Dilepton production in pion induced reactions

Zimnyi Winter school, Budapest, 04.12.2014

Gy. Wolf

in collaboration with M. Zétényi, P. Kovács

Wigner RCP, Budapest

- πN reaction
- πA reactions and quantum interference in nuclear matter
- Summary

$\pi + N \to N + e^+e^-$

- Coupled-channel approach

  K-matrix, Post, Leupold, Mosel,

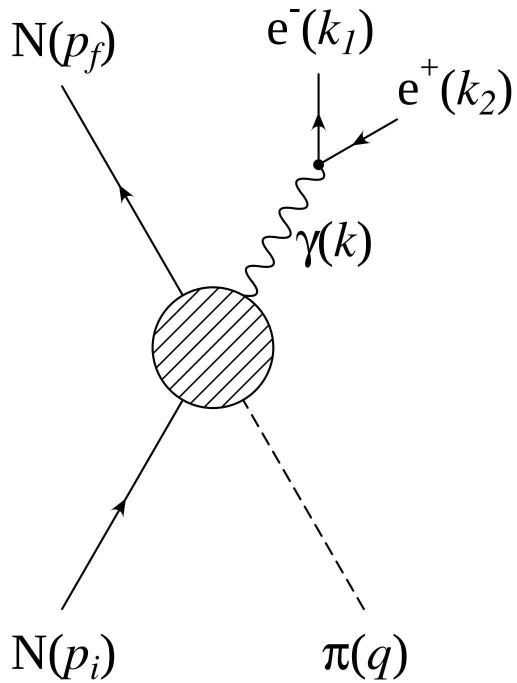
  Nucl. Phys. A689 (2001) 753

  Bethe-Salpeter, Lutz, Wolf, Friman,

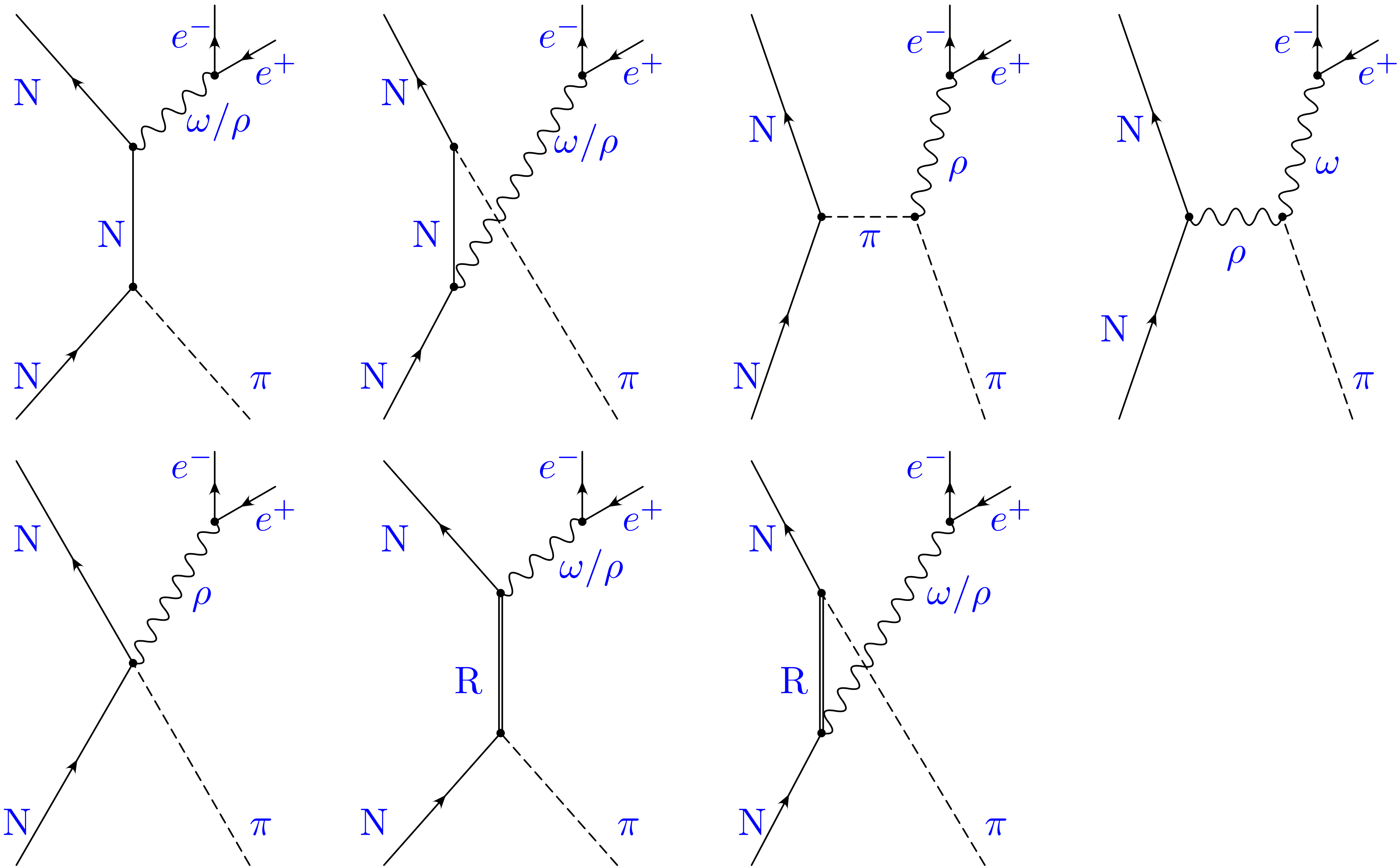
  Nucl.Phys. A706 (2002) 431

- Effective field theory

  Zétényi, Wolf, Phys. Rev. C86 (2012) 065209



Feynman diagrams for $\pi + N \rightarrow N + e^+e^-$



Effective Field Theory for $\pi + N \to N + e^+e^-$

$$\mathcal{L}_{\mathcal{N}\mathcal{N}\pi} = -\frac{\mathrm{f}_{\mathrm{NN}\pi}}{\mathrm{m}_\pi}\overline{\psi}_{\mathcal{N}}\gamma_5\gamma^\mu\vec{\tau}\psi_{\mathcal{N}} \cdot \partial_\mu\vec{\pi}.$$

$$\mathcal{L}_{\omega\rho\pi} = \frac{\mathrm{g}_{\omega\rho\pi}}{2}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\omega_\nu \mathrm{Tr}\left((\partial_\alpha\vec{\rho}_\beta \cdot \vec{\tau})(\vec{\pi}\cdot\vec{\tau})\right)$$

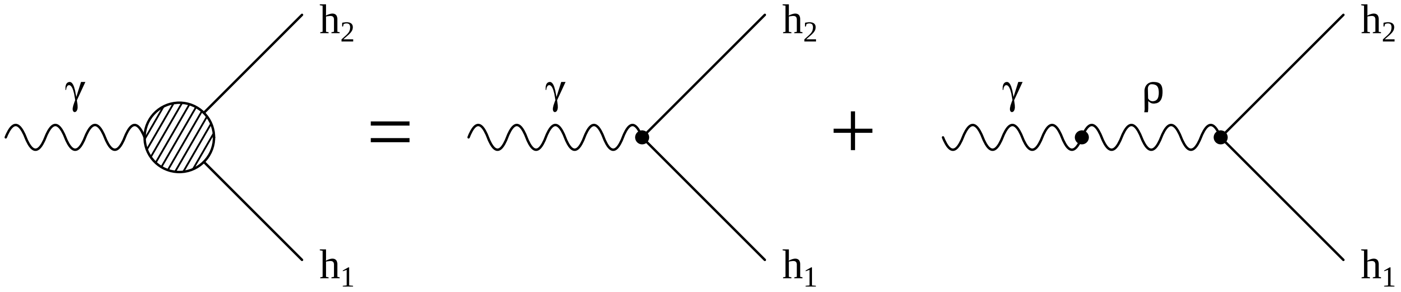
$$D_\mu = \partial_\mu + ieA_\mu Q - ig_\rho\vec{\rho}_\mu\vec{T} - ig_\omega\omega_\mu T_0^{1/2}$$

$$\mathcal{L}_{\mathcal{N}\mathcal{N}\rho} = \mathrm{g}_\rho\overline{\psi}_{\mathrm{N}}\left(\vec{\not\rho} - \kappa_\rho\frac{\sigma_{\mu\nu}}{4\mathrm{m}_{\mathrm{N}}}\vec{\rho}^{\mu\nu}\right)\cdot\vec{\tau}\psi_{\mathrm{N}}, \quad \mathcal{L}_{\mathcal{N}\mathcal{N}\omega} = \mathrm{g}_\omega\overline{\psi}_{\mathrm{N}}\left(\not\omega - \kappa_\omega\frac{\sigma_{\mu\nu}}{4\mathrm{m}_{\mathrm{N}}}\omega^{\mu\nu}\right)\psi_{\mathrm{N}}.$$

ρ_0 couples to $\bar{\psi}_N\tau_0\psi_N$ so to p and to n with different signs, while ω with the same sign

Considering $\pi^- p \to ne^+e^-$ and $\pi^+ n \to pe^+e^-$ in one of the channels constructive and in the other channel destructive interference

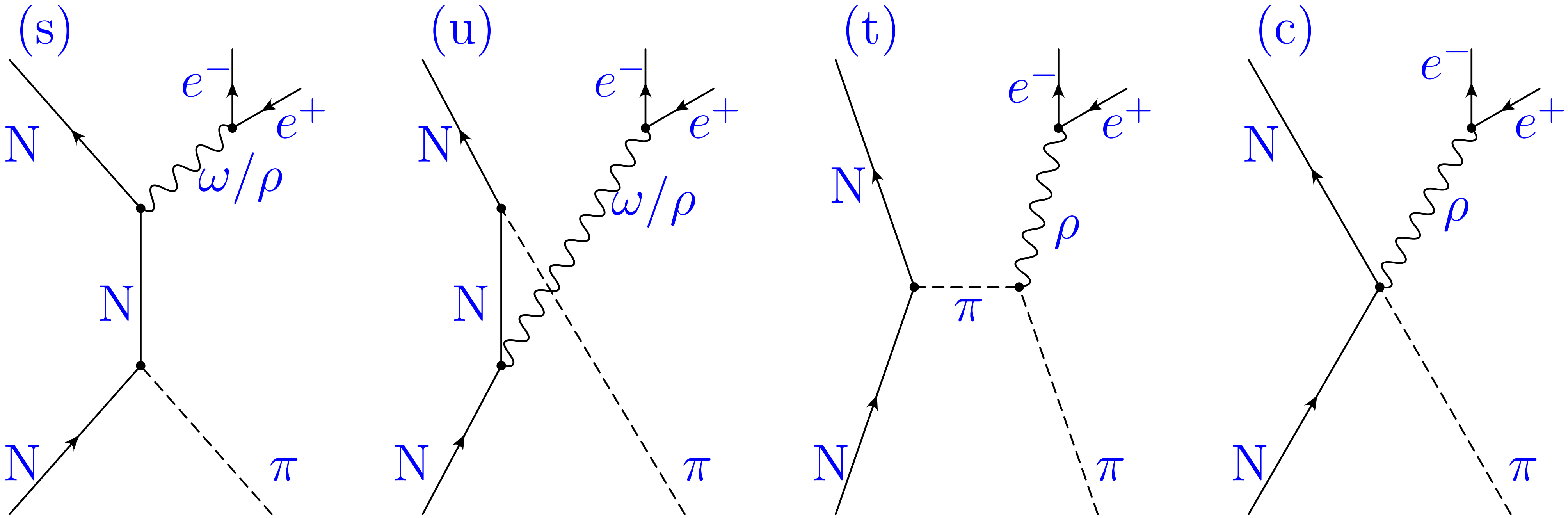
Vector meson photon coupling



- $\mathcal{L}_{VDM1} = -\frac{em_\rho^2}{g_\rho}\rho_\mu^0 A^\mu$
  The width of $R \to N\gamma$ and $R \to N\rho$ are not independent photons from ρ (ρ-width taken from PDG) overestimate the γ-width (also photon gets a mass, not gauge invariant; can be cured)
- $\mathcal{L}_{VMD2} = -\frac{e}{2g_\rho}F^{\mu\nu}\rho_{\mu\nu}^0$
  From ρ-width the contribution to the photonic decay can be obtained by multiplying it with $\frac{e}{g_\rho}\frac{k^2}{m_\rho^2 - k^2 - iq\Gamma_\rho(k^2)}$
  Decay through ρ does not contribute to the real photonic width.
  We use VMD2. The final result depend on the choice, the ratio: M_{dil}^2/m_ρ^2

Current conservation (Gauge invariance)

Photon (even virtual one) couples to conserved current



Only the sum of s, t, u and contact channels is gauge invariant
using different form factors for different diagrams spoils gauge invariance

Form factors

$$\mathcal{M}_i^\mu \longrightarrow \mathcal{M}_i^\mu \frac{\Lambda_i^4}{\Lambda_i^4 + (i - m_i^2)^2} \equiv \mathcal{M}_i^\mu F_i \qquad i = s, u, t$$

Add an extra term (combination of the 3 form factors) to keep gauge invariance

$$\hat{F} = F_s + F_u + F_t - F_s F_u - F_s F_t - F_u F_t + F_s F_u F_t$$

$$\Delta \mathcal{M}_s^\mu = (\hat{F} - F_s) C_s \gamma_5 \frac{2p_f^\mu + k^\mu}{s - m_N^2}$$

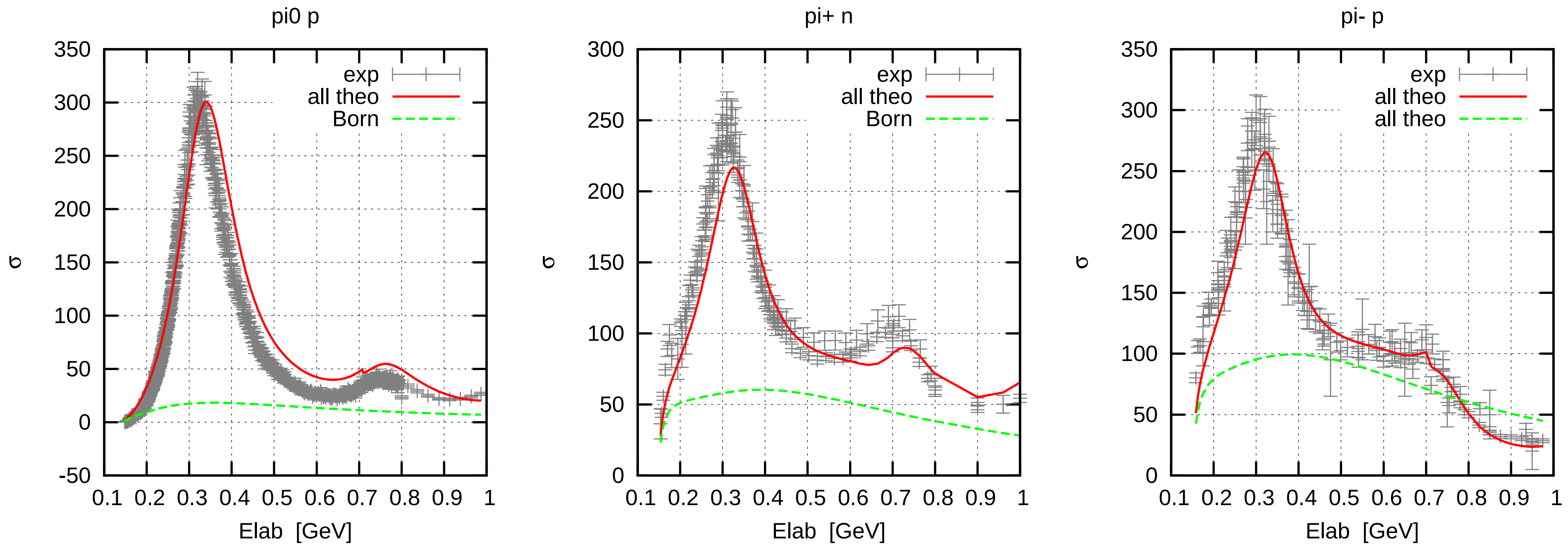
The coefficient of $\hat{F} - F_s$ is chosen so, that multiplied with k_μ it should be equal to $\mathcal{M}_s^\mu k_\mu$

$\Delta \mathcal{M}_s^\mu$ has no pole, it can be generated by a contact term

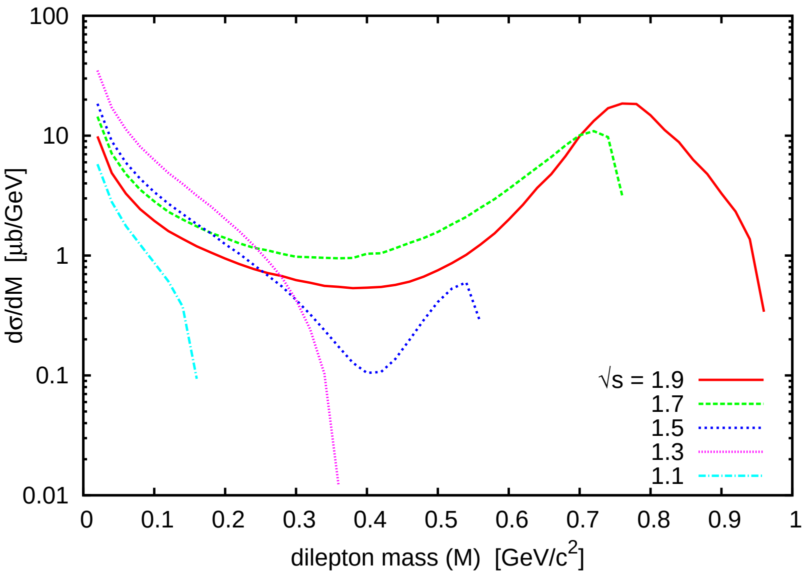
$$(\hat{F} - F_s) \sim (1 - F_s) \sim s - m_N^2$$

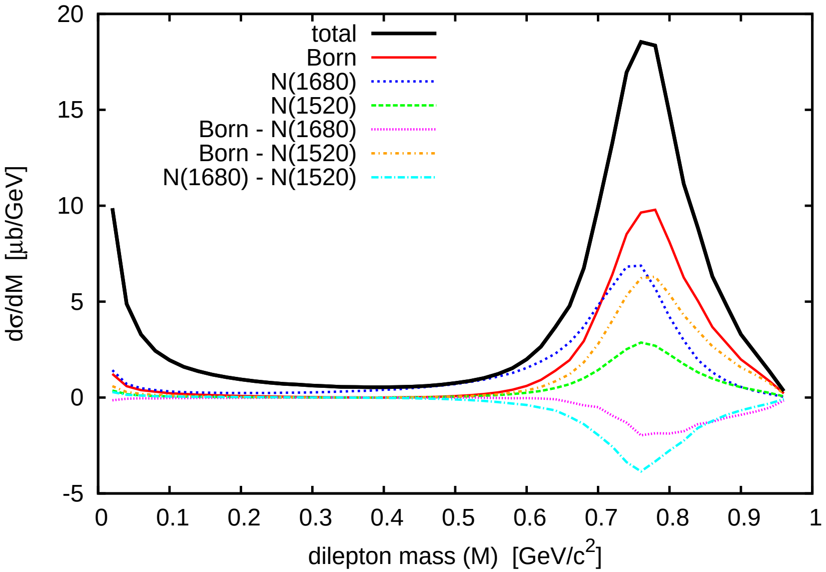
Parameters of the model

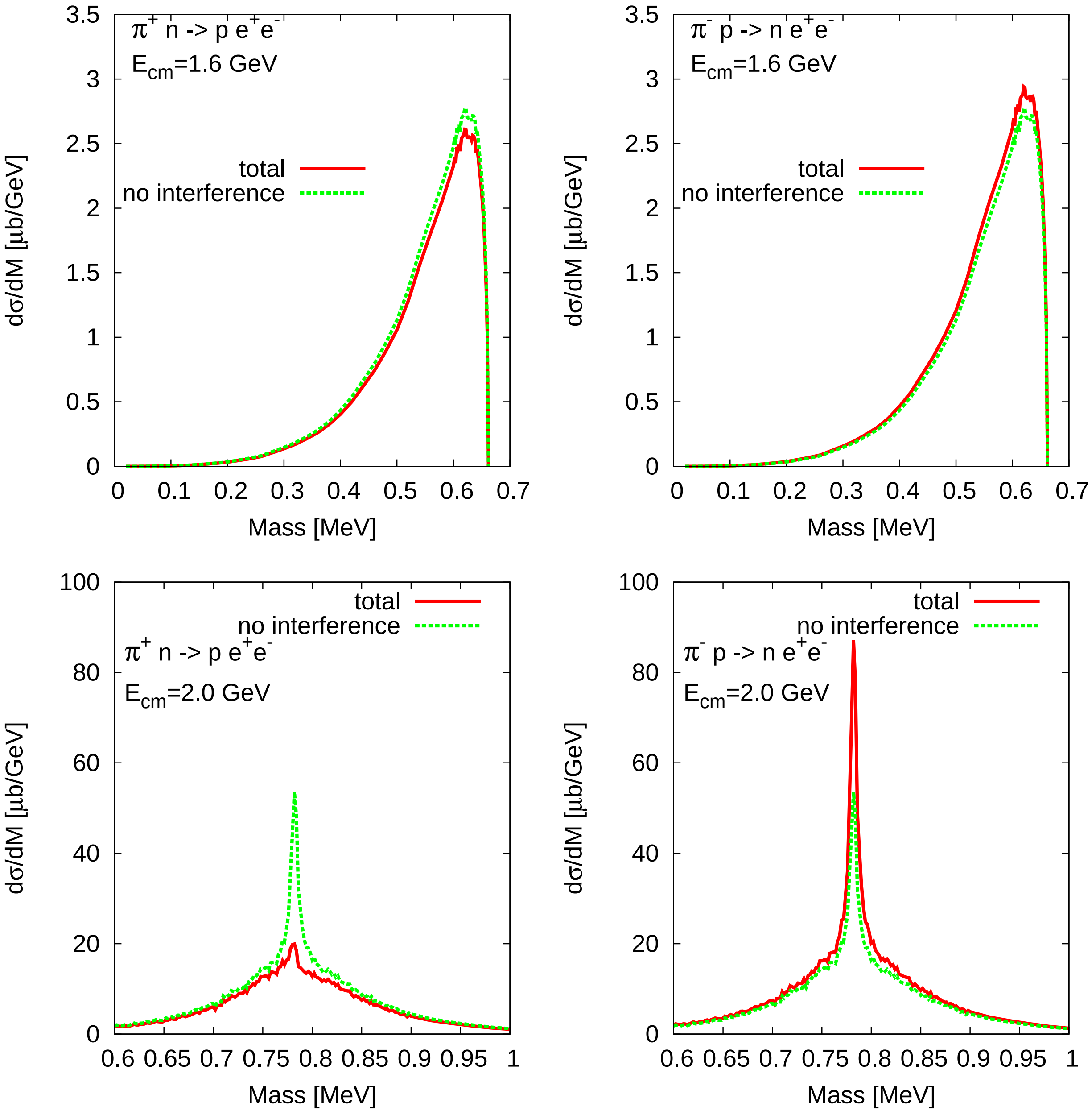
RNπ and RNρ from measured partial decay widths, RNγ is fitted to the pion photoproduction, ω couplings were taken from the literature



Dilepton production in πN (without ω)






π+ n -> p e+e-
Ecm=1.6 GeV
total
no interference
dσ/dM [μb/GeV]
Mass [MeV]
π- p -> n e+e-
Ecm=1.6 GeV
total
no interference
dσ/dM [μb/GeV]
Mass [MeV]
total
no interference
π+ n -> p e+e-
Ecm=2.0 GeV
dσ/dM [μb/GeV]
Mass [MeV]
total
no interference
π- p -> n e+e-
Ecm=2.0 GeV
dσ/dM [μb/GeV]
Mass [MeV]

Dileptons in pion-nucleus collisions

- $\dfrac{\frac{d\sigma}{dM}^{\pi^- p \to n e^+ e^-}(m_\omega)}{\frac{d\sigma}{dM}^{\pi^+ n \to p e^+ e^-}(m_\omega)} \approx 4$ because of the interference
- The effect is strong if cross secton through ρ and ω are similar
- coupling constants of ω were taken from the literature, we plan to make our fit
- The same problem was studied in M.F.M. Lutz, B. Friman, M. Soyeur, Nucl. Phys. A713 (2003) 97 and A.I. Titov, B Kämpfer, EPJ A 12 (2001) 217. They had smaller ρ cross section, so the effect was strong at lower $\sqrt{s}$
- How much of this coherence survive in a nucleus?
- In a nucleus coherence is lost if one of the vector meson collides

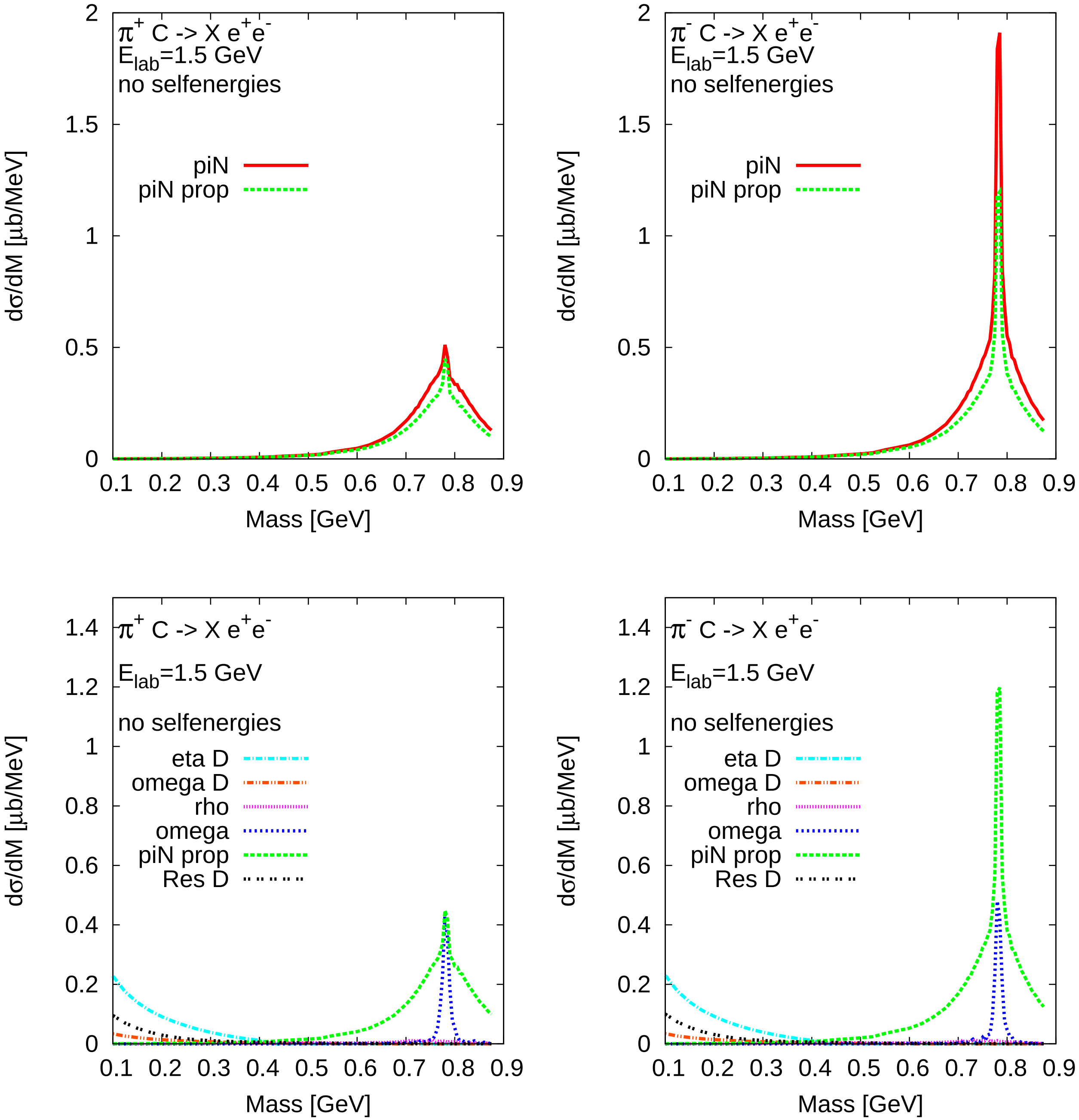
Simulation of π A collisions

- Same as usually except for $\pi N \to N e^+ e^-$
- in case of a πN collision several "doublets" are created.
  (The original π and N do not change their state.)
  A doublet consists of 2 perturbative particles ρ and ω with their cross sections and the "cross section" of the interference term. ρ and ω are created with the same position, momentum and mass.
- They propagate, decay and can be absorbed. The interference term contribute to the "decays".
- Propagation: perturbative ρ's and ω's propagate in the surounding medium
- Absorption: ρ's and ω's can be absorbed by a nucleon

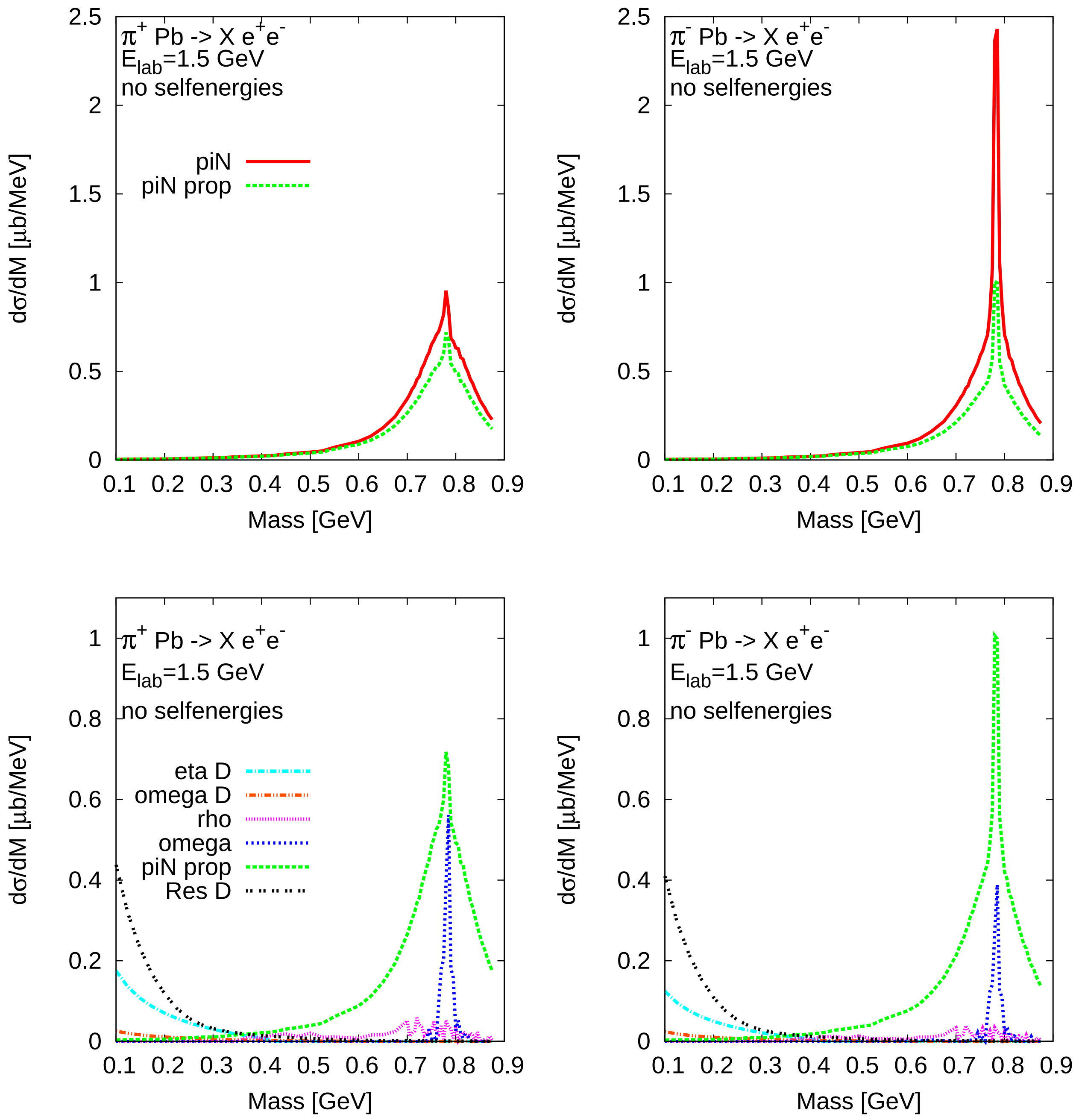
Decays

- Denote the probability that the ρ and ω decay in the ith time step as α_i and β_i, respectively.
- $\sum \alpha_i \leq 1$. At the end it is 1 if not absorbed. The same is for ω.
- In the nth timestep the ρ contribution to the dilepton yield:
  $\alpha_n \, \sigma^{\pi N \to N \rho_0 \to N e^+ e^-} \left(\approx \alpha_n \, \Gamma_\rho^{N e^+ e^-} / \Gamma_\rho^{tot} \, \sigma^{\pi N \to N \rho_0} \right)$. Similarly for ω.
  The contribution of the interference term is:
  $\sum_{ij} \alpha_i \beta_j \, \sigma^{\pi N \to N \rho - \omega \to N e^+ e^-}$.
  In vacuum it reproduces the original cross section.

π C, 1.5 GeV, no selfenergies, Preliminary results



π Pb, 1.5 GeV, no selfenergies, Preliminary results



Dileptons in pion-nucleus collisions

- $$\frac{\frac{d\sigma}{dM}^{\pi^- p\to n e^+ e^-}(m_\omega)}{\frac{d\sigma}{dM}^{\pi^+ n\to p e^+ e^-}(m_\omega)} \approx 4$$
  $$\frac{\frac{d\sigma}{dM}^{\pi^- C^{12}\to X e^+ e^-}(m_\omega)}{\frac{d\sigma}{dM}^{\pi^+ C^{12}\to X e^+ e^-}(m_\omega)} \approx 2.9$$
  $$\frac{\frac{d\sigma}{dM}^{\pi^- Pb^{207}\to X e^+ e^-}(m_\omega)/N_p}{\frac{d\sigma}{dM}^{\pi^+ Pb^{207}\to X e^+ e^-}(m_\omega)/N_n} \approx 2.0$$
  In case of complete decoherence these ratios should be 1.
- Experimentally the decoherence can be observed in strongly interacting matter.

Summary

- Dilepton production in πN and πA an unique way to study quantum interference inside strongly interacting matter by measuring on nucleon, on light and on heavy nuclei.
- Make own fit to vector meson production (including the resonances and their interference)

BUU

- Boltzmann-Ühling-Uhlenbeck equation

$$\frac{\partial F}{\partial t} + \frac{\partial H}{\partial \mathbf{p}}\frac{\partial F}{\partial \mathbf{x}} - \frac{\partial H}{\partial \mathbf{x}}\frac{\partial F}{\partial \mathbf{p}} = \mathcal{C}, \quad H = \sqrt{(m_0 + U(\mathbf{p},\mathbf{x}))^2 + \mathbf{p}^2}$$

- potential: momentum dependent, soft: K=215 MeV

$U^{nr} = A\frac{n}{n_0} + B\left(\frac{n}{n_0}\right)^\tau + C\frac{2}{n_0}\int \frac{d^3p'}{(2\pi)^3}\frac{f_N(x,p')}{1+\left(\frac{\mathbf{p}-\mathbf{p}'}{\Lambda}\right)^2},$

S. Teis, W. Cassing, M. Effenberger, A. Hombach, U. Mosel, Gy. Wolf, Z. Phys. A359 (1997) 297-304,
Gy. Wolf et al., Phys.Atom.Nucl. 75 (2012) 718-720, Open Nucl.Part.Phys.J. 3 (2010) 1-15

- testparticle method

$F = \sum_{i=1}^{N_{test}} \delta^{(3)}(\mathbf{x} - \mathbf{x}_i(t))\delta^{(4)}(p - p_i(t)).$

Collision term

- NN $\leftrightarrow$ NR, NN $\leftrightarrow$ $\Delta\Delta$
- baryon resonance can decay via 9 channels
  R $\leftrightarrow$ Nπ, Nη, Nσ, Nρ, Nω, $\Delta\pi$, N(1440)π, KΛ, KΣ
- 24 baryon resonances + Λ and Σ baryons
  $\pi, \eta, \sigma, \rho, \omega$ and kaons
- $\pi\pi \leftrightarrow \rho$, $\pi\pi \leftrightarrow \sigma$, $\pi\rho \leftrightarrow \omega$
- for resonances: energy dependent with
- $\frac{d\sigma^{X \to NR}}{dM_R} \sim A(M_R)\lambda^{0.5}(s, M_R^2, M_N^2)$

Spectral equilibration

- medium effects on the spectrum of hadrons (vector mesons)
- how they get on-shell (energy-momentum conservation)
- Field theoretical method (Kadanoff-Baym equation)
  B. Schenke, C. Greiner, Phys.Rev.C73:034909,2006
- Off-shell transport
  W. Cassing, S. Juchem, Nucl.Phys. A672 (2000) 417
  S. Leupold, Nucl.Phys. A672 (2000) 475
- Spectral equilibration: Markov or memory effect

Off-shell transport

- Kadanoff-Baym equation for retarded Green-function
  Wigner-transformation, gradient expansion
- transport equation for $F_\alpha = f_\alpha(x,p,t) A_\alpha$
  $A(p) = -2ImG^{ret} = \frac{\hat{\Gamma}}{(E^2 - \mathbf{p}^2 - m_0^2 - \mathrm{Re}\Sigma^{ret})^2 + \frac{1}{4}\hat{\Gamma}^2}$,

  W. Cassing, S. Juchem, Nucl.Phys. A672 (2000) 417
  S. Leupold, Nucl.Phys. A672 (2000) 475
- testparticle approximation

Transport equations

- $\frac{d\vec{X}_i}{dt} = \frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[2\,\vec{P}_i + \vec{\nabla}_{P_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{Im\Sigma^{ret}_{(i)}} \vec{\nabla}_{P_i} Im\Sigma^{ret}_{(i)} \right]$

  $\frac{d\vec{P}_i}{dt} = -\frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} Re\Sigma^{ret}_i + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{Im\Sigma^{ret}_{(i)}} \vec{\nabla}_{X_i} Im\Sigma^{ret}_{(i)} \right]$

  $\frac{d\epsilon_i}{dt} = \frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial Re\Sigma^{ret}_{(i)}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{Im\Sigma^{ret}_{(i)}} \frac{\partial Im\Sigma^{ret}_{(i)}}{\partial t} \right]$

- where $C_{(i)}$ renormalization factor

  $C_{(i)} = \frac{1}{2\epsilon_i} \left[\frac{\partial}{\partial \epsilon_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{Im\Sigma^{ret}_{(i)}} \frac{\partial}{\partial \epsilon_i} Im\Sigma^{ret}_{(i)} \right]$

- the last equation for homogenous system can be rewritten as

  $\frac{dM_i^2}{dt} = \frac{d(\epsilon_i^2 - P_i^2)}{dt} = \frac{dRe\Sigma^{ret}_{(i)}}{dt} + \frac{M_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{Im\Sigma^{ret}_{(i)}} \frac{dIm\Sigma^{ret}_{(i)}}{dt}$

Analytical solution for homogenous system

$$\frac{d}{dt}(M_i(t)^2 - M_0^2 - \mathrm{Re}\Sigma_\mathrm{i}) = \frac{\mathrm{M_i^2(t) - M_0^2 - Re\Sigma_i}}{\mathrm{Im\Sigma_i}} \frac{\mathrm{dIm\Sigma_i}}{\mathrm{dt}}$$

If the mass of the testparticle as just at the peak $(M_0^2 + \mathrm{Re}\Sigma_\mathrm{i})$ then it remains there

$$\frac{M_i^2(t) - M_0^2 - \mathrm{Re}\Sigma_\mathrm{i}(\mathrm{t})}{M_i^2(0) - M_0^2 - \mathrm{Re}\Sigma_\mathrm{i}(0)} = \frac{\mathrm{Im}\Sigma_\mathrm{i}(\mathrm{t})}{\mathrm{Im}\Sigma_\mathrm{i}(0)}.$$

If $A(M^2, t)$ is the spectral function of a Breit-Wigner form at t then

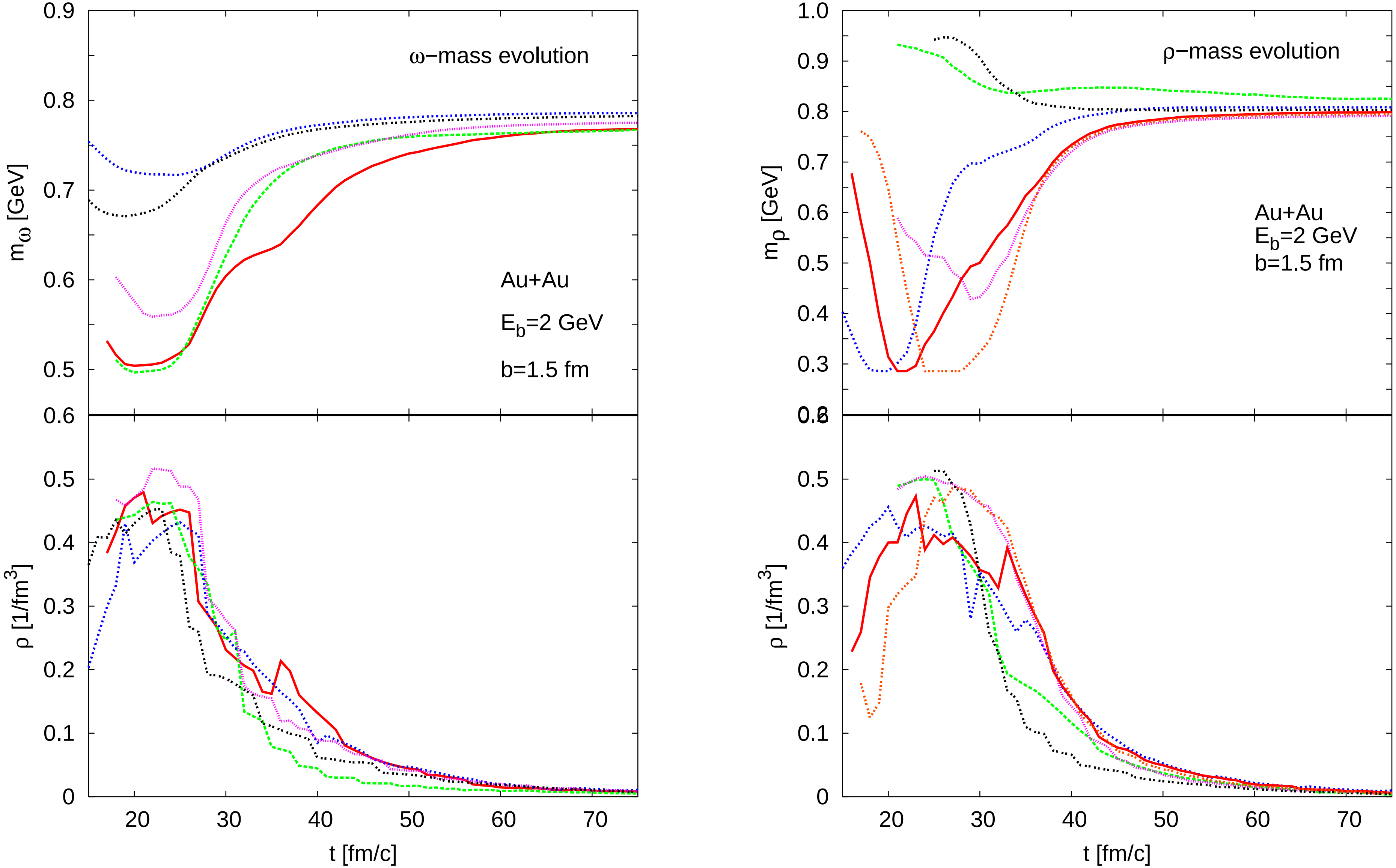
$$f(M'^2, t) = f(M^2, 0)\frac{dM^2}{dM'^2} = A(M^2, 0)\frac{dM^2}{dM'^2} = A(M^2, 0)\frac{\mathrm{Im}\Sigma(0)}{\mathrm{Im}\Sigma(\mathrm{t})} = A(M'^2, t)$$

The mass distribution agrees always with the spectral function at that point.

Medium effects

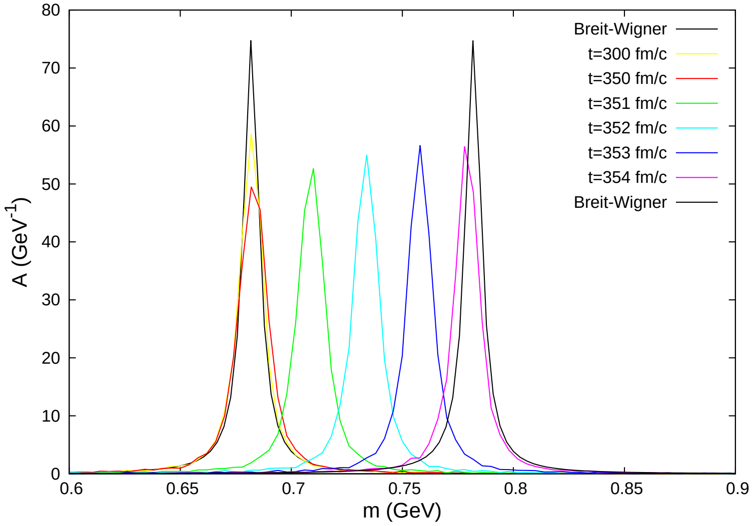
- imaginary part (collisional broadening):
  $\Gamma = \Gamma_{vac} + nv\sigma\gamma$
- real part (mass shift)
  $M = M_{vac} + n/n_o \Delta M$
  $\Delta M_\omega = -50$ MeV, $\Delta M_\rho = -120$ MeV
- danger of double counting
  collision term already contains partly the mixing of mesons with resonance-hole excitations
  but sum up only to finite order

Evolution of masses in heavy ion collisions

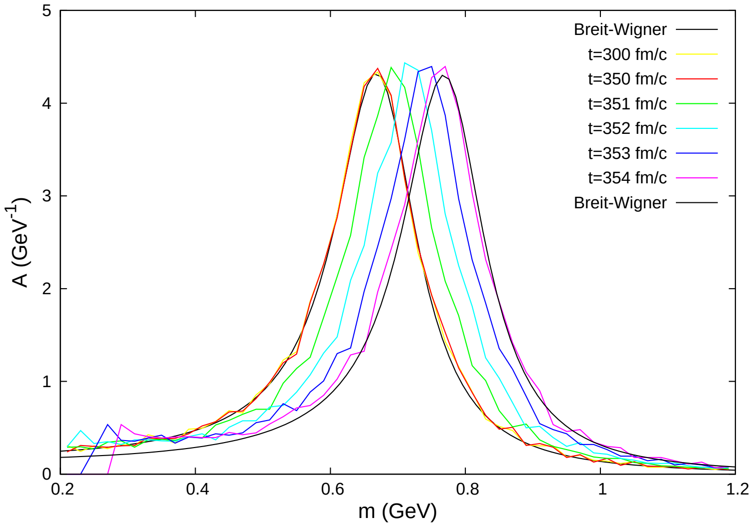


Evolution of mass distribution in a box

the vector meson masses are shifted linearly with density, and change the density linearly from ρ_0 to 0 in 4 fm/c:



ω



ρ

Cross sections

Elastic baryon-baryon cross section is fitted to the elastic pp data

Meson absorption cross sections are given by

$$\sigma_{\pi N \to R} = \frac{4\pi}{p^2} (spinfactors) \frac{\Gamma_{in}\Gamma_{tot}}{(s - m_R^2) + s\Gamma_{tot}^2}$$

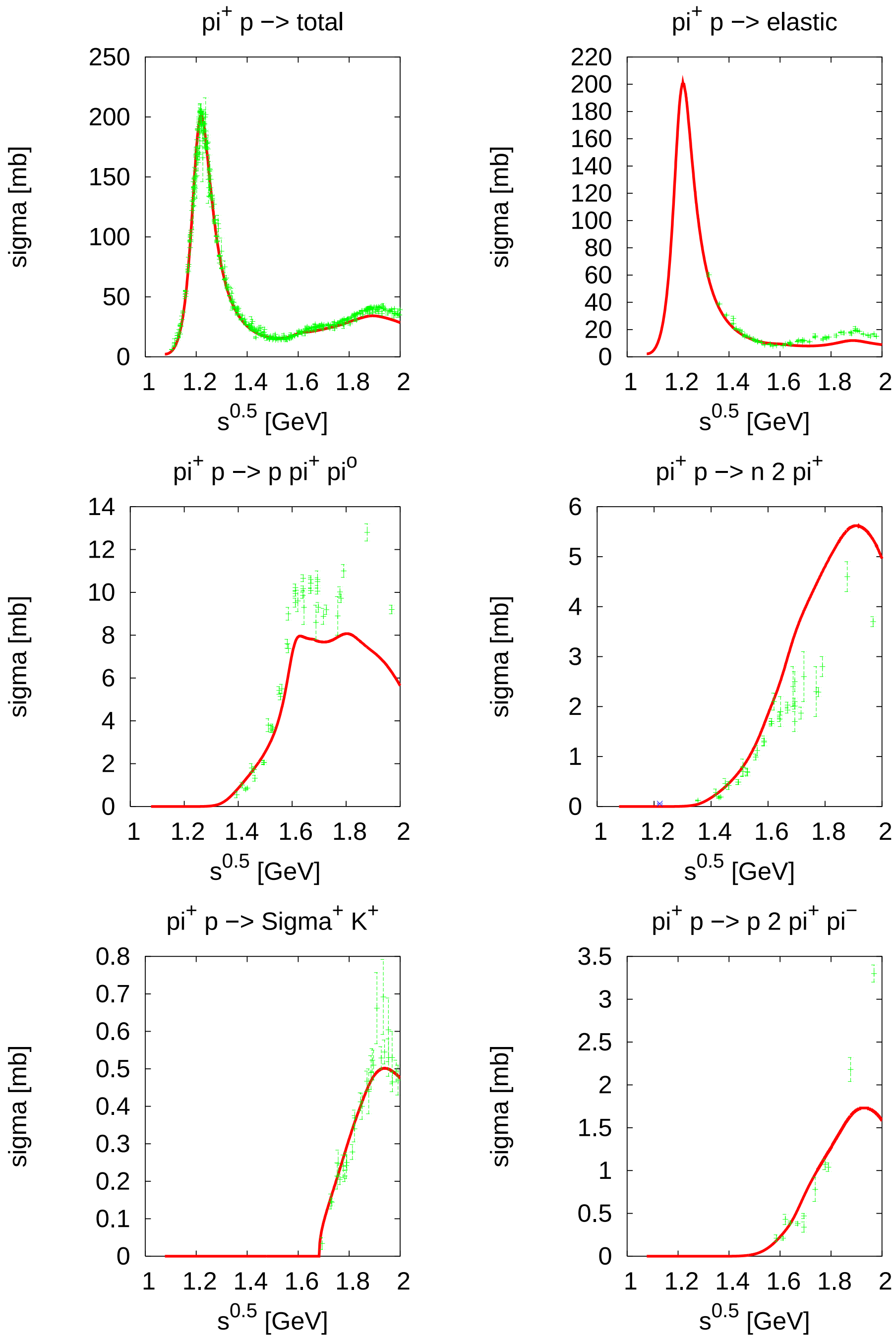
Baryon resonance parameters: mass, width, branching ratios are fitted by describing the meson production channels in πN collisions:

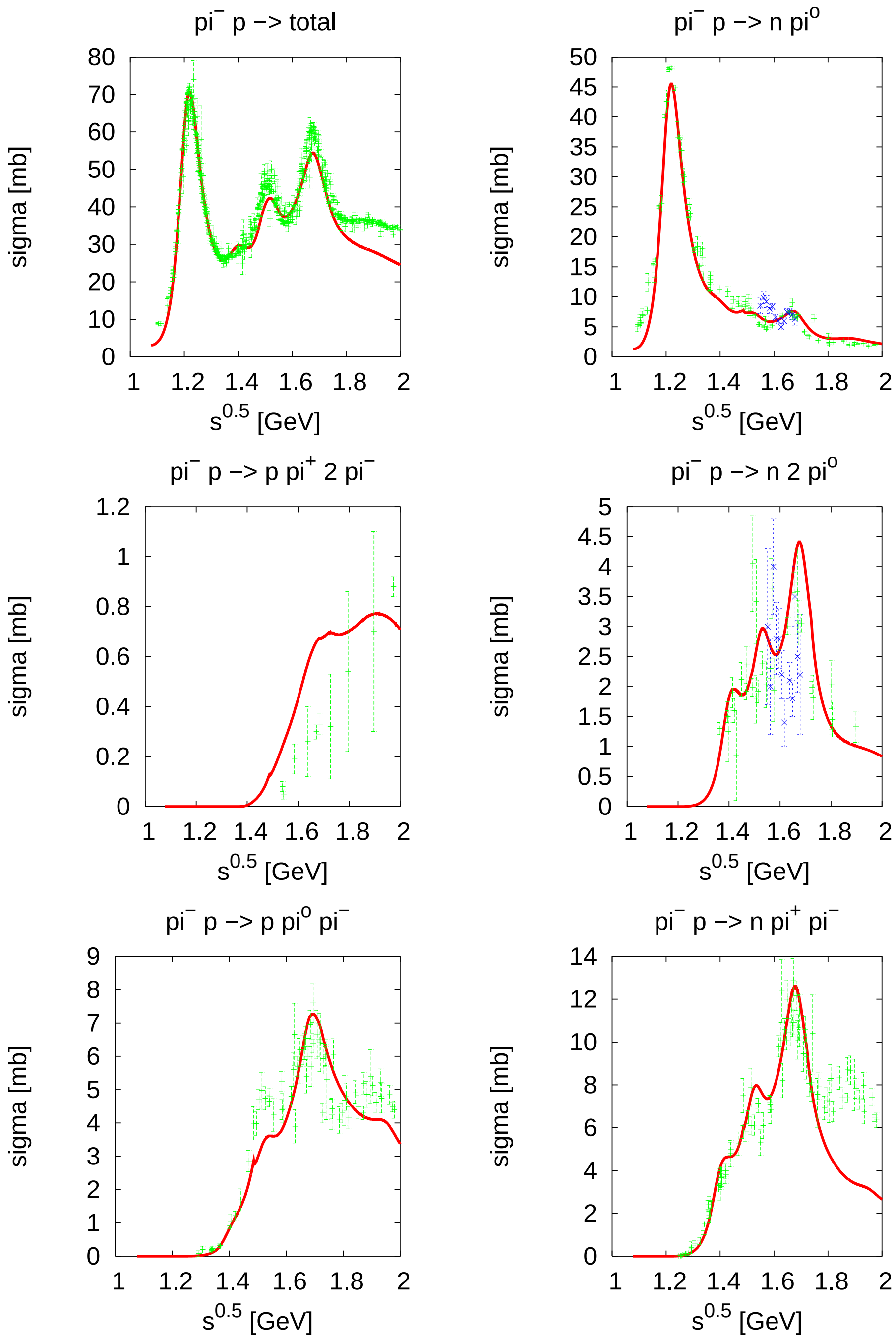
$$\sigma_{\pi N \to NM} = \sum_R \sigma_{\pi N \to R} \frac{\Gamma_{R \to NM}}{\Gamma_{tot}}$$

Resonance production cross section $NN \to NR$ is given by the fit of

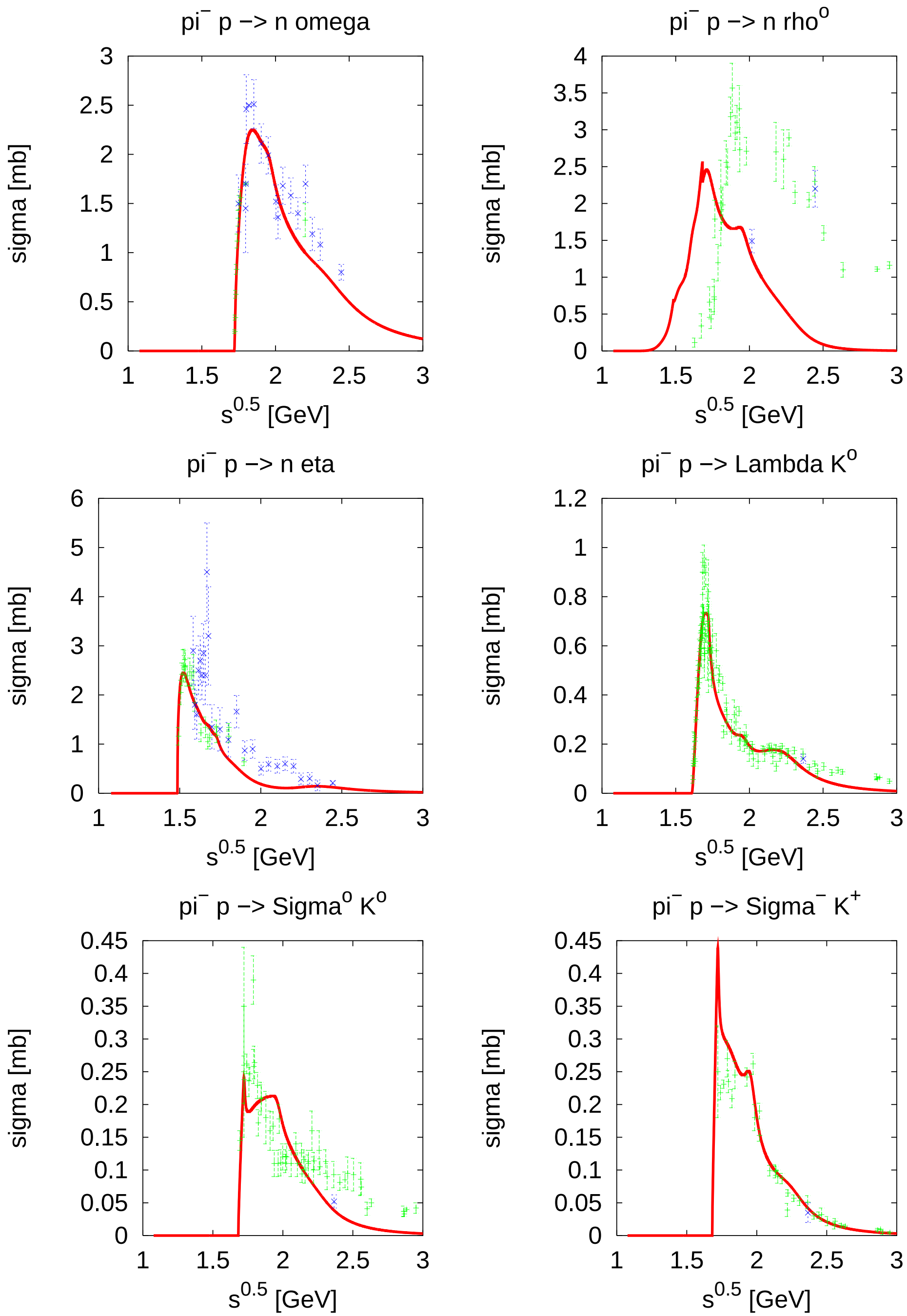
$$\sigma_{NN \to NM} = \sum_R \sigma_{NN \to NR} \frac{\Gamma_{R \to NM}}{\Gamma_{tot}}$$

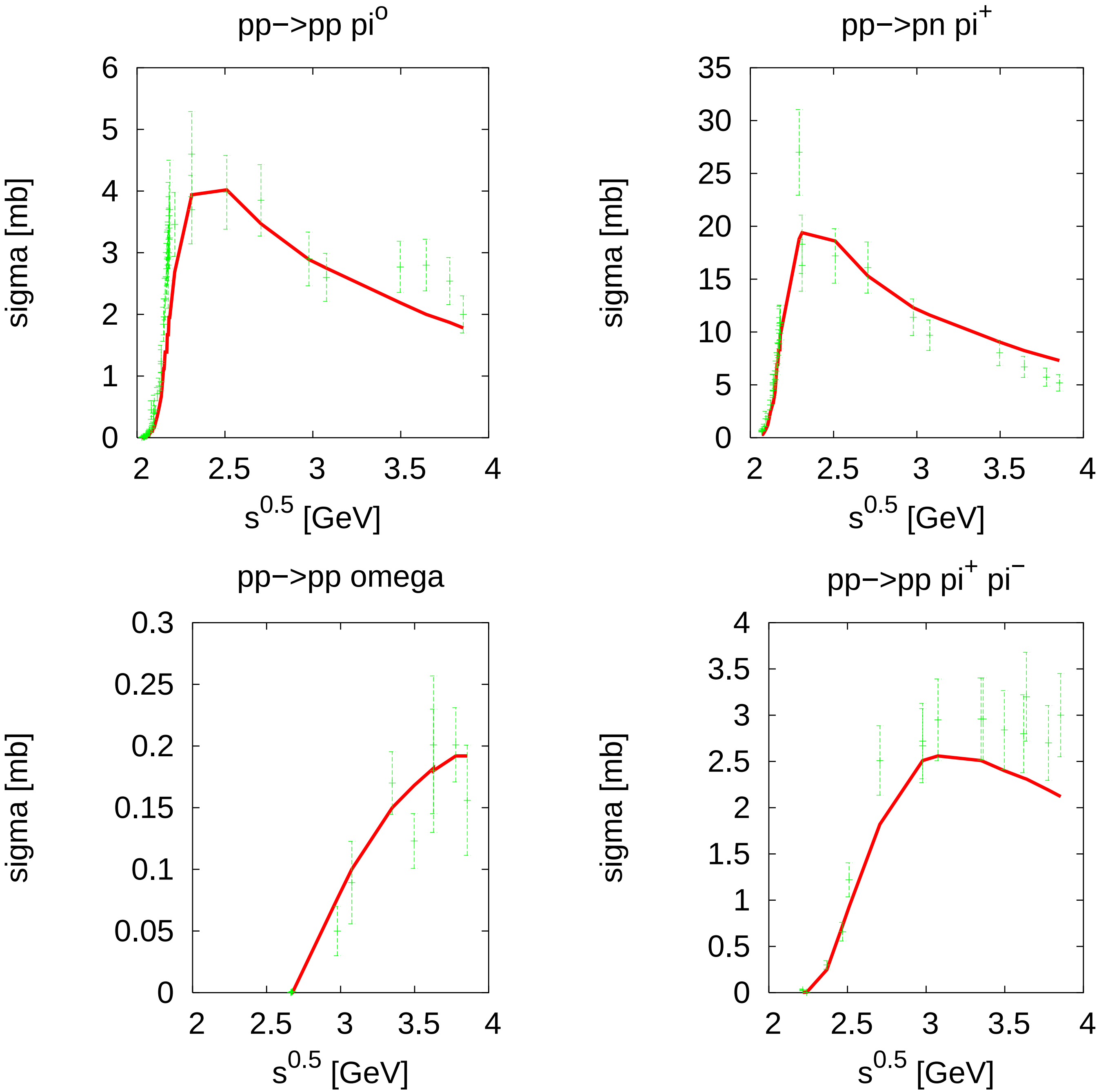
27 baryons, 6 mesons. Fit is done by the Minuit package (CERN)


pi+ p −> total
sigma [mb]
s0.5 [GeV]
pi+ p −> elastic
sigma [mb]
s0.5 [GeV]
pi+ p −> p pi+ pio
sigma [mb]
s0.5 [GeV]
pi+ p −> n 2 pi+
sigma [mb]
s0.5 [GeV]
pi+ p −> Sigma+ K+
sigma [mb]
s0.5 [GeV]
pi+ p −> p 2 pi+ pi−
sigma [mb]
s0.5 [GeV]



pi⁻ p -> total
sigma [mb]
s^0.5 [GeV]
pi⁻ p -> n pi⁰
sigma [mb]
s^0.5 [GeV]
pi⁻ p -> p pi⁺ 2 pi⁻
sigma [mb]
s^0.5 [GeV]
pi⁻ p -> n 2 pi⁰
sigma [mb]
s^0.5 [GeV]
pi⁻ p -> p pi⁰ pi⁻
sigma [mb]
s^0.5 [GeV]
pi⁻ p -> n pi⁺ pi⁻
sigma [mb]
s^0.5 [GeV]


pi⁻ p -> n omega
pi⁻ p -> n rho⁰
pi⁻ p -> n eta
pi⁻ p -> Lambda K⁰
pi⁻ p -> Sigma⁰ K⁰
pi⁻ p -> Sigma⁻ K⁺
sigma [mb]
s^0.5 [GeV]


pp->pp pi^{o}
pp->pn pi^{+}
pp->pp omega
pp->pp pi^{+} pi^{-}
sigma [mb]
s$^{0.5}$ [GeV]
0
1
2
3
4
5
6
10
15
20
25
30
35
0.05
0.1
0.15
0.2
0.25
0.3
0.5
1.5
2.5
3.5

Kadanoff-Baym Equation

Schwinger-Dyson equation:

$$G = G_0 + G_0 \Sigma G$$

$$\begin{aligned}
G^{11}(1,2) &= G^{T}(1,2) = \langle T(\phi(1)\phi(2))\rangle \\
G^{22}(1,2) &= G^{AT}(1,2) = \langle \tilde{T}(\phi(1)\phi(2))\rangle \\
G^{21}(1,2) &= G^{<}(1,2) = \langle \phi(2)\phi(1)\rangle \\
G^{12}(1,2) &= G^{>}(1,2) = \langle \phi(1)\phi(2)\rangle \\
G^{r}(1,2) &= \theta(t1-t2)(G^{>}(x1,t1;x2,t2) - G^{<}(x1,t1;x2,t2)) \\
G^{a}(1,2) &= \theta(t2-t1)(G^{<}(x1,t1;x2,t2) - G^{>}(x1,t1;x2,t2))
\end{aligned}$$

After some manipulation: *Kadanoff-Baym equation*:

$$\begin{aligned}
(i\hbar\partial_{t1} - H_0(1))G^{<}(1,2) &= \int d3\, \Sigma^{r}(1,3)G^{<}(3,2) + \int d3\, \Sigma^{<}(1,3)G^{a}(3,2) \\
(i\hbar\partial_{t1} - H_0(1))G^{r}(1,2) &= \delta^4(1,2) + \int d3\, \Sigma^{r}(1,3)G^{r}(3,2)
\end{aligned}$$

Wigner-transformation

- Retarded propagator is not a distribution function
- Wigner transform:

$$r = x1 - x2 \qquad , \qquad R = x1 + x2$$

R (center of mass) dependence of propagators and selfenergies are weaker than the r dependence

$$G^r(R,P) = \int d^4r \, G^r(X+r, X-r)$$

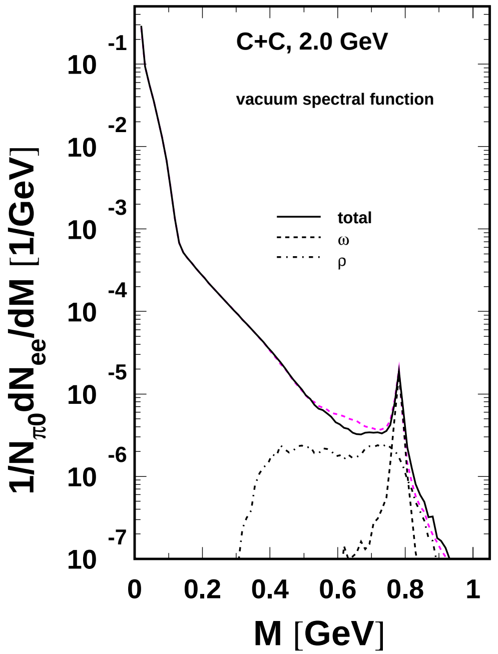
- Gradient expansion in r. Neglect all terms with more than one derivative in R
- transport equation for $F_\alpha = iG^<(R,P) = f_\alpha(x,p,t)A_\alpha$

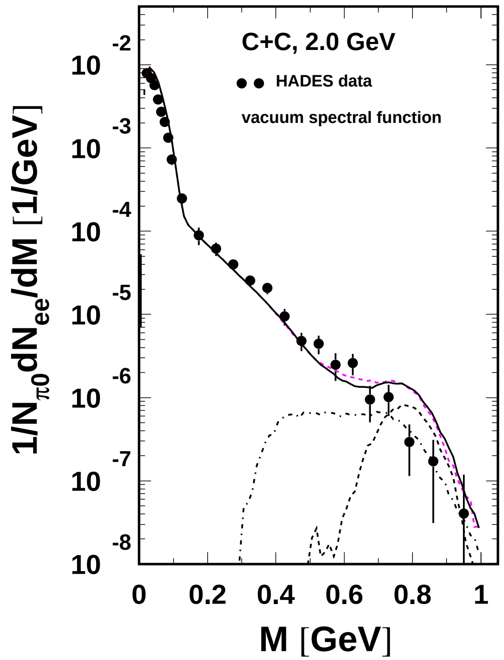
$A(p) = -2ImG^r = \frac{\hat{\Gamma}}{(E^2 - \mathbf{p}^2 - m_0^2 - \mathrm{Re}\Sigma^r)^2 + \frac{1}{4}\hat{\Gamma}^2}$,

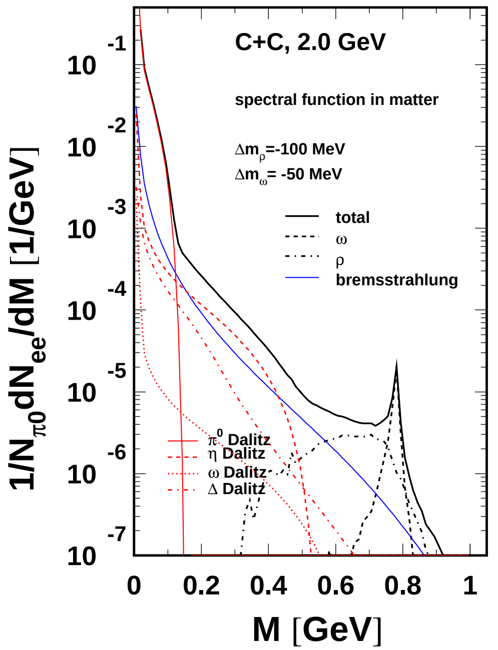
Cassing, Juchem (2000) and Leupold (2000)

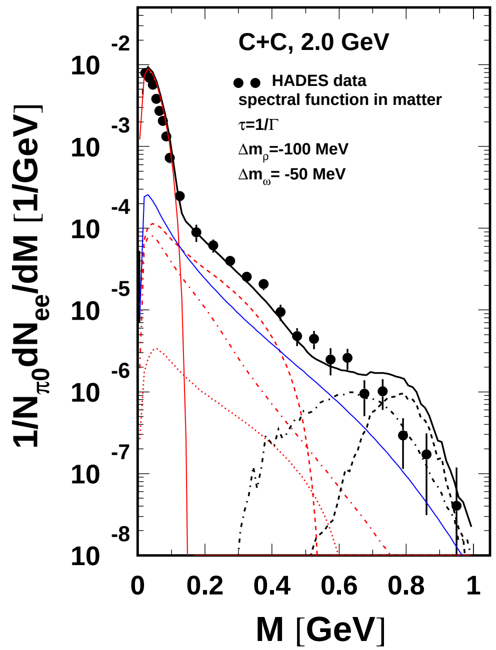
- testparticle approximation

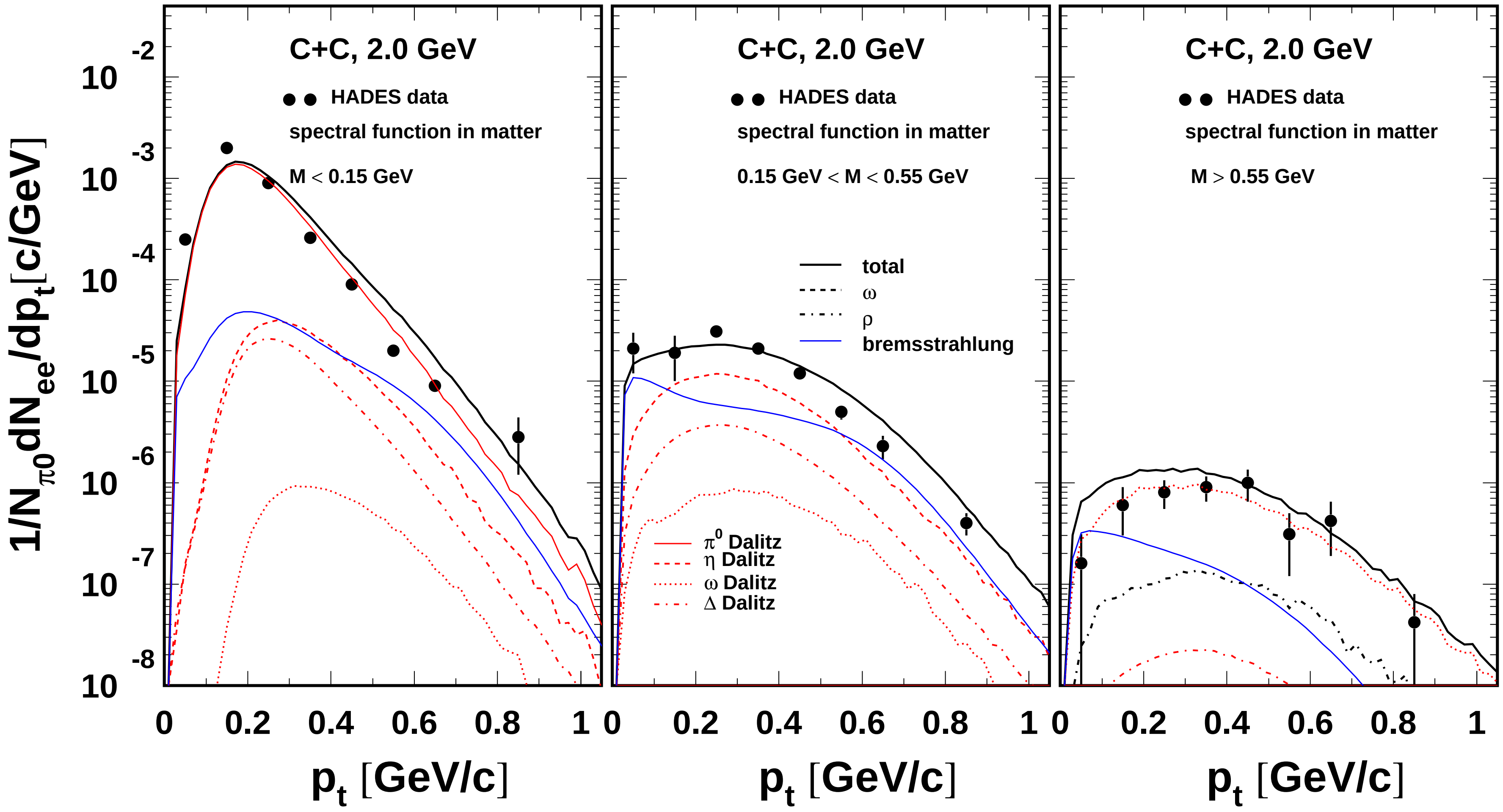
C + C 2 GeV


C+C, 2.0 GeV
vacuum spectral function
total
ω
ρ
1/N$_{\pi 0}$dN$_{ee}$/dM [1/GeV]
M [GeV]

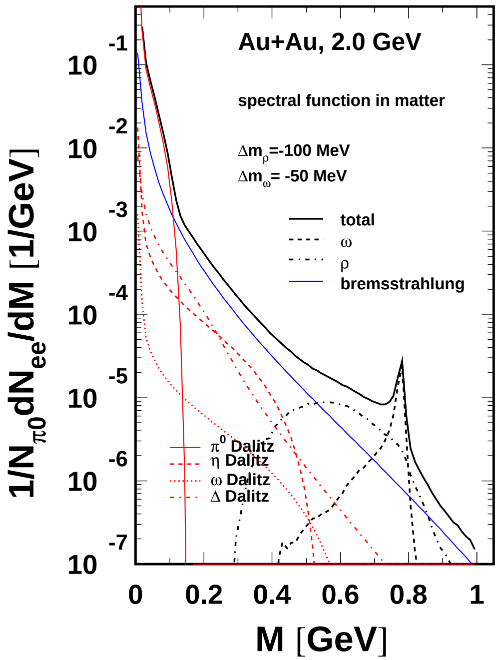

C+C, 2.0 GeV
HADES data
vacuum spectral function
1/N$_{\pi 0}$dN$_{ee}$/dM [1/GeV]
M [GeV]

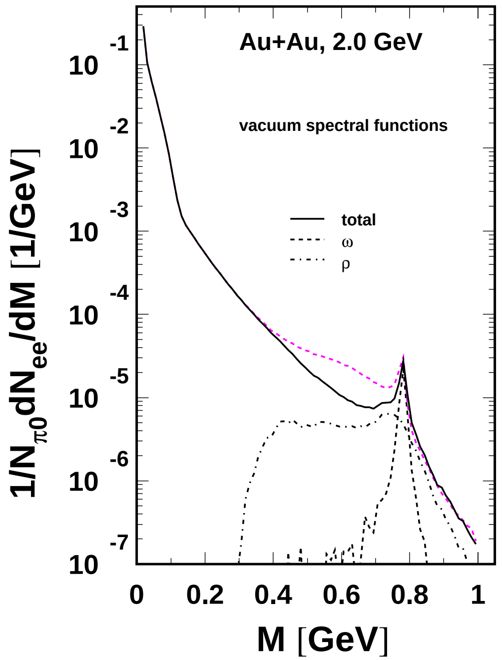

C+C, 2.0 GeV
spectral function in matter
Δm$_\rho$=-100 MeV
Δm$_\omega$= -50 MeV
total
ω
ρ
bremsstrahlung
π^0 Dalitz
η Dalitz
ω Dalitz
Δ Dalitz
1/N$_{\pi 0}$dN$_{ee}$/dM [1/GeV]
M [GeV]

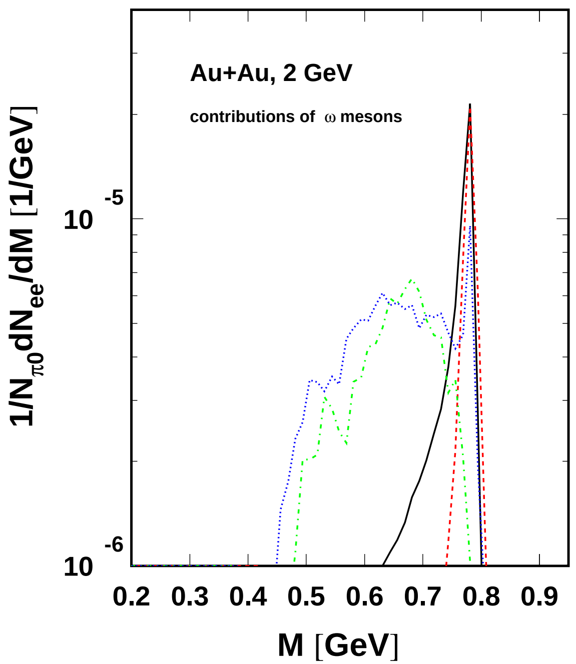

C+C, 2.0 GeV
HADES data
spectral function in matter
τ=1/Γ
Δm$_\rho$=-100 MeV
Δm$_\omega$= -50 MeV
1/N$_{\pi 0}$dN$_{ee}$/dM [1/GeV]
M [GeV]

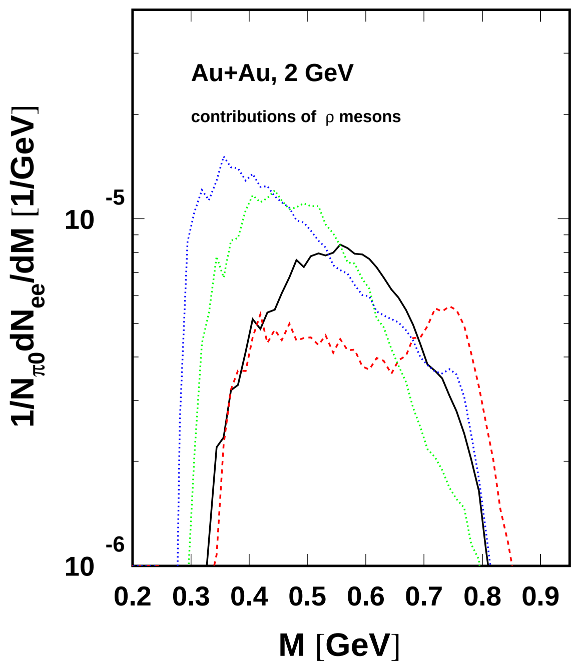

C+C, 2.0 GeV
HADES data
spectral function in matter
M < 0.15 GeV
0.15 GeV < M < 0.55 GeV
M > 0.55 GeV
total
ω
ρ
bremsstrahlung
π0 Dalitz
η Dalitz
ω Dalitz
Δ Dalitz
1/Nπ0dNee/dpt [c/GeV]
pt [GeV/c]

Au + Au 2 GeV







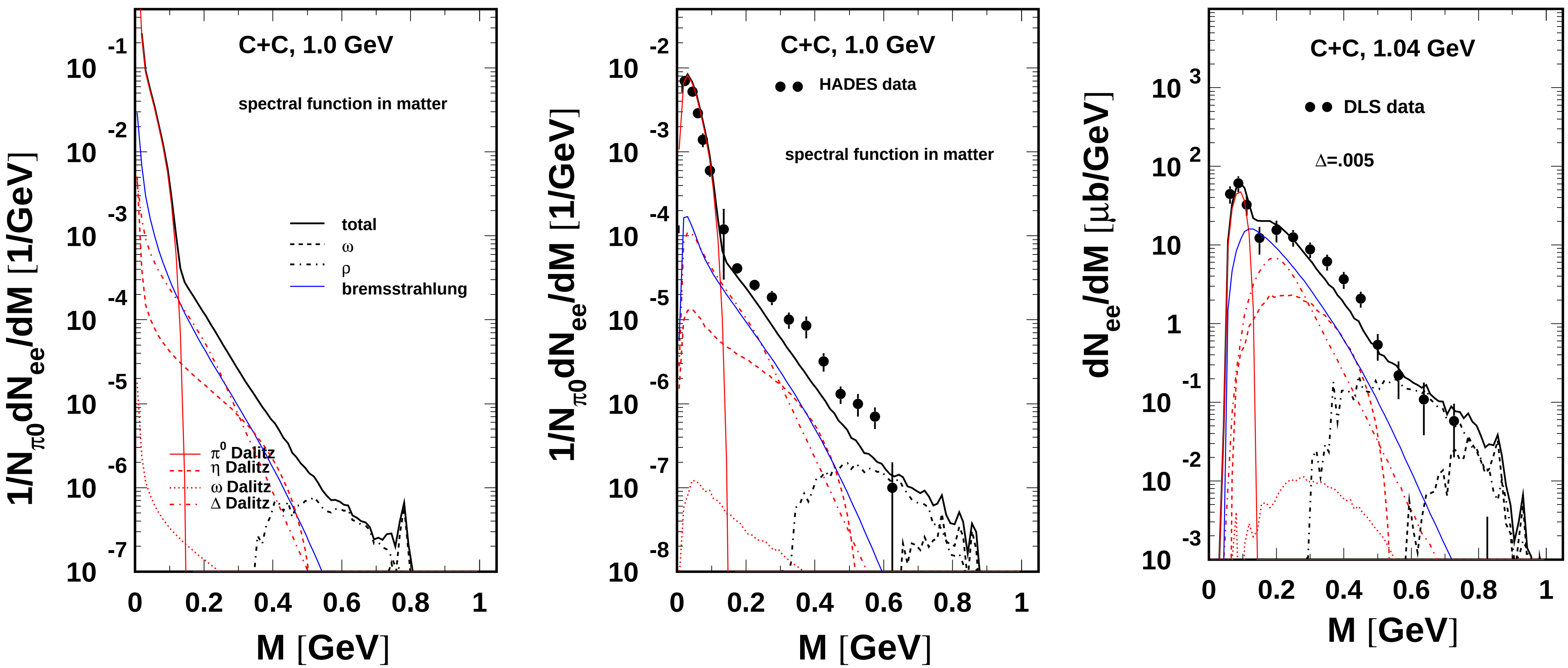


Vacuum

Matter

Static

C + C 1 AGeV


C+C, 1.0 GeV
spectral function in matter
total
ω
ρ
bremsstrahlung
π0 Dalitz
η Dalitz
ω Dalitz
Δ Dalitz
1/Nπ0 dNee/dM [1/GeV]
M [GeV]
C+C, 1.0 GeV
HADES data
spectral function in matter
1/Nπ0 dNee/dM [1/GeV]
M [GeV]
C+C, 1.04 GeV
DLS data
Δ=.005
dNee/dM [μb/GeV]
M [GeV]