

Future Femtoscopy Studies from LHCb

Tadeusz Lesiak

Institute of Nuclear Physics
Polish Academy of Sciences,
Kraków, Poland



Intensity Interferometry (HBT)

Hanbury-Brown
& Twiss

- ✓ quantum interference effect between indistinguishable particles, emitted by a finite-size source,
- ✓ useful tool to probe the spatial and temporal structure of the hadron emission volume,

Total wave function	Correlation
symmetrization	Bose-Einsten (BEC)
antisymmetrization	Fermi-Dirac (FDC)

- ✓ the two particle **correlation function**: $C_2(q_1, q_2) = \frac{\mathcal{P}(q_1, q_2)}{\mathcal{P}(q_1)\mathcal{P}(q_2)} = \frac{\mathcal{P}(q_1, q_2)}{\mathcal{P}_{ref}(q_1, q_2)}$

q_1, q_2 - four-momenta of two indistinguishable particles emitted from a common source

$\mathcal{P}(q_1), \mathcal{P}(q_2)$ - probability density distributions (PDF) of a single particle

$\mathcal{P}(q_1, q_2)$ - PDF of two particles (1,2)

$\mathcal{P}_{ref}(q_1, q_2)$ - PDF of two particles of the reference sample (see below)

Correlation Function

- ✓ THE relevant invariant - **four-momentum difference**:

$$Q = \sqrt{-(q_1 - q_2)^2} = \sqrt{M^2 - 4\mu^2}$$

M - the invariant mass of the pair (1,2)
 μ - mass of the particle 1(2)

- ✓ The correlation function in terms of Q:

$$C_2(q_1, q_2) = C_2(Q) = \frac{P(Q)}{P_{ref}(Q)}$$

the reference sample - see the next slides

- ✓ **Goldhaber parametrization (GP)**:

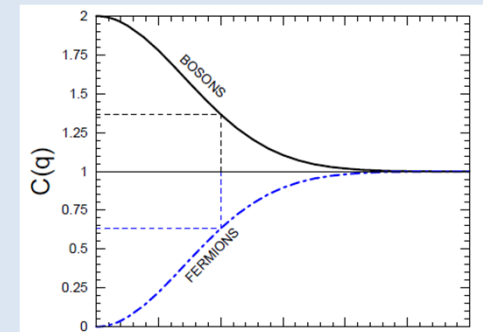
$$C_2(Q) = N(1 \pm \lambda e^{-R^2 Q^2})$$

plus-sign for bosons;
 minus-sign for fermions
 N - overall normalization factor

R - radius of the spherical source

λ - chaoticity parameter:

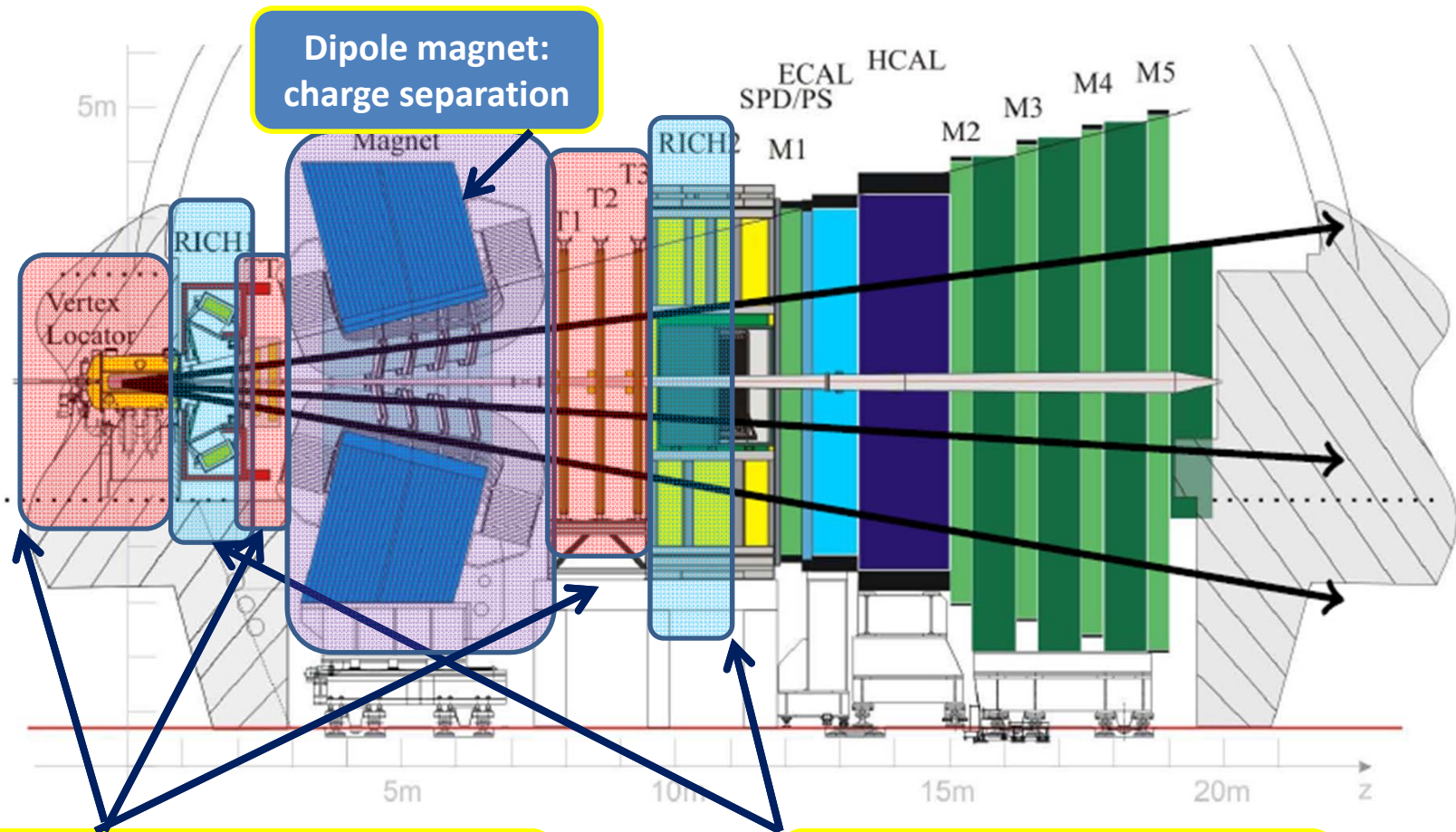
$0 \leq \lambda \leq 1$
 fully coherent source \leftarrow \rightarrow completely chaotic source



- Extensions of the GP e.g. to include long range correlations.
- Alternative parametrisations e.g. Levy distr. OR $C_2(Q) \propto e^{-RQ}$.

LHCb detector

LHCb is a forward arm spectrometer with a unique pseudorapidity range $2 < \eta < 5$

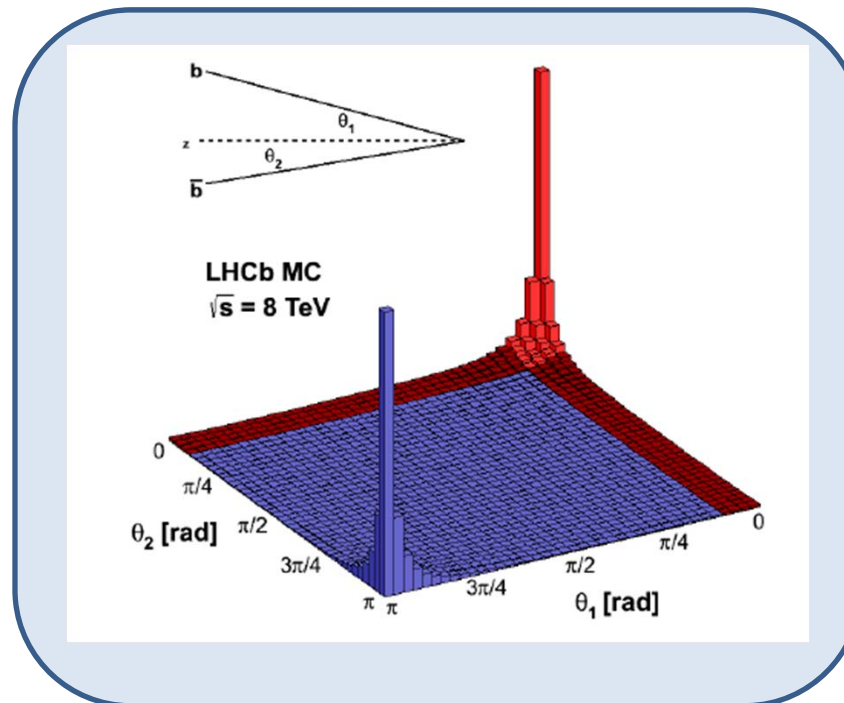


Excellent vertexing, tracking, time and momentum resolution

Two RICH detectors crucial for K/ π separation

Topology of LHCb events

$$p + p \rightarrow b\bar{b}$$



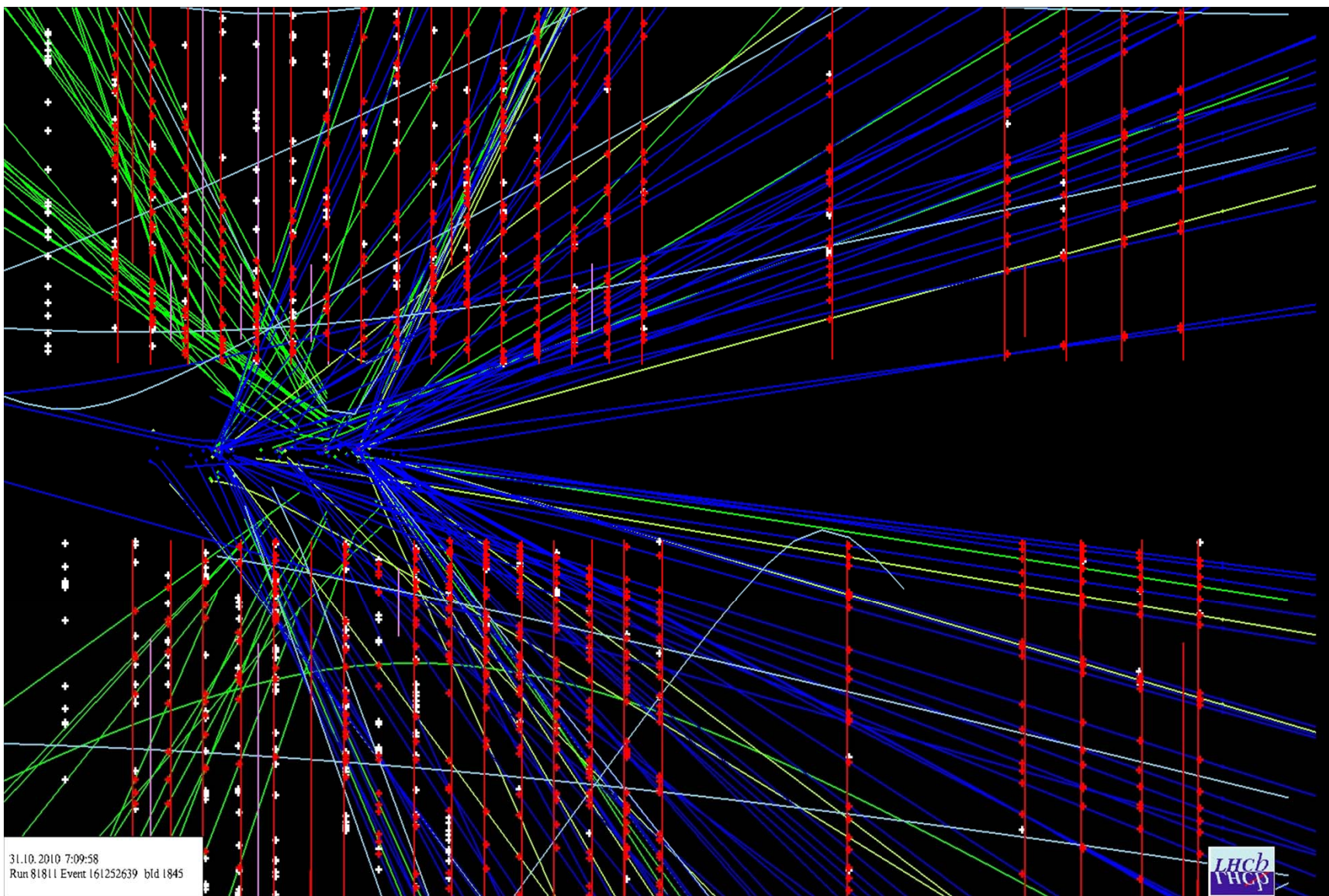
2011 & 2012 data taking:

On average $N_{\text{Int}} = 1.4$ interactions per bunch crossing.

Pile-up: $N_{\text{int}} = 2, 3, 4, \dots$

With the presence of pile-up one ends up with more than one primary vertex (PV)...

Topology of LHCb events



Reference Samples

- ✓ The denominator(s) in $C_2(Q) = \frac{P(Q)}{P_{ref}(Q)}$
- ✓ The reference sample should be free from BEC (or FDC) while possessing all other correlations, related to e.g. charge, energy-momentum, baryon number conservation, particle decays, event topology etc.
- ✓ The three reference samples (tried in the LHCb studies for pion pairs):



event – mix

- ✓ Pairs of identical charged hadron each originating from different event.

PV – mix

- ✓ Pairs of identical charged hadron each originating from different primary vertices (PV) of the same event.

Common features
of both
event-mix
and **PV-mix**
reference samples:

- The reference sample is derived directly from data. 
- The random choice of a pair eliminates not only the BEC (FDC) but also many other correlations... 

Reference Samples

unlike – sign

✓ Pairs of unlike-sign charged hadron e.g. $\pi^+\pi^-$



- The reference samples is derived directly from data.



- Unlike-sign pairs may originate from resonances.
- Effects of electrical attraction of opposite charges of the pair.
- Troublesome choice of the normalization point for the data far away from the interference region.

Double ratio

$$R(Q) = \frac{C_2(Q)^{data}}{C_2(Q)^{MC}} = \frac{\frac{\mathcal{P}(Q)^{data}}{\mathcal{P}_{ref}(Q)^{data}}}{\frac{\mathcal{P}(Q)^{MC}}{\mathcal{P}_{ref}(Q)^{MC}}}$$

Motivation: to reduce possible biases in the construction of the reference sample.

BEC and FDC in Two and Three Dimensions

- ✓ Leaving aside the assumption that the source shape is a sphere with a Gaussian distribution.
- ✓ → still the correlations for a non-spherical source can be studied in terms of the components of the particles' four momenta difference.

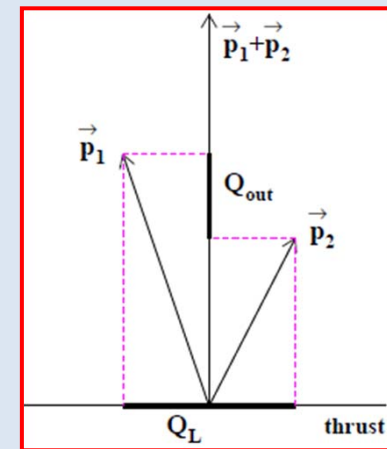
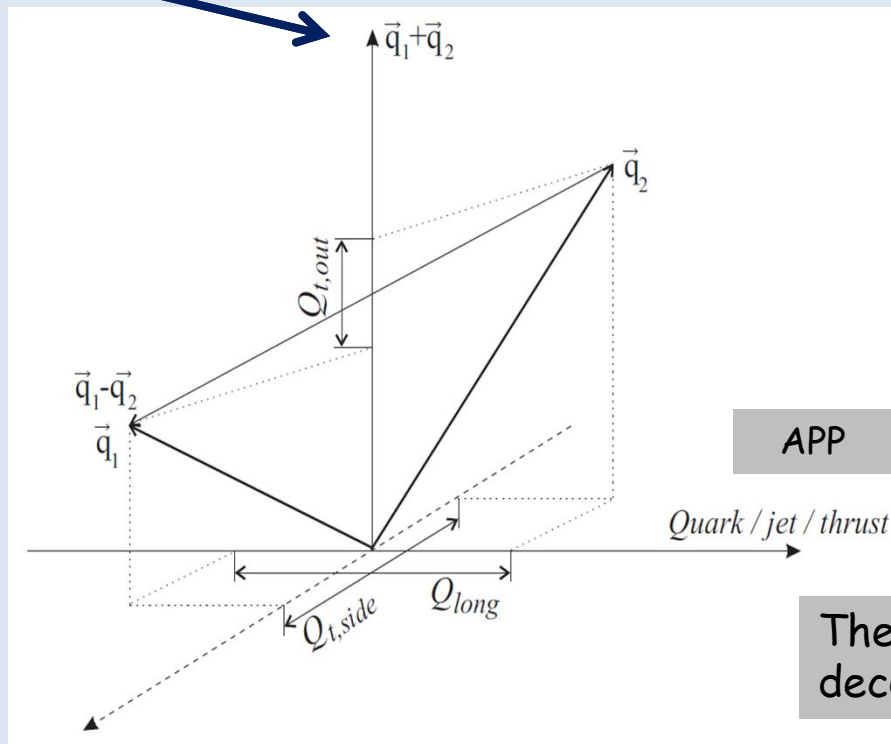
- ✓ In such case the decomposition of the particles' four momenta difference is usually performed in **the Longitudinal CM System (LCMS)**.
- ✓ The LCMS → allows for separation of spatial and temporal coordinates (see next slides).

BEC and FDC in Two and Three Dimensions

Boost each pair of particles along the axis of the physical process (APP) e.g. jet, thrust, z-axis... in such a way that the sum of momenta $\vec{q}_1 + \vec{q}_2$ is orthogonal to the APP

The two-particle system is then rotated in the plane perpendicular to $\vec{q}_1 + \vec{q}_2$ until the sum of longitudinal component of \vec{q}_1 and \vec{q}_2 amounts to zero.

APP = z coordinate for LHCb



The variable $\vec{Q} = \vec{q}_1 - \vec{q}_2$ is decomposed into three components:

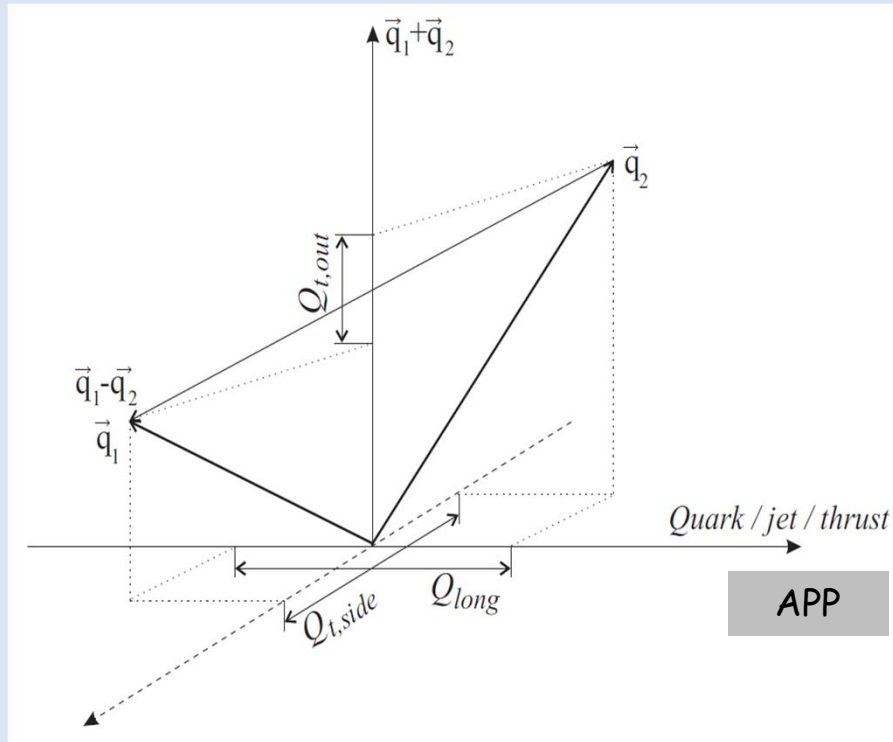
$$Q_{t,out}$$

$$Q_{t,side}$$

$$Q_{long} \equiv Q_z$$

$$C_2(Q_{t,out}, Q_{t,side}, Q_{long}) = 1 + \lambda \exp \left(-Q_{t,out}^2 \cdot R_{t,out}^2 - Q_{t,side}^2 \cdot R_{t,side}^2 - Q_{long}^2 \cdot R_{long}^2 \right)$$

BEC and FDC in Two and Three Dimensions



By construction of the LCMS

the projection of $\vec{q}_1 + \vec{q}_2$
on both Q_{long} and $Q_{t,side}$
vanishes.

→ By Heisenberg Principle
these variables couple only
to spatial dimensions of the source.

Q_{long} $Q_{t,side}$

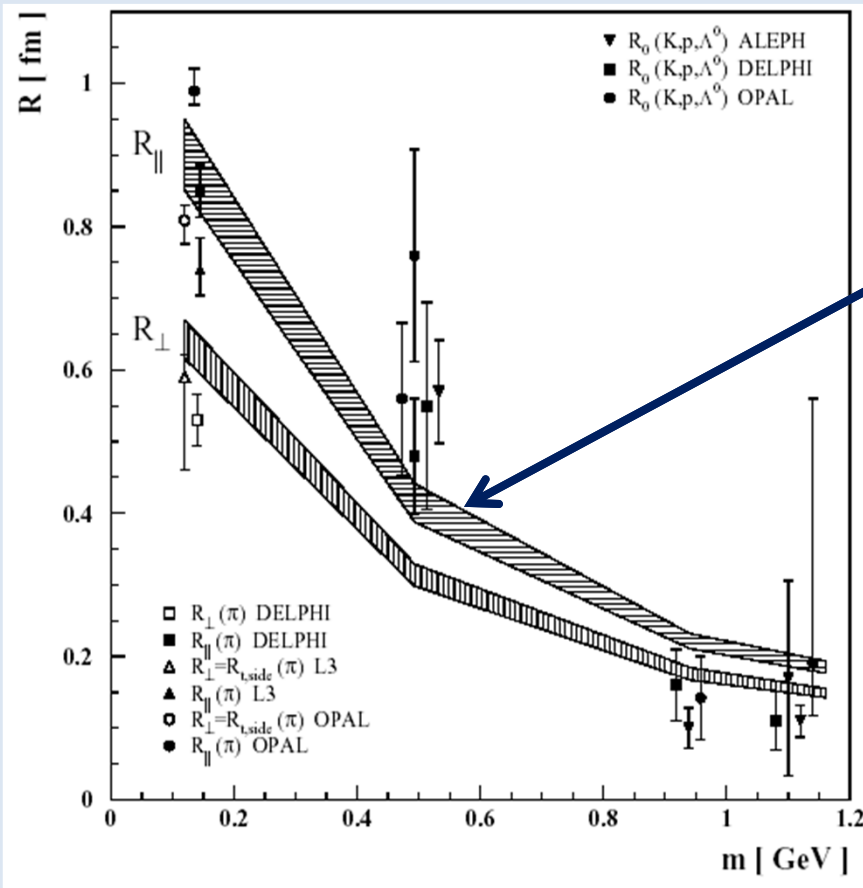
- reflect ONLY SPATIAL dimensions of the source

$Q_{t,out}$

- reflect A MIXTURE OF SPATIAL AND TEMPORAL
dimensions of the source

Correlation Radius vs the Hadron Mass

LEP data:



KK, pp, $\Lambda\Lambda$ - 1 dim. analyses

$\pi\pi$ (DELPHI) - 2 dim analysis

$\pi\pi$ (L3, OPAL) - 3 dim. analyses

A.Białas, M.Kucharczyk,
H.Pałka, K. Zalewski
[Phys. Rev. D 62 \(2000\) 114007](#)

- Controversy:
do we observe the
diminishment of
the correlation radius
vs
the hadron mass ?
- Few theoretical models
on the market:
 - semiclassical (G.Alexander),
 - quantum-mechanical approach
(A.Białas, K.Zalewski),
 -

Femtoscopia Studies @ LHCb:

✓ **BEC for pion and kaons like-sign pairs**

- determination of the correlation radii for 1-dim. case,
- extension to three-dimensional analysis,
- separate studies for 7 and 8 TeV CMS data for pp collisions,
- analysis of p-Pb data (5 TeV, 1.6 nb⁻¹).

✓ **FDC for protons** (also with 2010 min-bias data)

- feasibility studies ongoing, if successful
→ the same strategy as for pions and kaons.

✓ **FDC for Λ^0**

- feasibility study → determination of the spin composition.

✓ **BEC for charmed mesons**

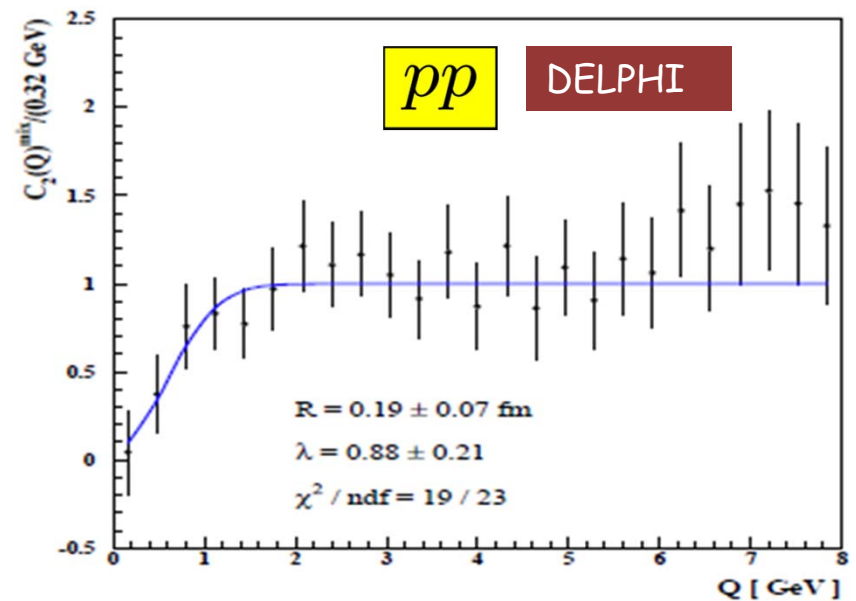
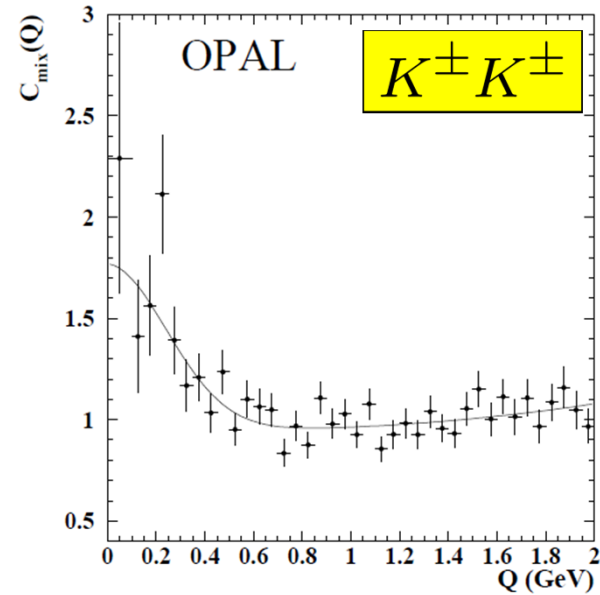
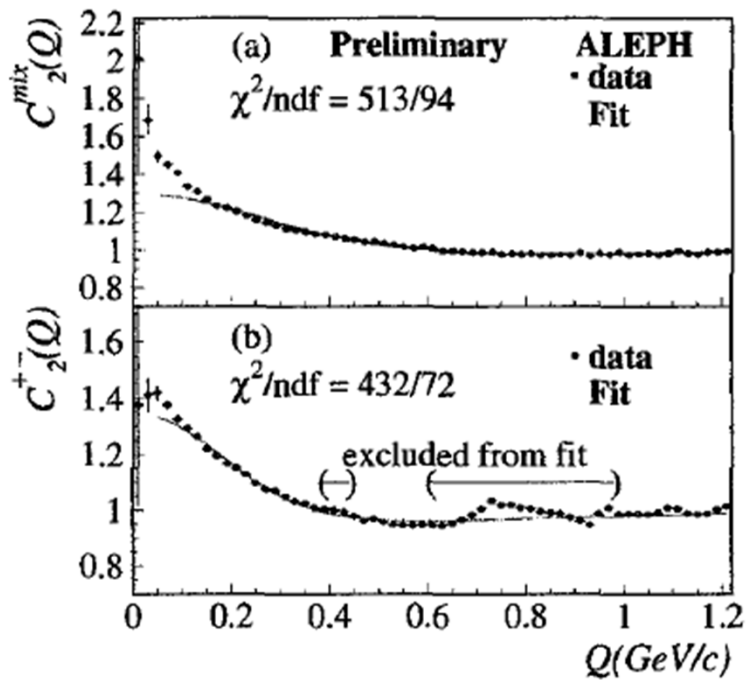
- feasibility study; statistics seems to be sufficient.

✓ **Three particle correlations for pions**

- input from theory needed to define the best observables.

Femtoscscopy of $\pi\pi$, KK and pp pairs

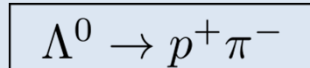
$\pi\pi$



Λ and $\bar{\Lambda}$ spin composition

- ✓ Studies of pairs of identical baryons produced in hadronization.
- ✓ **Observe the decrease of the $S = 1$ state contribution due to the Pauli exclusion principle** → determination of of the baryon emitter dimension.

✓ **The variables:**



θ_p - Helicity angle of the proton direction in its parent Λ rest frame (w.r.t. the parent momentum in LAB)

y^* - cosine of the angle between the two hyperons' decay protons, each measured in its parent Λ rest frame

✓ **The Wigner-Eckart theorem:**
relation between averages
of these angular distributions:

$$\frac{\langle y^* \rangle}{\langle \cos \theta_{p1} \rangle \langle \cos \theta_{p2} \rangle} = \begin{cases} -3 & \text{for } S = 0 \\ +1 & \text{for } S = 1 \end{cases}$$

✓ **The distribution of θ_p angle:**

$$\frac{dN}{d \cos \theta_p} = 1 - \alpha_\Lambda \cdot \cos \theta_p$$

The decay parameter:

$$\alpha_\Lambda = 0.642 \pm 0.013$$

$$\alpha_{\bar{\Lambda}} = -\alpha_\Lambda$$

Λ and $\bar{\Lambda}$ spin composition

✓ The y^* angular distributions

(valid at threshold but also for non-relativistic di-hyperon CMS energies):

$\Lambda\bar{\Lambda}$ & $\bar{\Lambda}\Lambda$

$$S = 0$$

$$\left. \frac{dN}{dy^*} \right|_{S=0} = 1 + \alpha_{\Lambda}^2 \cdot y^*$$

$$S = 1$$

$$\left. \frac{dN}{dy^*} \right|_{S=1} = 1 - \frac{\alpha_{\Lambda}^2}{3} \cdot y^*$$

$\Lambda\Lambda$ & $\bar{\Lambda}\bar{\Lambda}$

$$S = 0$$

$$\left. \frac{dN}{dy^*} \right|_{S=0} = 1 - \alpha_{\Lambda}^2 \cdot y^*$$

$$S = 1$$

$$\left. \frac{dN}{dy^*} \right|_{S=1} = 1 + \frac{\alpha_{\Lambda}^2}{3} \cdot y^*$$

ϵ - fraction of the $S = 1$ state contribution to the di-hyperon system.

$$\frac{dN}{dy^*} = (1 - \epsilon) \cdot \left. \frac{dN}{dy^*} \right|_{S=0} + \epsilon \cdot \left. \frac{dN}{dy^*} \right|_{S=1}$$

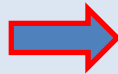
G.Alexander, H.Lipkin
Phys. Lett. B352 (1995) 162

Expectation: $\epsilon = \frac{3}{4}$ for a statistical spin mixture ensemble
(each spin probability is weighted by the factor $2S+1$).

Λ and $\bar{\Lambda}$ spin composition

Simple corollary:

$$\epsilon = \frac{3}{4}$$



$$\frac{dN}{dy^*} = (1 - \epsilon) \cdot \left. \frac{dN}{dy^*} \right|_{S=0} + \epsilon \cdot \left. \frac{dN}{dy^*} \right|_{S=1}$$

Flat distribution

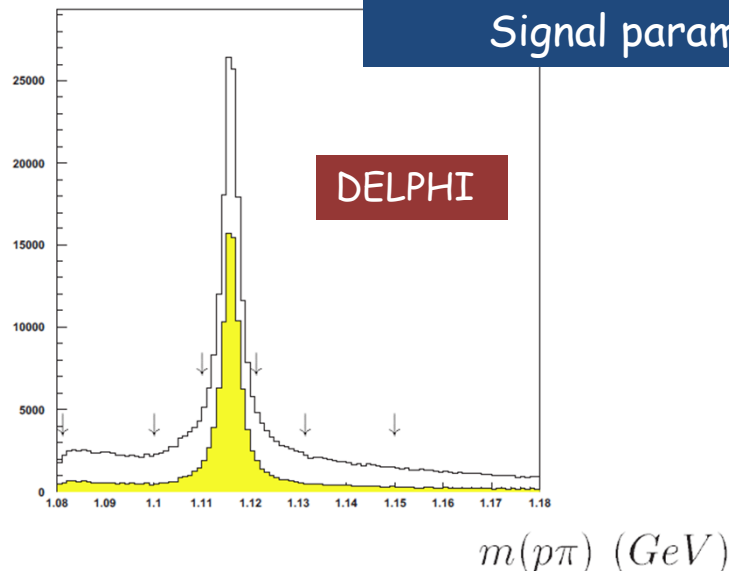
In the presence of background:

$$\frac{dN}{m(p\pi) \cdot 0.001}$$

$$\frac{dN}{dy^*} = P \cdot \left\{ (1 - \epsilon) \cdot \left. \frac{dN}{dy^*} \right|_{S=0} + \epsilon \cdot \left. \frac{dN}{dy^*} \right|_{S=1} \right\} + (1 - P) \cdot (1 + \kappa y^*)$$

Signal parametrization

Background parametrization



P - signal purity of Λ samples
(see next slide)

Parameters to be determined:

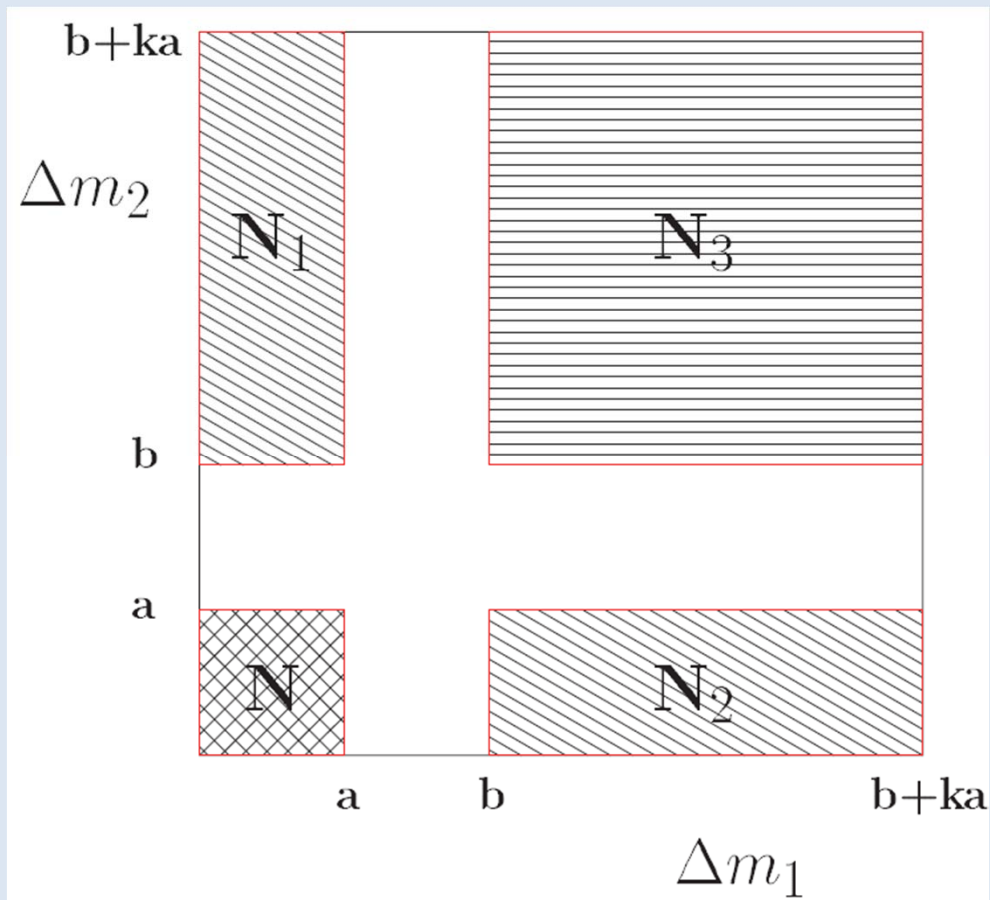
ϵ - spin composition parameter

κ - slope of the linear background

Λ and $\bar{\Lambda}$ spin composition

P-signal purity of Λ samples evaluated from data:

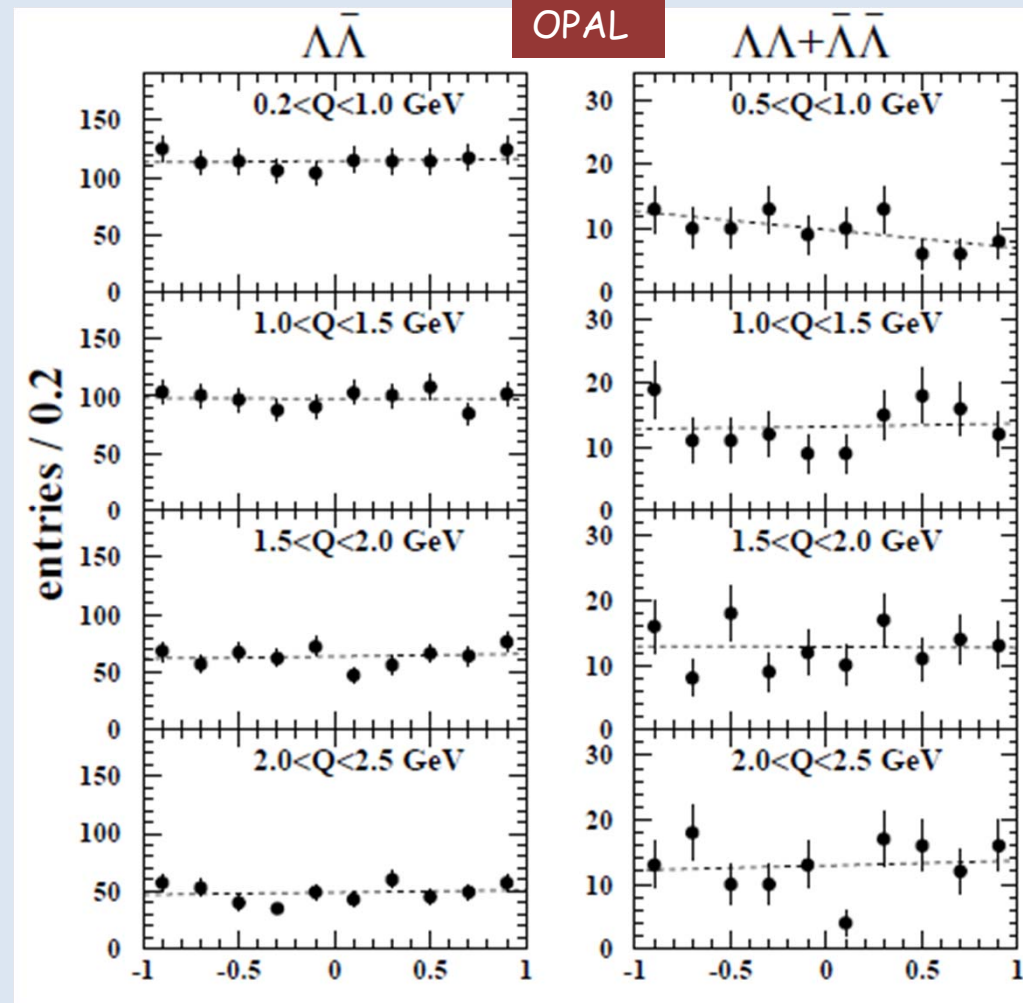
$$\Delta m_i = m_{\Lambda_i} - m_{\Lambda}^{\text{PDG}}, \quad i = 1, 2$$



Λ and $\bar{\Lambda}$ spin composition

Fits to the y^* distributions in several bins of

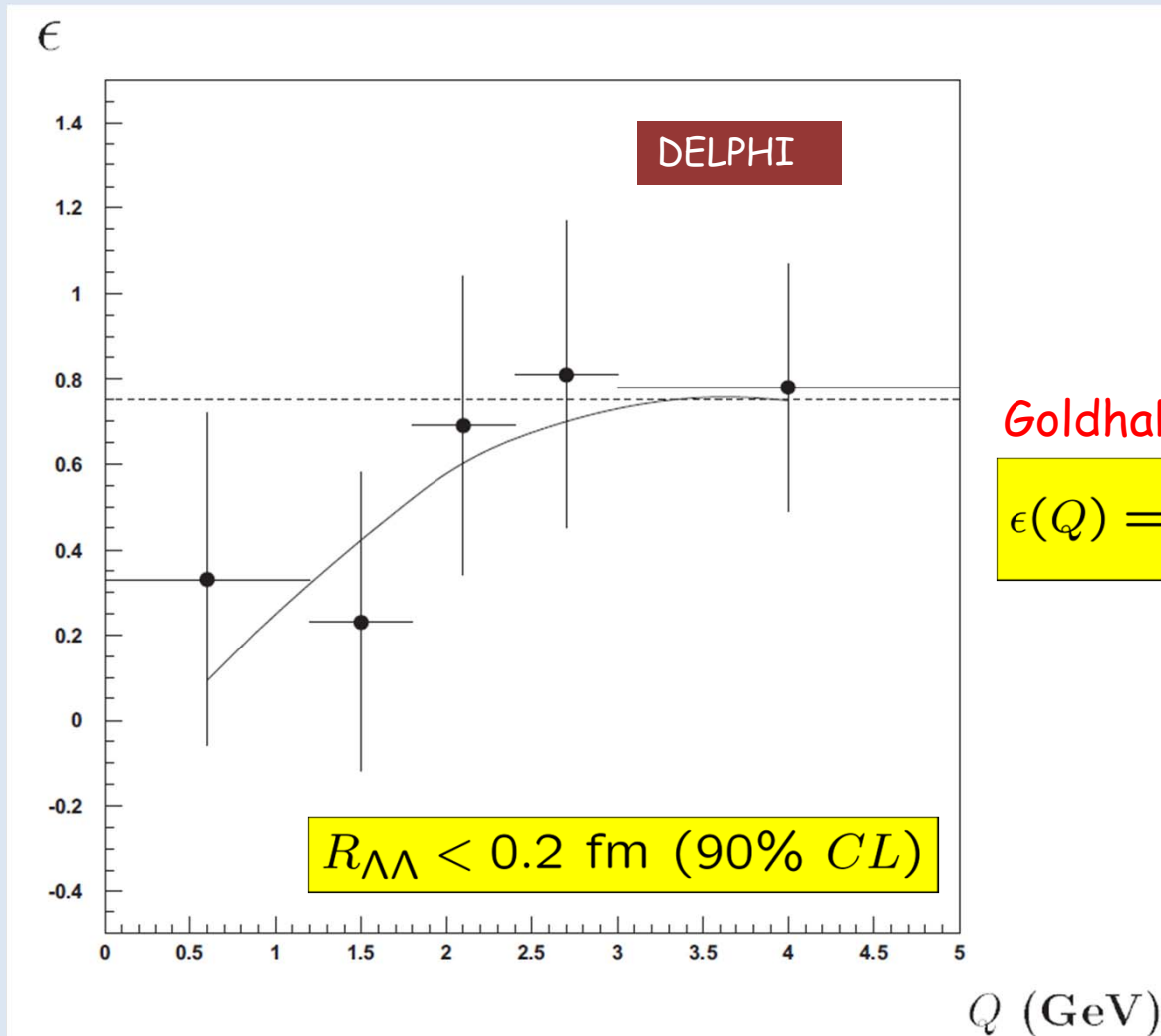
$$Q = \sqrt{m_{\Lambda\Lambda}^2 - 4m_{\Lambda}^{\text{PDG}}}$$



y^*

Λ and $\bar{\Lambda}$ spin composition

Fit results in bins of Q :



Goldhaber parametrization:

$$\epsilon(Q) = \frac{3}{4} \cdot \left(1 - e^{-R_{\Lambda\Lambda}^2 \cdot Q^2}\right)$$

T.Lesiak, H.Pałka
CERN-OPEN-99-460

Λ Correlations at LHCb

LHCb: $\Lambda^0 \rightarrow p + \pi^-$ invariant mass spectrum

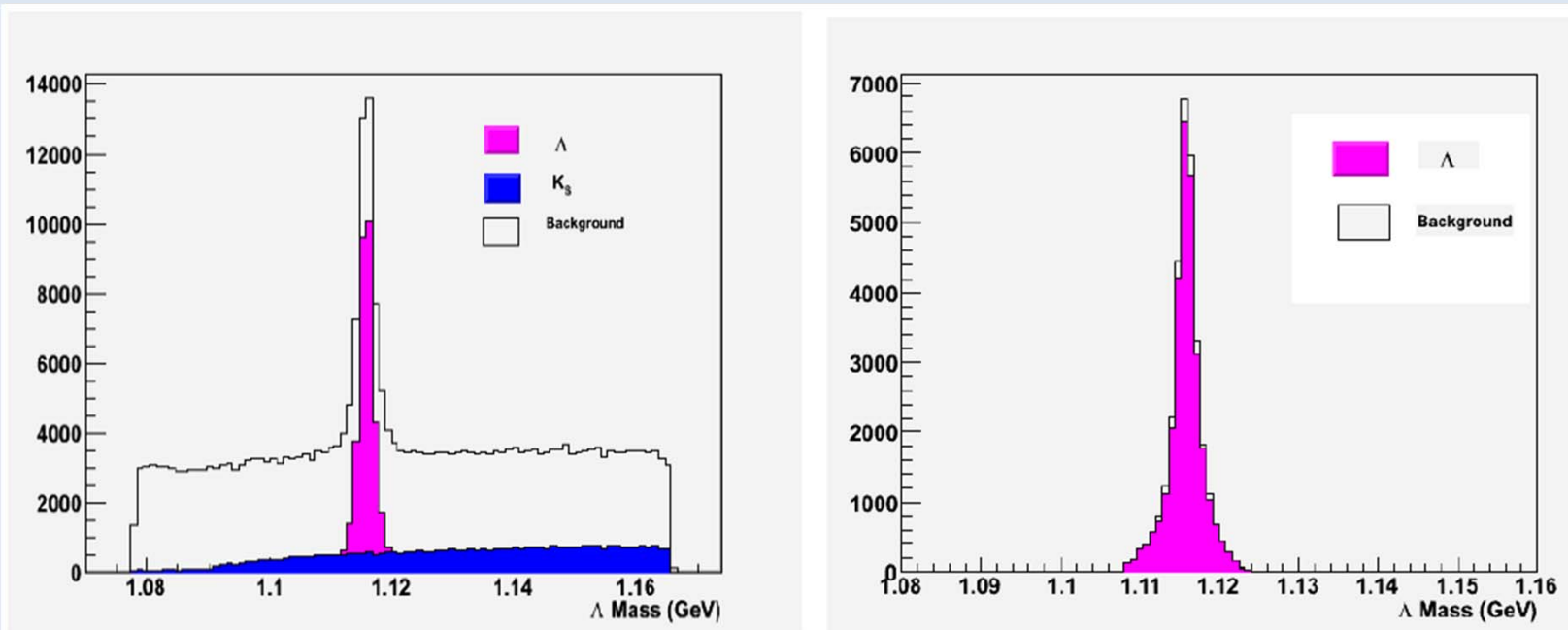


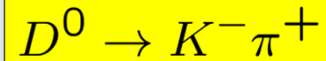
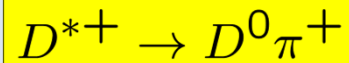
Figure: $p\pi$ -invariant mass before (left) and after (right) the selection

<http://lphe.epfl.ch/publications/diplomas/rm.master.pdf>

BEC for Charmed Mesons at LHCb

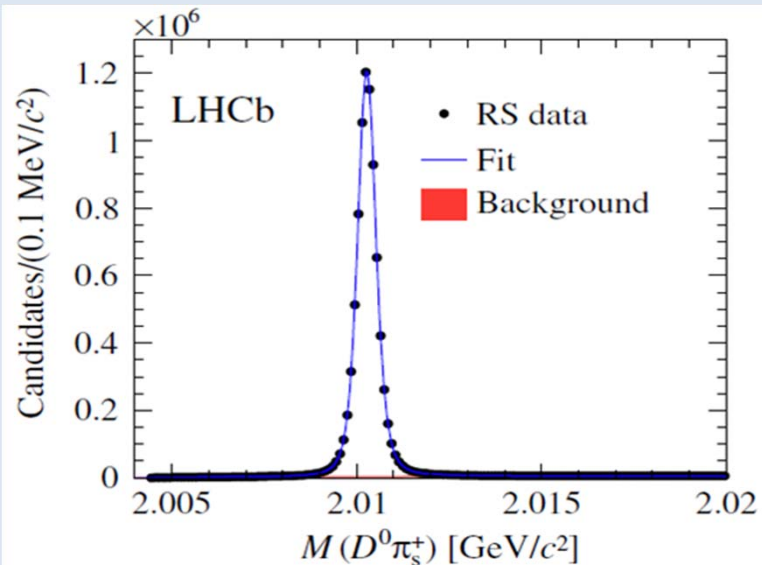
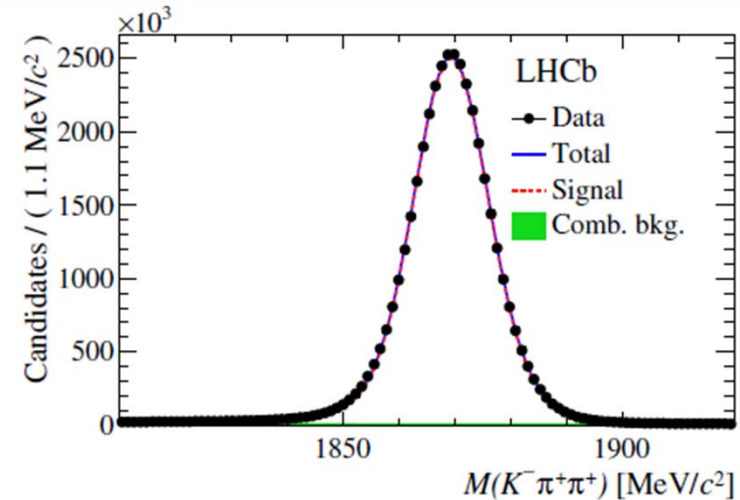


14×10^6 decays / 1fb^{-1}



8.4×10^6 decays / 1fb^{-1}

Seems promising and feasible



Three Particle Correlations for Pions

Assuming static source density $\rho(x)$ in space time:

$$G(Q) = \int e^{iQx} \rho(x)$$

2-particle correlation function

$$C_2(Q_{12}) = \frac{P(Q_{12})}{P_{ref}(Q_{12})} = 1 + \lambda |G_{12}(Q_{12})|^2$$

$$Q_{12}^2 = Q^2 = M_{12}^2 - 4\mu^2$$

3-particle correlation function

$$C_3(Q_3) = \frac{P(Q_3)}{P_{ref}(Q_3)} = 1 + \lambda (|G_{12}(Q_3)|^2 + |G_{13}(Q_3)|^2 + |G_{23}(Q_3)|^2)$$

2-particle BEC correlations

$$+ 2\lambda^{1.5} \Re [G(Q_{12}) \cdot G(Q_{13}) \cdot G(Q_{23})]$$

Genuine 3-particle BEC correlations

$$Q_3^2 = Q_{12}^2 + Q_{13}^2 + Q_{23}^2 = M_{123}^2 - 9\mu^2$$

Remark about Systematic Uncertainties

- ✓ Coulomb interactions.
- ✓ PV reconstruction.
- ✓ Overall tracking uncertainties.
- ✓ Clones and ghost tracks.
- ✓ Particle identification.
- ✓ Magnet polarity.
- ✓ Minijet effect.
- ✓ Fit range, binning etc.
- ✓ Different correlation functions.
- ✓ Different generator tunings.

Remark about Coulomb Effect

- ✓ Coulomb repulsion between two same-charge hadron
→ reduction of the BEC enhancement in the correlation function:

$$C_2^{eff}(Q) = C_2(Q) \times G_2(Q)$$

Gamov penetration factor:

$$G_2(Q) = \frac{2\pi\alpha m \epsilon_1 \epsilon_2}{Q} \cdot \frac{1}{\exp(2\pi\alpha m) - 1}$$

m - hadron mass
 ϵ_1, ϵ_2 - hadron charges
 α - fine structure constant

- ✓ **Coulomb Effect subtraction for $\pi\pi$, KK and pp pairs:**
check the difference between in the Q distributions for unlike-sign pairs of hadrons coming from the same and different PV's.



Summary

- ✓ Rich program of future studies of BEC and FDC correlations at LHCb experiments.
- ✓ First results to be issued in few months from now.
- ✓ Strong feedback from theorists (A. Białas, K.Zalewski, T.Csörgö...)



BACKUP

LHCb Data Samples

✓ Data

- Collected in 2011,
- $\sqrt{s} = 7 \text{ TeV} \int \mathcal{L} dt = 1 \text{ fb}^{-1}$
- Sample of $\approx 40 \times 10^6$ minimum bias events

✓ Monte Carlo

- p-p minimum bias events generated using PYTHIA 8 2011 (tuned for LHCb),
- BEC and FDC switched off,
- Sample of $\approx 20 \times 10^6$ minimum bias events



The source function S = Fourier transform of the source density matrix in momentum space

$$\rho(q, q') = \int d^4 X e^{iQX} S(P, X),$$

= source density matrix in momentum space

The Standard Wigner function:

$$\rho(\vec{q}, \vec{q}') = \int d^3 \vec{X} e^{-i\vec{Q} \cdot \vec{X}} W(\vec{P}, \vec{X}).$$

Wigner function relates to the particle wave functions at different positions but at the same moment of time

S is a generalised Wigner function. S describes the particle production amplitudes at different moments of time and at different positions

One of the possible realizations of the source function:

$$S(P, X) = W(\vec{P}, \vec{X}) \delta(X = 0)$$

Such source function describes only the situation when all particles are produced simultaneously a 4-point $X = 0$