Flow patterns for classical fields

Elliptic flow based on Jacobi-Anger Formula

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Introduction

- Illusory Flow by Unruh type radiation in $dN/d\eta$
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  (1401.1987 –> EPJ A 2014)
  - Accelerating, radiating point charge
  - Hydrodynamic-like photon rapidity distribution
- Present talk: Jacobi-Anger Formula delivers Elliptic Flow
  - Two point-charge radiation
  - v_n coefficients

Outline

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1. Non-spherical Flow from Waves
   - Jacobi-Anger Formula
   - The v_n coefficients
2. Averages and Fits
3. Summary

Interference Term in 1-Photon Yield

Jacobi-Anger Formula

$$e^{ix\cos\Theta} = J_0(x) + 2\sum_{n=1}^{\infty} i^n J_n(x)\cos(n\Theta). \tag{1}$$

The yield is proportional to

$$Y \propto \left| A_1 e^{ik_\perp \frac{d}{2}\cos(\alpha-\psi)} + A_2 e^{-ik_\perp \frac{d}{2}\cos(\alpha-\psi)} \right|^2 \tag{2}$$

Detector angle α, distance angle ψ, distance d result in

Higher Flow coefficients

$$\Theta = \alpha - \psi$$

Flow coefficients are defined by relative amplitudes of $\cos(n\Theta)$ terms to the zeroth order term.

$$v_n = \frac{2 R_n J_n(k_\perp d)}{|A_1|^2 + |A_2|^2 + R_0 J_0(k_\perp d)} \tag{3}$$

with

$$R_n := 2\,\mathrm{Re}\,(i^n A_1 A_2^*) = 2\,|A_1|\,|A_2|\,\cos\left(\Delta\varphi + n\frac{\pi}{2}\right).$$

In relative ratios of this *Young* interference $k_\perp$ powers cancel in the ratio of A-squares!!!

Higher Flow coefficients

The complex amplitudes may differ in a further phase $\Delta\varphi$ due to longitudinal and time positions at the start and at the end of deceleration.

We define the interference ratio:

$$r_n := \frac{2\,|A_1|\,|A_2|}{|A_1|^2 + |A_2|^2} \cos\left(\Delta\varphi + \frac{n\pi}{2}\right). \tag{4}$$

We get

$$v_n = \frac{2 r_n\, J_n(k_\perp d)}{1 + r_0\, J_0(k_\perp d)} \tag{5}$$

Behaviour of v2

For $n = 2$ the (in)famous v_2 is

$$v_2 = \frac{-2\,\varepsilon\, J_2(k_\perp d)\, cos(\Delta\varphi)}{1 + \varepsilon\, J_0(k_\perp d)\, cos(\Delta\varphi)} \tag{6}$$

with

$$\varepsilon = \frac{2\,|A_1|\,|A_2|}{|A_1|^2 + |A_2|^2} \leq 1. \tag{7}$$

For small $k_\perp d$ the J-Bessel behave like power, so v_2 would go like $k_\perp^2$.

Outline

Longitudinal Phase averaged v_2 formula

Considering a longitudinal phase difference $\Delta\varphi$

$$v_n = \frac{2\varepsilon J_n \cos\left(\Delta\varphi + n\frac{\pi}{2}\right)}{1 + \varepsilon J_0 \, \cos(\Delta\varphi)}. \tag{8}$$

Integrating over $\Delta\varphi$ uniformly we obtain

$$\langle\, v_n \,\rangle = 2\cos(n\frac{\pi}{2})\,\frac{J_n}{J_0}\,\left(1 - \frac{1}{\sqrt{1-\varepsilon^2 J_0^2}}\right) \tag{9}$$

The v_2 vs amplitudes

formulas

v_2 coefficient expressed by the amplitude ratio:

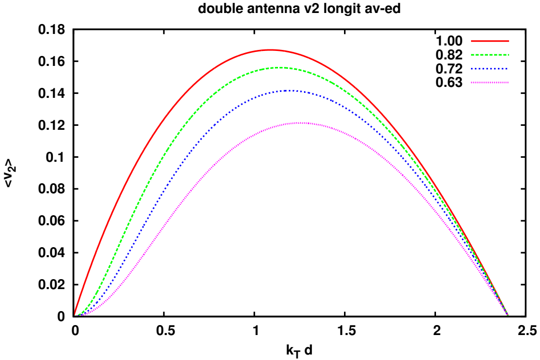
$$v_2 = F_2 \frac{2J_2(x)}{J_0(x)} \left(\frac{1}{\sqrt{1 - \varepsilon^2 J_0^2(x)}} - 1 \right) \quad (10)$$

with $x = k_\perp d$,

$$\varepsilon = \frac{2\,|A_1|\,|A_2|}{|A_1|^2 + |A_2|^2} \quad (11)$$

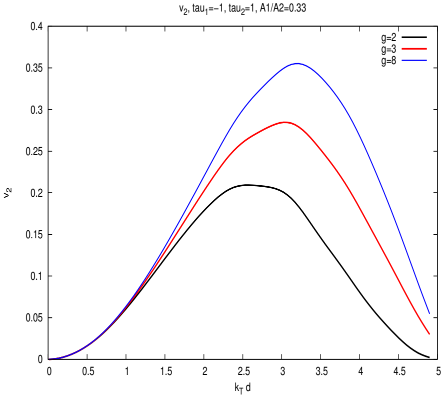
and F_2 depending on centrality, but not on $k_\perp$.

v_2 dependence on amplitude ratio $|A_1|/|A_2|$



It starts linear only for equal amplitudes, otherwise quadratic. For all curves $F_2 = 1$ [by T.S. Biro].

Numerical v2 results



Numerical data for v_2 with $|A_1|/|A_2| = 0.37$

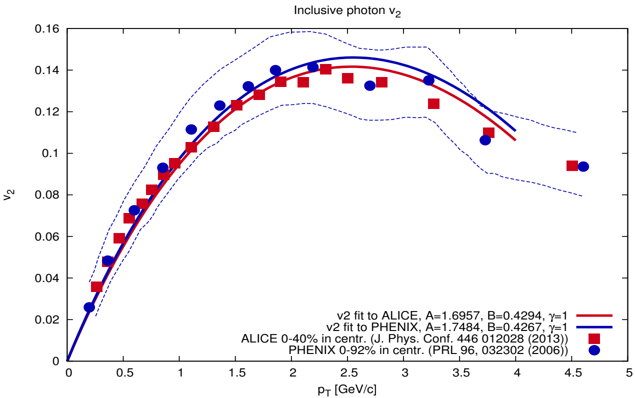
Fit parameters

In the simplest (two-antenna arrays) scenario we fit:

- $\varepsilon = \frac{2\gamma}{1+\gamma^2}, \quad \gamma = |A_1|/|A_2|$ magnitude ratio parameter
- $B = d$ antenna distance parameter
- $A = F_2$ geometric form factor

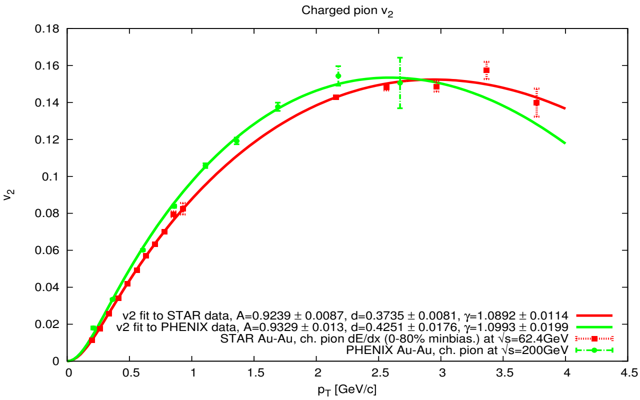
We assume that F_2 depends on centrality, but not on the momentum $k_\perp$.

Photon v2



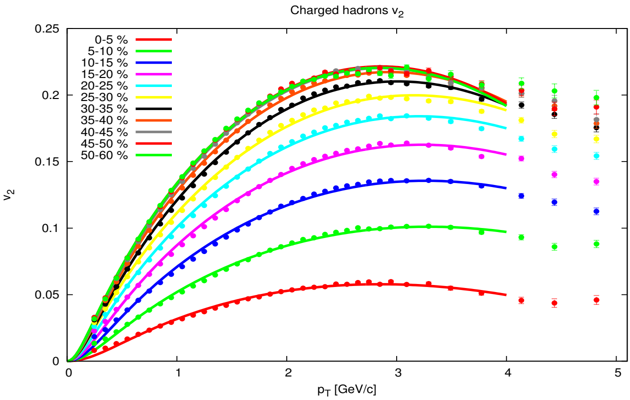
v_2 fits to photon data [by M. Horvath], data from PRL 96:032302, J. Phys. Conf. 446,01208

Pion v2



v_2 fits to charged pion data [by M. Horvath], data from PRC 75:054906, PRL 91:1802301

Charged hadrons v2



v_2 fits for charged hadron data [by M. Horvath], data from arxiv:10003.5586v2

Fit conclusions

- Even a simple model comes close to data
- No need for hydrodynamics or initial state fluctuations
- Fits to amplitude ratio and characteristic distance are stable

The shape of v_2 vs $k_\perp$ is explained well!

Outline

Summary

- Radiation of one accelerating charge delivers Bjorken and Gaussian-like $\frac{dN}{d\eta}$
- Radiation from two point charge:
  - v_n parameters can be calculated through Jacobi-Anger formula
  - Avaraged v_2 coefficient fits experimental data
  - We need centrality-dependent form-factors
  - Numerical data goes with analitical results