v2 and Jet Fragmentation @LHC & LEP

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Outline

- v2 & Spectra @ LHC & RHIC (Tsallis)

- Statistical Jet Fragmentation @ LEP & LHC
  - Scale evolution,
  - Application in Parton Model Calculations

- Interesting N and s dependence in pp collisions

Outline

- *v2 & Spectra @ LHC & RHIC (Tsallis)*

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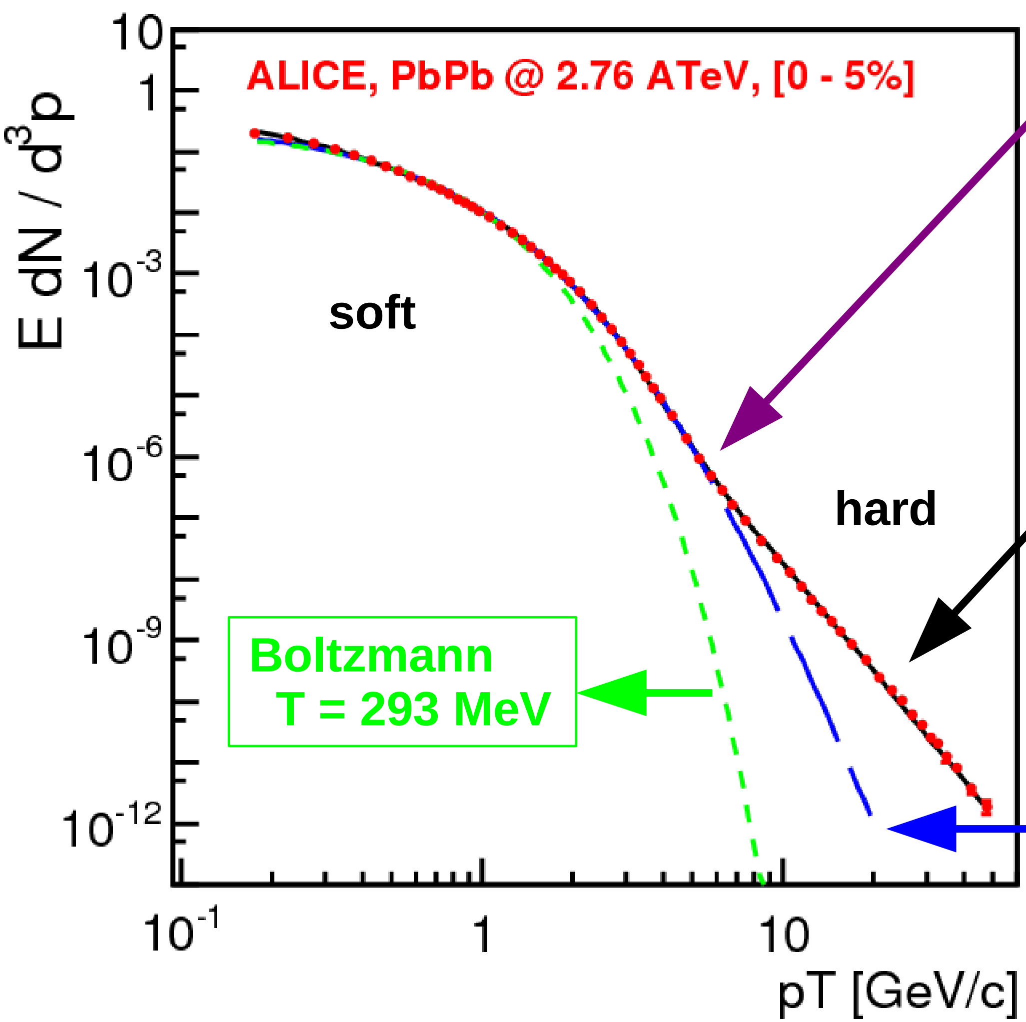
- *Interesting N and s dependence in pp collisions*

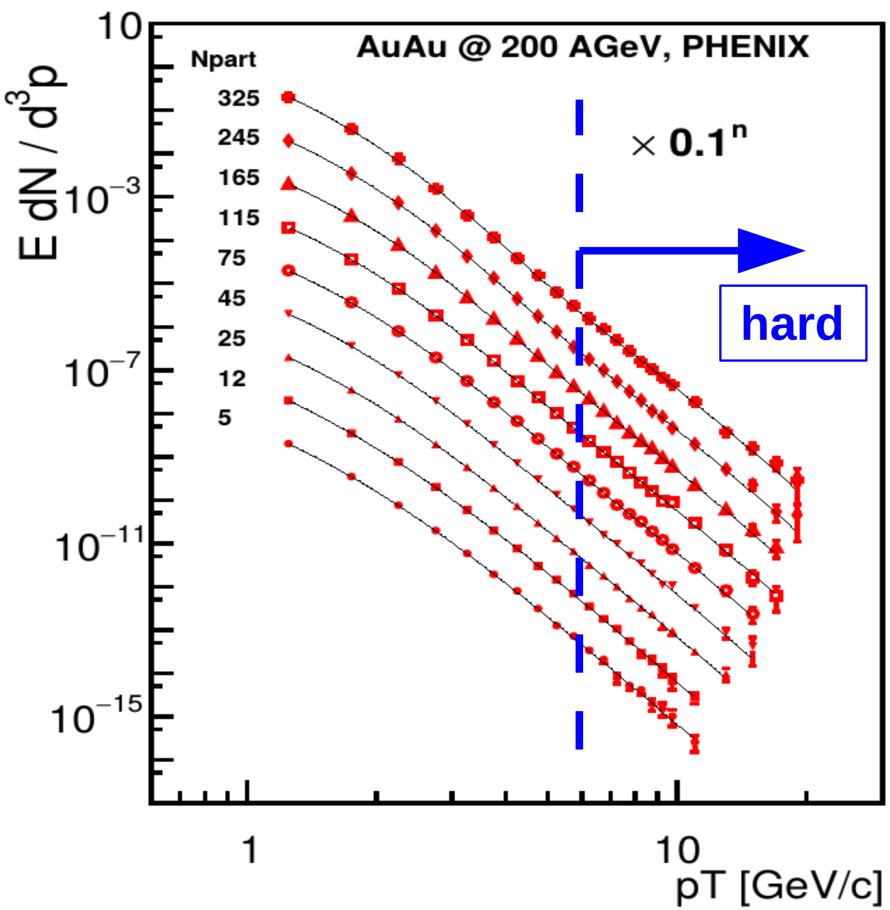
The power of the spectrum changes drastically at pT ~ 6 GeV/c.

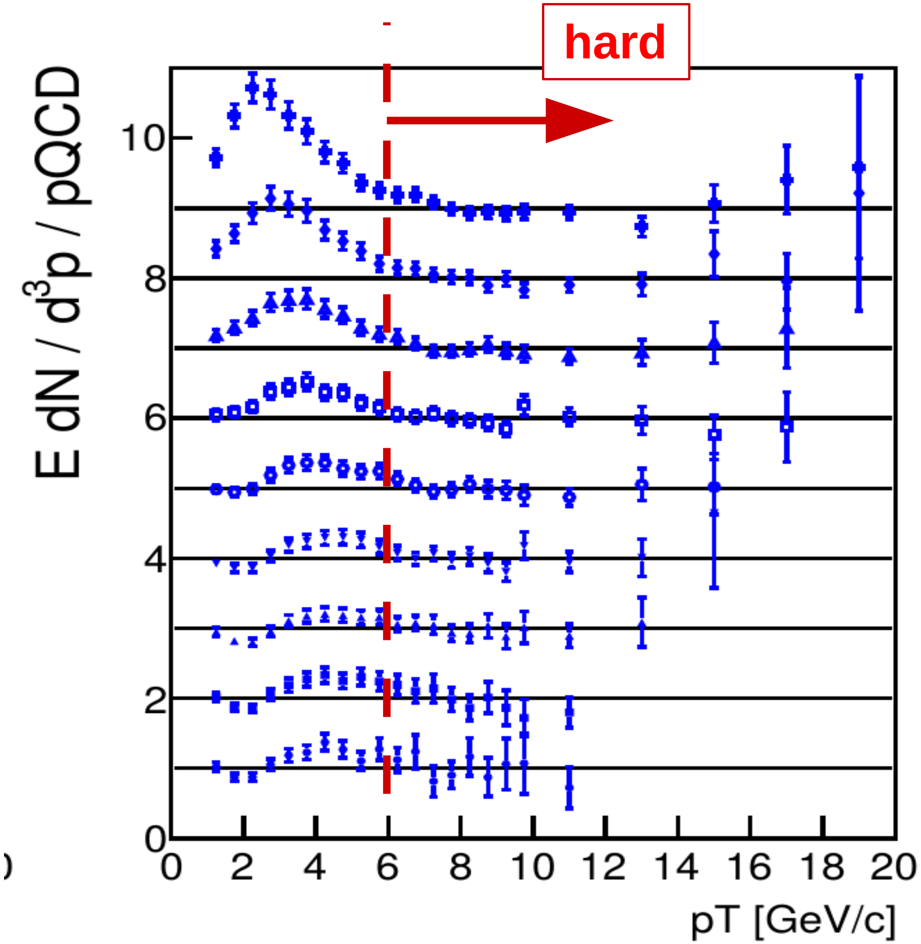


$\sim \; p_T^{-6.08}$

Tsallis $\sim \; p_T^{-13.7}$

The *hard* yields: *pQCD*

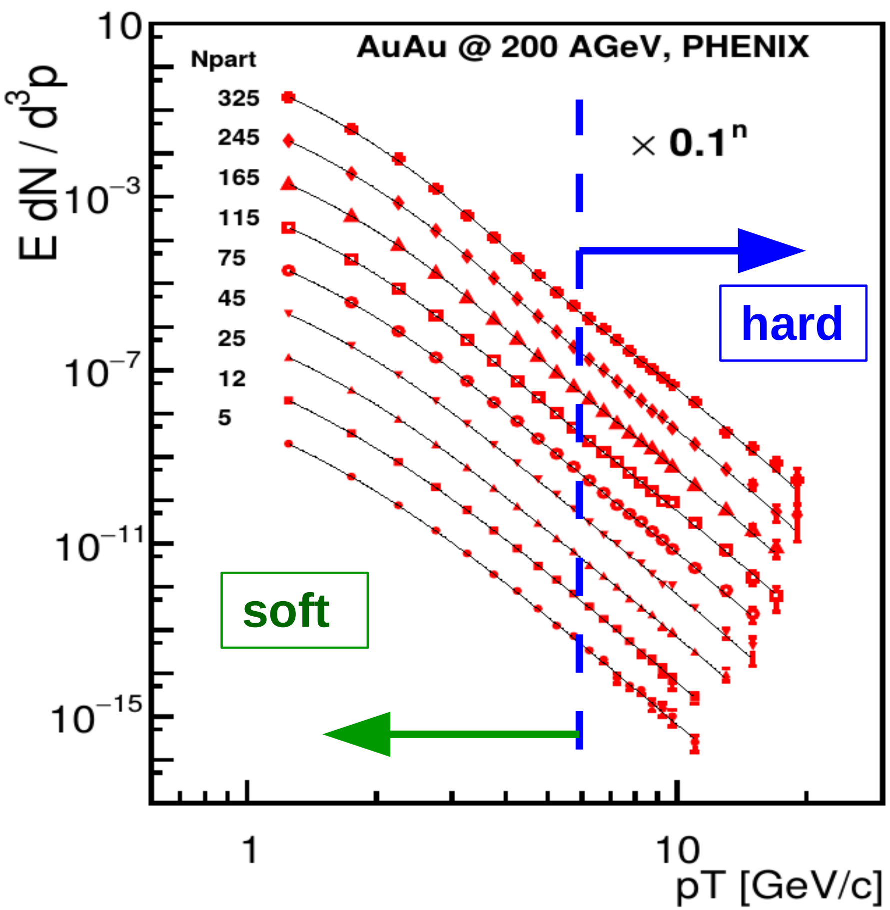


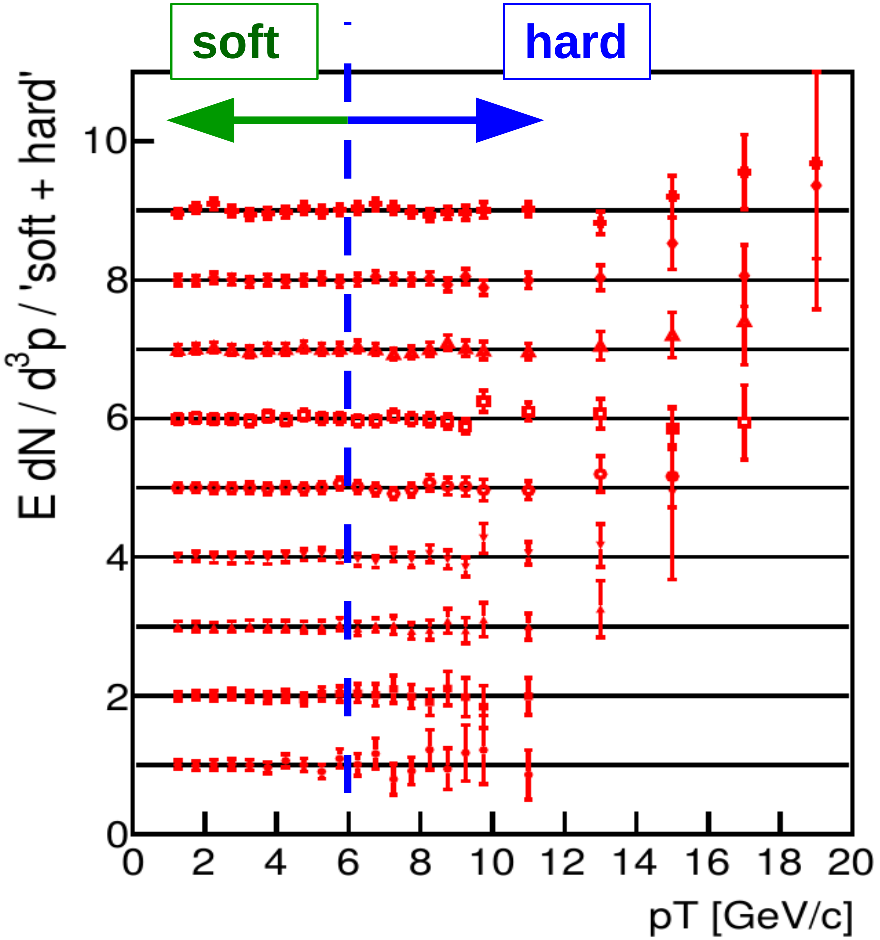


Extrapolation to low-pT by fitted Tsallis:

$$E\frac{dN}{d^3\boldsymbol{p}}^{hard} = A\left[1 + \frac{q-1}{T}[E^{co} - m]\right]^{-1/(q-1)} \qquad E^{co} = \gamma(m_T - v\, p_T)$$

The *whole* yield: *soft + hard*

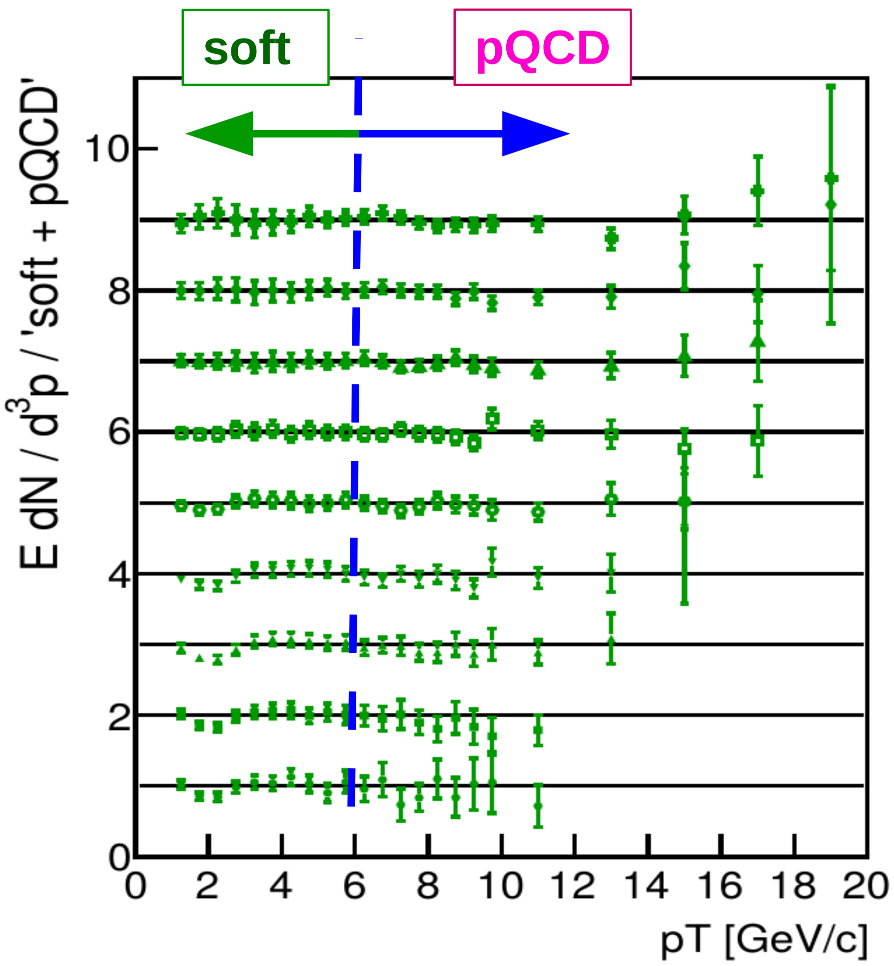


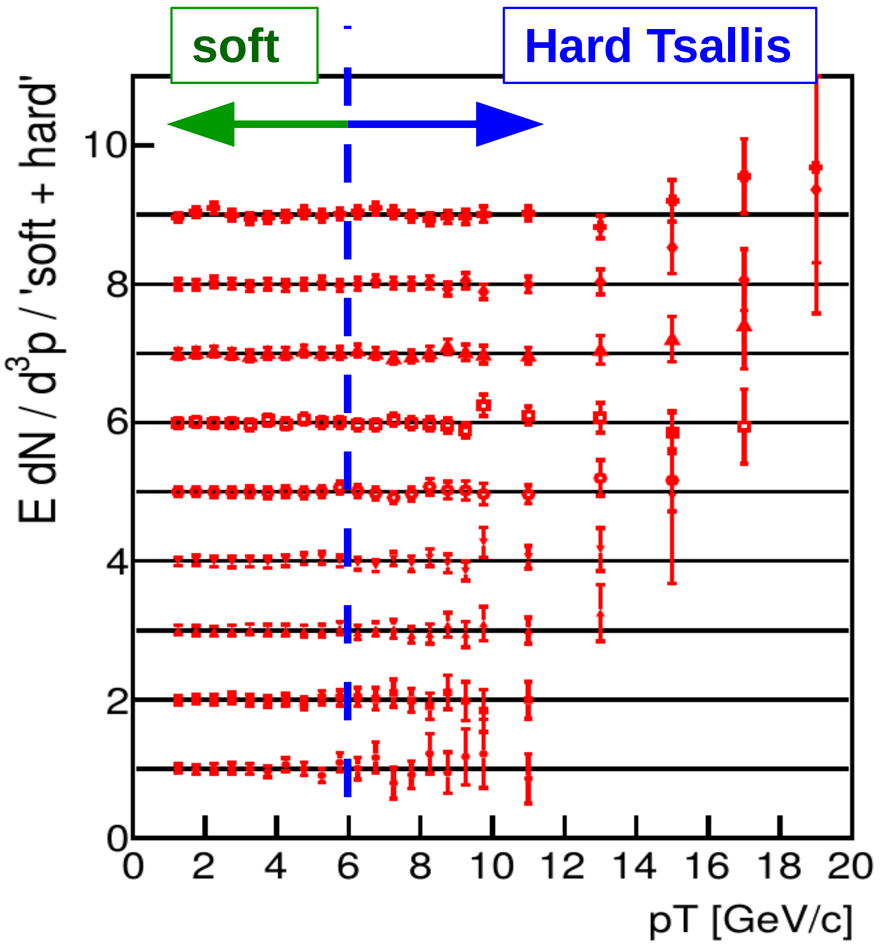


Both **the *soft* and the *hard* yields are *Tsallis*:**

$$E\frac{dN}{d^3\mathbf{p}}^{soft/hard} = A\left[1 + \frac{q-1}{T}\left[E^{co} - m\right]\right]^{-1/(q-1)} \qquad E^{co} = \gamma\left(m_T - v\,p_T\right)$$

There is no much difference between the
'soft + pQCD' **and the** ***'soft + hard'***
scenarios





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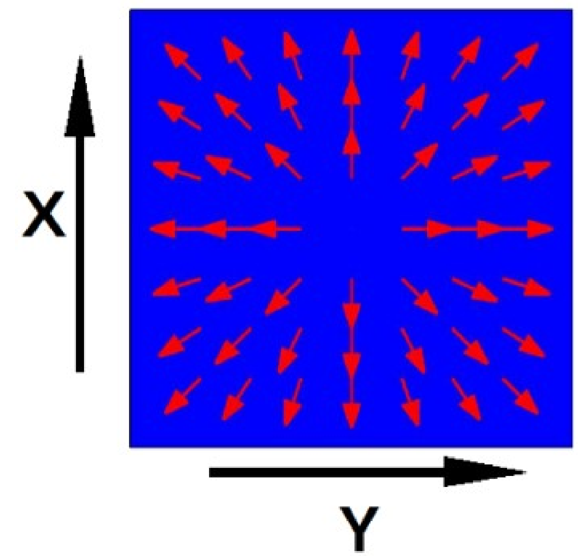
(1) Statistical description of hadron spectra:

$$E\frac{dN}{d^3\mathbf{p}} = \sum_{sources} f\left[u_\mu p^\mu\right]$$

(2) Space-time dependence only through $u_\mu(x)$

$$v(\alpha) = v_0 + \sum_1^N \delta v_m \cos(m\alpha)$$



Then, the spectrum and the v2 are

$$\frac{dN}{p_T dp_T dy}_{y=0} \propto f\left[E(v_0)\right] + O(\delta v^2)$$

$$v_2 \propto \delta v_2 (v_0 m_T - p_T)\frac{f'\left[E(v_0)\right]}{f\left[E(v_0)\right]} + O(\delta v^2)$$

$$E(v_0) = \gamma_0 (m_T - v_0 p_T)$$

arXiv:1405.3963

Then, the spectrum and the v2 are

$$\left.\frac{dN}{p_T\,dp_T\,dy}\right|_{y=0} \propto f\left[E(v_0)\right] + O(\delta v^2) \qquad E(v_0) = \gamma_0 (m_T - v_0 p_T)$$

$$v_2 \propto \delta v_2 (v_0 m_T - p_T) \frac{f'\left[E(v_0)\right]}{f\left[E(v_0)\right]} + O(\delta v^2)$$

Boltzmann-distribution:

$$f\left[E(v_0)\right] \propto e^{-E(v_0)/T}$$

$$v_2 \propto p_T - v_0 m_T$$

Tsallis-distribution:

$$f\left[E(v_0)\right] \propto \left[1+(q-1)\frac{E(v_0)-m}{T}\right]^{-1/(q-1)}$$

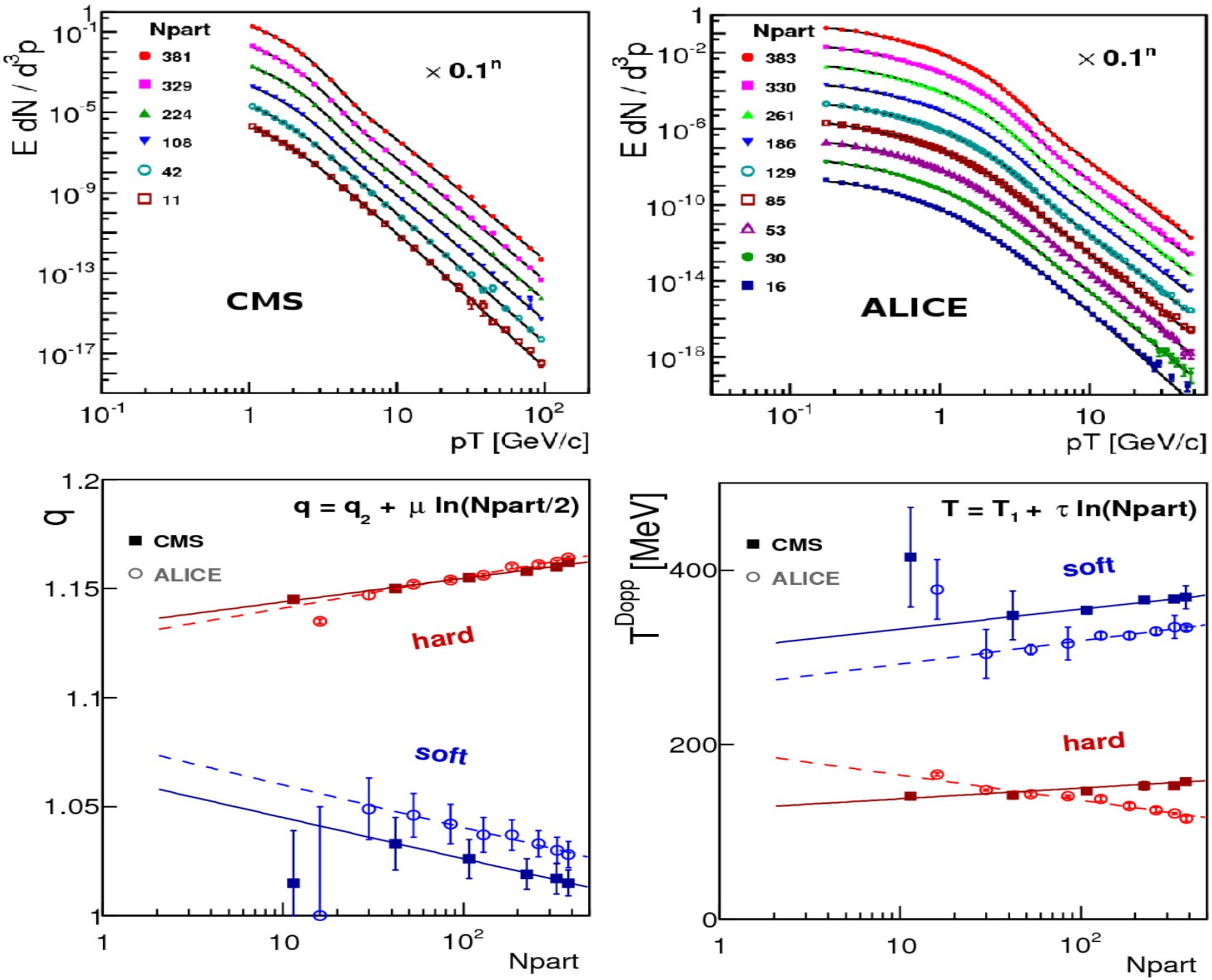
$$v_2 \propto \frac{p_T - v_0 m_T}{1+\frac{q-1}{T}\left[\gamma_0 (m_T - v_0 p_T) - m\right]}$$

arXiv:1405.3963

1.

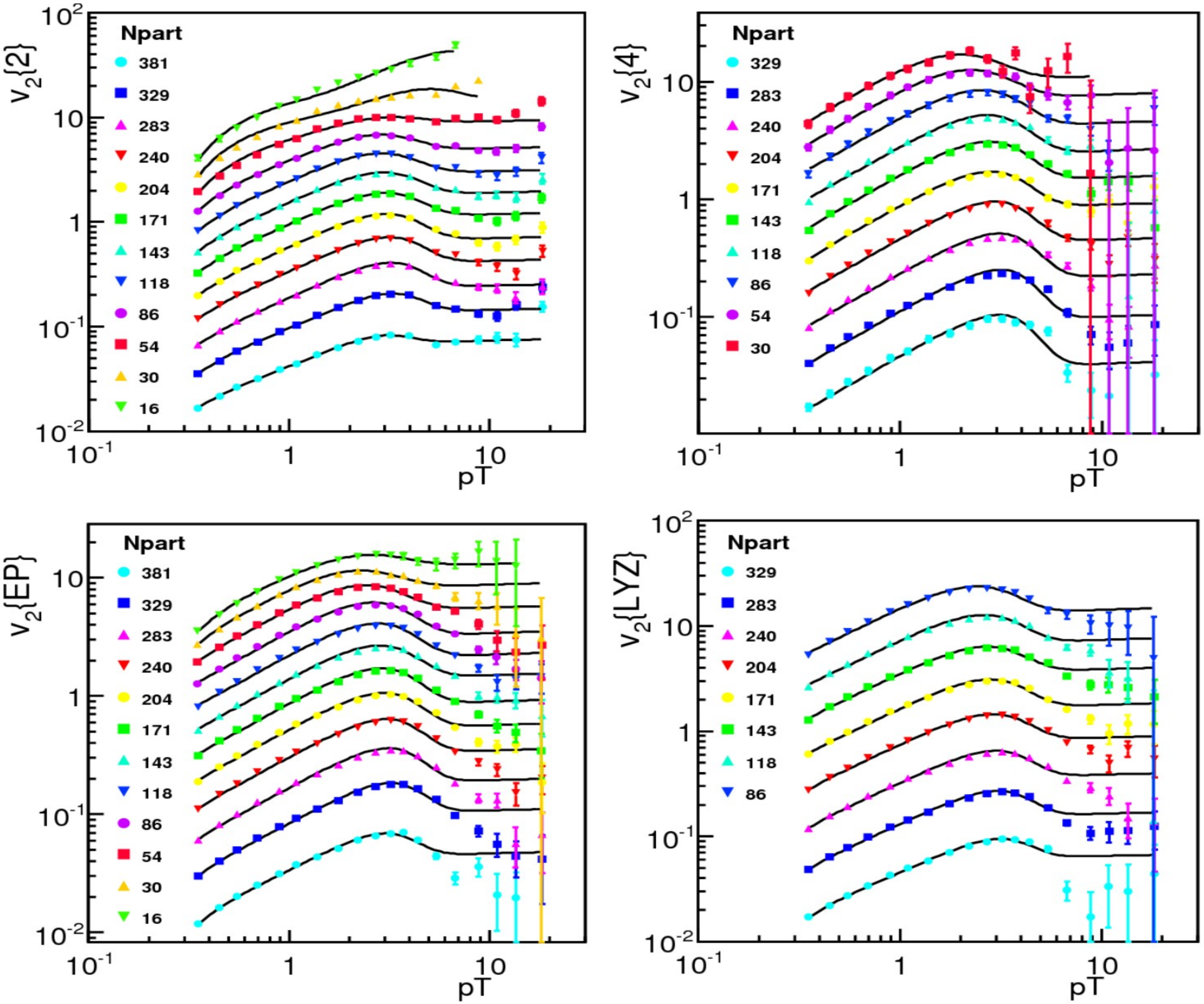
PbPb → $h^{\pm}$

CMS



arXiv:1405.3963

v2 of $h^{\pm}$



arXiv:1405.3963

Outline

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Jet-fragmentation in e^+e^- & pp collisions

The cross-section of the creation of hadrons $h_1, \dots, h_N$ in a jet of N hadrons

$$d\sigma^{h_1,\dots,h_N} = |M|^2 \delta^{(4)}\left(\sum_i p^{\mu}_{h_i} - P^{\mu}_{tot}\right) d\Omega_{h_1,\dots,h_N}$$

- **If the jet is very narrow, hadrons fly *nearly co-linearly* (*quasi 1 Dim*).**
- ***If* we neglect hadron masses ($m_i = 0$), the conservation of P_μ reduces to the conservation of E**
- **If $|M| \approx$ *constans*, *we arrive at a 1D microcanonical ensemble*:**

$$d\sigma^{h_1,\dots,h_N} \propto \delta\left(\sum_i \epsilon_{h_i} - E_{tot}\right) d\Omega_{h_1,\dots,h_N} \propto E_{tot}^{N-1}$$

- U.K. et.al., *Phys. Lett. B*, **718**, 125-129, (2012)
- U.K. et.al., *Phys.Lett.B*, **701,** 111-116 (2011)
- U.K. et.al., *Acta Physica Polonica B*, **5**:(2), pp. 363-368, (2012)
- T.S. Biró et.al., *Acta Physica Polonica B*, **43**:(4), pp. 811-820, (2012)

Jet-fragmentation in e^+e^- & pp collisions

Thus, the ***energy distribution*** of hadrons in a jet of ***N*** (massless) hadrons is:

$$f_N(z) = A_N(1-z)^{N-2}, \quad z = \epsilon_h / E_{jet}$$

- **The *hadron multiplicity* in a jet *fluctuates* as *(exp. observation):***

$$p(N) \propto (N-N_0)^{\alpha-1} e^{-\beta(N-N_0)}$$ ***or NBD***

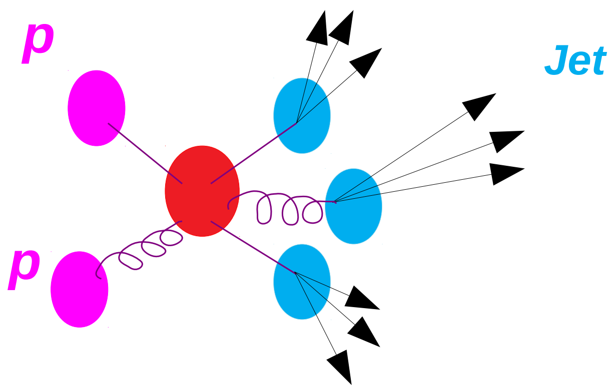
- **Thisway, the *multiplicy averaged hadron distribution* (*fragmentation function*):**

$$\frac{d\sigma}{dz} = \sum_{N=N_0}^{\infty} f_N(z) N\, p(N) \propto \frac{(1-z)^{\nu(N_0)}}{\left(1 - \frac{(q-1)}{T/E_{jet}} \ln(1-z)\right)^{1/(q-1)}}$$

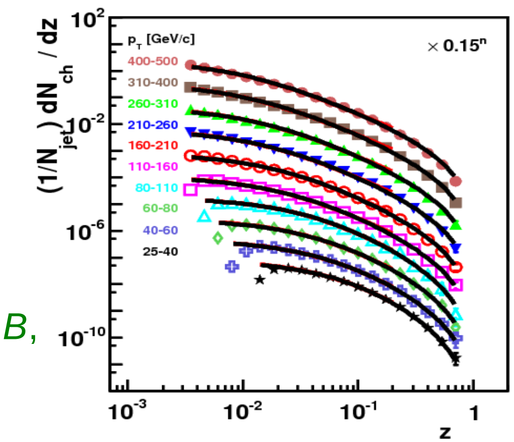
$$q = 1 + 1/(\alpha+2), \; T = E_{jet}\beta/(\alpha+2), \; \nu = N_0 - 2$$

Confrontation with measurements

pp → jets @LHC (pT = 25–500 GeV/c)

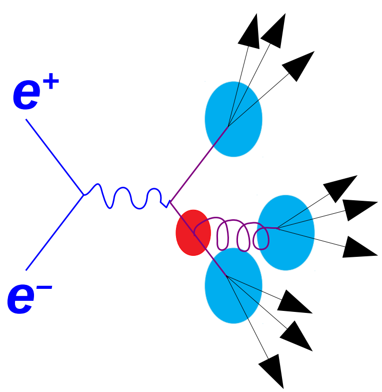


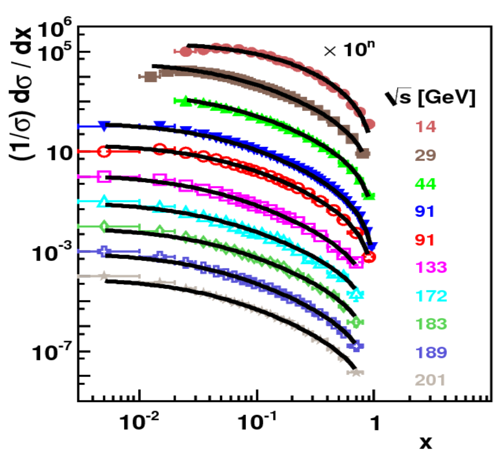
$$\frac{dN}{dz} \propto \left[1 - a\ln(1-z)\right]^{-b}$$



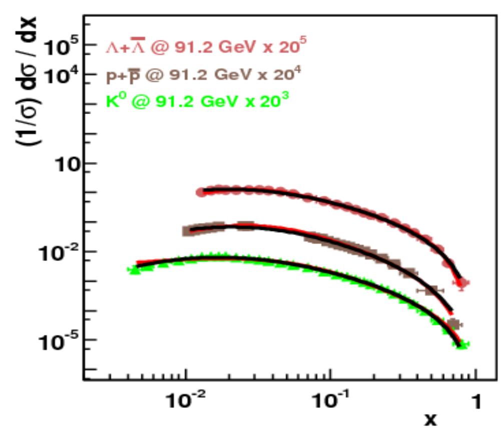
Urmossy et.al. *Phys. Lett. B*, **718**, 125-129, (2012)

e⁺e⁻ annihilation @LEP (√s = 14–200 GeV)

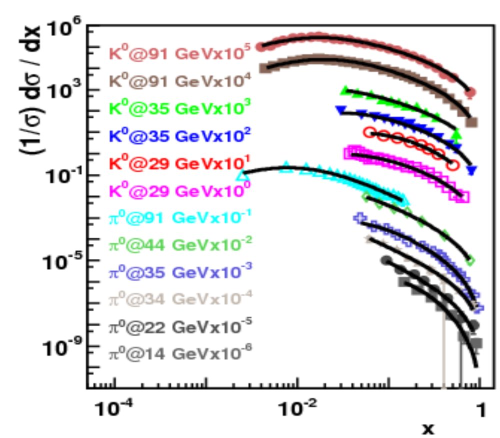




Urmossy et. al., *Phys. Lett. B*, **701**, 111-116 (2011)



Urmossy et.al., *Acta Phys. Polon. Supp.* **5** (2012) 363-368



T. S. Biró et.al., *Acta Phys. Polon. B*, **43** (2012) 811-820

Generalisation to 3D

The cross-section with conserved P^μ :

$$d\sigma^{h_1,\dots,h_N} \propto \delta^4\left(\sum_i p_i^\mu - P_{tot}^\mu\right) d\Omega_{h_1,\dots,h_N} \propto M^{2(N-2)} \quad , \quad M^2 = P_\mu P^\mu$$

Thus, the *energy distribution* of hadrons in a jet of *N* (massless) hadrons is:

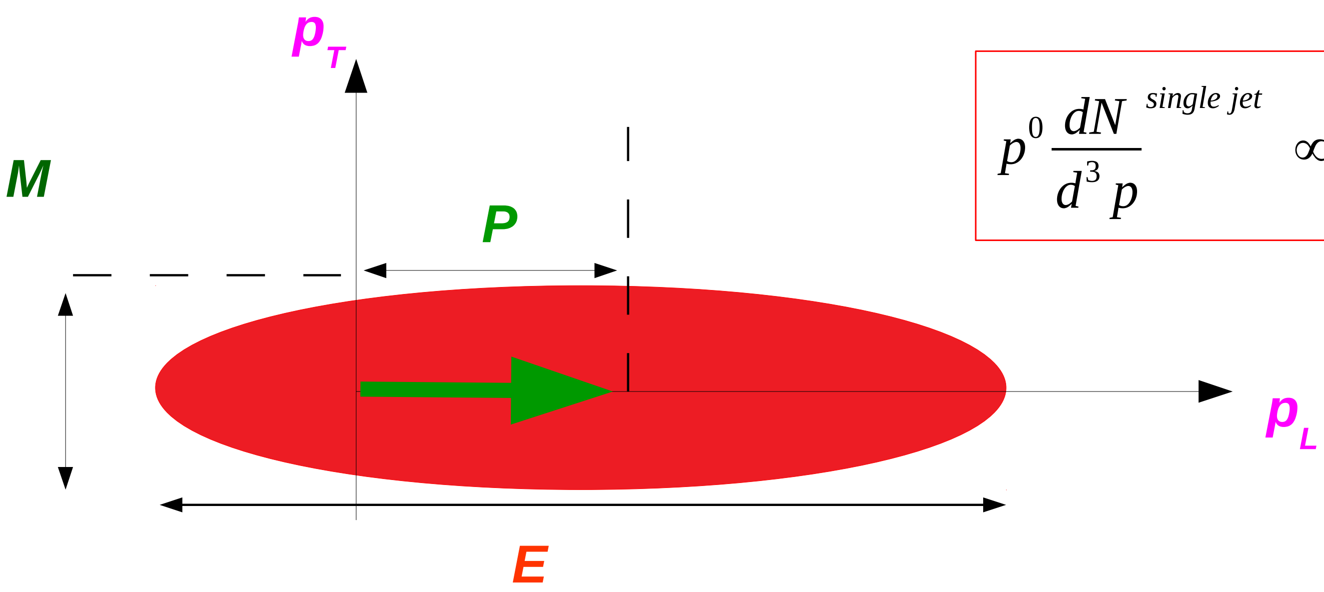
$$f_N(z) = A_N\left(1 - \frac{u_\mu p^\mu}{M/2}\right)^{N-3} , \quad u_\mu = P_\mu / M$$

• **Thisway, the *multiplicy averaged hadron distribution* (*fragmentation function*):**

$$p^0 \frac{d\sigma}{d^3p} = \sum_{N=N_0}^{\infty} f_N(z) N p(N) \propto \left(1 + \frac{q-1}{T}\frac{u_\mu p^\mu}{M/2}\right)^{-1/(q-1)} - \sum_{N=0}^{N_0-1} a_N \left(1 - \frac{u_\mu p^\mu}{M/2}\right)^{N-3}$$

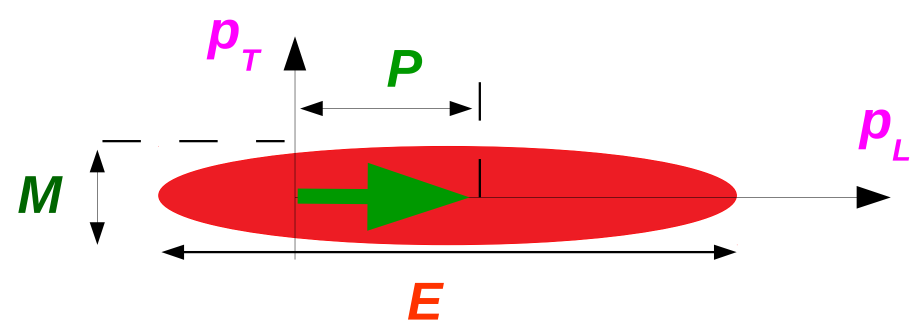
microcanonical cut @ maximal momentum

Hadrons in a Single Jet



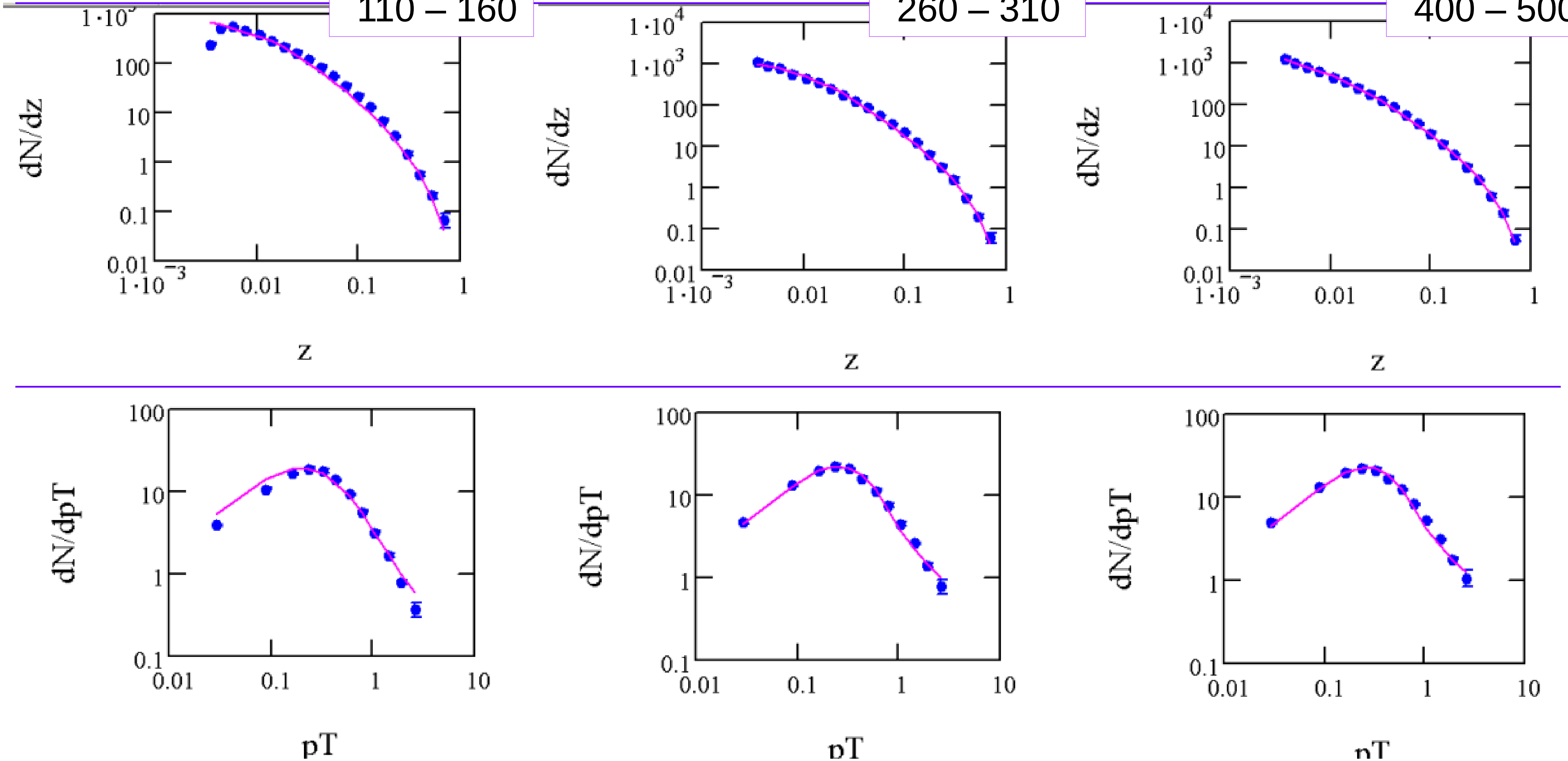
$$p^0 \frac{dN}{d^3 p}^{\text{single jet}} \propto \left[1 - \frac{\gamma \left(1 - v \cos \vartheta \right) p^0}{M/2} \right]^{N-3}$$

ATLAS data in pp@7 TeV

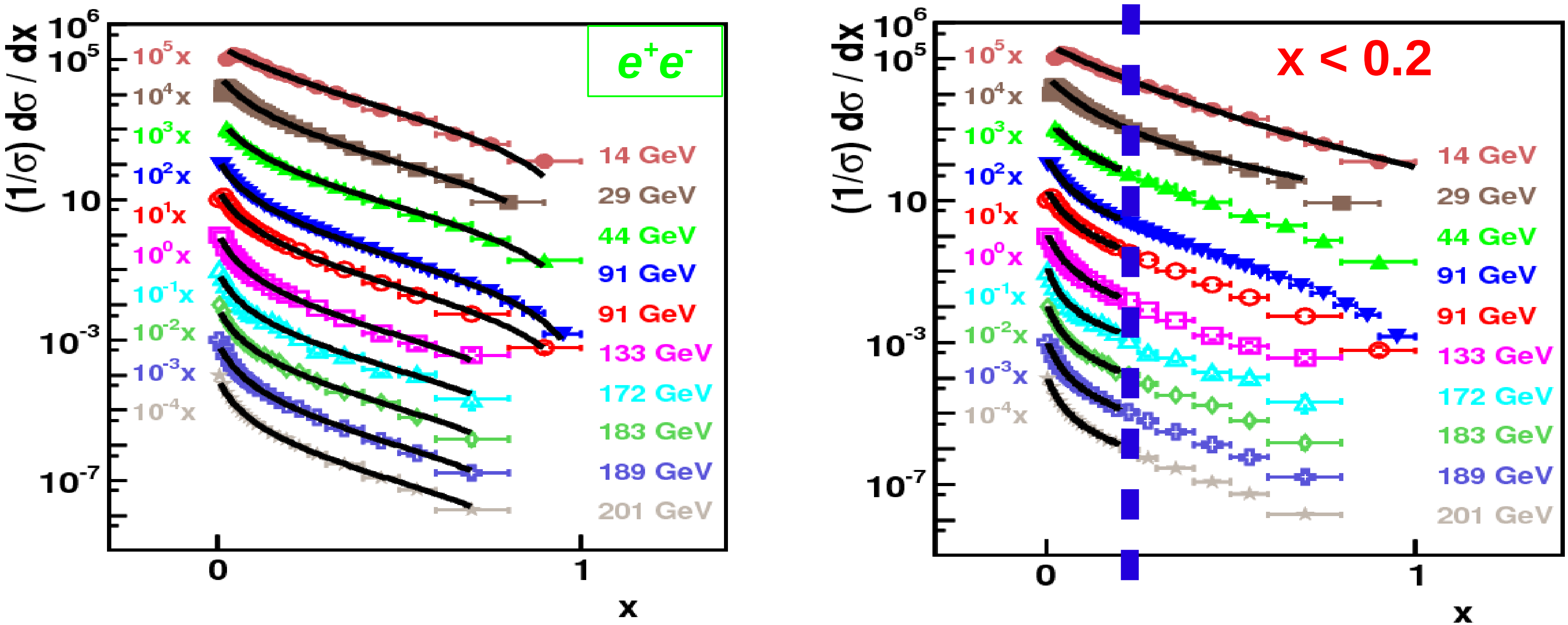


P_{jet} [GeV/c]:

110 – 160 | 260 – 310 | 400 – 500



x << 1 limit: microcanonical → canonical Tsallis distribution



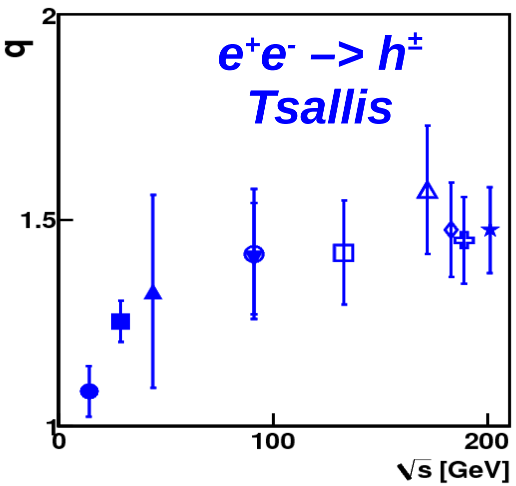
$$\frac{d\sigma}{dx} \propto \left(1 - \frac{q-1}{T/(\sqrt{s}/2)} \ln(1-x)\right)^{-1/(q-1)} \quad \Rightarrow \quad \left(1 + \frac{q-1}{T/(\sqrt{s}/2)} x\right)^{-1/(q-1)}$$

Urmossy et.al., *Phys.Lett.B*, **701,** 111-116 (2011), arXiv:1101.3023

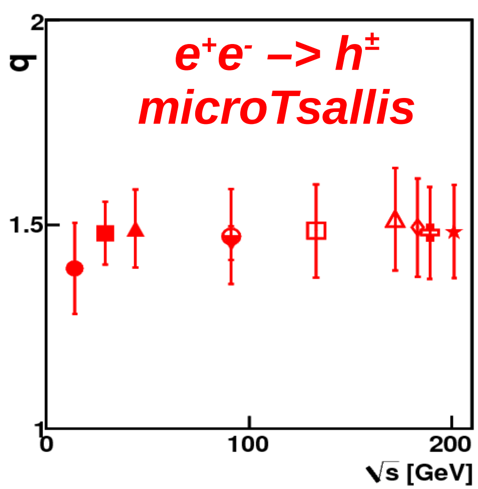
Scale Evolution

Fitts:

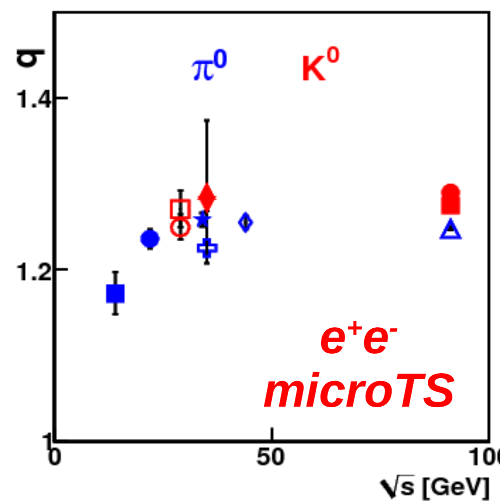
1)



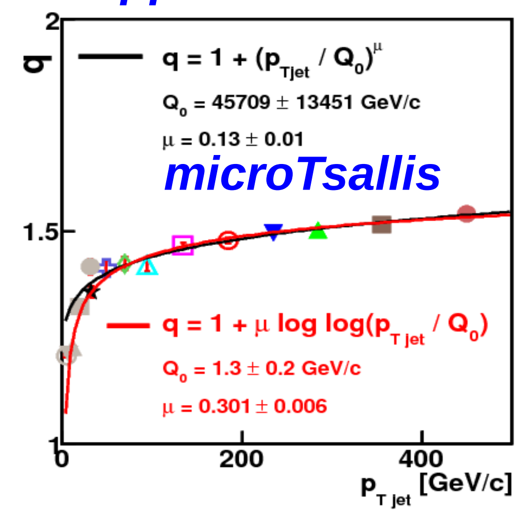
2)



3)

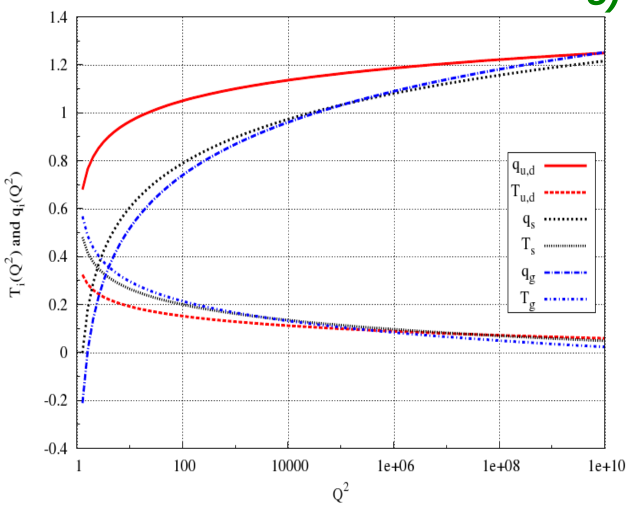


pp –> Jet –> $h^\pm$ 4)



Theory: Scale evolution of *q*, *T* ***from fits to AKK Frag. Funcs:***

5)



$$D_{p_i}^{\pi^+}(z) \sim \left(1+(q_i-1)z/T_i\right)^{-1/(q_i-1)}$$

$$q_i = q_{0i} + q_{1i}\ln\left(\ln\left(Q^2\right)\right)$$

1-2) U.K. etal., *Phys.Lett. B*, **701** (2011) 111-116

3) T. S. Biró etal., *Acta Phys.Polon. B*, **43** (2012) 811-820

4) U.K. etal., *Phys.Lett. B*, **718** (2012) 125-129

5) Barnaföldi etal., *Gribov-80 Conf*: C10-05-26.1, p.357-363

Scale Evolution

Fitts:

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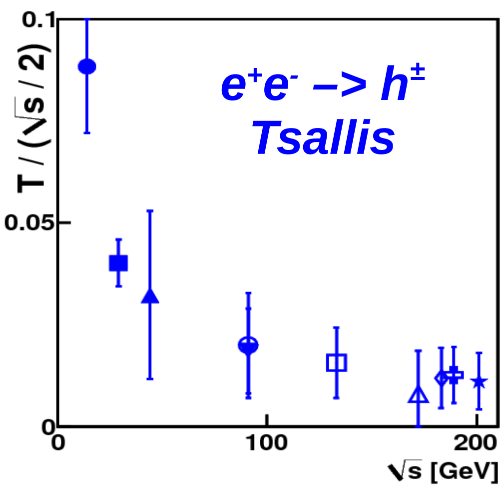
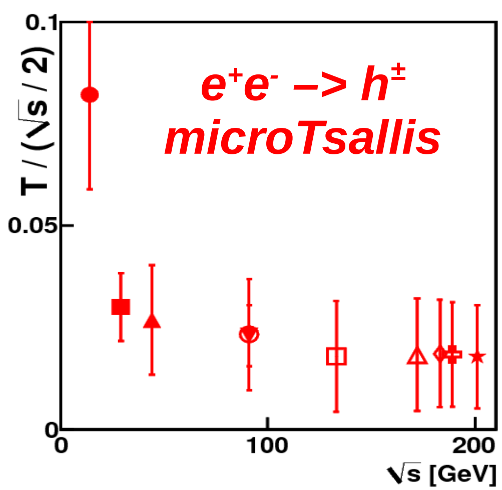
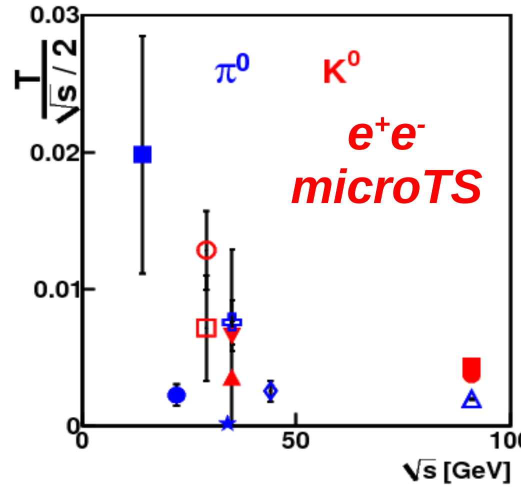


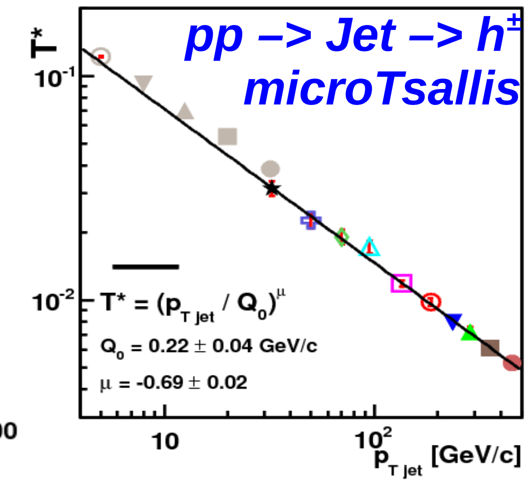
2)



3)



4)



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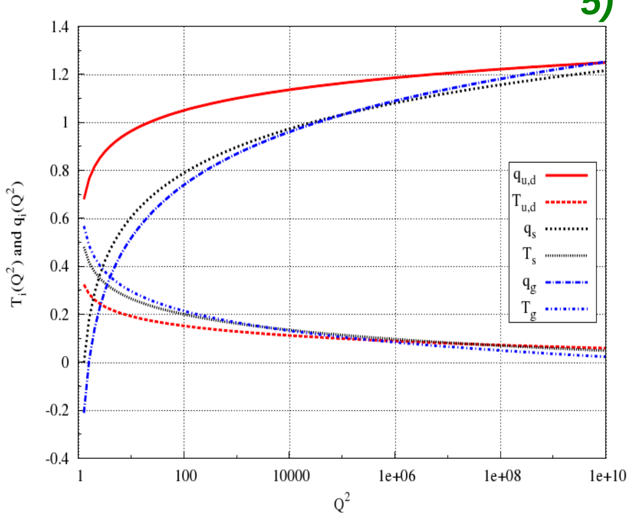


$$D^{\pi^+}_{p_i}(z) \sim \left(1+(q_i-1)z/T_i\right)^{-1/(q_i-1)}$$

$$T_i = T_{0i}+T_{1i}\ln\left(\ln\left(Q^2\right)\right)$$

1-2) U.K. etal., *Phys.Lett. B,* **701** (2011) 111-116

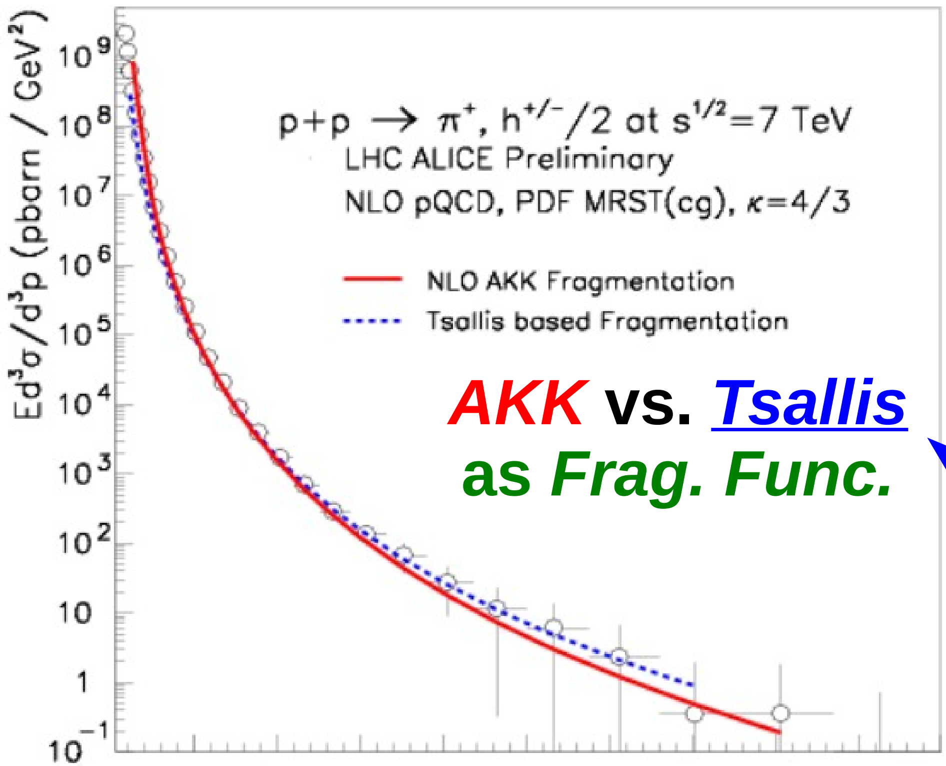
3) T. S. Biró etal., *Acta Phys.Polon. B*, **43** (2012) 811-820

4) U.K. etal., *Phys.Lett. B*, **718** (2012) 125-129

5) Barnaföldi etal., *Gribov-80 Conf*: C10-05-26.1, p.357-363

Application in a pQCD calculation

π^+ spectrum in pp --> $\pi^\pm$X @ √s=7 TeV (NLO pQCD)



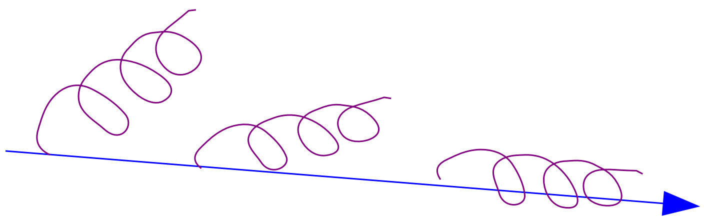
AKK vs. Tsallis as Frag. Func.

$$D_{p_i}^{\pi^+}(z) \sim \left(1+(q_i-1)\,z/T_i\right)^{-1/(q_i-1)}$$

Barnaföldi et. al., *Proceedings of the Workshop Gribov '80* (2010)

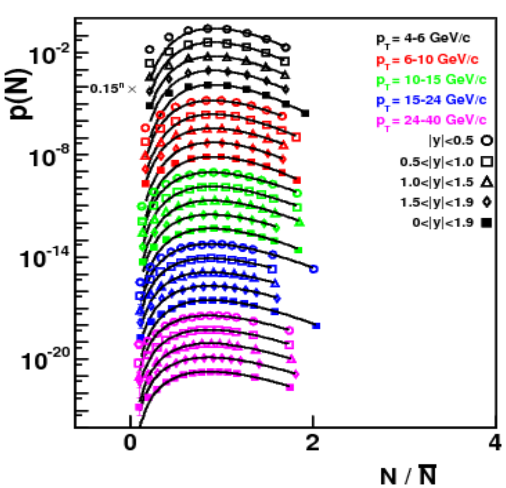
Motivationtion for such N distribution

- **Small angle coherent radiation of n gauge bosons causes**
  - ***scale evolution,***
  - ***large log-s and***
  - ***Poissonian n distribution***



$$\propto \frac{(\alpha_s/2\pi)^n}{n!}\log^n\left(\frac{Q^2}{Q_0^2}\right) \propto \frac{\lambda^n}{n!}$$

- ***However, in jets, we see NBD / Γ(n) type n fluctuations:***



pp → jets @ 7 TeV

Urmossy et. al., *PLB*, **718**, 125-129, (2012)

- ***This may be resolved by:***

$$f(\lambda) \propto \left(\frac{\lambda}{\lambda_0}\right)^{b-1} e^{-b\lambda/\lambda_0}$$

$$\rightarrow \quad \left\langle \frac{\lambda^n}{n!} e^{-\lambda} \right\rangle_\lambda \propto \left(\frac{n}{n_0}\right)^{a-1} e^{-an/n_0}$$

What is T?

If in a single event / jet, we have equipartition:

$$1\,event: \qquad \frac{E_{event}}{N_{event}} = D\,T_{event}$$

On the average, we have:

$$\frac{E}{N} = \frac{\int \epsilon f_{TS}(\epsilon)}{\int f_{TS}(\epsilon)} = \frac{D\,T}{1-(q-1)(D+1)}$$

(m ≈ 0 particles)

Conclusion

Jet-fragmentation might be statistical?

It could be checked by measuring the dN/dz fragmentation function in jets of fix multipliciy.

Outline

- ***v2 @ LHC***
- ***Statistical Jet Fragmentation @ LEP & LHC***
  - ***- Scale evolution,***
  - ***- Application in Parton Model Calculations***
- ***Interesting N and s dependence in pp collisions***

Transverse spectra in pp

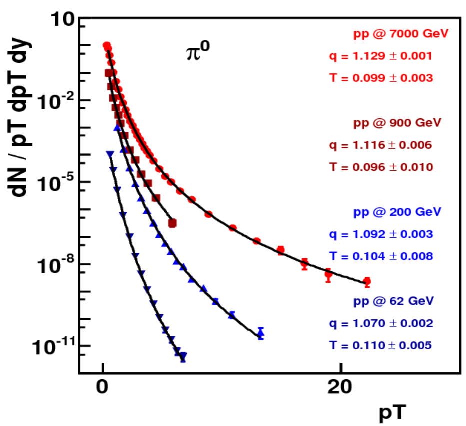
Hadron spectra in *pp* collisions can be described by the *Tsallis distribution*:

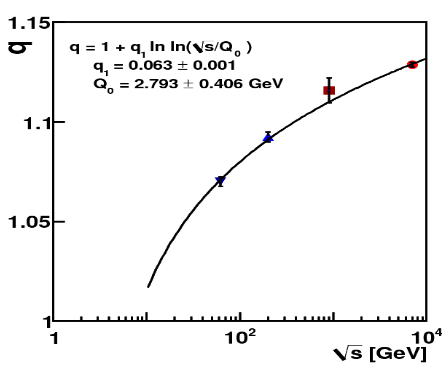
$$\frac{dN}{d^3\boldsymbol{p}} \propto \left[1 + \frac{q-1}{T}(m_T - m)\right]^{-1/(q-1)} .$$

π* spectra in *pp* collisions depends *similarly on √s* and on the multiplicity *N

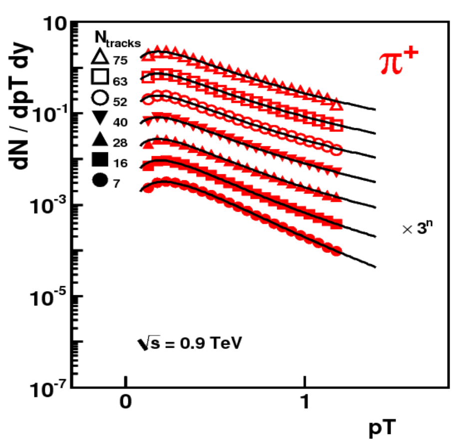
$$q(s) = 1 + q_1 \ln\ln(\sqrt{s}/Q_0) ,$$
$$q(N) = 1 + \mu \ln\ln(N/N_0) .$$

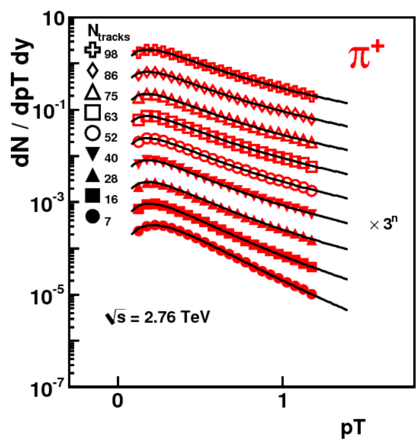
√s = fix

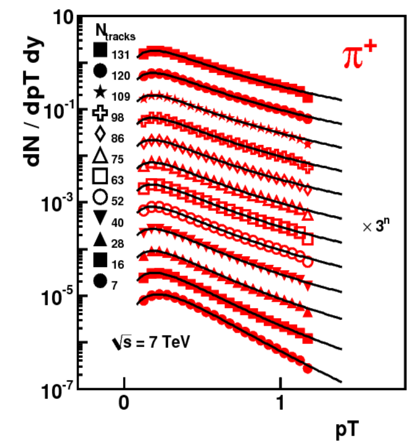


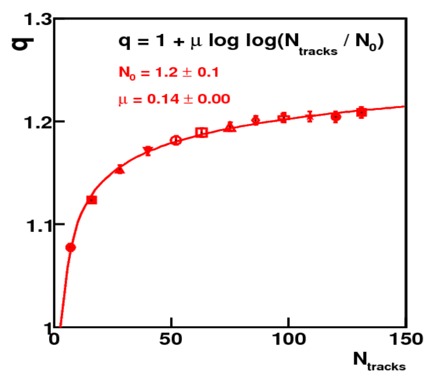


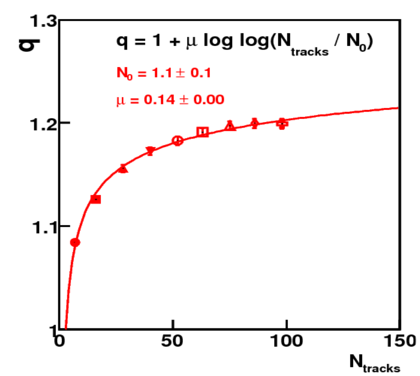
N = fix

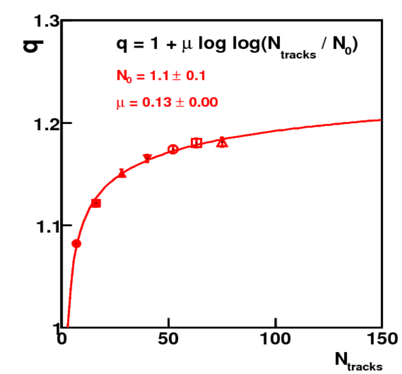












Conclusion

Jet-fragmentation might be statistical?

It could be checked by measuring the dN/dz fragmentation function in jets of fix multipliciy.

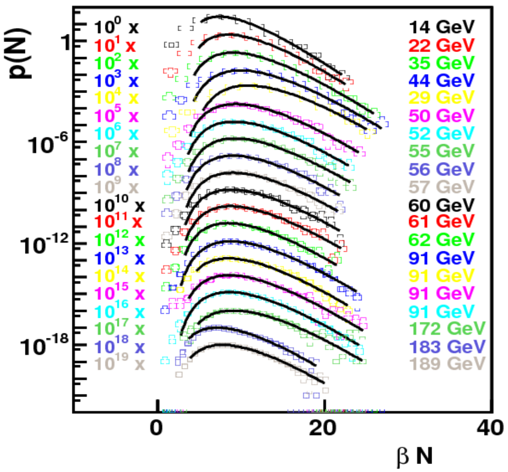
What causes the transition of Poissonian multiplicity distributions at the parton level in jets to NBD/Gamma-type multiplicity distributions observed at the hadronic level?

Back-up Slides……………………..

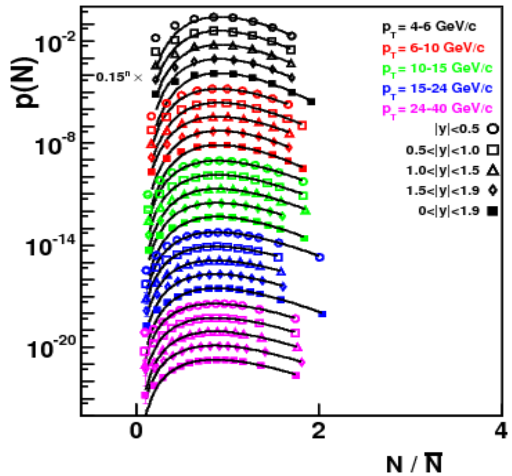
Motivation

Particle Multiplicity fluctuates according to the Gamma-distribution

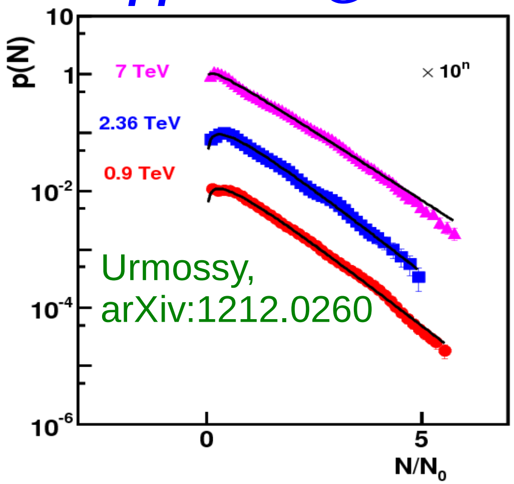
$e^-e^+ \rightarrow h^\pm$



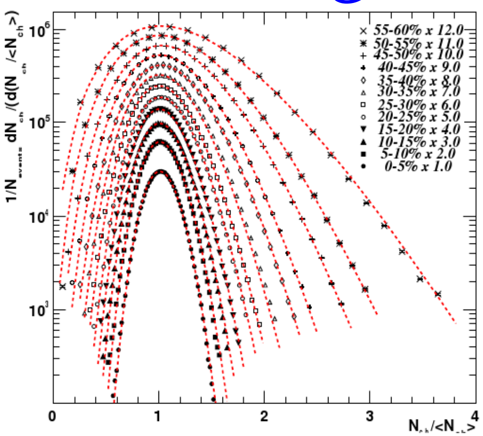
pp → jets @ 7 TeV



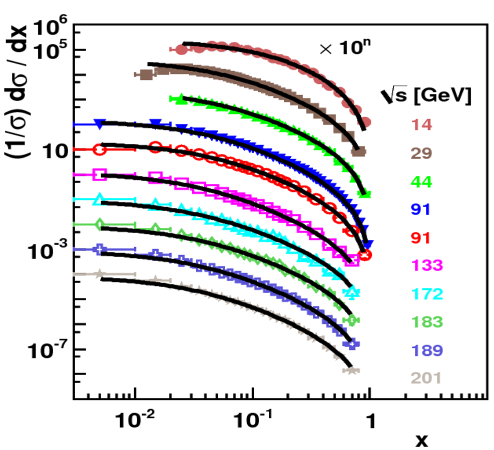
pp → h± @ LHC



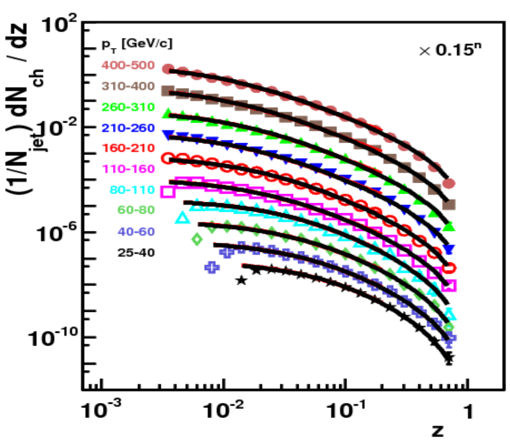
AuAu → h± @RHIC



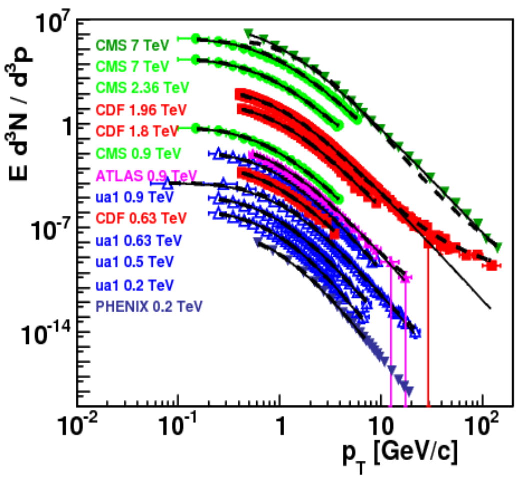
Power-law hadron spectra



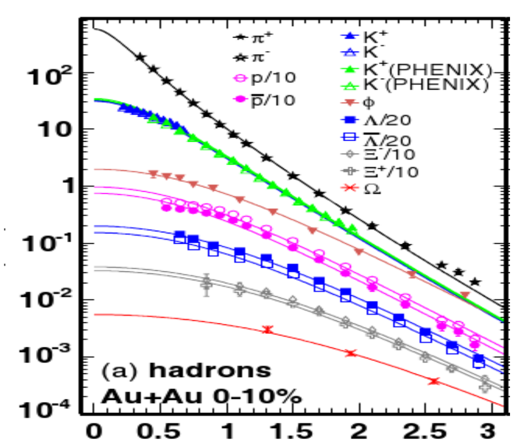
Urmossy et.al.,*PLB*, **701**: 111-116 (2011)



Urmossy et. al., *PLB*, **718**, 125-129, (2012)



Barnaföldi etal, *J. Phys.: Conf. Ser.*, **270,** 012008 (2011)



J. Phys. G: Nucl. Part. Phys. **37** 085104 (2010),

***KNO – scaling* (Koba-Nielsen-Olesen) of Multiplicity distributions :**

$$p(N) = \frac{1}{\langle N(s)\rangle}\Psi\left(\frac{N-N_0}{\langle N(s)\rangle}\right)$$

- A. Rényi, Foundations of Probability, Holden-Day (1970).
- A. M. Polyakov, Zh. Eksp. Teor. Fiz. 59, 542 (1970).
- Z. Koba, H. B. Nielsen, P. Olesen, Nucl. Phys. B 40, 317 (1972).
- S. Hegyi, Phys. Lett. B: 467, 126-131, 1999.
- S. Hegyi, Proc. ISMD 2000, Tihany, Lake Balaton, Hungary, 2000
- Yu.L. Dokshitzer, Phys. Lett. B, 305, 295 (1993); LU-TP/93-3 (1993).

A functional form that is consistent with experiments:

$$p(N) \propto (N-N_0)^{\alpha-1} e^{-\beta(N-N_0)}$$

The average hadron distributions are

$$\frac{d\sigma}{d^D x} = \sum f_N(x) N\, p(N)$$

Tsallis from N fluctuations

If in a collision, the hadron distribution is *Boltzmann-Gibbs*,

$$f(\epsilon) = A\exp(-\beta\epsilon), \qquad E/n = DT$$

But *multiplicity fluctuates from collision to collision, while E = constant*

$$p(n) \propto n^{\alpha-1}\exp(-\alpha n/\langle n\rangle)$$

the *average spectrum is the Tsallis* distributions:

$$\frac{dN}{d^3p} = \int dn\, p(n) f_n(\epsilon) \propto \left(1+\frac{D\langle n\rangle}{\alpha E}\epsilon\right)^{-(\alpha+D+1)}$$

$$\propto \left(1+\frac{q-1}{T}\epsilon\right)^{-1/(q-1)}$$

Tsallis Distribution from Fluctuations

Moreover,

If the hadron distribution is *Boltzmann-Gibbs*,

$$f(\epsilon) = A\exp(-\beta\epsilon), \qquad\qquad E/n = DT$$

but the *total transverse energy fluctuates event-by-event while n = fix*

$$p(E) \propto E^{-\alpha-2}\exp(-\alpha\langle E\rangle/E)$$

the *average distribution* becomes the *Tsallis* distribution:

$$\frac{dN}{d^3p} = \int dE\, p(E) f_E(\epsilon) \propto \left(1 + \frac{Dn}{\alpha\langle E\rangle}\epsilon\right)^{-(\alpha+D+1)}$$

Tsallis Distribution from Fluctuations

SuperStatistics ***(C. Beck, G. Wilk,*** *Eur. Phys. J. A*, **40**, 267 and 299-312, (2009)***):***

If the hadron distribution is *Boltzmann-Gibbs*,

$$f(\epsilon) = A \exp(-\beta \epsilon)$$

but the *temperature fluctuates event-by-event* or *position-to-position* as

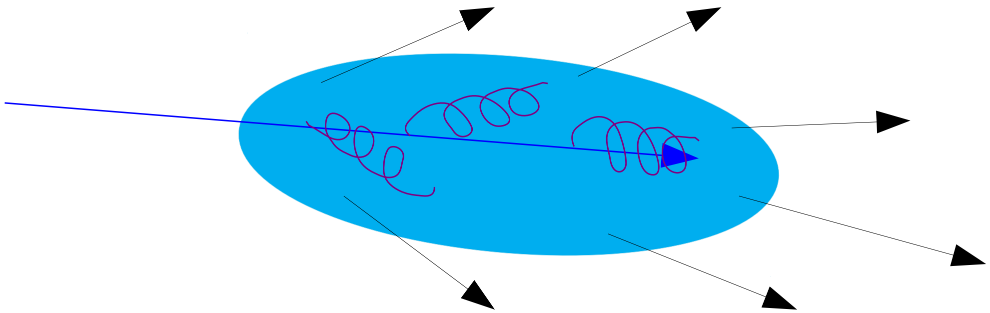
$$p(\beta) \propto \beta^{\alpha-1} \exp(-\alpha \beta / \langle \beta \rangle)$$

the *average distribution* becomes the *Tsallis* distribution:

$$\frac{dN}{d^3 p} = \int d\beta\, p(\beta) f_{\beta}(\epsilon) \propto \left(1 + \frac{\langle \beta \rangle \epsilon}{\alpha}\right)^{-(\alpha + D + 1)}$$

Motivation

- ***If the hadronisation of a single jet was statistical (microcanonical),***

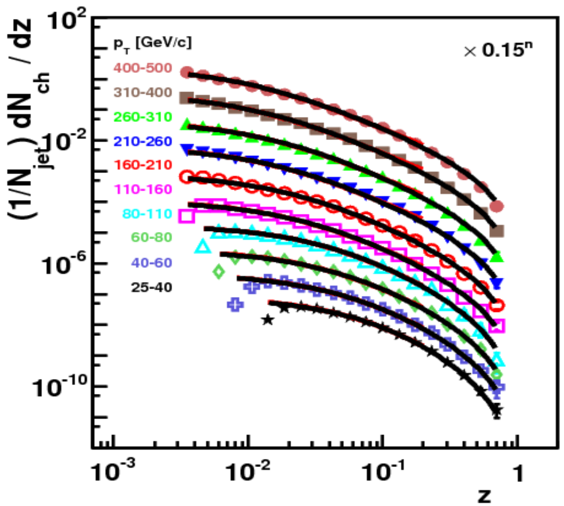


- ***N fluctuations of the form of***

$$p(n) \propto \left(\frac{n}{n_0}\right)^{a-1} e^{-an/n_0}$$

turn statistical distributions into cut-power laws:

$$\frac{d\sigma}{dz} \propto \left(1-\frac{q-1}{T}\ln(1-z)\right)^{-1/(q-1)}$$



pp → jets @ 7 TeV

Urmossy et. al., *PLB*, **718**, 125-129, (2012)