Shear viscosities from a generalized NJL model

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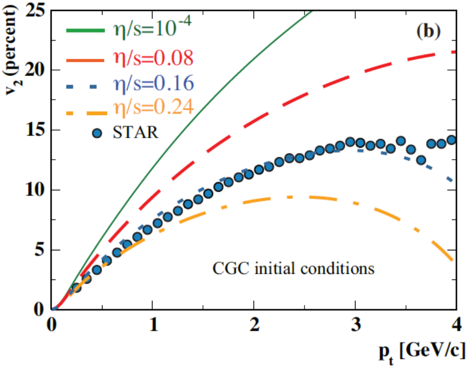
Motivation

Quark-gluon plasma is almost-perfect fluid

elliptic flow v_2 $\oplus$ hydrodynamic simulations $\oplus$ CGC/Glauber

$\Downarrow$

measurement/extraction of a **constant** ratio η/s



[Lacey et al. Phys.Rev.Lett **98** 092301 (2007)]

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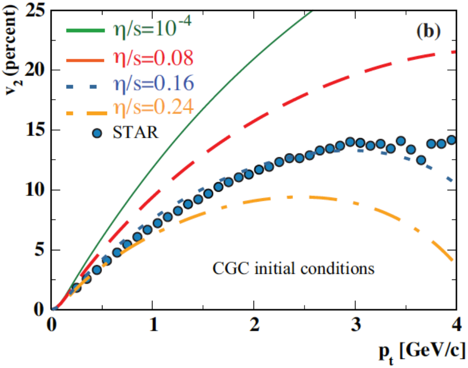
From the energy-momentum tensor

$$T^{\mu\nu} = u^\mu u^\nu (\epsilon + P) - P g^{\mu\nu} + \tau^{\mu\nu}$$

the **shear viscosity** $\boldsymbol{\eta}$ parameterizes the **traceless part** of τ.



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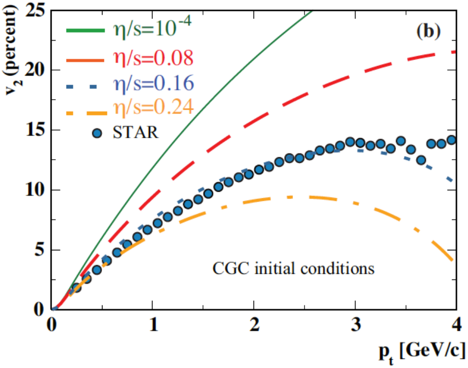
Aim of my work:
Microscopic calculation of the (T, μ)-dependence of η/s within a large-N_c Nambu–Jona-Lasinio (NJL) model



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Snapshot of the NJL model and its large-N_c scaling

- **two-flavor Lagrangian** with scalar/pseudoscalar interactions (parameters m_0, G, Λ):

$$\mathcal{L}_{\mathrm{NJL}}^{2\mathrm{f}} = \bar{\psi}\left(\mathrm{i}\not{\partial} - m_0\right)\psi + \frac{G}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}\mathrm{i}\gamma_5\vec{\tau}\psi)^2\right]$$

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- $\mathcal{L}_{\mathrm{NJL}}^{\mathrm{2f}}$ **imitates** the (approximate) symmetries of QCD:

$$\mathcal{CPT} \times \mathrm{SU}(3)_\mathsf{c} \times \mathrm{SU}(2)_\mathsf{L} \times \mathrm{SU}(2)_\mathsf{R} \times \mathrm{U}(1)_\mathsf{V}$$

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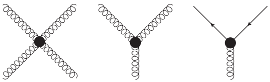
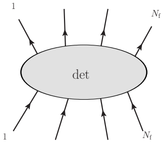
- QCD building blocks for NJL vertices:



V_4 **four-gluon vertices**,
V_3 **three-gluon vertices**,
V_g **quark-gluon vertices**,
L **momentum loops**.



$\Rightarrow$ **connected vertex** with $2N_\mathsf{f}$ external quarks:

$$\boxed{V_4 + \frac{1}{2}(V_3 + V_g) = N_\mathsf{f} - 1 + L}$$

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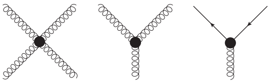

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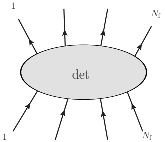

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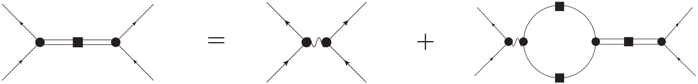
From the **running coupling in QCD**, $\alpha_{\rm s} = \alpha_{\rm s}(N_{\mathsf{c}})$:

N_{c}-scaling for $2N_{\mathsf{f}}$ NJL vertex: $\frac{1}{N_{\mathsf{c}}^{N_{\mathsf{f}}-1+L-l}} \leq \frac{1}{N_{\mathsf{c}}^{N_{\mathsf{f}}-1}}$, e.g. four vertex $G \sim \frac{1}{N_{\mathsf{c}}}$

Rediscover standard many-body approaches

GAP $\mathcal{O}(1)$:

BSE $\mathcal{O}(N_c^{-1})$:



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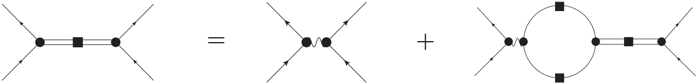

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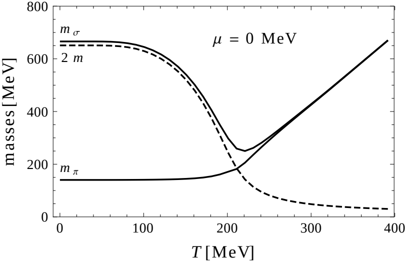
Solving the **Gap equation** (GAP) and **Bethe-Salpeter equation** (BSE) leads to

- thermal quark masses m
- thermal meson masses m_π and m_σ
- ! coupling by **meson clouds**



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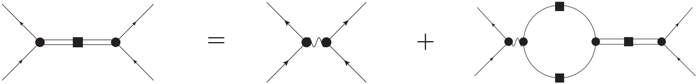

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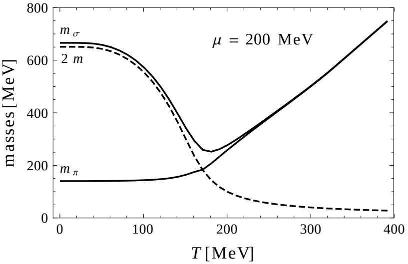

Shear viscosity from Kubo formalism

- **energy-momentum tensor** in the NJL model: $T_{\mu\nu} = \mathrm{i}\bar{\psi}\gamma_\mu\partial_\nu\psi$
- **Kubo formula for (static) shear viscosity**

$$\eta := \lim_{x\to 0}\lim_{\omega\to 0}\eta(\omega,\vec{x},x_0) = \frac{1}{T}\int_0^\infty \mathrm{d}t\ \mathrm{e}^{\mathrm{i}\omega t}\int \mathrm{d}^3 r\ (T_{21}(\vec{r},t), T_{21}(\vec{x},x_0))$$

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- 4-point (Green's) function and **Optical Theorem**

$$\eta = -\left.\frac{\mathrm{d}}{\mathrm{d}\omega}\,\mathrm{Im}\ \Pi^{\mathrm{R}}(\omega)\right|_{\omega=0}$$

with **retarded correlator** $\Pi^{\mathrm{R}}(\omega) = -\mathrm{i}\int_0^\infty \mathrm{d}t\ \mathrm{e}^{\mathrm{i}\omega t}\int \mathrm{d}^3r\ \langle[T_{21}(\vec{r},t),T_{21}(0)]\rangle$

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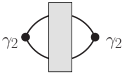
$\Pi(\omega_n) =$ 

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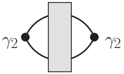
$\Pi(\omega_n) =$ 

$= \mathcal{O}(N_\mathrm{c}^1) + \mathcal{O}(N_\mathrm{c}^0) + \dots$

$$\Rightarrow \quad \boxed{\eta = \frac{N_\mathrm{c} N_\mathrm{f}}{8\pi^2 T} \int_{-\infty}^{\infty} \mathrm{d}\epsilon \int \mathrm{d}^3 p \; p_x^2 \, n_\mathrm{F}^+(\epsilon)[1 - n_\mathrm{F}^+(\epsilon)] \, \mathrm{Tr}_\mathrm{D} \left[\gamma_2 \rho(\epsilon, \vec{p}) \gamma_2 \rho(\epsilon, \vec{p})\right]}$$

Study of the shear viscosity - I

Ansatz for the full quark propagator:

$$G_{\mathrm{R}}(p_0,\vec{p}) = \frac{1}{\not{p} - m + \mathrm{i}\,\Gamma(p)}\;, \qquad \Rightarrow \rho = -\frac{1}{\pi}\mathrm{Im}\;G_{\mathrm{R}}$$

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Convergence criterion for η

The asymptotic behavior of $\Gamma(p)$ is **forbidden to converge too rapidly to zero**:

$$\eta[\Gamma(p)] < \infty \quad \Leftrightarrow \quad p^{7/2}\mathsf{e}^{-\beta p/2} \in \mathrm{o}(\Gamma(p))$$

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Possible parameterizations of the spectral width:

$$\begin{aligned}
\Gamma_{\mathrm{const}} &= 100\ \mathrm{MeV}\\
\Gamma_{\mathrm{exp}}(p) &= \Gamma_{\mathrm{const}}\,\mathrm{e}^{-\beta p/8}\\
\Gamma_{\mathrm{Lor}}(p) &= \Gamma_{\mathrm{const}}\frac{\beta p}{1 + (\beta p)^2}\\
\Gamma_{\mathrm{div}}(p) &= \Gamma_{\mathrm{const}}\sqrt{\beta p}
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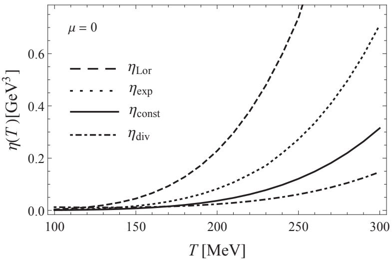

no momentum cutoff

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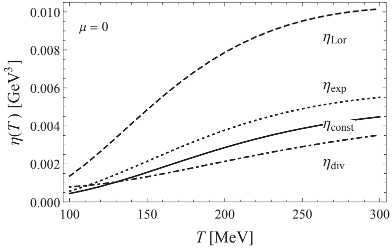
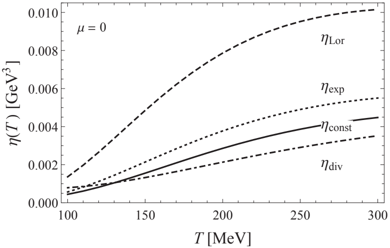


momentum cutoff $\Lambda = 651$ MeV

Study of the shear viscosity - II



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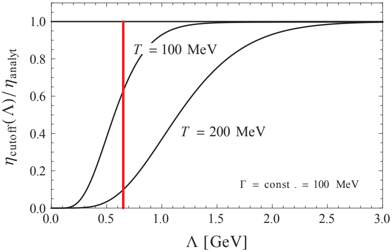

strong cutoff dependence

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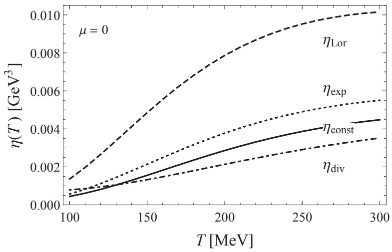

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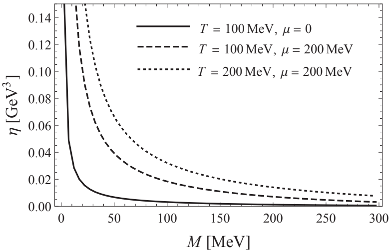
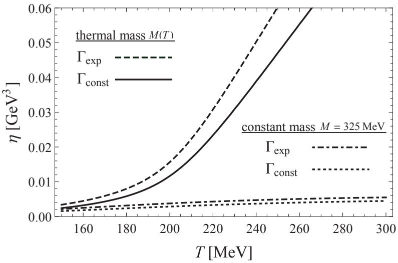
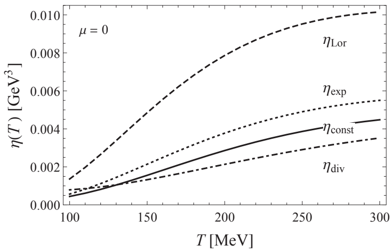


mass dependence



thermal vs. constant quark masses

Study of the shear viscosity - II



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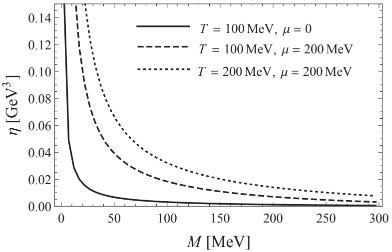

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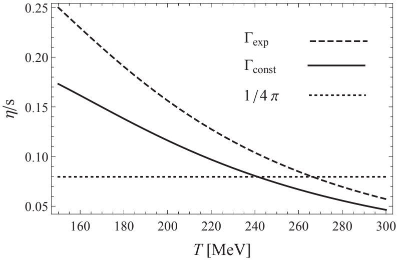

constant quark mass

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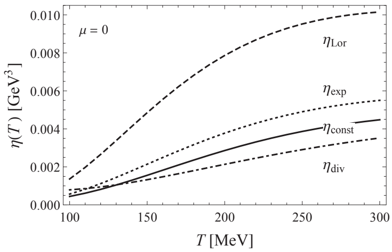

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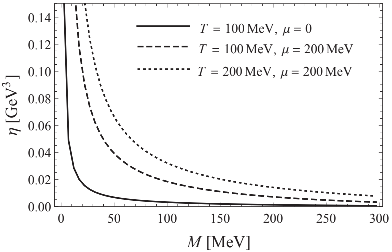

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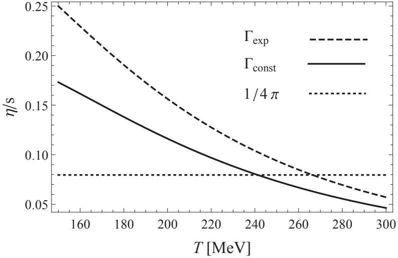

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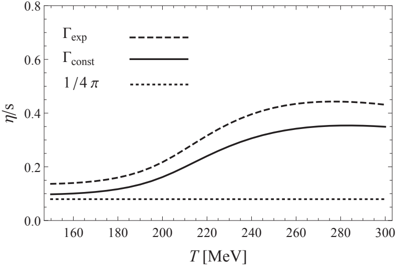
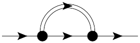
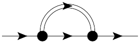


thermal quark mass

Shear viscosity from mesonic fluctuations - I

Self energy at $\mathcal{O}(N_c^{-1})$: $\Sigma_\beta^{\pi/\sigma}(\vec{p},\nu_n) =$  $= \mp m\Sigma_0 - \vec{p}\cdot\vec{\gamma}\,\Sigma_3 + \nu_n\gamma_4\,\Sigma_4$

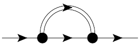
Shear viscosity from mesonic fluctuations - I

Self energy at $\mathcal{O}(N_c^{-1})$: $\Sigma_\beta^{\pi/\sigma}(\vec{p},\nu_n) =$  $= \mp m\Sigma_0 - \vec{p}\cdot\vec{\gamma}\,\Sigma_3 + \nu_n\gamma_4\,\Sigma_4$

Determination of the new pole: $G_\mathrm{R}(p_0,\vec{p}) = \dfrac{1}{\not{p} - m - \Sigma_\mathrm{R}} = \dfrac{1}{0}$

$$\Rightarrow \quad p_0^2 = \underbrace{\vec{p}^{\,2} + m^2}_{\text{free part}} + \Omega\,, \quad \text{with} \quad \boxed{\Omega = \vec{p}^{\,2}\,(2\Sigma_3 - 2\Sigma_4) + m^2\,(\pm 2\Sigma_0 - 2\Sigma_4)}$$

Shear viscosity from mesonic fluctuations - I

Self energy at $\mathcal{O}(N_\mathsf{c}^{-1})$: $\Sigma_\beta^{\pi/\sigma}(\vec{p},\nu_n) =$  $= \mp m\Sigma_0 - \vec{p}\cdot\vec{\gamma}\,\Sigma_3 + \nu_n\gamma_4\,\Sigma_4$

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Modified dispersion relation $\Rightarrow$ old ansatz for G_R is invalid, i.e. $G_\mathrm{R} \neq \frac{1}{\not{p} - m + \mathrm{i}\,\Gamma(p)}$

$$\boxed{\Rightarrow \eta[\{\rho_j\}] = \frac{2N_\mathsf{c}N_\mathsf{f}}{3\pi^3 T}\int_{-\infty}^{\infty}\mathsf{d}\epsilon\int_0^{\Lambda}\mathsf{d}p\,\frac{p^4}{D^2}\left[-m^2A^2 - \frac{3}{5}\vec{p}^{\,2}C^2 + p_0^2B^2\right]}$$

with $\rho_j = \mathrm{Im}\,\left(3\Sigma_j^\pi + \Sigma_j^\sigma\right)$ for $j = 0, 3, 4$,
and A, B, C, D being lengthy expressions in ρ_j.

Shear viscosity from mesonic fluctuations - II

$\rightarrow$ **lengthy and complex self energies** Σ_j, but their imaginary parts are **analytical**

$$\mathrm{Im}\,\Sigma_0(p_0,p) = -\frac{g^2_{\mathrm{Mqq}}T}{16\pi p}\ln\left(\frac{n^-_{\mathrm{F}}(E_{\mathrm{max}})\,n_{\mathrm{B}}(E_{\mathrm{min}}+p_0)}{n^-_{\mathrm{F}}(E_{\mathrm{min}})\,n_{\mathrm{B}}(E_{\mathrm{max}}+p_0)}\right)$$

$$\mathrm{Im}\,\Sigma_3(p_0,p) = \left(1+\frac{m^2}{2p^2}\right)\mathrm{Im}\,\Sigma_0 + \frac{g^2_{\mathrm{Mqq}}p_0T}{16\pi p^3}\;H_{34}(E)|^{E_{\mathrm{max}}}_{E_{\mathrm{min}}}$$

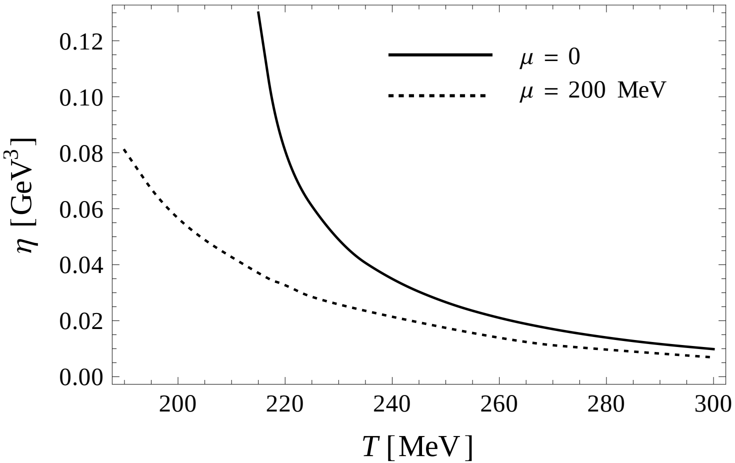
$$\mathrm{Im}\,\Sigma_4(p_0,p) = \mathrm{Im}\,\Sigma_0 + \frac{g^2_{\mathrm{Mqq}}T}{16\pi p p_0}\;H_{34}(E)|^{E_{\mathrm{max}}}_{E_{\mathrm{min}}}$$

with the auxiliary functions

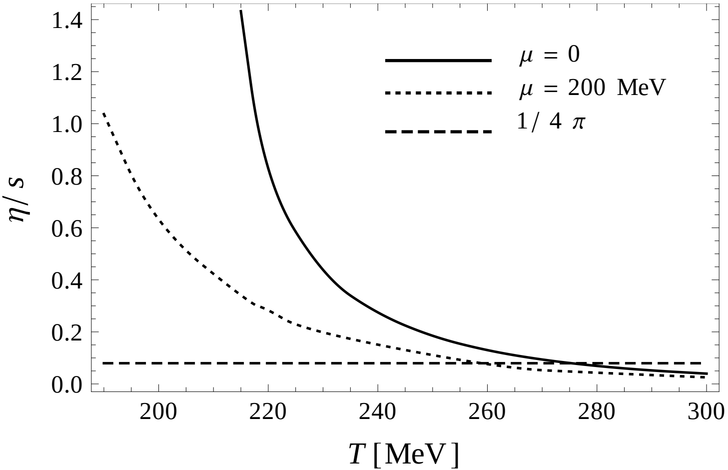
$$H_{34}(E) = (E+p_0)\ln n^-_{\mathrm{F}}(E) - T\,\mathrm{Li}_2\left(-\frac{1}{n_{\mathrm{B}}(E+p_0)}\right) - T\,\mathrm{Li}_2\left(1-\frac{1}{n^-_{\mathrm{F}}(E)}\right),$$

$$E_{\mathrm{max/min}} = \frac{1}{2m^2}\left[\left(m^2_{\mathrm{M}}-2m^2\right)\sqrt{m^2+p^2}\pm p m_{\mathrm{M}}\sqrt{m^2_{\mathrm{M}}-4m^2}\right]$$

Shear viscosity from mesonic fluctuations - Results



Shear viscosity from mesonic fluctuations - Results



entropy density at LO in $1/N_c$, i.e. free Fermi gas with thermal quark masses $m(T,\mu)$

Summary

In this talk:

- From HIC: **quark-gluon plasma** is almost-perfect fluid
- **Shear viscosity** in NJL model from **Kubo formalism**
  $\Rightarrow$ Large-N_c formalism for bookkeeping
- (Scalar) shear viscosity is highly **sensitive** to the NJL parameters, e.g. Λ
- **New Kubo formula** for Σ_R with full Dirac structure
- In the NJL model **Shear viscosity decreases** as function of T and μ undershooting the AdS/CFT benchmark $1/4\pi$

Thank YOU for your attention!

This work has been supported by:

Backup: Expressions for full Dirac shear viscosity

$$N_1 = p_0^2\left(1-\rho_4^2\right) - \vec{p}^{\,2}\left(1-\rho_3^2\right) - m^2\left(1-\rho_0^2\right),$$
$$N_2 = p_0^2\rho_4 - \vec{p}^{\,2}\rho_3 - m^2\rho_0\ ,$$

$$A = \rho_0 N_1 - 2N_2\ ,$$
$$B = \rho_4 N_1 - 2N_2\ ,$$
$$C = \rho_3 N_1 - 2N_2\ ,$$
$$D = N_1^2 + 4N_2^2\ .$$

One has in general for the **quark spectral width**:

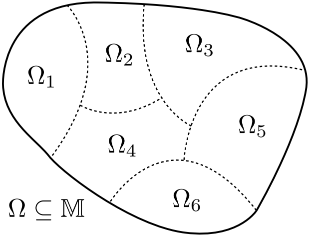
$$\rho = -\frac{1}{\pi}\mathrm{Im}\ G_{\mathrm{R}} = -\frac{1}{\pi D}\left[mA + p_0\gamma_0\, B - \vec{p}\vec{\gamma}\, C\right]$$

and the **off-shell** relation:

$$-m^2A^2 + p_0^2B^2 - \vec{p}^{\,2}C^2 = \left(-m^2\rho_0^2 - \vec{p}^{\,2}\rho_3^2 + p_0^2\rho_4^2\right)\cdot D$$

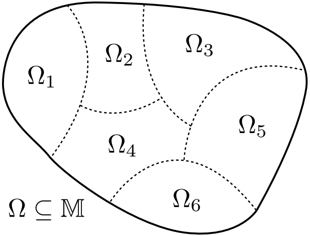
Backup: Dissipative hydrodynamics

$$\mathring{T}^{\mu\nu} = \mathrm{diag}\,(\epsilon, P, P, P)$$



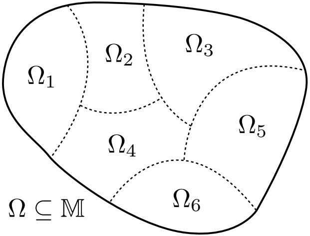
Backup: Dissipative hydrodynamics

$$T^{\mu\nu} = u^{\mu}u^{\nu}(\epsilon + P) - Pg^{\mu\nu}$$



Backup: Dissipative hydrodynamics

$$T^{\mu\nu} = u^\mu u^\nu(\epsilon + P) - Pg^{\mu\nu} + \tau^{\mu\nu}$$



1. $u_\mu \tau^{\mu\nu} = 0$, with $u^\mu(x) = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \gamma(x)(1, \boldsymbol{v}(x))$
2. $\tau^{\mu\nu}$ is a function of first-order derivatives
3. 2^{nd} law of thermodynamics: $\partial_\mu s^\mu = \frac{1}{T}\tau_{\mu\nu}\partial_\perp^\mu u^\nu \geq 0$

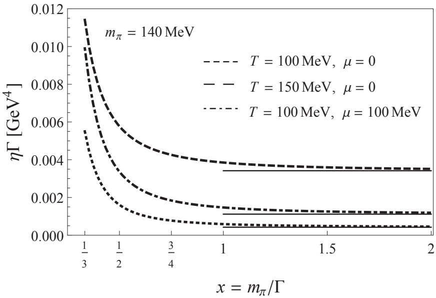
$$\Rightarrow \tau^{\mu\nu} = \eta\left[\partial_\perp^\mu u^\nu + \partial_\perp^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}(\partial_\perp \cdot u)\right] + \xi\Delta^{\mu\nu}(\partial_\perp \cdot u)$$

with

$$\partial_\perp^\mu := \partial^\mu - u^\mu(u \cdot \partial), \qquad \Delta^{\mu\nu} := g^{\mu\nu} - u^\mu u^\nu$$

Backup: Shear viscosity from Kubo formalism

$$\eta[\Gamma(p)] = \frac{16N_c N_f}{15\pi^3 T}\int_{-\infty}^{\infty} d\varepsilon \int_0^{\infty} dp\, p^6 \frac{m^2\,\Gamma^2(p)\,n_F(\varepsilon)(1-n_F(\varepsilon))}{\left[(\varepsilon^2 - p^2 - m^2 + \Gamma^2(p))^2 + 4m^2\Gamma^2(p)\right]^2}$$



- N_c large
- LO ladder diagrams at small compared to NLO ring diagrams

$$\eta[\Gamma] = \frac{A_{-1}}{\Gamma} + A_0 + A_1\Gamma + A_2\Gamma^2 + \ldots$$

$\Rightarrow$ NJL model for $\Gamma \ll m_\pi$ **perturbative**

ladder-diagram resummation $\Leftrightarrow$ perturbative NJL model

Backup: Shear viscosity vs. resistance in a parallel electric circuit

N electric resistances in parallel circuits

$$R_{\text{tot}}^{-1} = \sum_{i=1}^{N} R_i^{-1} \,, \qquad \Rightarrow \quad R_{\text{tot}} \longrightarrow \frac{R}{N} \quad (N \to \infty, R \equiv R_i)$$

$$\Updownarrow$$

N uncorrelated dissipative processes

$$\Gamma_{\text{tot}} = \sum_{i=1}^{N} \Gamma_i \,, \qquad \eta[\Gamma] \sim \frac{1}{\Gamma} \quad \text{(perturbative approach)}$$

$$\eta_{\text{tot}}^{-1} = \sum_{i=1}^{N} \eta_i^{-1} \,, \qquad \Rightarrow \quad \eta_{\text{tot}} \longrightarrow \frac{\eta}{N} \quad (N \to \infty, \eta \equiv \eta_i)$$

Backup: Kinetic approach vs. Kubo formalism

Kinetic theory

$$\eta = \frac{\beta}{15} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{p^4}{E^2} \, \tau f_0 (1 \pm f_0)$$

- δf small
- τ large
- no spatial dependences

Kubo formalism

$$\eta = \frac{1}{15} \int_0^\infty \mathrm{d}t \; \mathrm{e}^{\mathrm{i}\omega t} \int \mathrm{d}^3 r \, (T_{\mu\nu}(\vec{r}, t), T^{\mu\nu}(0))$$

$$\eta|_{\lambda\phi^4} = \frac{\beta}{30} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{p^4}{E^2 \, \Gamma(p)} n_\mathrm{B}(E) \, [1 + n_\mathrm{B}(E)]$$

- linear response theory
- $\langle B \rangle \ll \langle A \rangle$
- perturbative $\Gamma \sim \lambda^2 \ll 1$

BUT: Within the (diagrammatic) Kubo formalism at LO in $\lambda\phi^4$ theory ladder diagram resummation is necessary due to existence of pinch poles.

$\Rightarrow$ Kinetic approach and resummed LO Kubo formalism are equivalent.
[Aarts, Hidaka, Jeon, ...]

Backup: Entropy of non-interacting quarks

In the limit $N_\mathsf{c} \to \infty$ the NJL model becomes a **free Fermi theory**! $(G \sim N_\mathsf{c}^{-1})$

$$\ln Z_0 = 2N_\mathsf{c}N_\mathsf{f}V \int \frac{\mathrm{d}^3p}{(2\pi)^3}\left[\beta\omega + \ln\left(1+\mathrm{e}^{-\beta(\omega-\mu)}\right) + \ln\left(1+\mathrm{e}^{-\beta(\omega+\mu)}\right)\right]$$

$$\Rightarrow \quad P_0 = \frac{\ln Z_0}{\beta V}\ , \quad s_0 = \frac{\partial P_0}{\partial T}$$

In the massless (Boltzmann) limit $m_\mathrm{q} = 0$ $(\Rightarrow \omega = p)$:

$$s(T,\mu) = -\frac{12T^2}{\pi^2}\left[\mu\left(\mathrm{Li}_3(-\mathrm{e}^{-\beta\mu}) - \mathrm{Li}_3(-\mathrm{e}^{\beta\mu})\right) + 4T\left(\mathrm{Li}_4(-\mathrm{e}^{-\beta\mu}) + \mathrm{Li}_4(-\mathrm{e}^{\beta\mu})\right)\right]$$

$$= \frac{14\pi^3}{15}T^3 + 2T\mu^2$$

$$\mathrm{Li}_n(z) = \sum_{k=1}^{\infty}\frac{z^k}{k^n} \quad \text{(Polylog)}$$

$$\mathrm{Li}_n(1) = \zeta(n)$$

$$\mathrm{Li}_n(-1) = (2^{1-n}-1)\zeta(n)$$

$\Rightarrow$ Polynomial structure of

$$\mathrm{Li}_3(-\mathrm{e}^{-x}) - \mathrm{Li}_3(-\mathrm{e}^{x})$$

$$\mathrm{Li}_4(-\mathrm{e}^{-x}) + \mathrm{Li}_4(-\mathrm{e}^{x})$$

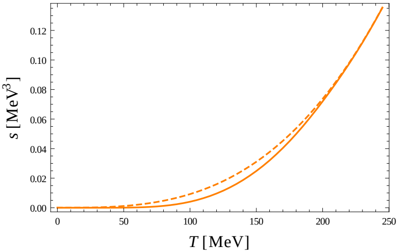
Remark

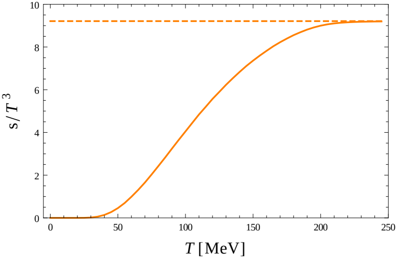
There is no Feynman representation of Z_0 (or $\ln Z_0$)!

foundations and discussion of "perturbative" theories:

[R.C. Helling arXiv:1201.2714]

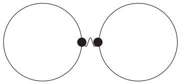
Backup: Entropy of interacting quarks ($\mu = 0$)





Figures: Boltzmann limit (dashed) vs. thermal-mass entropy density (solid); soft cutoff scheme used

$$\ln Z = \ln Z_0 + \ln Z_{\text{int}} = \ln Z_0 + \ln Z_1 + \ln Z_2 + \dots \qquad \text{where } \ln Z_i = \beta V \Phi^{(i-1)}$$



$$\ln Z_1 = \frac{G\beta V}{2}\left[\sum_{n=-\infty}^{\infty}\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\,\mathrm{Tr}\,S(\omega_n,\vec{p})\right]^2 =$$

$$s_1 = \frac{\partial}{\partial T}\frac{\ln Z_1}{\beta V} \qquad = \frac{2G\beta V}{\pi^4}N_\mathrm{c}^2 N_\mathrm{f}^2 m^2\left[\int_0^\Lambda \mathrm{d}p\,\frac{p^2}{\omega}\left(\frac{1}{2} - n_\mathrm{F}(\omega)\right)\right]^2$$

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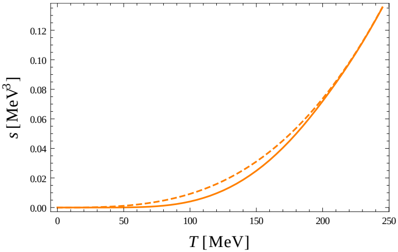

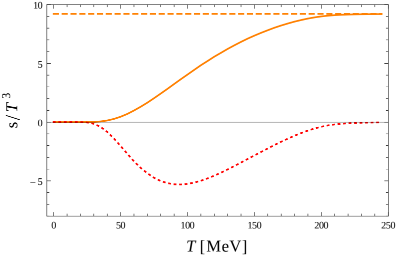

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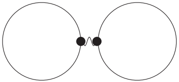

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Backup: Overview of perturbative corrections to the entropy density

[J.I. Kapusta, C. Gale: Cambridge 2006]

- ϕ^4 theory

$$s(T) = \frac{2\pi^2 T^3}{45}\left[1 - \frac{15\lambda}{8\pi^2}\right] + \dots$$

- QED

$$s(T) = \frac{11\pi^2 T^3}{45}\left[1 - \frac{25\alpha}{22\pi}\right] + \dots$$

- QCD

$$s(T) = 4 d_{\mathrm{A}} T^3 \left[\frac{1}{5}\left(1 + \frac{7 d_{\mathrm{F}}}{4 d_{\mathrm{A}}}\right) - \frac{\alpha_{\mathrm{s}}^2}{4\pi}\left(C_{\mathrm{A}} + \frac{5}{2} S_{\mathrm{F}}\right)\right] + \dots$$

- χPT [N. Kaiser] [P. Gerber and H. Leutwyler, Nucl.Phys.B **321**, 387 (1989)]

$$\begin{aligned} s(T) =& \frac{T}{2\pi^2}\left[4T^2 h_5(\beta m_\pi) + 3 m_\pi^2 h_3(\beta m_\pi)\right] \\ & - \frac{3 m_\pi^2 T}{16\pi^4 f_\pi^2} h_3(\beta m_\pi)\left[2T^2 h_3(\beta m_\pi) + m_\pi^2 h_1(\beta m_\pi)\right] + \dots \end{aligned}$$

$$\text{where} \quad h_n(\xi) = \int_\xi^\infty \mathrm{d}x\, \frac{(x^2 - \xi^2)^{\frac{n}{2}-1}}{\mathrm{e}^x - 1} > 0$$