Impact of transverse HOM on beam stability in the HL-LHC

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HL-LHC WP 2.4 Meeting

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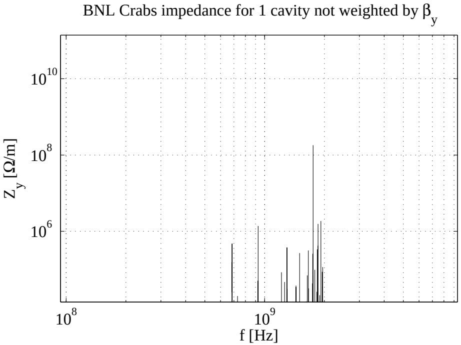
Outline

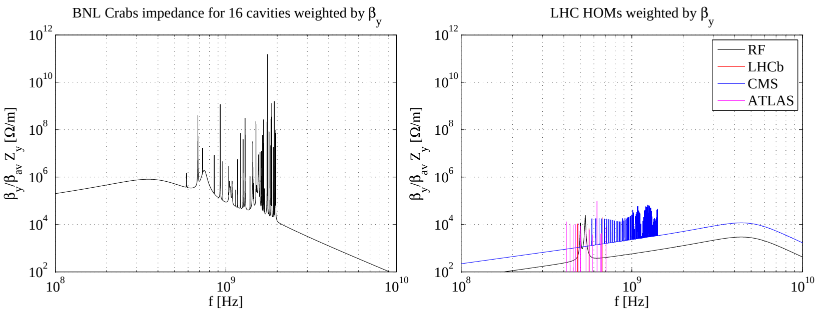
- Crab cavities exhibit HOMs from ≃500 MHz up to 2 GHz in a location where the transverse β is $\simeq 3000$.
- Some HOM reach the level of $R_s = 10\,\mathrm{G}\Omega/\mathrm{m} \rightarrow$ it might drive transverse CB instabilities!

Example for BNL cavities:



f (GHz)	Q	R (Ohm/m) per half cavity	plane	Deltaf in kHz
0.400	6.66E+09	5.98E+12	V	-
0.685	1880	4.67E+05	H	365
0.927	8600	1.38E+06	H	108
1.30	6540	3.73E+05	V	200
1.50	10800	2.67E+05	H	140
1.66	24200	3.14E+05	H	69
1.75	5800	2.57E+05	H	301
1.75	4160000	1.81E+08	H	0.4
1.84	9990	3.34E+05	V	185
1.86	26400	4.17E+05	H	70
1.86	88200	1.56E+06	V	21
1.92	102000	1.85E+06	H	18
1.96	54400	1.15E+05	H	36

- Comparison of HLLHC (round $\beta^* = 15$cm) BNL-HOMs Vs main HOMs from nominal LHC ($\beta^* = 60$cm).

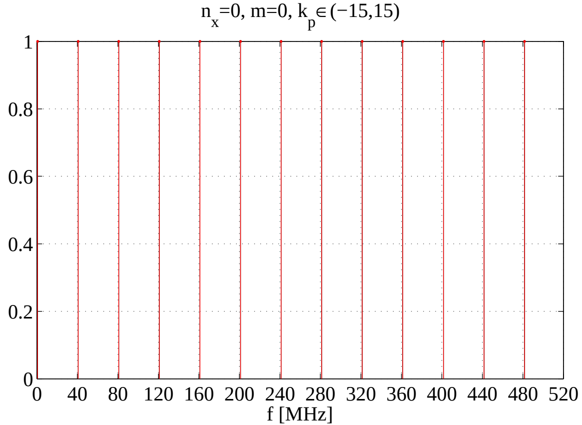


- We study an equivalent case with $R_s = 1.4\,\mathrm{G}\Omega/\mathrm{m}$ varying $Q = 1000$ and resonance frequency $f_r = 800\,\mathrm{MHz}$.
- How this applies to the HL-LHC? And what are the limits in terms of impedance?

scenario	HL-LHC 25 ns	
Energy	E	7 TeV
# bunch particles	N_b	2.2 10^{11} ppb
Bunch number	M	3564
Bunch length	σ_z^{rms}	7.55 cm
Main RF voltage	V	16 MV
Transverse Emittance	$\varepsilon_{x,y}^{norm.rms.}$	2.5000 μm
Average β_y	$\bar{\beta}_y$	71.5255 m
Average β_y	$\bar{\beta}_x$	65.9756 m
Hor. Tune	Q_{x0}	62.31
Ver. Tune	Q_{y0}	60.32
Sync. Tune	Q_s	0.002
Revolution frequency	f_0	11.245 kHz
Sync. frequency	f_s	22.994 Hz
Chromaticity	$\xi_{x,y}$	0

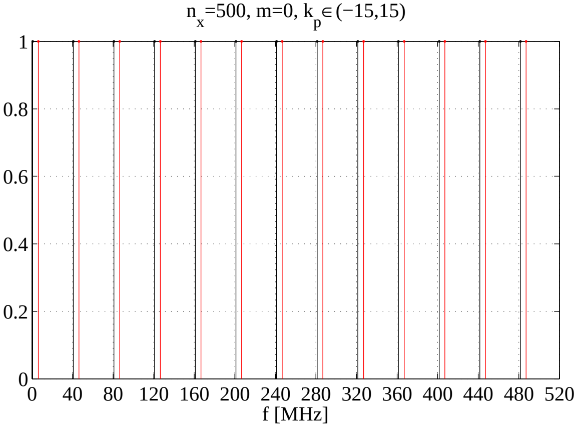
Main machine and beam parameters used for the stability estimations

- CB lines at: $f_p = \left(n_x + k_p M + m Q_s + Q_{y0}\right) f_0$ with $n_x \in (0..M-1)$ coupled bunch number, $m = 0, 1, 2, \ldots$ azimuthal bunch number, k_p line number.



Varying n_x: Lines spaced by $Mf_0 \simeq 40\,\mathrm{MHz}$ and shifted by $Q_{y0} f_0$.

- CB lines at: $f_p = \left(n_x + k_p M + m Q_s + Q_{y0}\right) f_0$ with $n_x \in (0..M-1)$ coupled bunch number, $m = 0, 1, 2, \ldots$ azimuthal bunch number, k_p line number.



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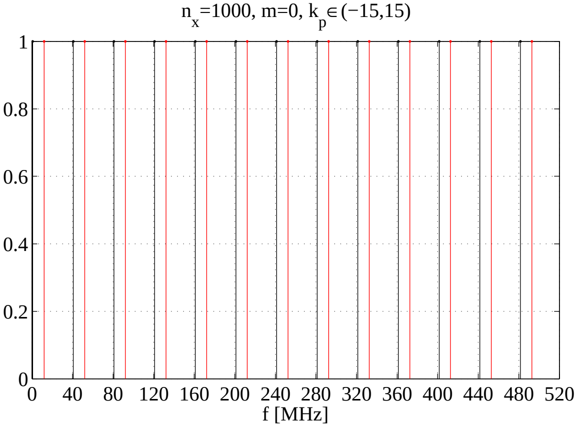
- CB lines at: $f_p = \left(n_x + k_p M + mQ_s + Q_{y0}\right) f_0$ with $n_x \in (0..M-1)$ coupled bunch number, $m = 0, 1, 2, \ldots$ azimuthal bunch number, k_p line number.



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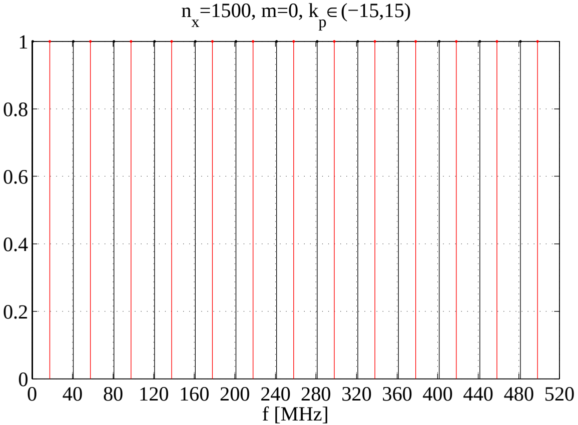

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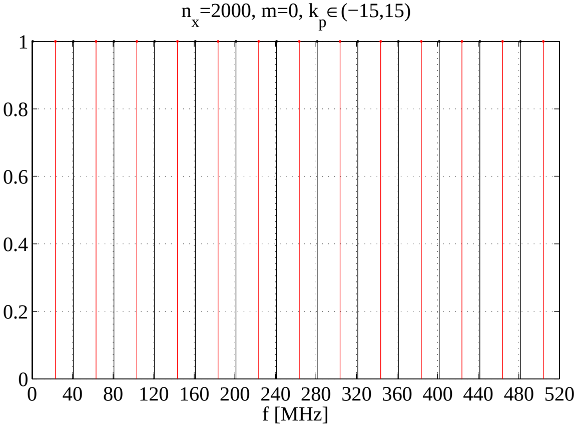

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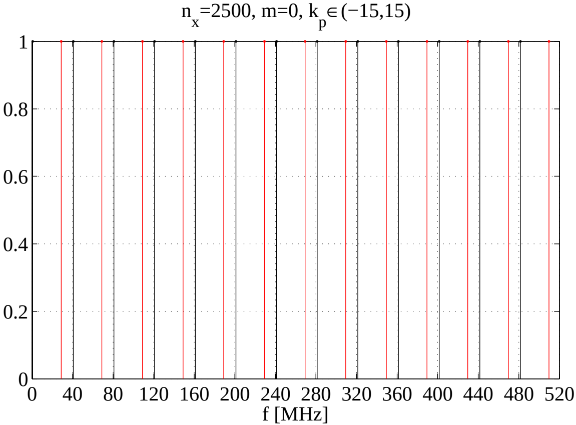

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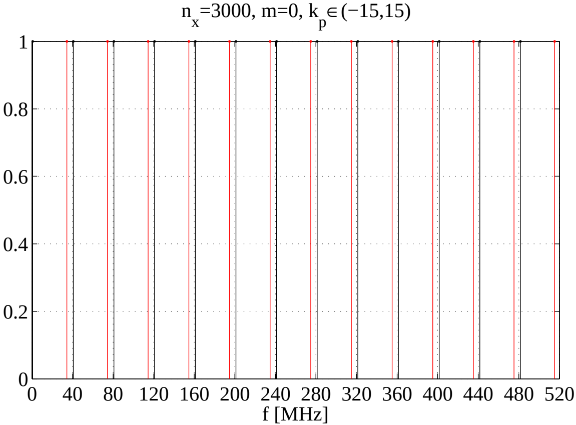
- CB lines at: $f_p = \left(n_x + k_p M + m Q_s + Q_{y0}\right) f_0$ with $n_x \in (0..M-1)$ coupled bunch number, $m = 0, 1, 2, \ldots$ azimuthal bunch number, k_p line number.



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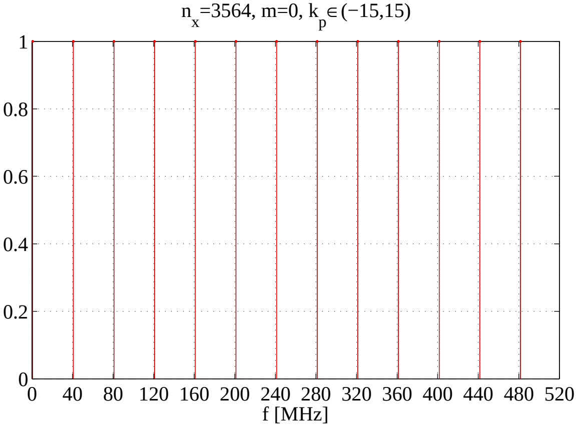

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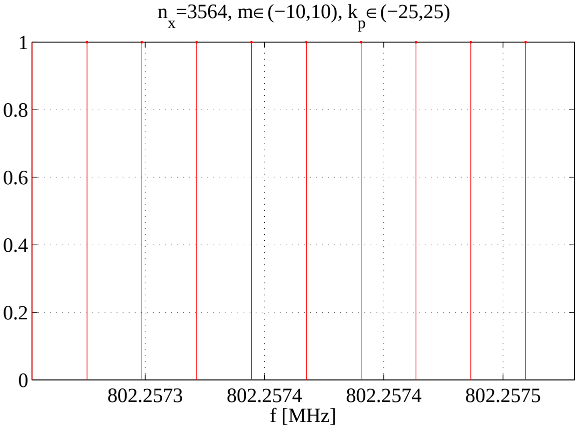
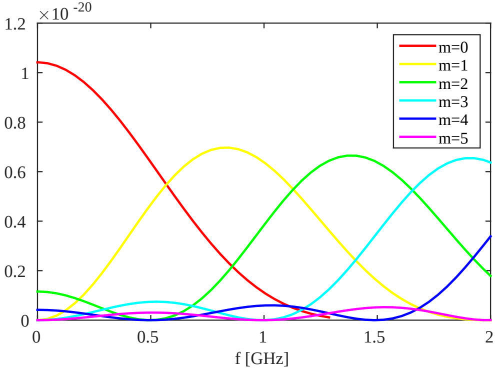


Varying m: around each CB line there is a cluster of lines spaced by $f_s \simeq$ 22Hz (Q_s=0.002).
Each azimuthal mode impacts differently: → Sacherer's sinusoidal modes

- CB lines at: $f_p = \left(n_x + k_p M + m Q_s + Q_{y0}\right) f_0$ with $n_x \in (0..M-1)$ coupled bunch number, $m = 0, 1, 2, \ldots$ azimuthal bunch number, k_p line number.
- Lines amplitude for each azimuthal mode is given by the Sacherer sinusoidal modes.



- Each CB line can be driven unstable in presence of an impedance.
- The rise-time and frequency shift can be approximately calculated by means of the Sacherer formula:

$$\Delta\omega_m^{x,y} = \frac{1}{|m|+1}\frac{jq\beta I_b}{2\,m_0\gamma Q_{x_0,y_0}\Omega_0 L_b}\left(Z_{x,y}^{eff}\right)_m$$

where $\Delta\omega_m^{x,y}$ the CB line complex frequency shift, q is the proton charge, m_0 the proton rest mass, $I_b = e\,N_b/T_0$ the beam current, T_0 the revolution frequency,ω_0 revolution radial frequency, Q_{x_0,y_0} the machine unperturbed tune, β and γ relativistic factors, $L_b = 4\sigma_z$ with σ_z the rms bunch length, $Z_{x,y}^{eff}$ the impedance weighted by the sinusoidal modes with

$$h(\omega) = \frac{8\tau_b^2}{\pi^4}(|m|+1)^2\,\frac{1+(-1)^{|m|}\cos(\omega\,4\tau_b)}{\left[(\omega\,4\tau_b/\pi)^2-(|m|+1)^2\right]^2}$$

and

$$Z_{x,y}^{eff} = \frac{\sum_{p=-\infty}^{+\infty} Z(\omega_p)_{x,y}h(\omega_p)}{\sum_{p=-\infty}^{+\infty} h(\omega_p)}$$

- The chromatic frequency $\omega_\xi = \dfrac{\omega\xi}{\eta}$ shifts the sinusoidal modes (replace $\omega \rightarrow \omega - \omega_\xi$).
- NB: No damper is considered here.

- A transverse HOM can be characterized by shut impedance R_s, merit factor Q and resonant frequency f_r:

$$Z(f) = \frac{f_r}{f} \frac{R_s}{1 - j\,Q\left(\frac{f_r}{f} - \frac{f}{f_r}\right)};$$

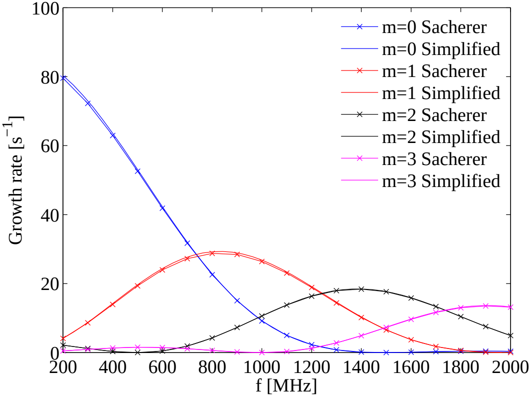
- If the bandwidth $\Delta f = \frac{f_r}{Q} \geq f_0$, the mode covers one or more CB lines that are driven unstable.
- If the bandwidth $\Delta f = \frac{f_r}{Q} \ll f_0$, the mode can fall between the CB lines → complicated situation due to the many azimuthal modes spaced by f_s.
  - If falling on the CB line we can simplify the Sacherer formula

$$\Delta\omega_m^{x,y} = \frac{1}{|m|+1} \frac{jq\beta I_b}{2\,m_0\gamma Q_{x_0,y_0}\Omega_0 L_b} R_s \frac{h(\omega_r)}{\sum_{p=-\infty}^{+\infty} h(\omega_p)}$$

  - If not falling on the CB line we are in principle stable.

Growth-rate Vs frequency

- We study an equivalent case with $R_s = 1.4\,\mathrm{G\Omega/m}$, $Q = 10^3$ and variable resonance frequency f_r.
- Study of growth rate Vs frequency when $f_r \simeq f_p$ (on top of CB line):
- $Q' = 0$



- Good agreement between Sacherer and the simplified formula.

Growth-rate Vs frequency

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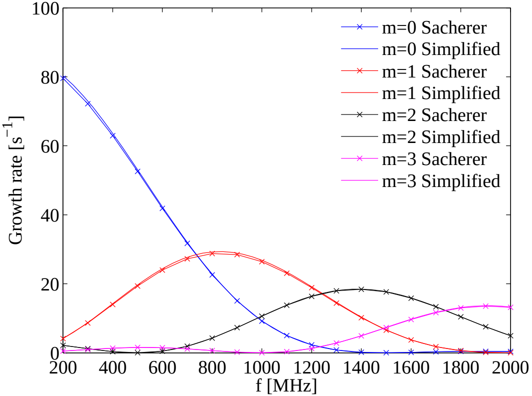
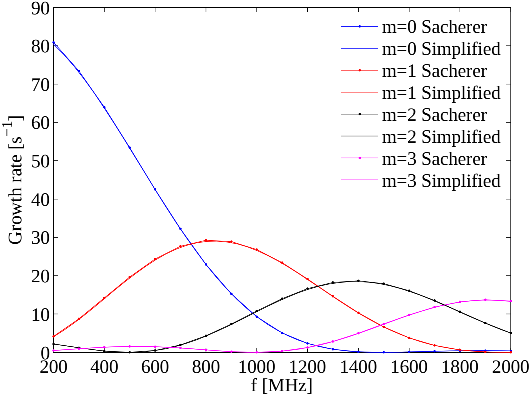
- $Q' = 0$



- Same curves since shortest $\Delta f = 200\mathrm{MHz}/1000 = 200\mathrm{kHz} > f_0$.

Growth-rate Vs frequency

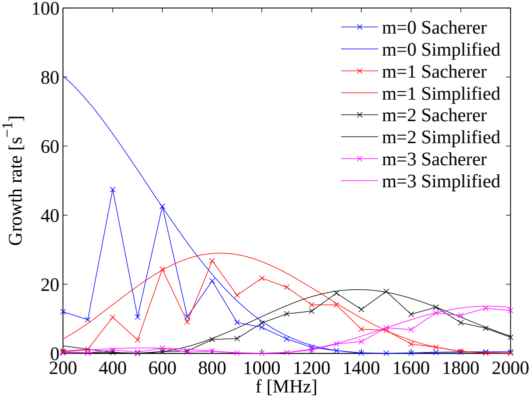
- We study an equivalent case with $R_s = 1.4\,\mathrm{G\Omega/m}$, $Q = 10^5$ and variable resonance frequency f_r.
- Study of growth rate Vs frequency when $f_r \simeq f_p$ (on top of CB line):
- $Q' = 0$



- Same curves as $Q = 10^3$ as expected.

Growth-rate Vs frequency

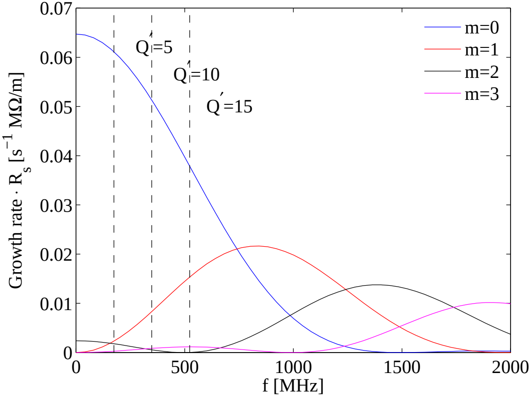
- We study an equivalent case with $R_s = 1.4\,\mathrm{G\Omega/m}$, $Q = 10^5$ and variable resonance frequency f_r.
- Study of growth rate Vs frequency when $f_r \simeq f_p$ (not on top of CB line):
- Varying $Q' = 0$



- Only high frequency matches theory. At low frequency $\Delta f = 200\mathrm{MHz}/10^5 = 2\mathrm{kHz} < f_0$.

Growth-rate Vs frequency

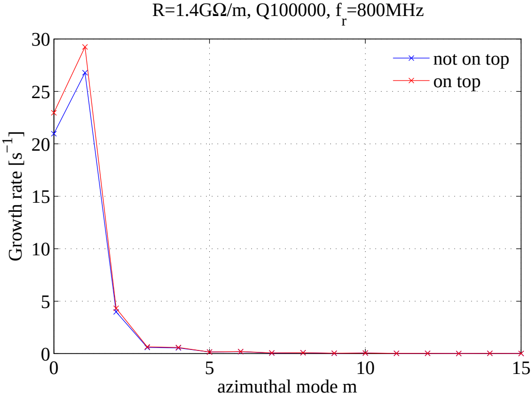
- We study an equivalent case with $R_s = 1.4\,\mathrm{G}\Omega/\mathrm{m}$, $Q = 10^5$ and variable resonance frequency f_r.
- Study of growth rate Vs frequency when $f_r \simeq f_p$ (on top of CB line). The growth rates curves are on the right of the chromatic frequency given by $f_\xi = f_0 Q'/\eta$.



- Only high frequency matches theory. At low frequency $\Delta f = 200\mathrm{MHz}/10^5 = 2\mathrm{kHz} < f_0$.

Growth-rate Vs azimuthal mode

- We study an equivalent case with $R_s = 1.4\,\mathrm{G\Omega/m}$, $Q = 10^5$ and resonance frequency $f_r = 800\,\mathrm{MHz}$.
- $Q' = 0$
- For a certain frequency, and a given limit rise-time, we can estimate the number of azimuthal modes m to consider for the stability studies.



- The case "on top" can be considered as a pessimistic case.

Conclusions and outlook

Conclusions

- Reviewed the CB line distribution for HL-LHC 25 ns beams.
- CB lines are spaced by f_0 with clusters of f_s around them.
- HOMs with $\Delta f \ll f_0$ can drive CB lines unstable if falling on top of a CB line, as we resumed in the Sacherer simplified formula → Growth rate depends only on R_s.
- HOMs with $\Delta f \ll f_0$ can be stable if falling within CB lines.
- HOMs with $\Delta f \geq f_0$ can drive many CB lines unstable but the effect tends to broaden → Growth rate depends on both R_s and Q.
- Given a minimum threshold growth-rate we can estimate how many CB azimuthal mode m we need to consider harmful.

Future plans

- Study of the effect of damper with DELPHI.
- Update of the BNL HOM tables and estimation of HOM tolerances.
- . . . ?

Thanks for your attention!