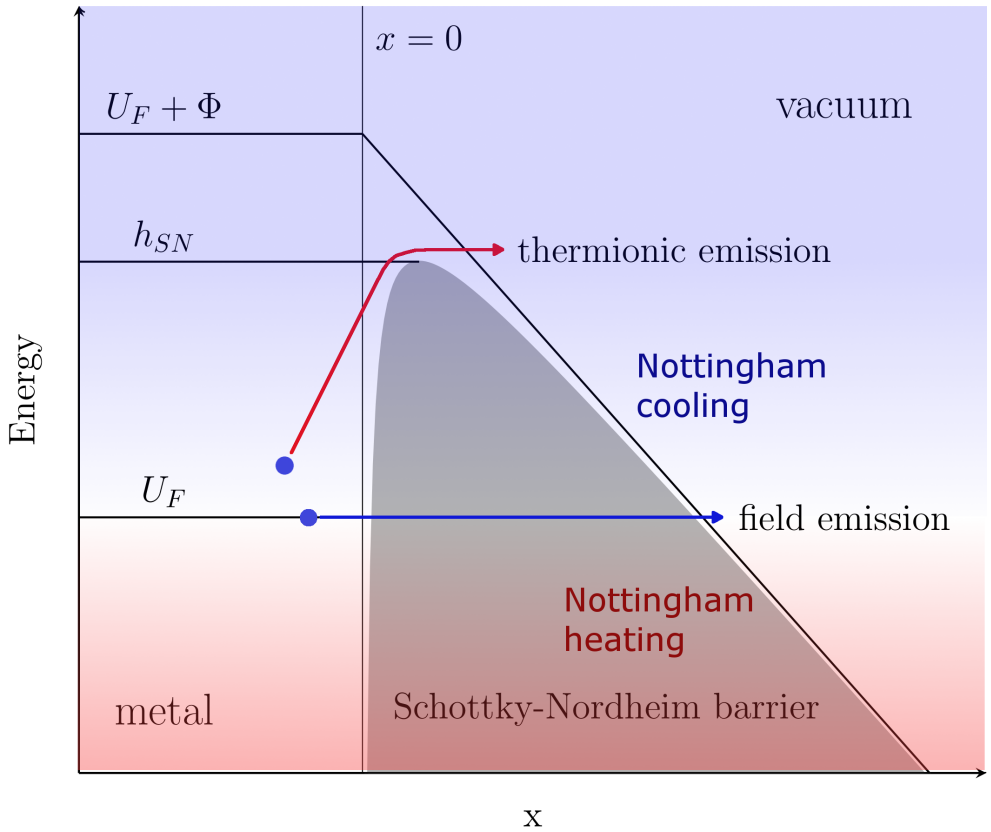
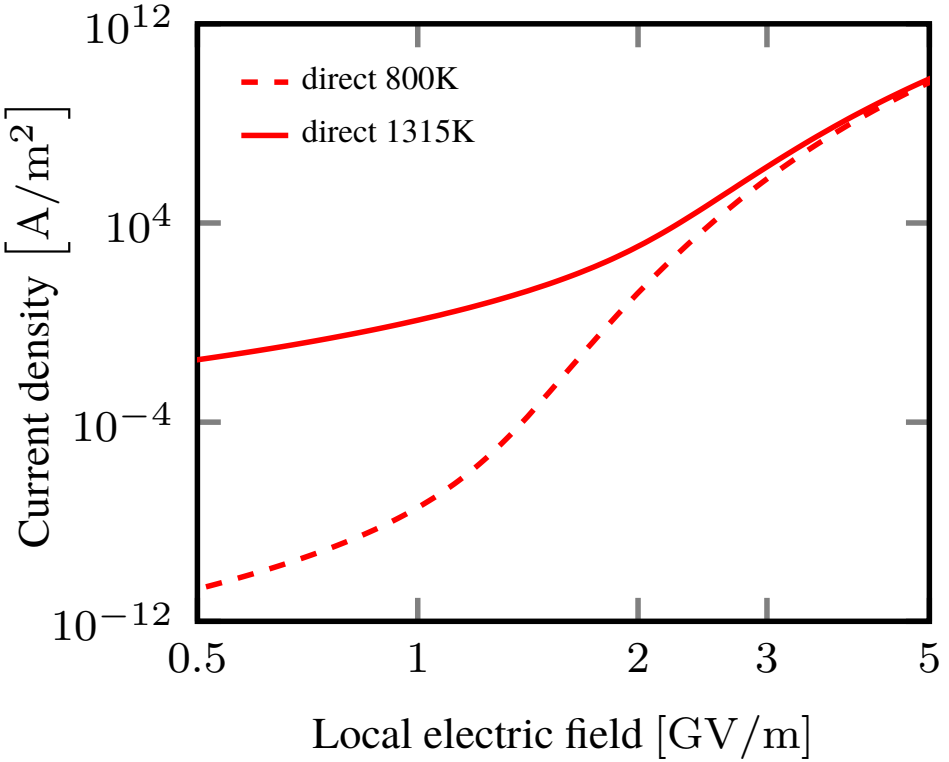


Direct integration

$$J(F,T) = q \int N(U) D(U) dU,$$

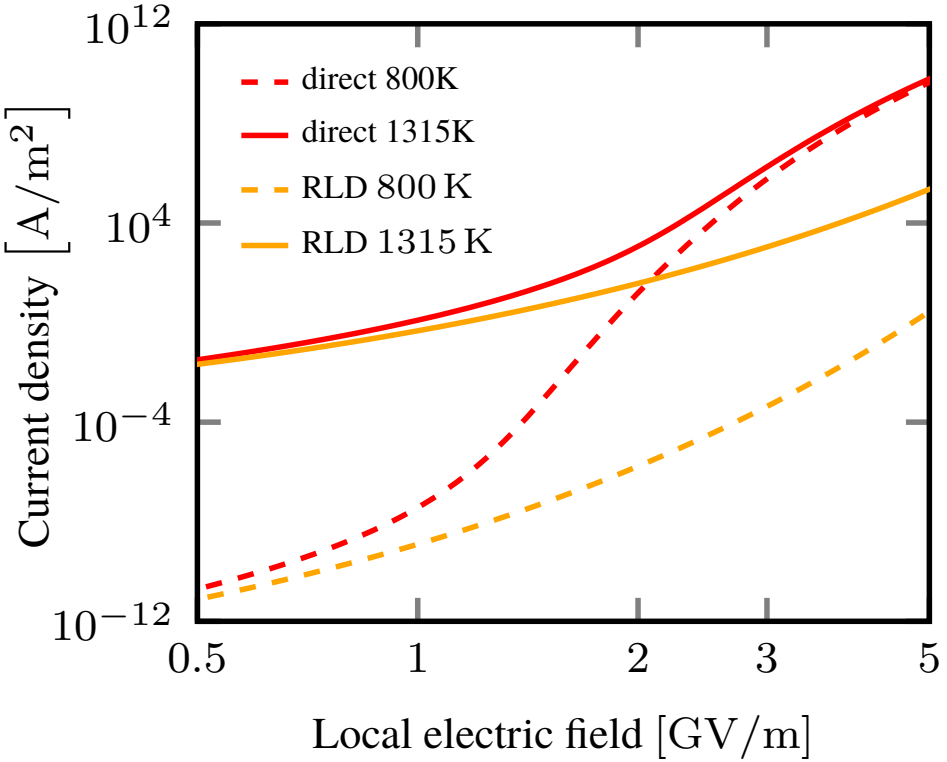
- $N(U,T)$ - supply function
- $D(U,F)$ - tunnelling probability
- in computationally intensive simulations, analytical expression is favourable



Richardson-Laue-Dushman equation

$$J_{\mathrm{RLD}}(F,T) = AT^2 \exp\left(\frac{-\Phi + \sqrt{\frac{q^3 F}{4\pi\epsilon_0}}}{k_B T}\right)$$

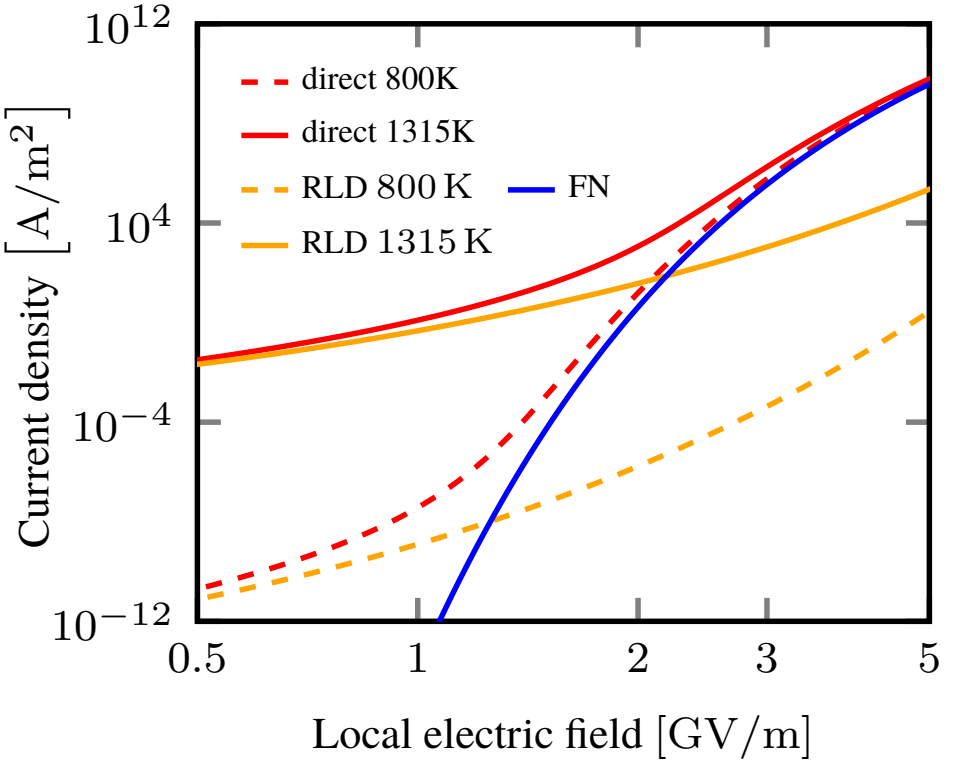
- only thermionic emission, no quantum tunnelling
- Schottky correction - barrier lowering



Fowler-Nordheim equation

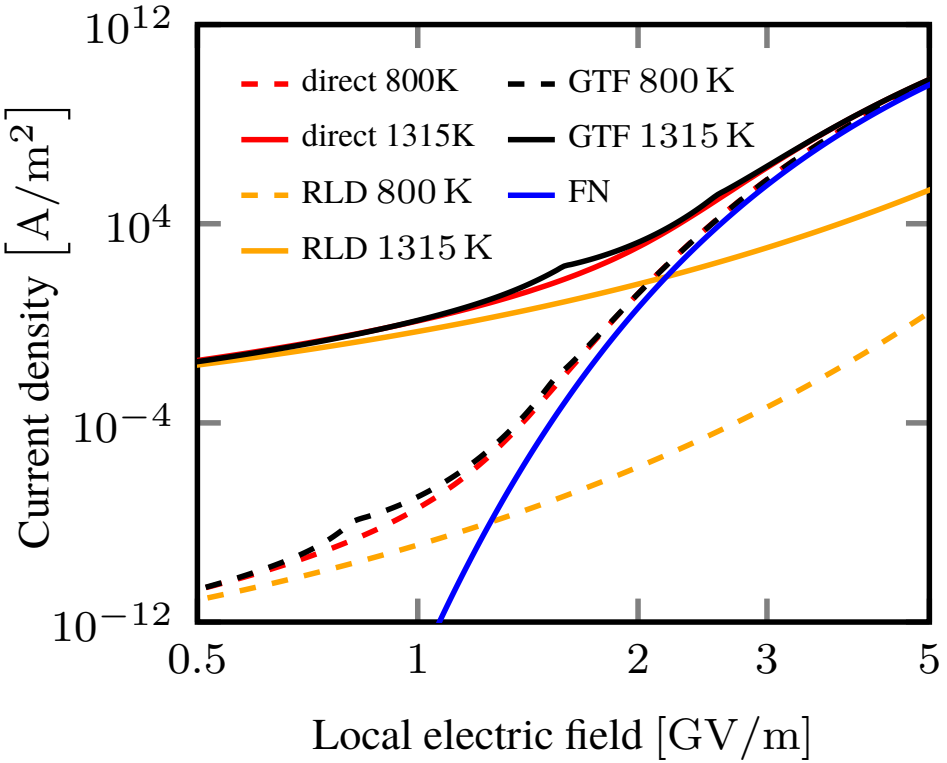
$$J_{FN}(F) = \frac{aF^2}{\Phi} \exp\left(-\frac{b\Phi^{\frac{3}{2}}}{F}\right)$$

- standard (Murphy and Good 1956) assumes 0 K
- takes only field emission into account, disregards thermionic emission



General thermal field emission equation

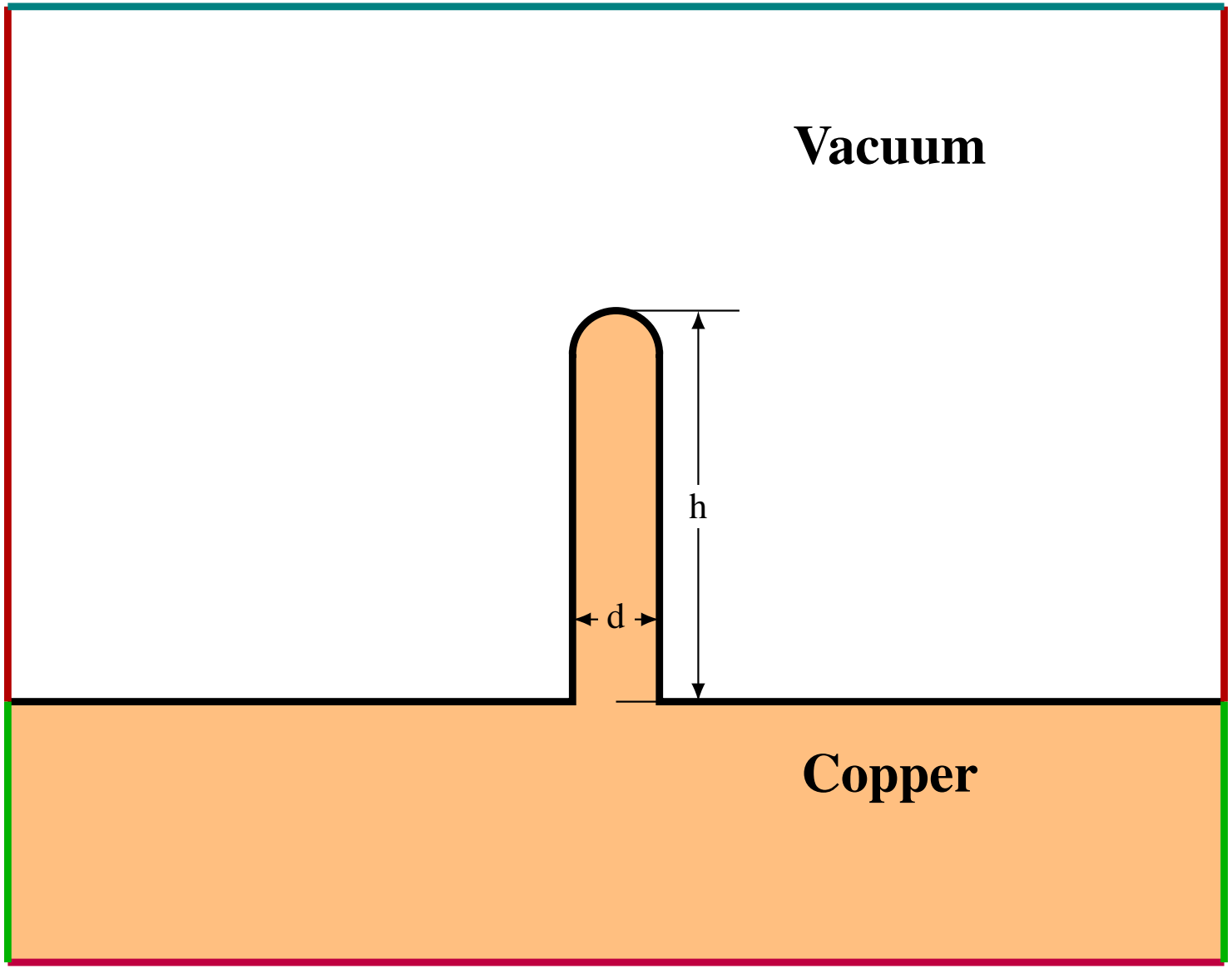
- linear expansion of WKB area near the integrand maximum
- describes the current well in thermionic, field and transition regions



$$J_{GTF}(F,T) = A_{RLD} T^2 N \left(\frac{\beta_T}{\beta_F(U_m)}, \beta_F(U_m)(U_o - \mu) \right)$$

K. L. Jensen, J. Appl. Phys. (2007)
K. Eimre *et al.*, J. Appl. Phys. (2015)

Simulated system



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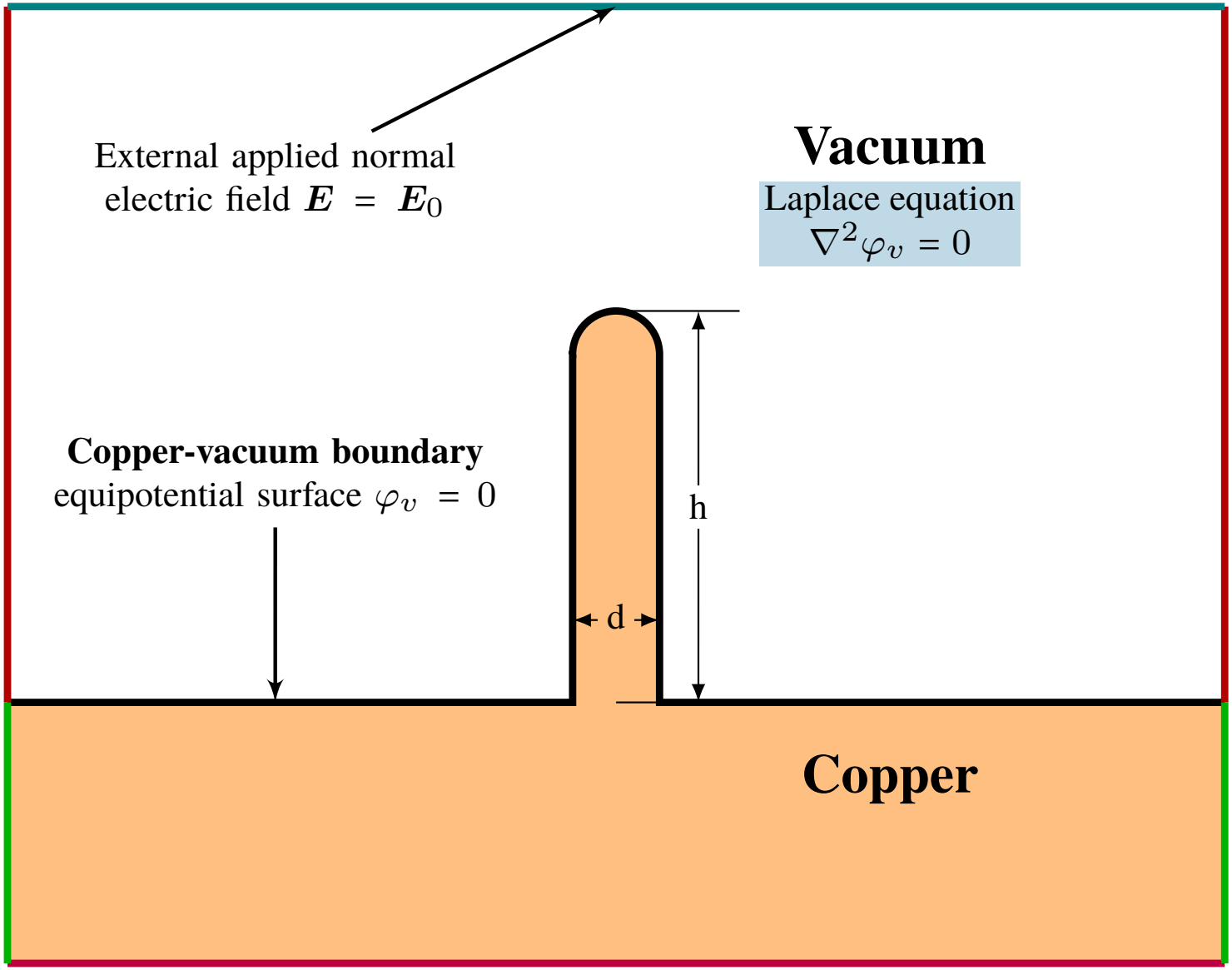

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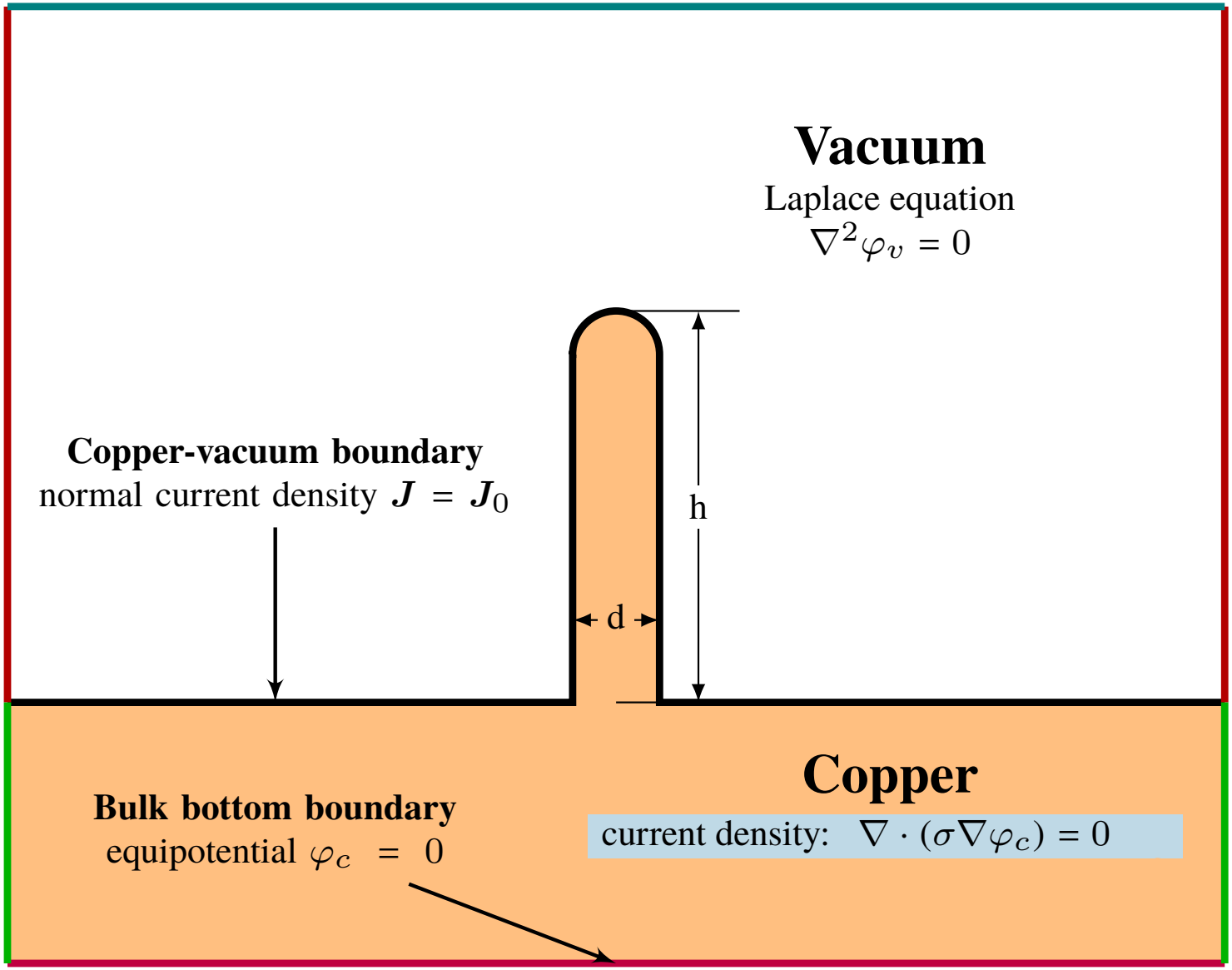

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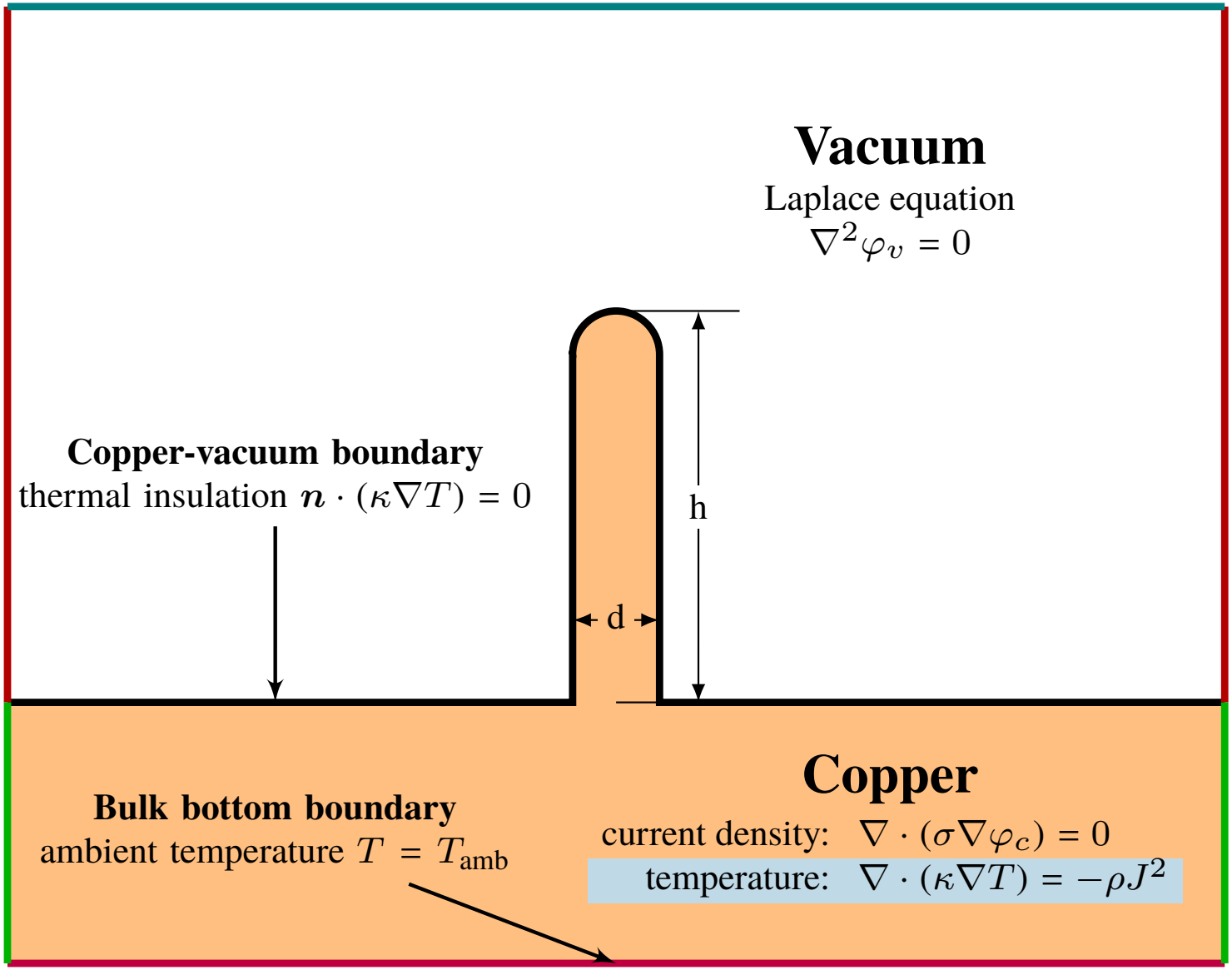

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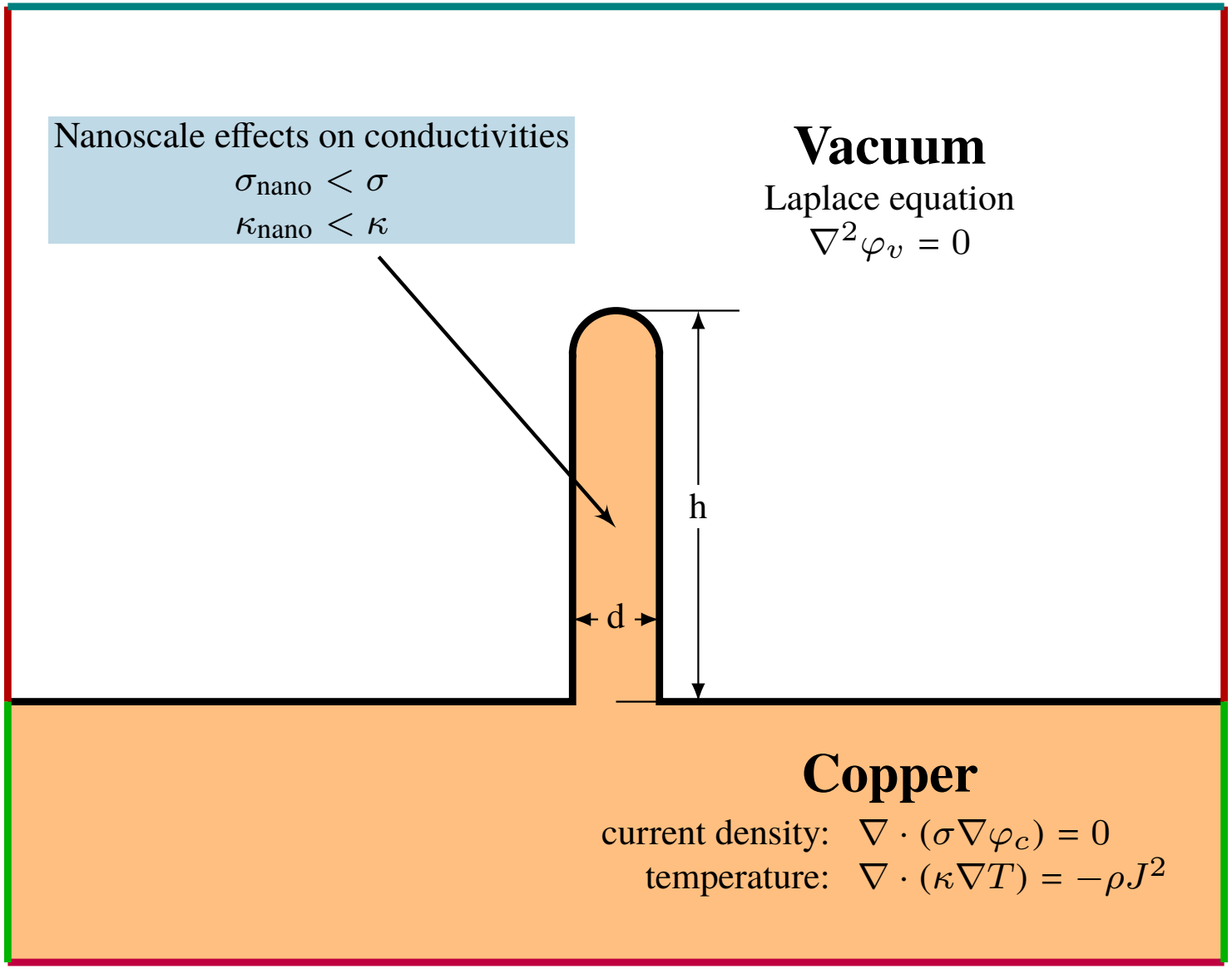
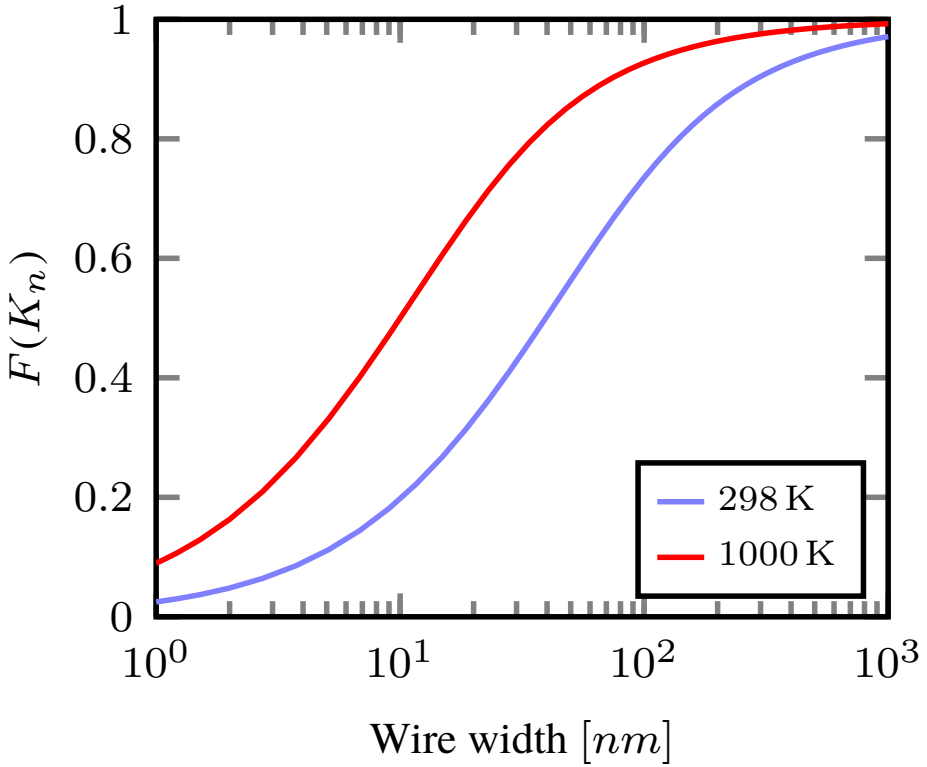
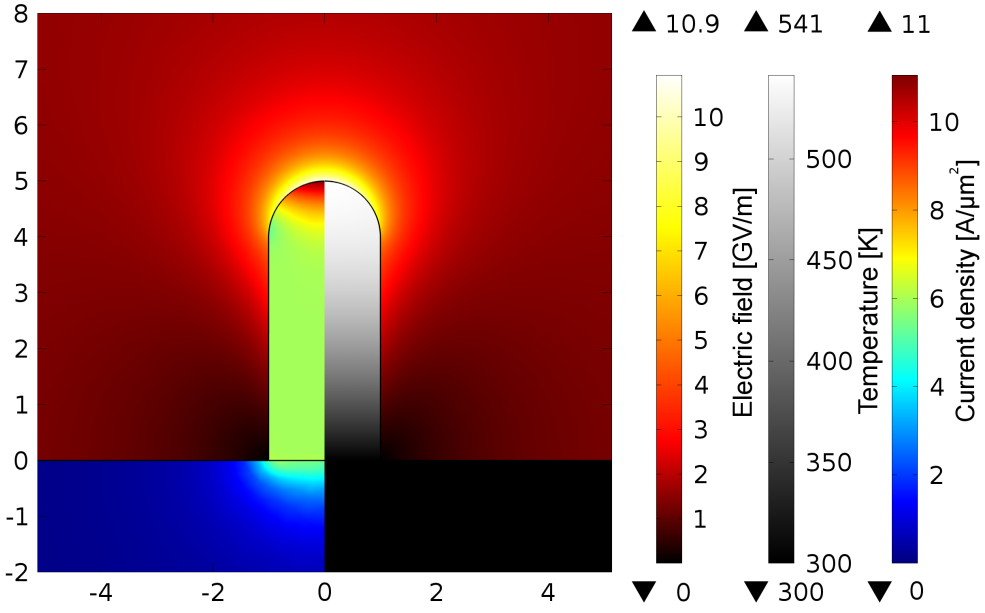


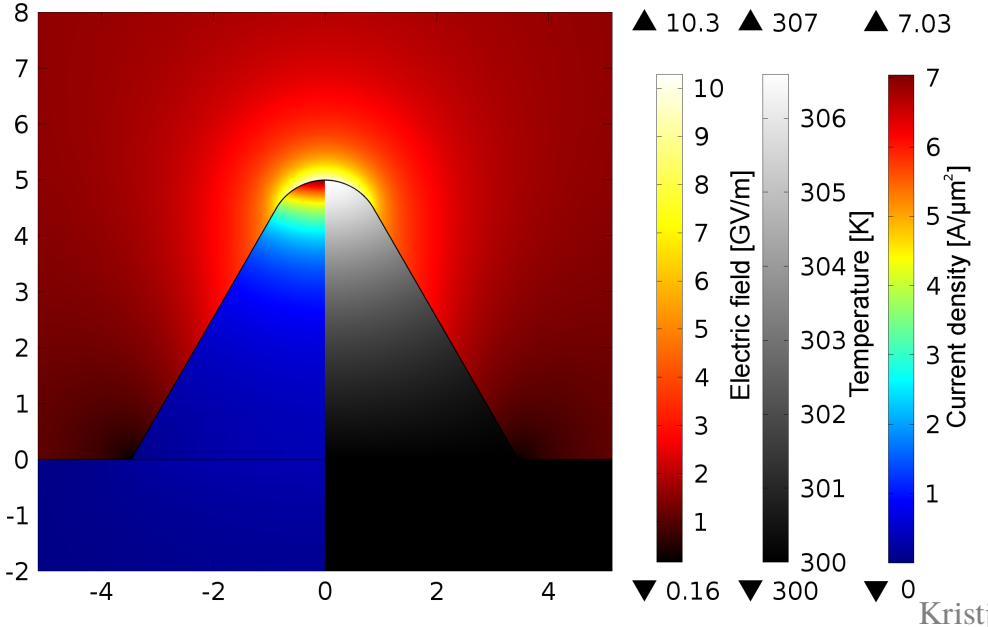
Finite size effects



- when the characteristic length becomes comparable with the mean free path, conductivities are reduced considerably
- $\sigma_{\text{nano}} = F(K_n)\sigma_{\text{bulk}}$
  $\kappa_{\text{nano}} = F(K_n)\kappa_{\text{bulk}}$
- $F(K_n) = F(K_n, T)$, as MFP is dependent on temperature

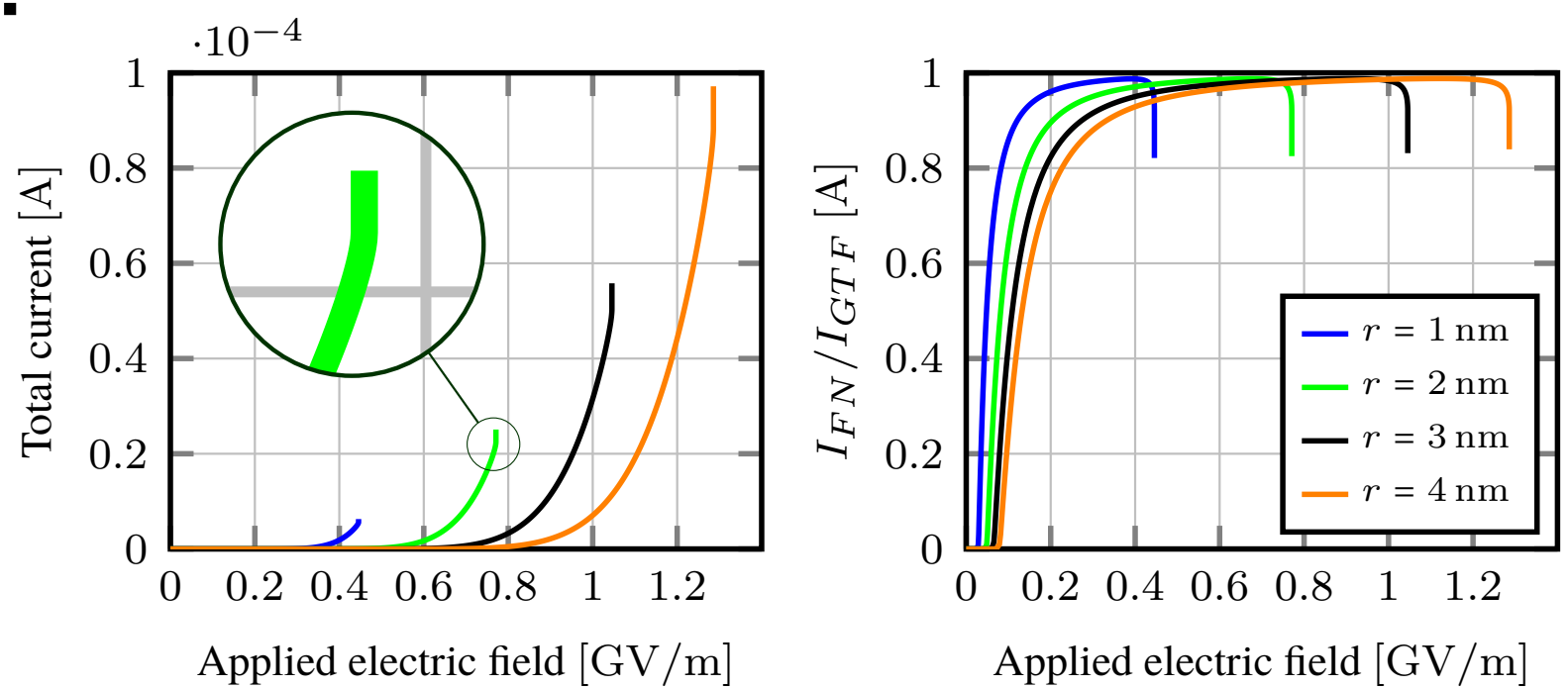
Field, current and temperature distributions





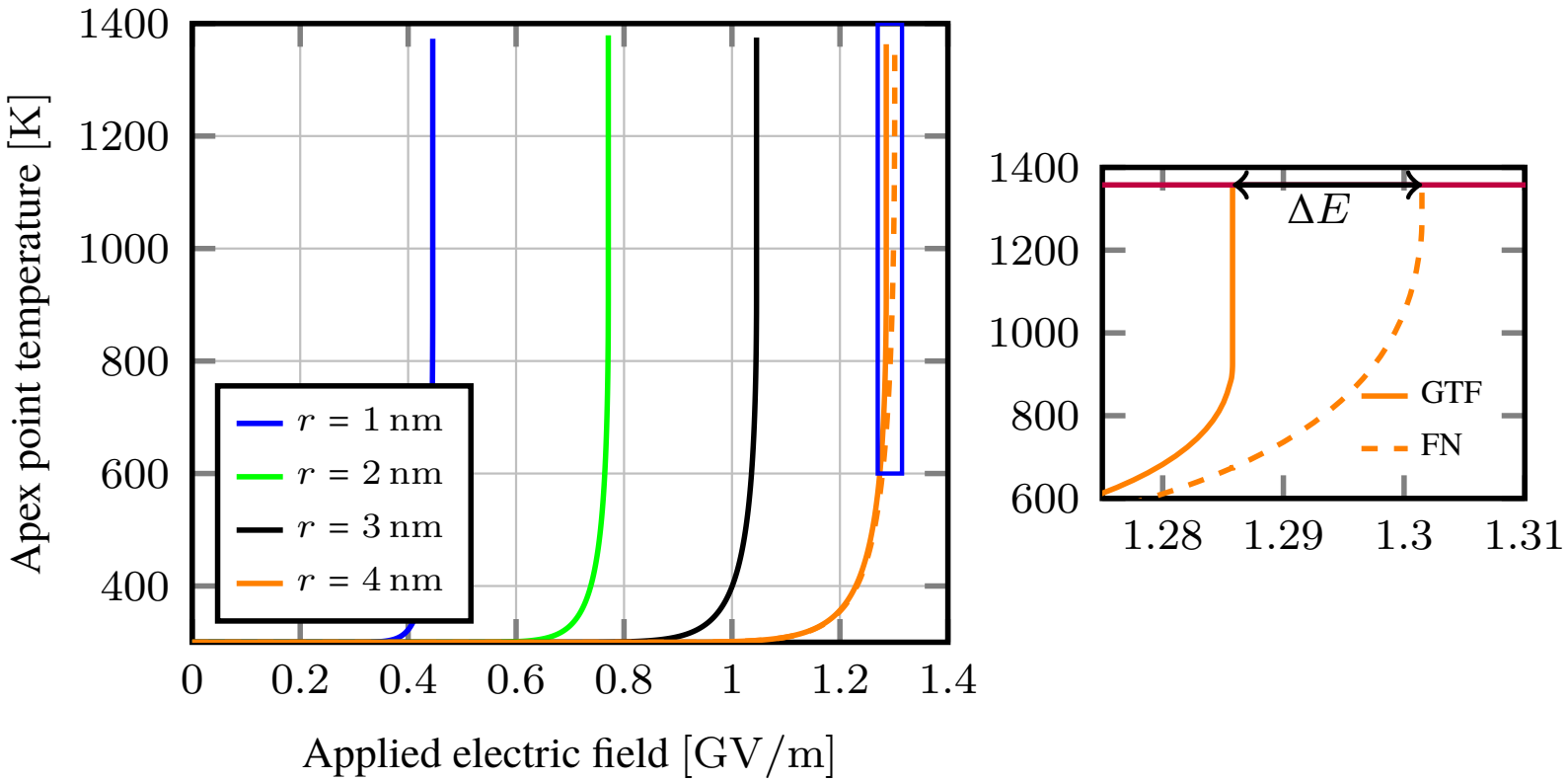
- $E = 1.5\,\mathrm{GV/m}$
- highest field, current and temperature at the apex
- conical emitter has smaller β, better thermal and electric conductivities
- general behaviour similar
- focus was on cylindrical emitter, as it is easier to compare with other models

Emission currents



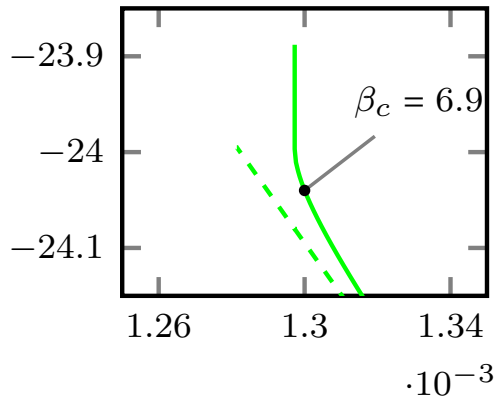
- $h = 20\,\mathrm{nm}$, lines end at melting temperature
- near melting temperature, GTF experiences field-independent increase in current
- Difference of $>15\,\%$ between GTF and FN currents near melting point

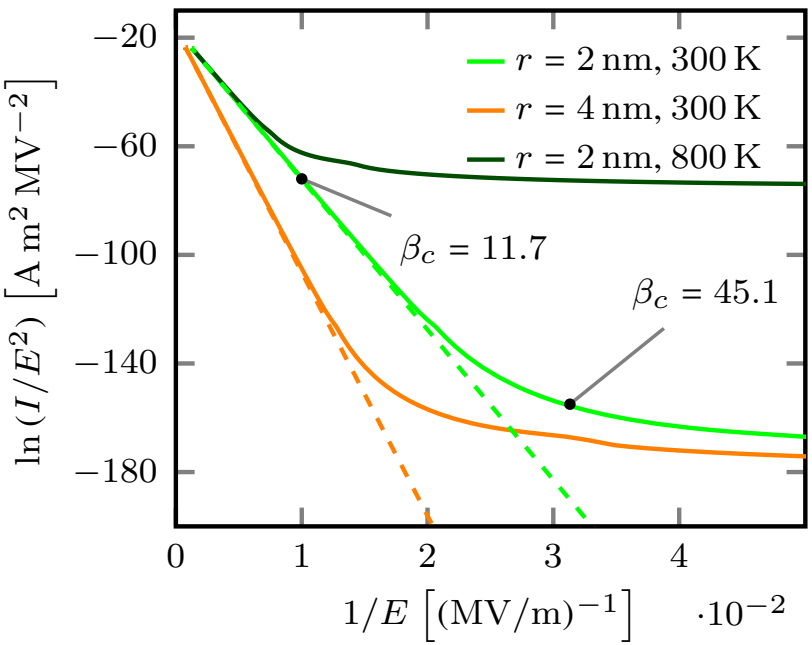
Thermal behaviour



- sudden temperature growth
- difference of up to $\sim 50\,\%$ between GTF and FN temperatures
- difference of melting fields $\sim 1.2\,\%$

Fowler-Nordheim plot

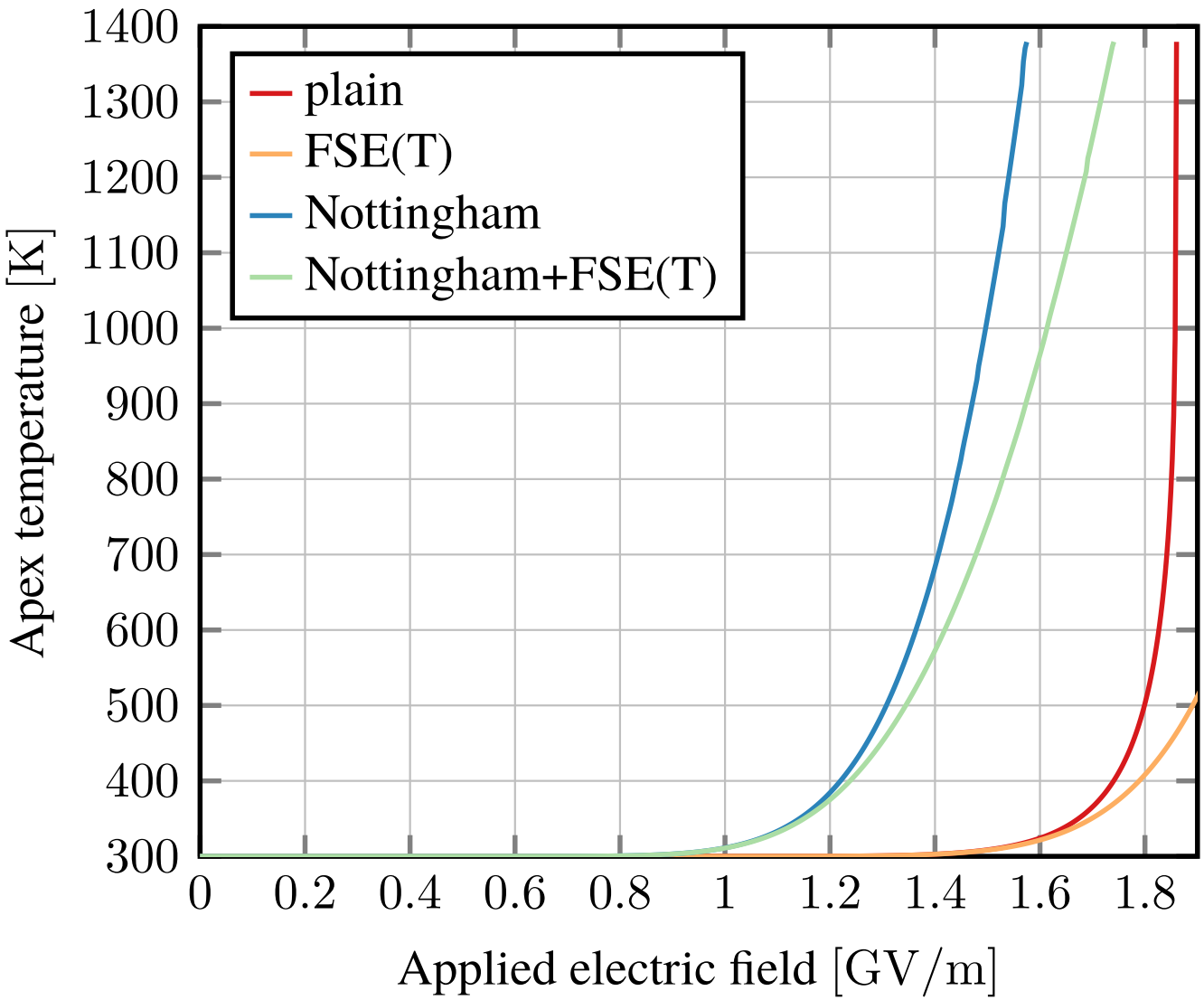




- FN plot slope can be used to estimate the field enhancement factor β_c
- analysis assumes that the FN equation holds
- if thermionic emission is non-negligible, FN plot deviates from linear
- at lower fields $\beta_c > \beta$
- near melting $\beta_c < \beta$

K. Eimre *et al.*, J. Appl. Phys. (2015)

Nottingham effect

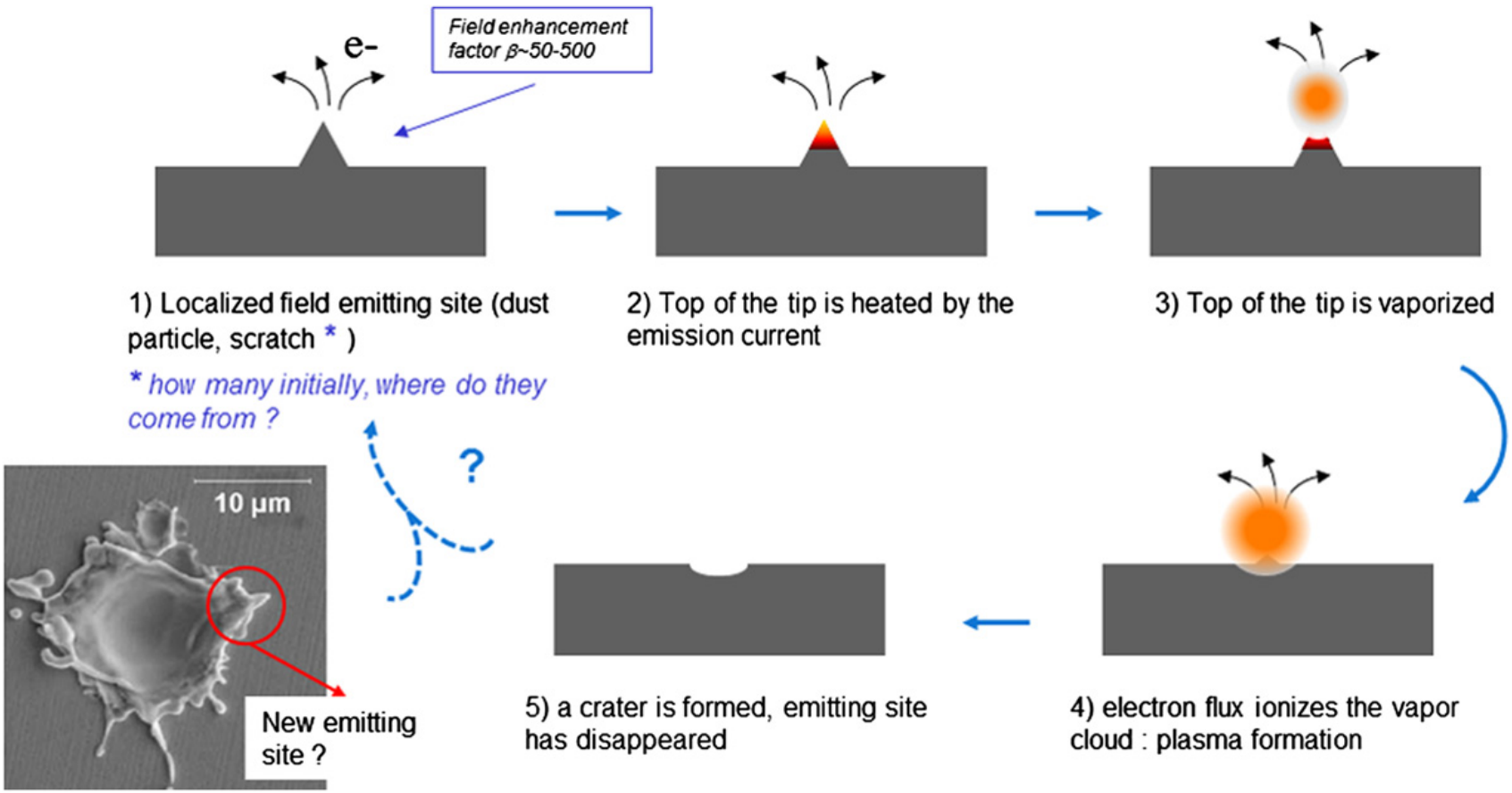


$h = 20\,\mathrm{nm}, r = 4\,\mathrm{nm}$

Summary and future directions

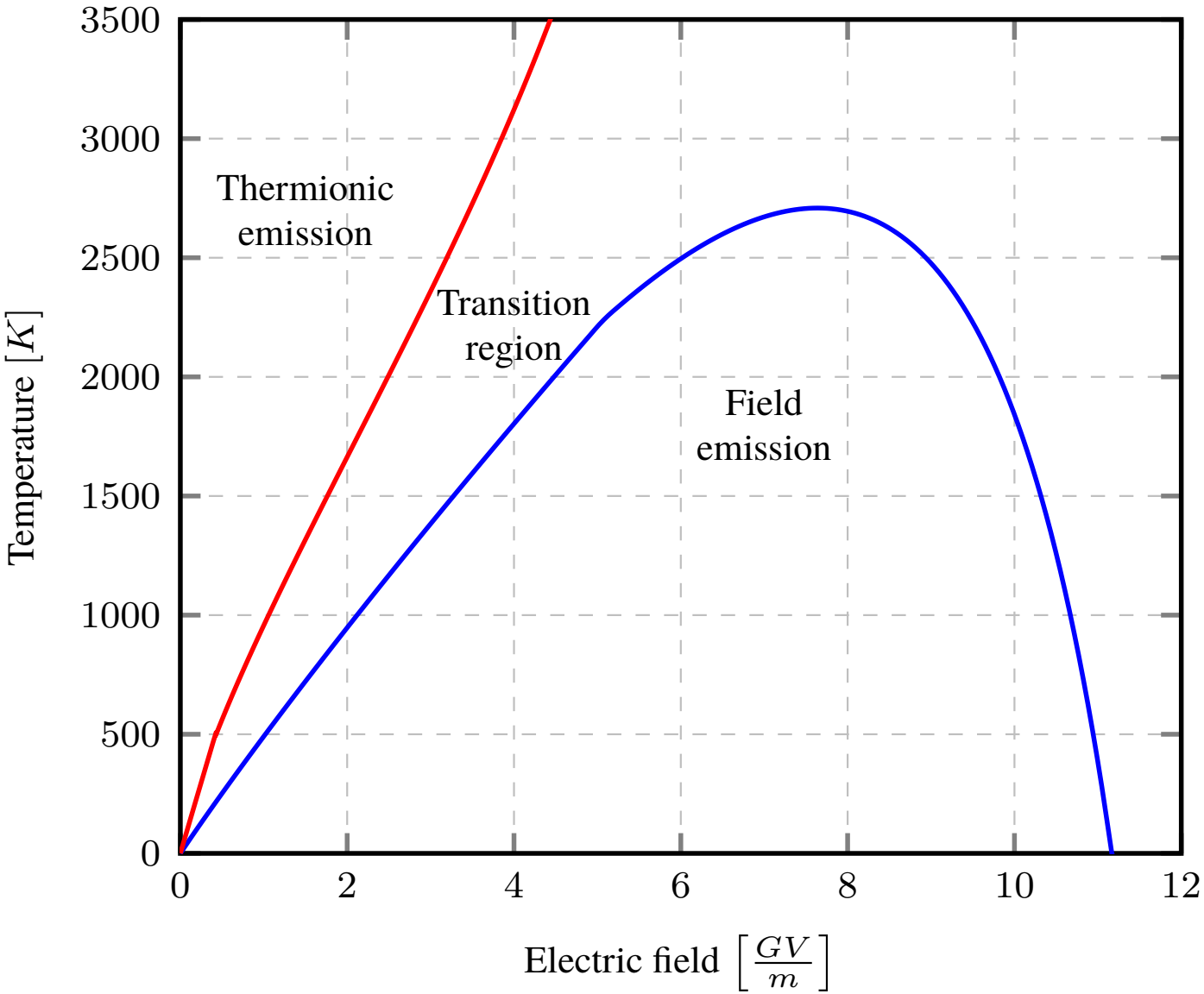
- FEM model of a nanoprotrusion utilizing the GTF equation was developed
- GTF and FN results were compared, notable differences near melting point and low fields were found
- thermal effects to the FN plot were found using the GTF equation
- the FEM model behaviour was compared with an ED-MD (HELMOD) model
- Nottingham effect was found to dominate over ohmic heating
- future plans:
  - further analysis of the Nottingham effect
  - add space charge screening
  - ...

Breakdown mechanism

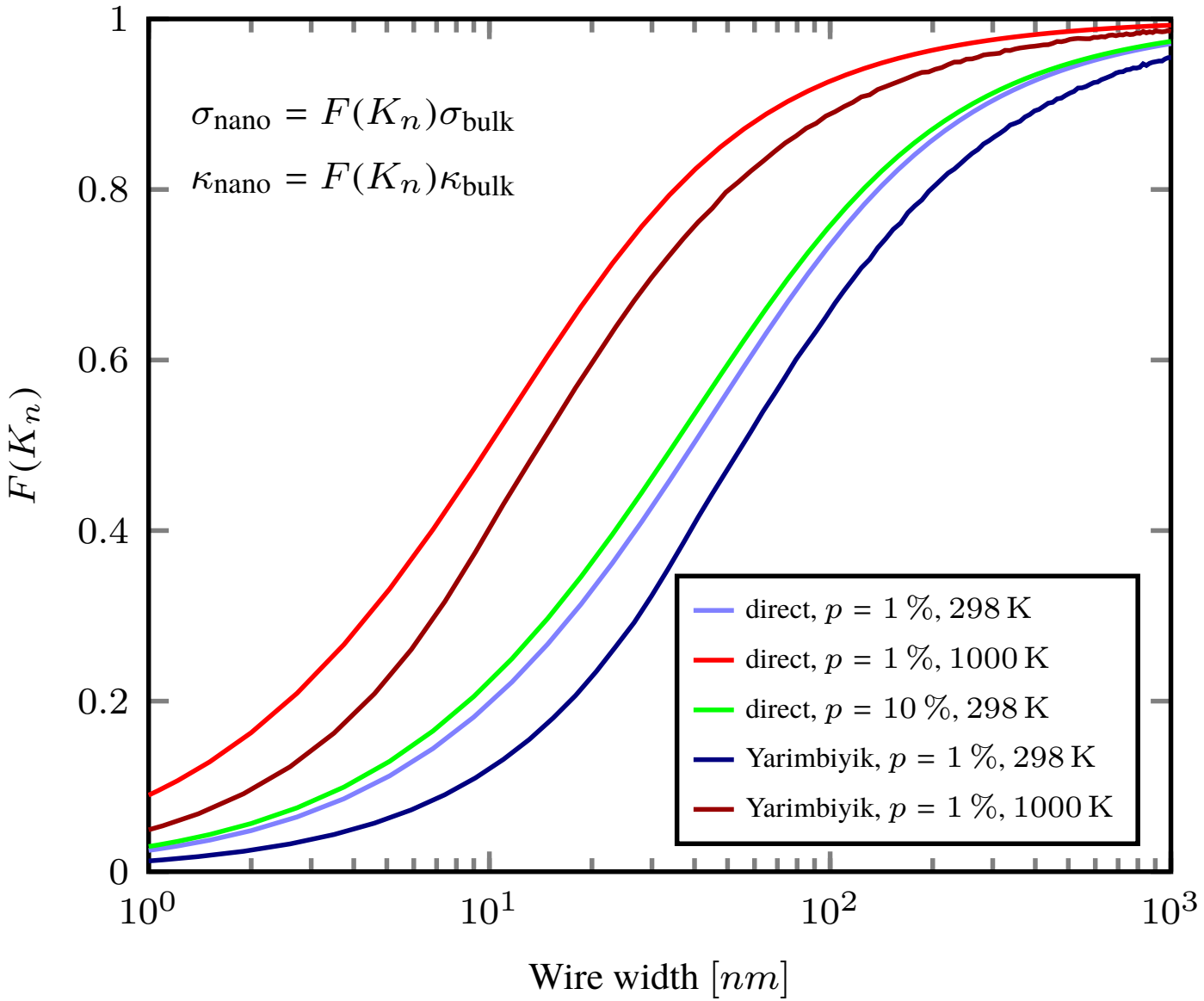


C. Z. Antoine et al (2012)

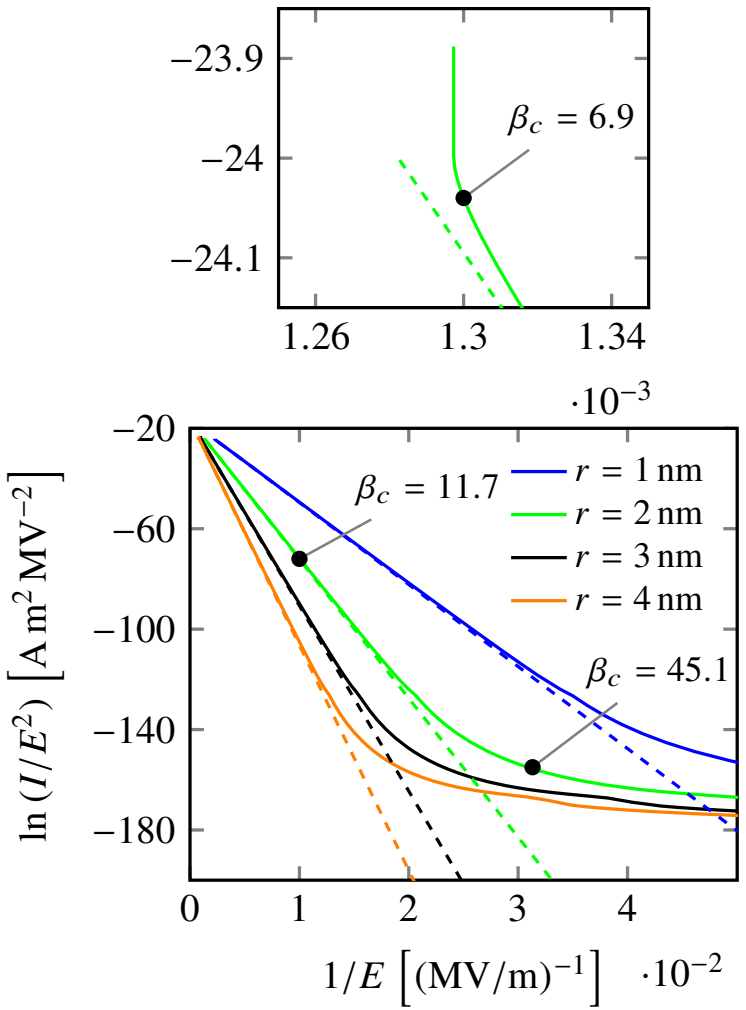
Validity regions of emission equations



Finite size effects detailed



Fowler-Nordheim plot 2



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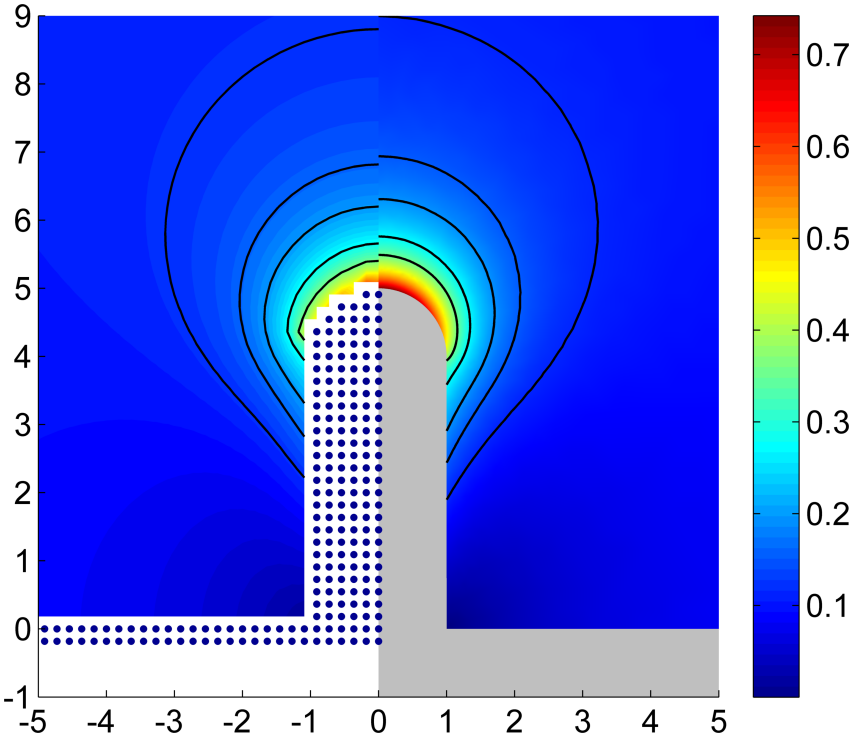
K. Eimre *et al.*, J. Appl. Phys. (2015)

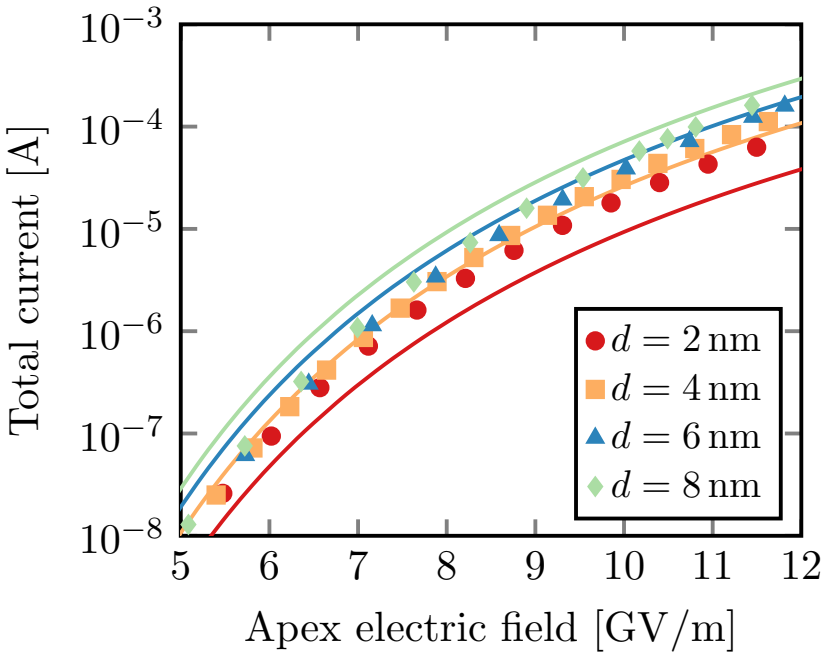
Comparison of FEM and ED-MD

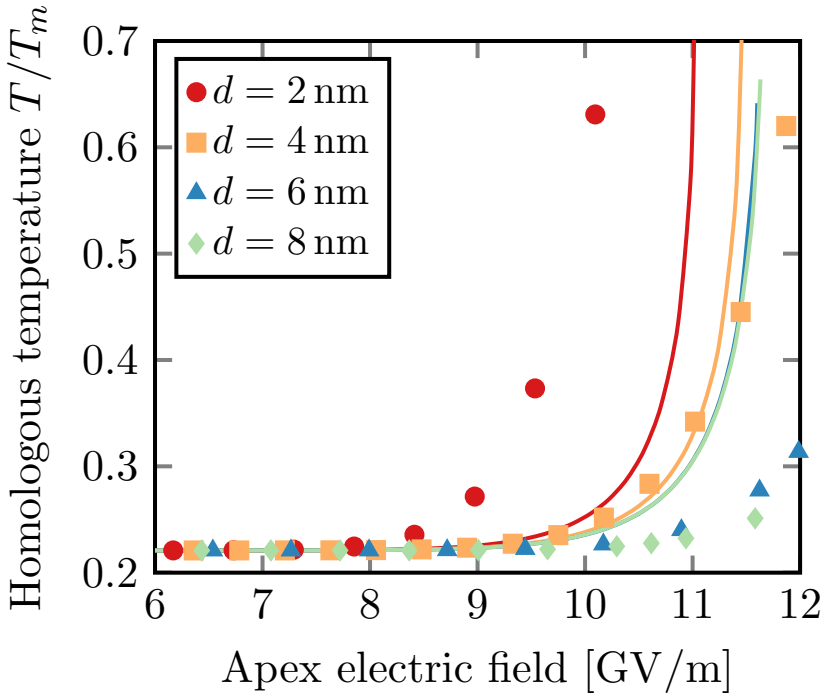


$E_{\text{appl}} = 0.1\,\text{GV/m}$

- HELMOD - molecular dynamics (MD) + finite difference method (FDM)
- GTF also implemented
- FEM surface is smooth and continuous; HELMOD's is discrete
- General field distribution similar, but at apex, the field is significantly lower for HELMOD

Comparison of FEM and ED-MD II





- currents are consistent
- general temperature behaviour is similar, but considerable differences are present