Systematic investigation of two-particle HBT correlations for Lévy type sources

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Outline

QGP or the perfect quark liquid

- Quark-Gluon-Plasma (QGP): ,,perfect fluid of quarks"
- Extremly localized in space-time
- Mapping of dynamic behavior of QGP is difficult but interesting task
- Single-particle observables: not completely satisfactory
- Measurement and analysis of Bose-Einstein correlations: a tool to explore the shape of the produced matter in a heavy ion collision

Correlations between particles

- after the hadronization process, correlations between identical particles can be observed
- momentum space correlations between bosons
- the origin of these correlations is quantum mechanical
- they are intrinsically connected to the space-time shape of the particle source (the produced QGP)

Calculation of Bose-Einstein correlation

- source function: $S(x, p)$
- symmetrized wave function of n particles: $\psi^{(n)}(\mathbf{x}_1 ... \mathbf{x}_n)$
- momentum distribution of n particles:

$$N_n(\mathbf{p}_1 ... \mathbf{p}_n) := \int d^3x_1 ... d^3x_n S(\mathbf{x}_1, \mathbf{p}_1) ... S(\mathbf{x}_n, \mathbf{p}_n) |\psi^{(n)}(\mathbf{x}_1 ... \mathbf{x}_n)|^2$$

- correlation function of n particles: $C_n(\mathbf{p}_1 ... \mathbf{p}_n) := \frac{N_n(\mathbf{p}_1 .. \mathbf{p}_n)}{\prod_{i=1}^{n} N_1(\mathbf{p}_i)}$

The simplest situation

- Two particles (symmetrization easy)
- Use of Gaussian distribution as source function:

$$S(\mathbf{x},\mathbf{k}) = \frac{K_0}{\sqrt{(2\pi R)^3}} \exp\left\{-\frac{x^2}{2R^2}\right\} \exp\left\{-\frac{k^2}{2mT}\right\}$$

- no final state interaction between particles:

$$\left|\psi^{(2)}(\mathbf{x}_1,\mathbf{x}_2)\right|^2 = 1 + \cos\left[(\mathbf{k}_1-\mathbf{k}_2)(\mathbf{x}_1-\mathbf{x}_2)\right]$$

- So we arrive at the correlation function as

$$C_2^0(\mathbf{k}_1,\mathbf{k}_2) = 1 + \exp\left\{-R^2(\mathbf{k}_2-\mathbf{k}_1)^2\right\}$$

In practice

- Fit the correlation function to measured data in the most easy situation (two particle, no interaction, Gaussian distribution as source function)
- the measured data don't fit $\rightarrow$ problems with the model
- Next step: to take into account the Coulomb interaction, and supplement the model with the core-halo picture
  T. Csörgő, B. Lörstad and J. Zimányi, Z.Phys. C71 (1996) 491
- This model can be fitted using an iterative method
- the iterated term: $C_2^{meas} \frac{C_2^0(\mathbf{k}_1,\mathbf{k}_2,\beta)}{C_2^c(\mathbf{k}_1,\mathbf{k}_2,\beta)} = C_2^c(\mathbf{k}_1, \mathbf{k}_2, \beta)$
- This takes the Coulomb interaction into account properly

Lévy distribution as source function

- The symmetric Lévy distribution has the form:
  $\mathcal{L}(R,\alpha,\mathbf{r}) = \frac{1}{(2\pi)^3}\int d^3q e^{i\mathbf{qr}} e^{-\frac{|qR|^\alpha}{2}}$
- Lévy distributions are *stable* under convolution, so they are limiting distributions (just as Gaussians in the finite variance case)
- the used source function in the core-halo picture:

$$\mathcal{L}(R_c, R_h, \alpha, \mathbf{X} \pm \frac{\mathbf{x}}{2}, \lambda, \alpha) =$$
$$\sqrt{\lambda}\mathcal{L}_c(R_c, \alpha, \mathbf{X} \pm \frac{\mathbf{x}}{2}) + (1-\sqrt{\lambda})\mathcal{L}_h(R_h, \alpha, \mathbf{X} \pm \frac{\mathbf{x}}{2})$$

- the convolution of two Lévy distributions in the variable

$$\mathbf{X} : \mathcal{L}_{12}(R_{cc}, R_{ch}, R_{hh}\alpha, \mathbf{x}, \lambda, \alpha) = \lambda\mathcal{L}_{cc}(R_{cc}, \alpha, \mathbf{x})$$
$$+2\sqrt{\lambda}(1-\sqrt{\lambda})\mathcal{L}_{ch}(R_{ch}, \alpha, \mathbf{x}) + (1-\sqrt{\lambda})^2\mathcal{L}_{hh}(R_{hh}, \alpha, \mathbf{x})$$

The two particle Coulomb wave function

- two-particles symmetrized wave function in Coulomb interacting case:

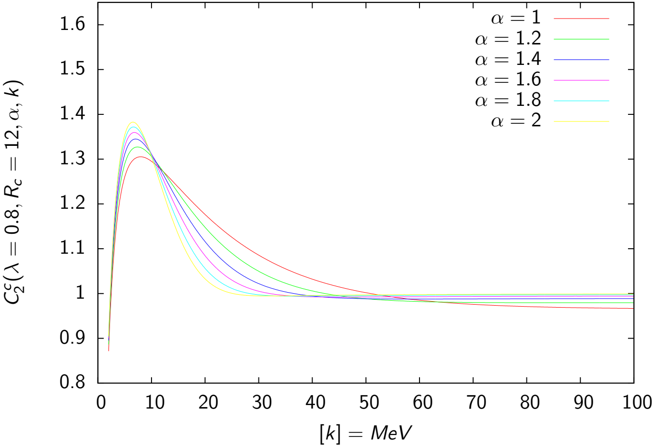
$$\psi^c(\mathbf{x}, \mathbf{X}) =$$
$$\frac{N}{\sqrt{2}} e^{i\mathbf{KX}} \left\{ e^{i\mathbf{kx}} F\left(-i\eta, 1, i(kx - \mathbf{kx})\right) - e^{-i\mathbf{kx}} F\left(i\eta, 1, i(kx + \mathbf{kx})\right) \right\}$$

- where $F(a, b, z)$ the confluent hypergeometric function is: $F(a, b, z) := \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b)}{\Gamma(b+n)\Gamma(a)} \frac{z^n}{n!}$
- the two-particle distribution have to calculate numerical: $N(\mathbf{k}_1, \mathbf{k}_2) = \int d^3x \mathcal{L}_{12}(R_{cc}, R_{ch}, R_{hh}\alpha, \mathbf{x}, \lambda, \alpha) \left| \psi^c(\mathbf{x}, \mathbf{X}) \right|$
- after reduction of the numerical problem the correlation function becomes: $C_2^c(\alpha, R_{cc}, \lambda) = N(\mathbf{k}_1, \mathbf{k}_2)$

The numerical method used by us

- calculate the value of integrals, and then save them
- not the most conventional method
- we can investigate the parameter space of the Lévy distribution
- fitting of the model to the measured data is faster (original iterative method very cumbersome for Lévy distributions)

The result of the calculations with $R = const$ and $\lambda = const$ for different α values



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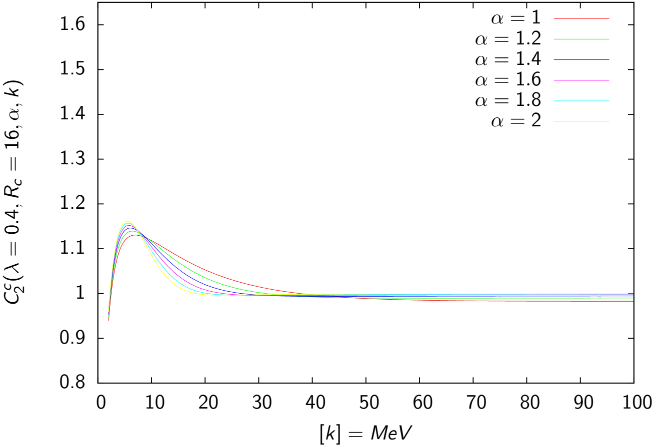
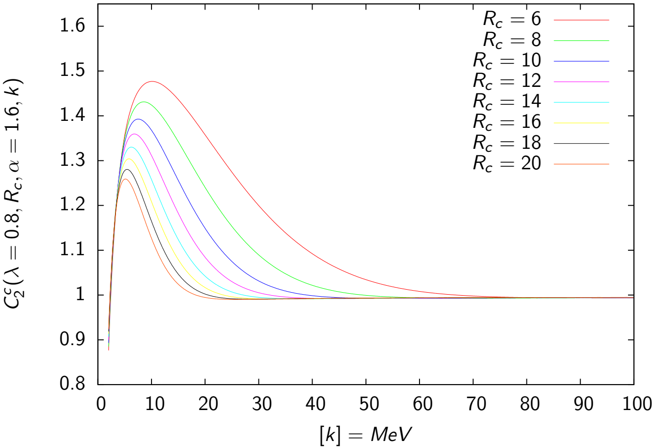


The result of the calculations with $\lambda = const$ and $\alpha = const$ for different R values



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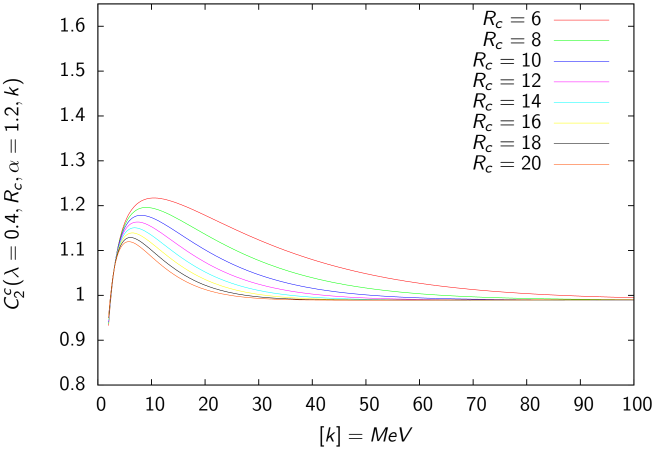

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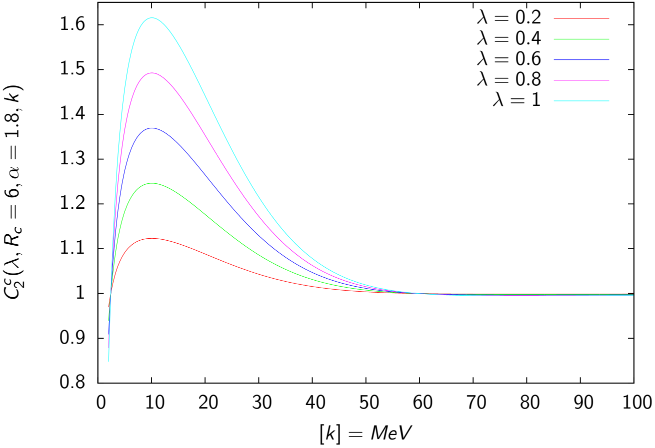

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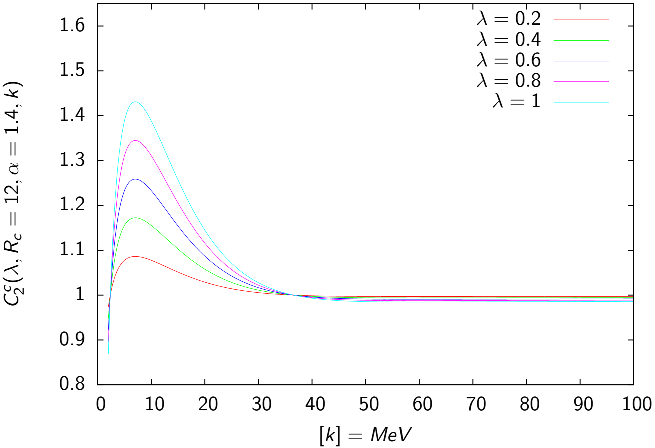

Summary of results

- We worked out a new method of the calculation of the HBT correlation functions for Lévy type sources
- Fitting of Lévy sources becomes significantly faster
- Lévy sources: generalization of Gaussian case, with long-range component in the source function
- Utilizing the method for real-life correlation function fitting is underway
- However, our systematic investigation suggests that fit results for parameters will be strongly correlated

Thank you for your attention.