# Fluctuation contribution to anisotropic flow in pA collisions at RHIC and LHC energies

## - the GLVB approach -

Péter Lévai (Wigner RCP, Budapest, Hungary)

On the basis of our mutual work

M. Gyulassy, P.L., I. Vitev, T.S. Biró,

Phys. Rev. D90 (2014) 054025. hep-ph/1805.7825

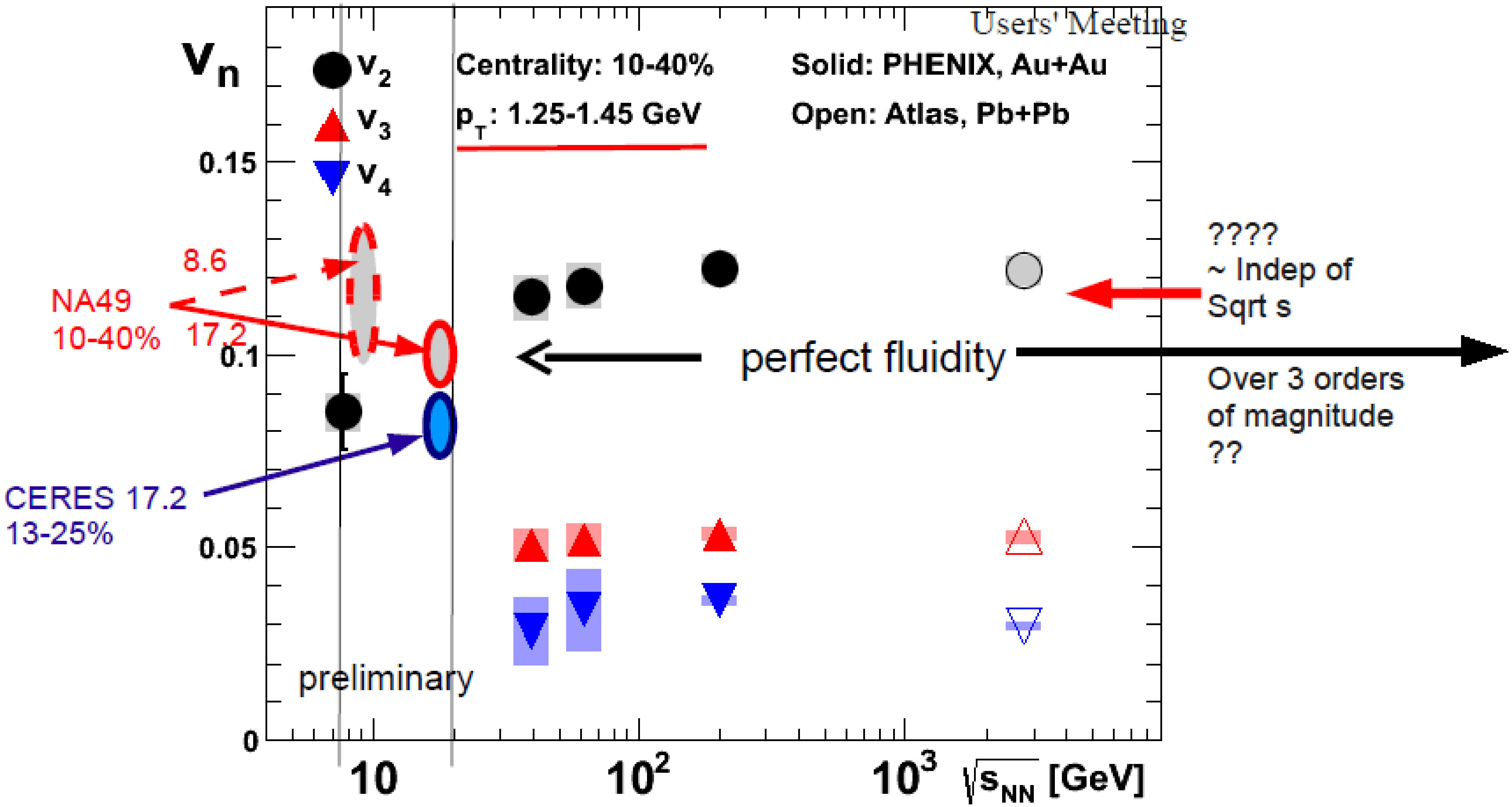
1 December 2014, Zimanyi School, Budapest

**QM12 shock 1:**

## ~ Beam Energy Independence of vn harmonics

## ?? Where is the dissipative HRG Corona ??

R.Pack, 2012 RHIC & AGS Annual Users' Meeting



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**Part 1. Introduction**

--- particle production, pQCD, jets
--- multiscattering, shadowing, jet energy loss
--- RHIC and LHC data, results (?)

**Part 2. Hydrodynamical description in pA (?)**
**vs. fluctuating multiple jet-interaction**

--- do we need perfect fluidity for explanation ?
--- scaling laws of v_n in pA

**Part 3. Short discussion**

**1. Introduction – building up a consistent description**

**--- particle production mechanism at high-pT**

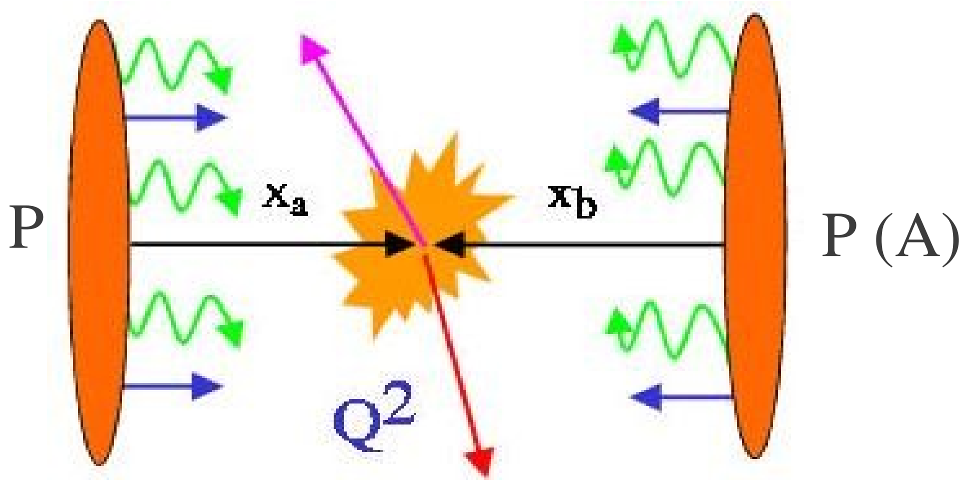
**--- pQCD description for high-pT**

**---> p+p: parton-parton collisions**

**---> p+A: multiscattering + shadowing**

**---> A+A: jet energy loss**

# Jets (high $p_T$ probes) in pp and AA collisions described by pQCD:

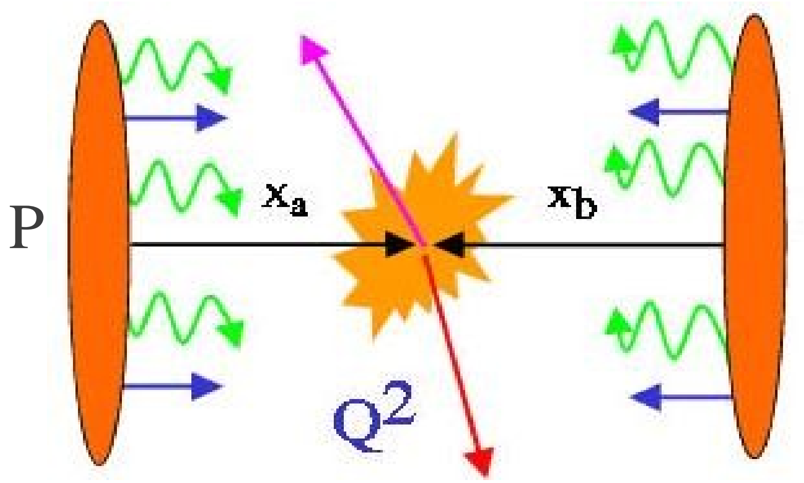



**Jet production in pp collision**
**("in vacuum"):**
**➣ pQCD description**

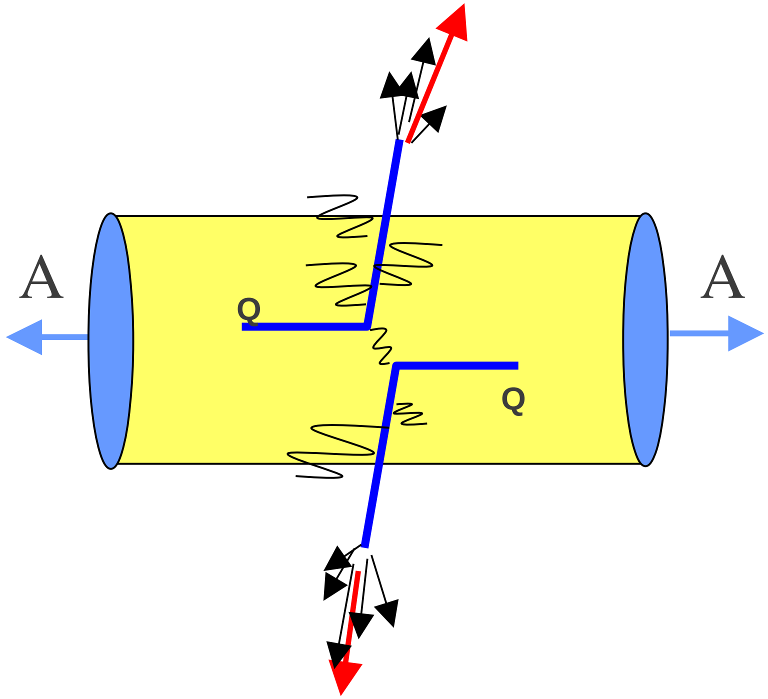
**Jet production in AA collision (" creating hot matter → Quark-Gluon Plasma")**
**➣ sofisticated PQCD description:**
**--- SHADOWING inside A**
**--- MULTISCATTERING/BROADENING penetrating A**
**--- ENERGY LOSS penetrating the "hot matter"**

**Basic questions:**
**Can we separate these mechanisms?**
**Can we determine them separately during analyzing data theoretically ?**
**We could learn a lot about QGP from high precision RHIC and LHC data !**

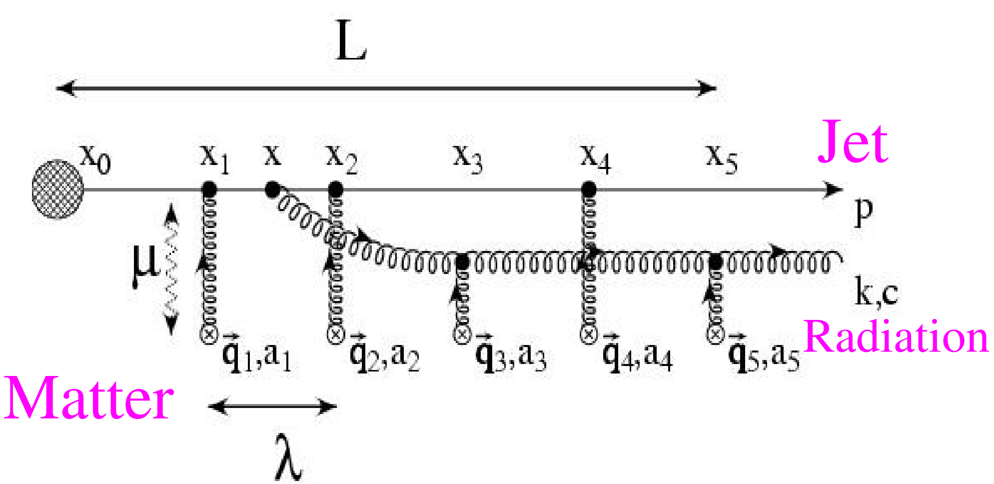
# *Jets in pp and in AA collisions:*




Jet production in pp collision
(in vacuum):
➣ pQCD description




Jet production and propagation
in AA collision (inside hot dense matter)
➣ induced gluon radiation in a
modified pQCD description
***JET-TOMOGRAPHY***

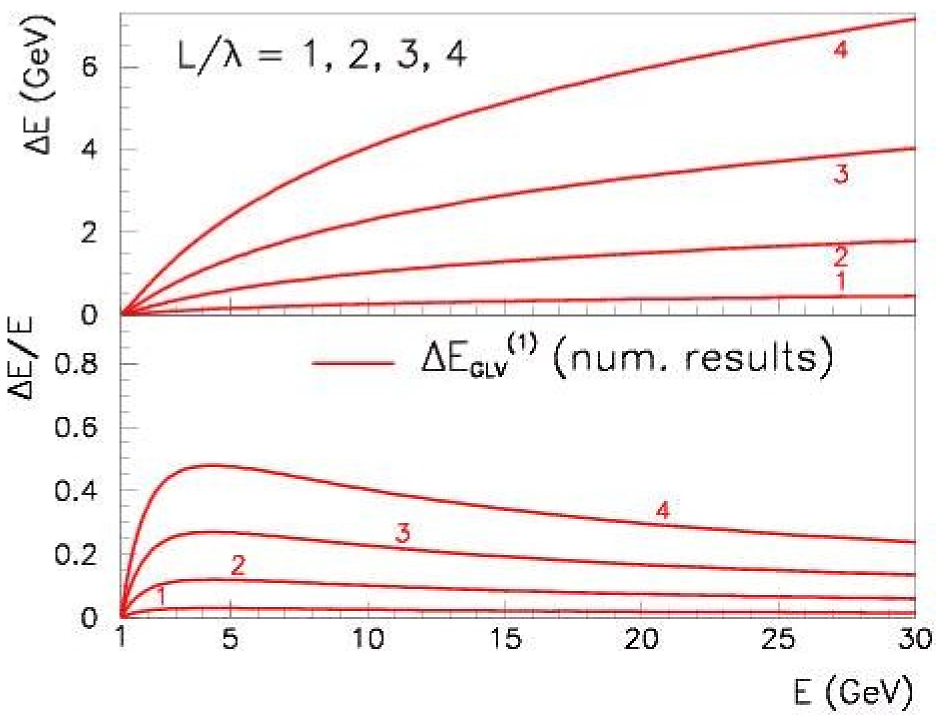


# 'Jet-quenching' : induced jet energy loss --- in thin colored matter

**M. Gyulassy, P. Levai, I. Vitev,**
**PRL85,5535(2000), NPB594,371(2001)**

GLV: time-ordered pQCD (Feyman diagrams)
+ OPACITY expansion (N = 1,2,3, ...)
+ kinematical cuts

$$\Delta E_{GLV} \approx \frac{C_R \alpha_s}{N(E)} \frac{L^2 \mu^2}{\lambda_g} \log \frac{E}{\mu}$$




**E-dependent ΔE energy loss**

**E-independent ΔE/E**
**in the window**
**3 < E < 10-15 GeV**

$$L/\lambda \rightarrow \int \tau \rho(\tau) d\tau$$

**Opacity ↣ Density**

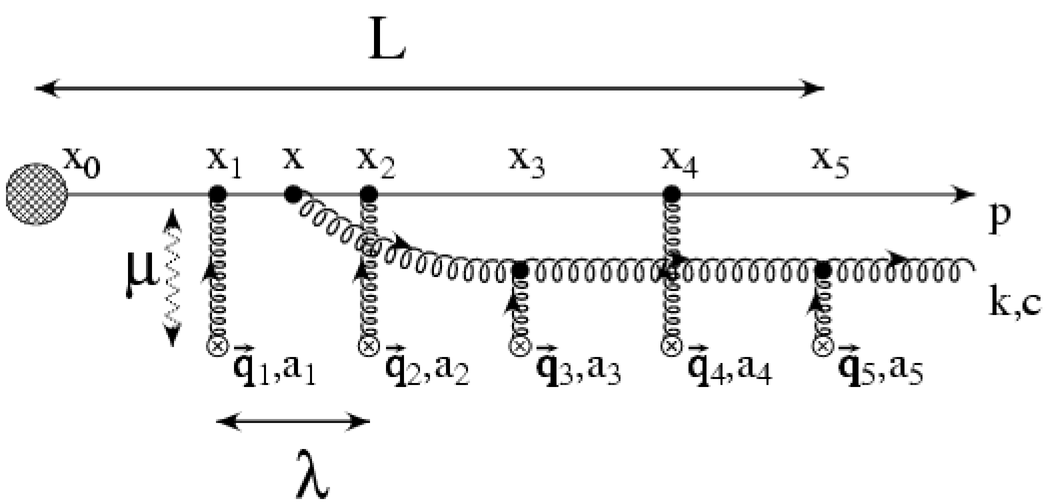
## Induced jet energy loss --- agreements and disagreements:

**BDMS, GW, GLV, Zakharov, Wiedemann, Salgado, ...**

1. $\Delta E_{loss} \sim L^2$ **non-abelian nature**

2. $\Delta E_{loss} \sim \hat{q}$ **transport coefficient:** $\hat{q} \approx \mu^2/\lambda \approx \int d^2 q_T \, q_T^2 \, d\sigma / dq_T^2$

3. $\Delta E_{loss} = C_R \alpha_S \hat{q} L^2 F[\ldots]$ **where F[...] depends on theories**




**Coherence & Interference**

# Induced jet energy loss in expanding matter:

**1. Averaged opacity ↣ time dependent color density:**

$$1/\lambda_{col} = \sigma_{el}\rho_{col} \rightarrow \frac{9/2\pi\alpha_S^2}{\mu^2}\,\frac{2}{L^2}\int_0^L \tau\rho_{col}(\tau)\,d\tau$$

**2. 1-DIM Bjorken expansion:**

$$\rightarrow \frac{9/2\pi\alpha_S^2}{\mu^2}\,\frac{2}{L^2}\,\frac{1}{A_T}\,\frac{dN^{col}}{dy}\,L$$

**3. Energy loss as the function of rapidity density:**

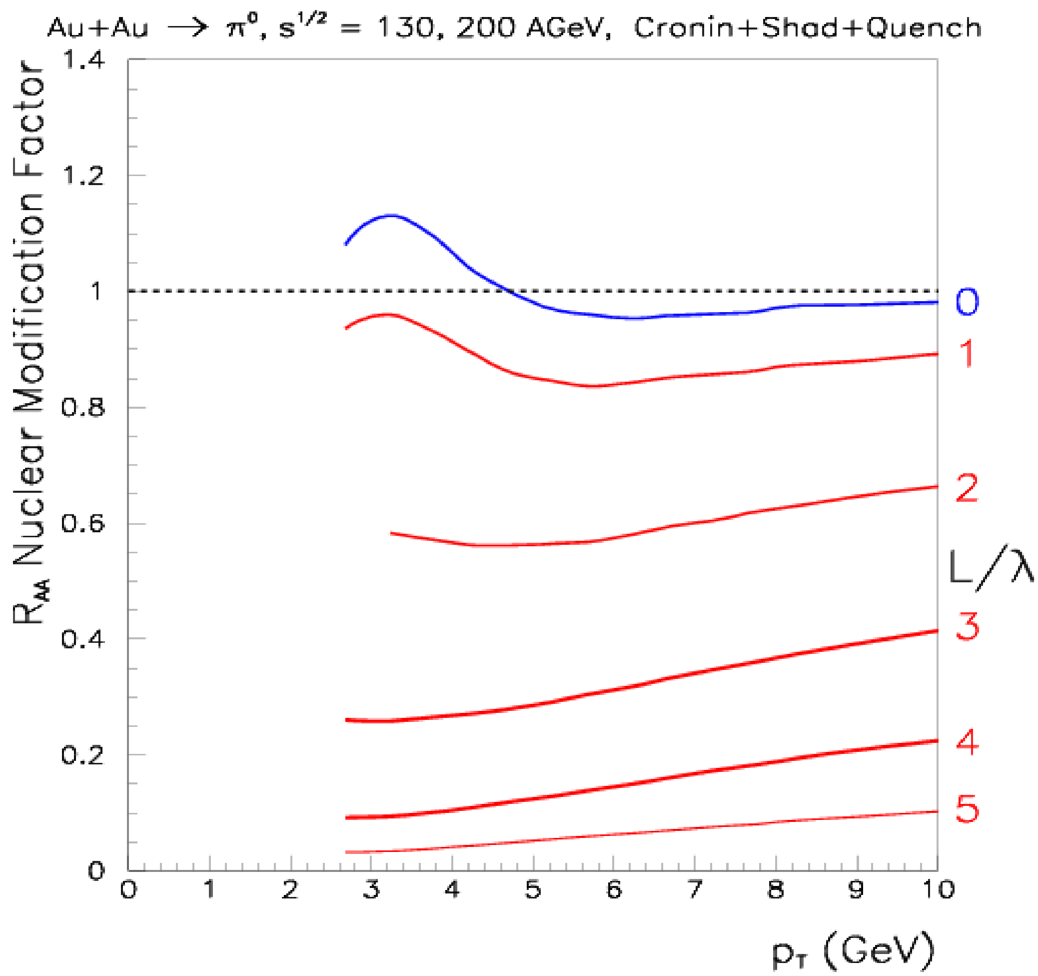
$$\Delta E_{GLV}^{1\mathrm{DIM}} \approx \frac{9C_R\pi\alpha_s^3}{4}\,\frac{1}{A_T}\,\frac{dN^{col}}{dy}\,L\,\log\frac{2E}{\mu^2 L}$$

# Hard physics: pion production in AA collision at high- $p_T$

**Perturbative QCD calculations in NLO for heavy ion collisions:**

**geometrical overlap + shadowing, multiscattring, jet-quenching, ...**

$$E_\pi \frac{d\sigma^{AB}}{d^3 p_\pi} = \int d^2 b \, d^2 r \;\; t_A(\vec{r}) \, t_B(|\vec{b} - \vec{r}|) \;\; E_\pi \frac{d\sigma^{pp}}{d^3 p_\pi} \otimes S(\ldots) \otimes M(\ldots) \otimes Q(\ldots)$$




**RHIC**
**200 A GeV**
**(130 A GeV)**

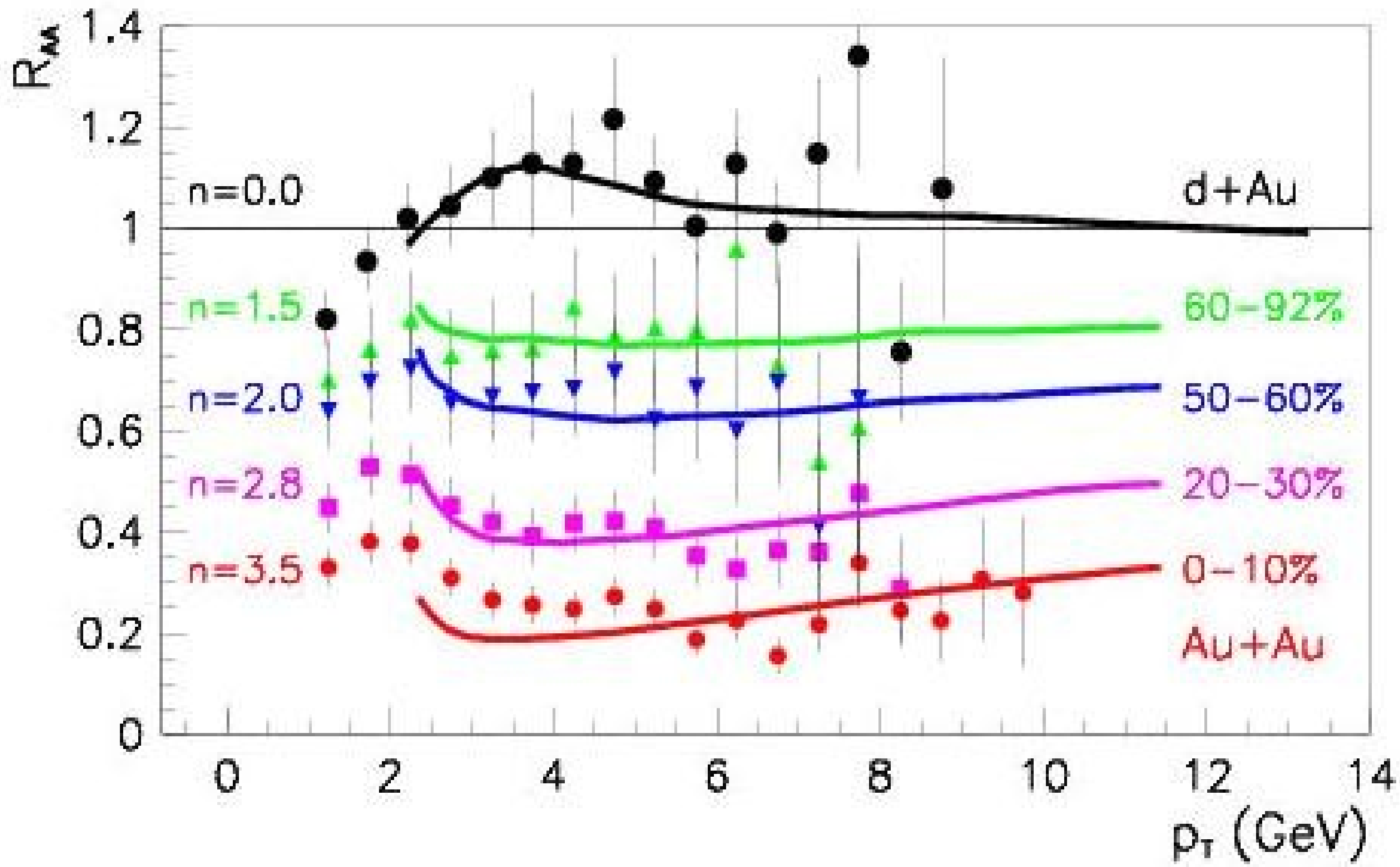
**Au+Au**

# Hard physics: pion production in AA collision at high- $p_T$

**Perturbative QCD calculations in NLO for heavy ion collisions:**

**geometrical overlap + shadowing, multiscattring, jet-quenching, ...**

$$E_\pi \frac{d\sigma^{AB}}{d^3 p_\pi} = \int d^2b\, d^2r\;\; t_A(\vec{r})\, t_B(|\vec{b}-\vec{r}|)\;\; E_\pi \frac{d\sigma^{pp}}{d^3 p_\pi} \otimes S(\ldots) \otimes M(\ldots) \otimes Q(\ldots)$$




**RHIC**
**200 GeV/N**

d+Au

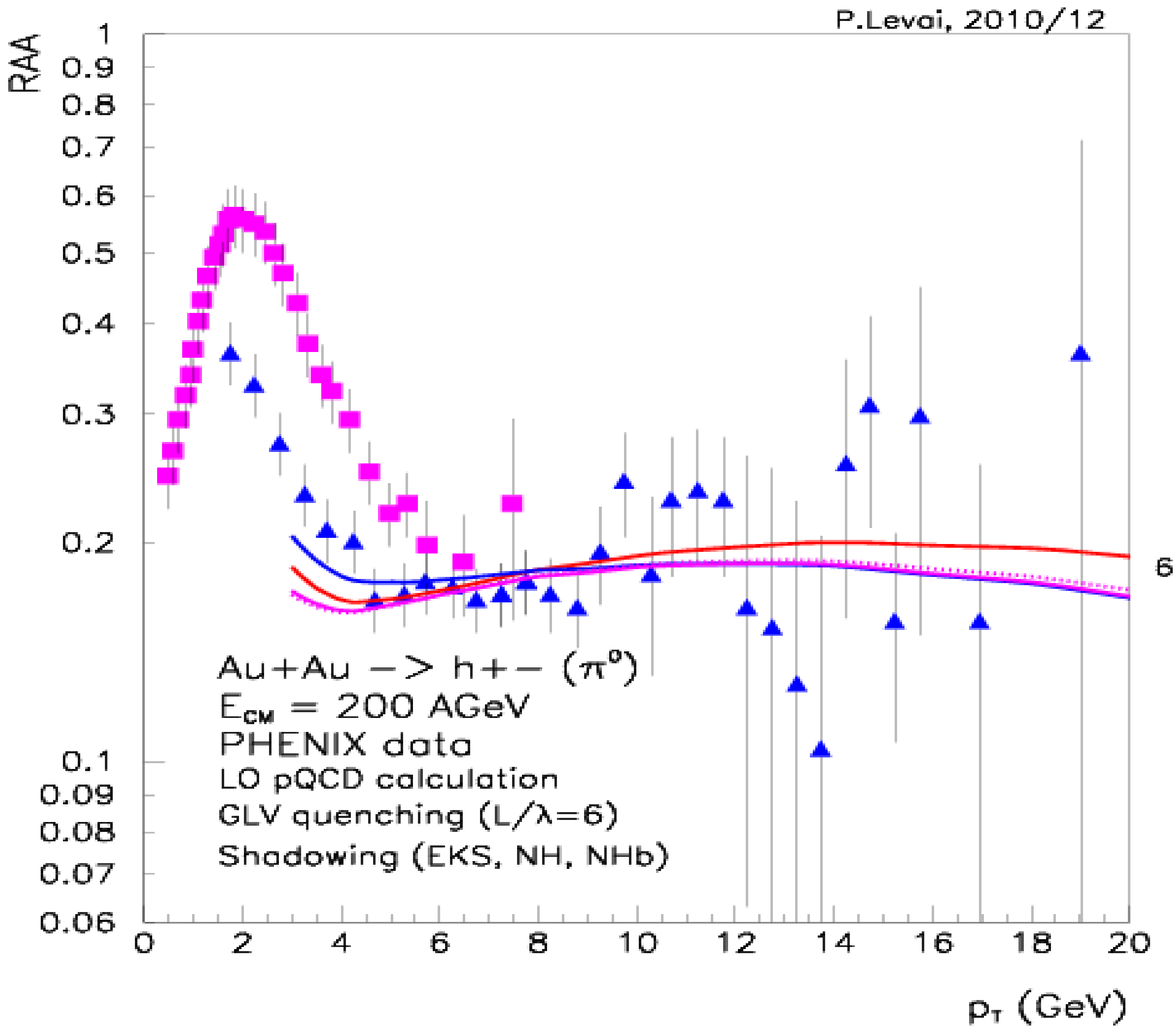
Au+Au

$\eta = 0$

**G.G. Barnaföldi, PL, EPJ, 2006**

# Most central Au+Au collisions (5%) at RHIC 200 AGeV

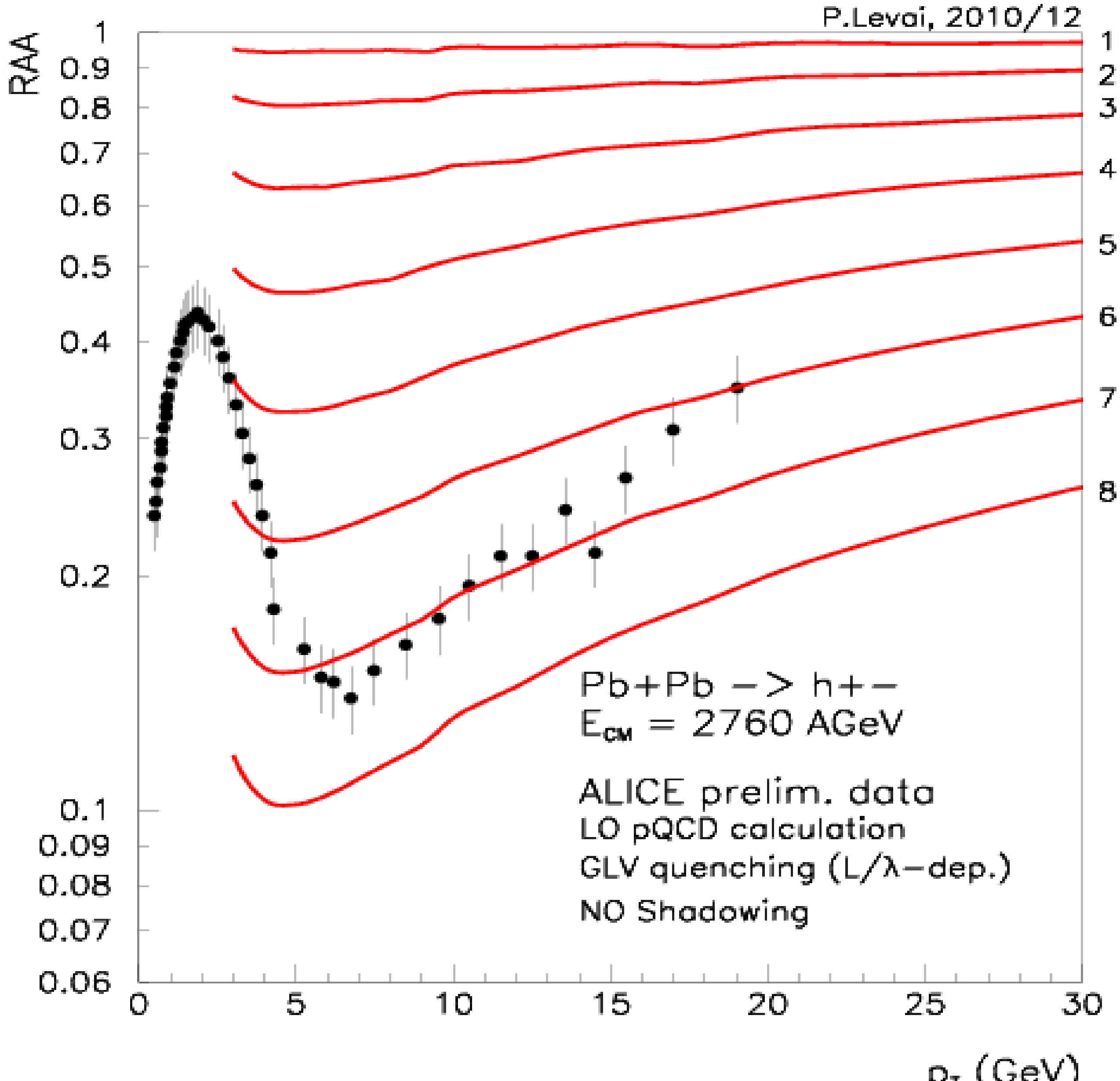
# "Quenching at L/λ=6 + Shadowing"




**Shadowing: small effect at RHIC**

# Most central Pb+Pb collisions (5%) at LHC 2.76 ATeV

## "Quenching only" - NOT FLAT at high-pT !!!! ($p_T$ > 7 GeV)



## Theoretical conclusion for jet energy loss:

1. pQCD model frame with jet quenching and nuclear shadowing is a very useful description at LHC energies

2. LHC data are very interesting after theoretical interpretation

3. Detailed investigations are needed

→ → → RHIC results: QGP has been produced "perfect fluid"

→ → → LHC results: many characteristics are the same !!??

## Part 2. Hydrodynamical description in pA (?) vs. fluctuating multiple jet-interaction

--- do we need perfect fluidity for explanation ?

--- scaling laws of higher harmonic flow (v_n) in pA

Last 10 years: → QGP is "perfect fluid"
Produced in AA collisions at RHIC/LHC

Proof: Hydro explanation of v_n anisotrop flow in AA

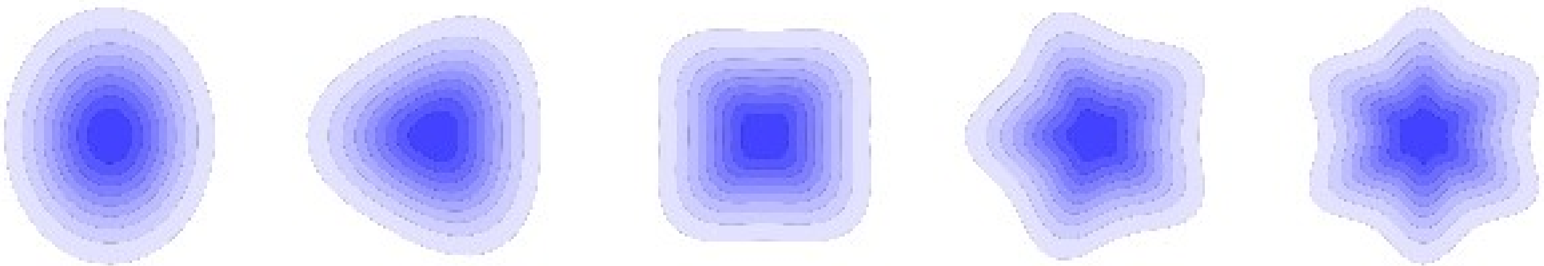
Problem: pA looks very similar !!!
Do we have hydro in pA ??!!

slide of B. Schenke QM12

## Higher harmonic flow **Fourier starts at n=1**

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + \sum_{n=2}(2v_n\cos[n(\phi - \psi_n)]\right)$$

When including fluctuations, all moments appear:



$n = 2$ $n = 3$ $n = 4$ $n = 5$ $n = 6$

also $v_1$ and $n > 6$

Compute $v_n = \langle\cos[n(\phi - \psi_n)]\rangle$

with the event-plane angle $\psi_n = \frac{1}{n}\arctan\frac{\langle\sin(n\phi)\rangle}{\langle\cos(n\phi)\rangle}$

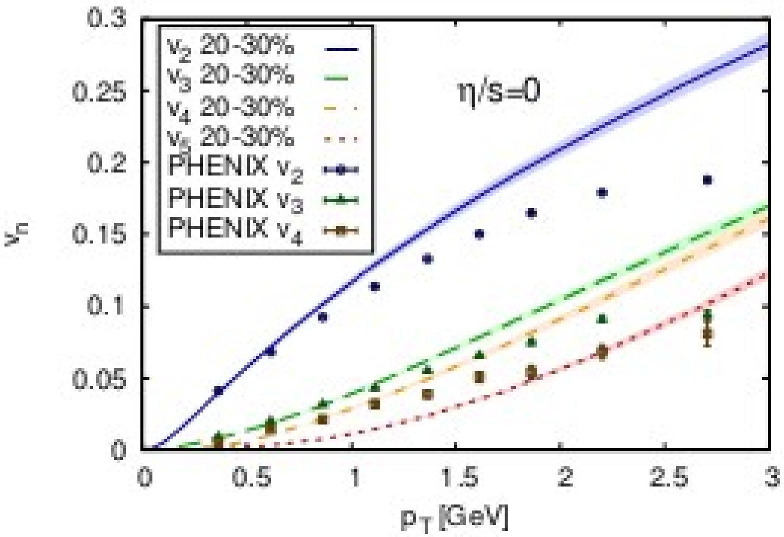
# Part 1: case for perfect fluidity with CGC IS

## Using higher harmonics to determine $\eta/s$

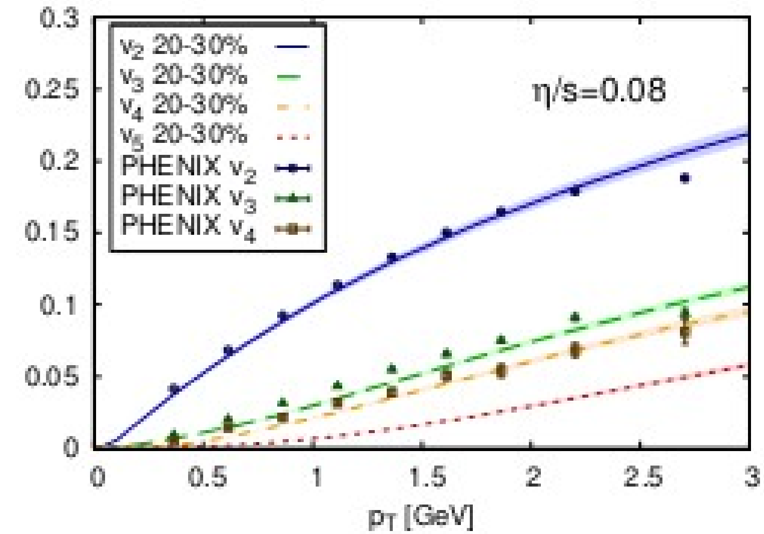
Viscocity/entropy
Of perfect fluid QGP

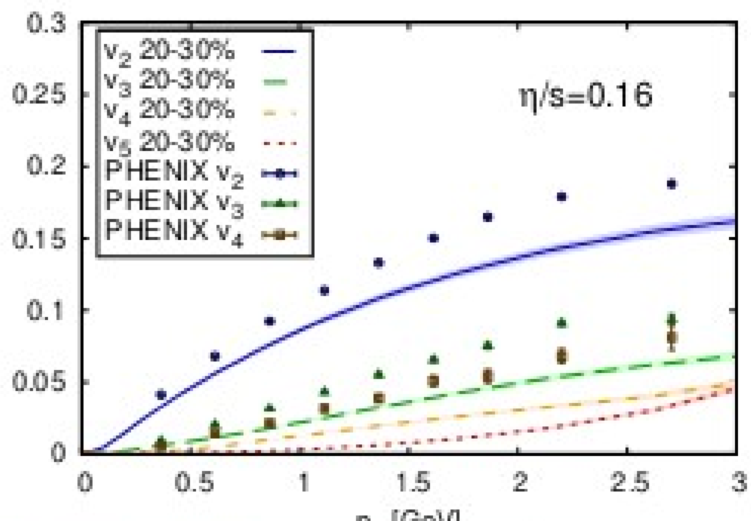
B. Schenke, S. Jeon, C. Gale, arXiv:1109.6289

Data is from event-plane method. Calculations are $\sqrt{\langle v_n^2 \rangle}$.



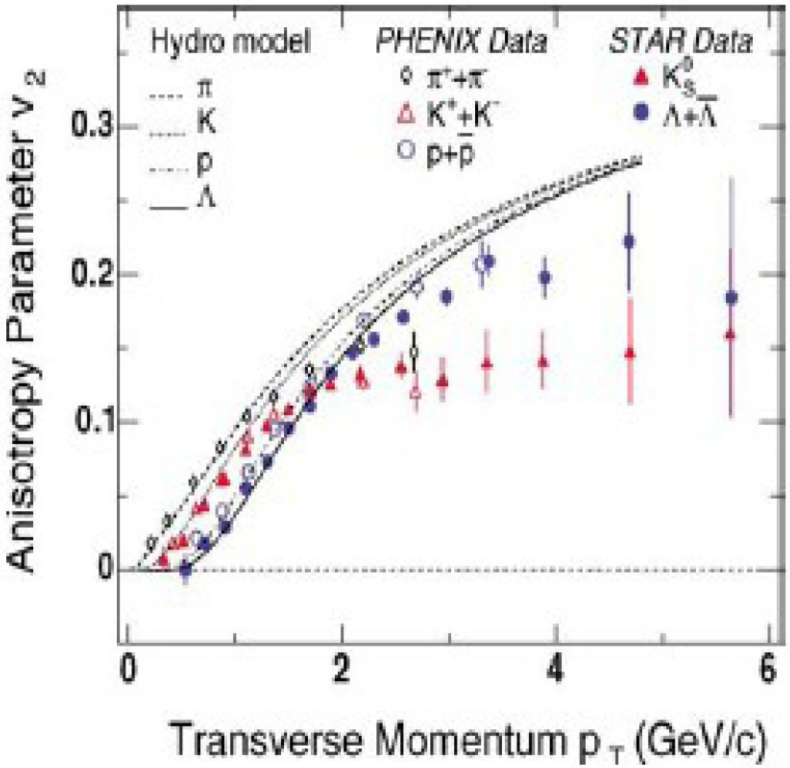

MC-Glauber initial conditions




This is promising.
Need systematic study of all $v_n$ as function of initial conditions, granularity, $\eta/s$, ...

Experimental data: PHENIX, arXiv:1105.3928



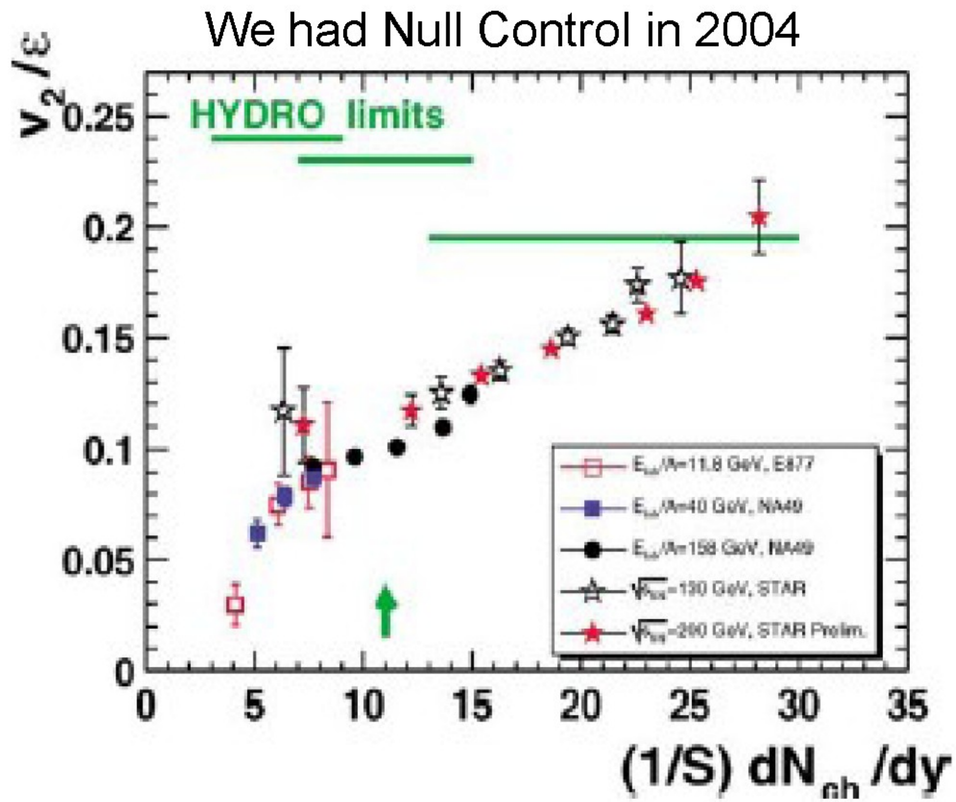
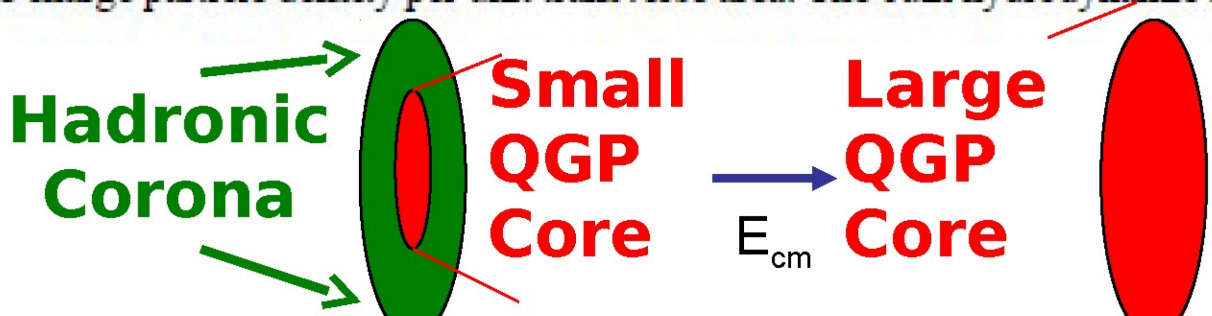
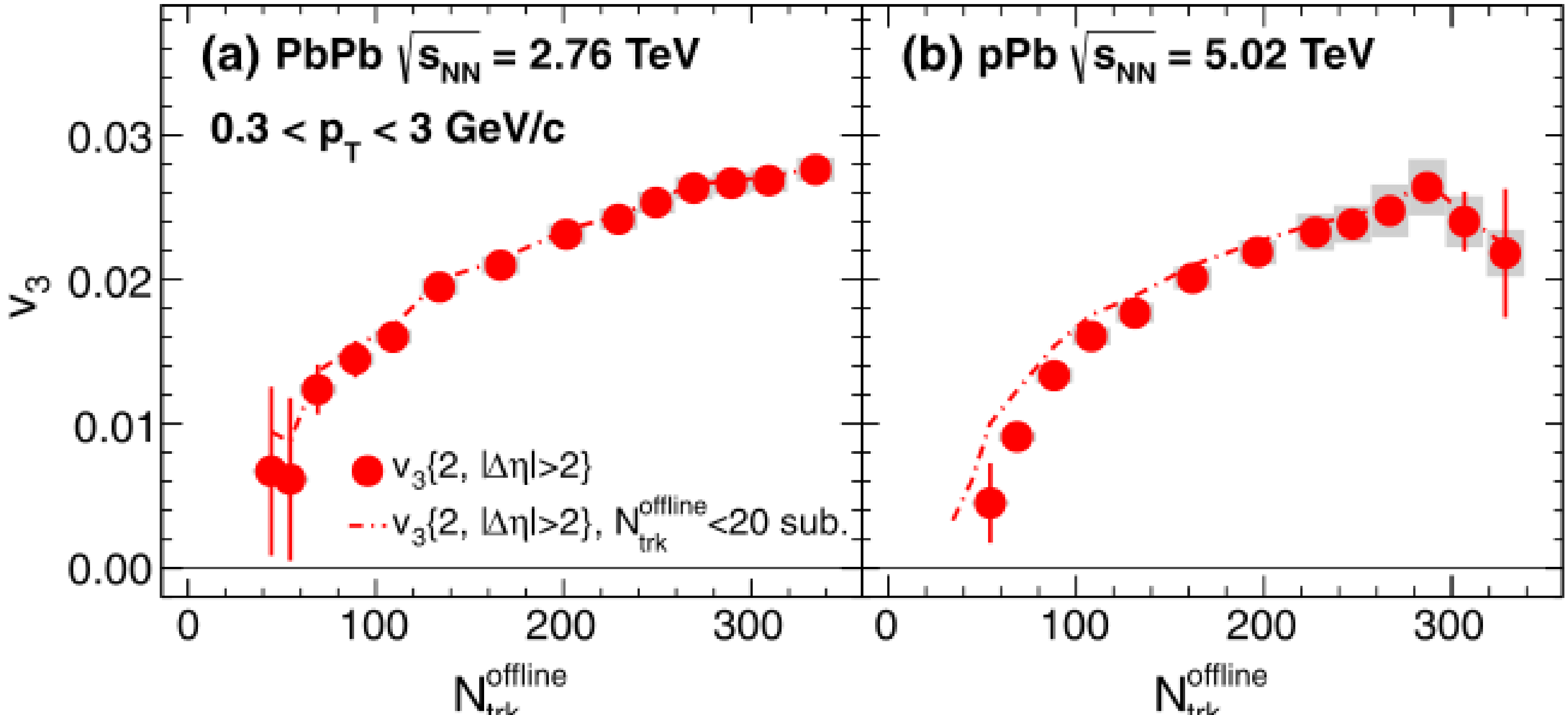
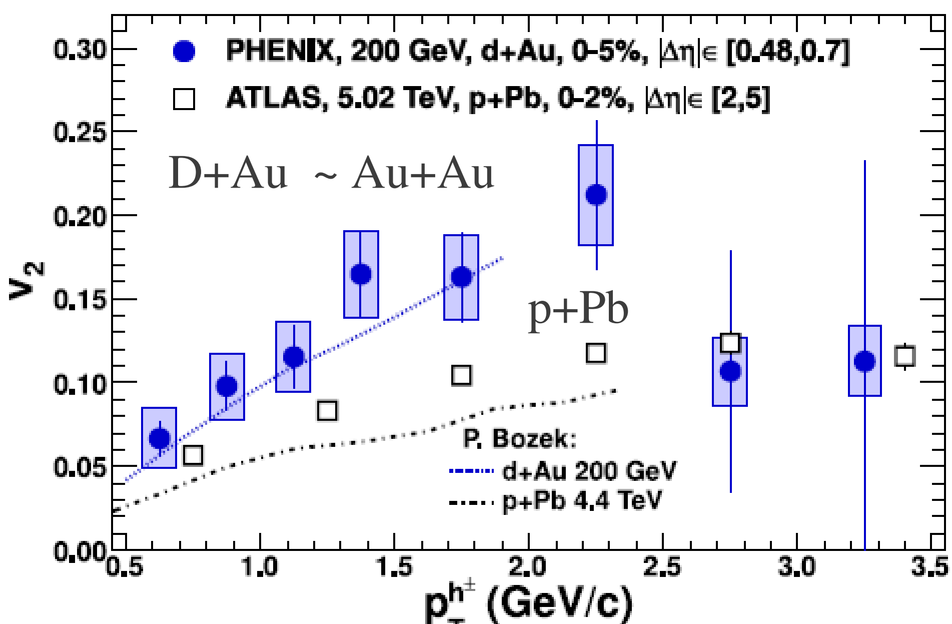



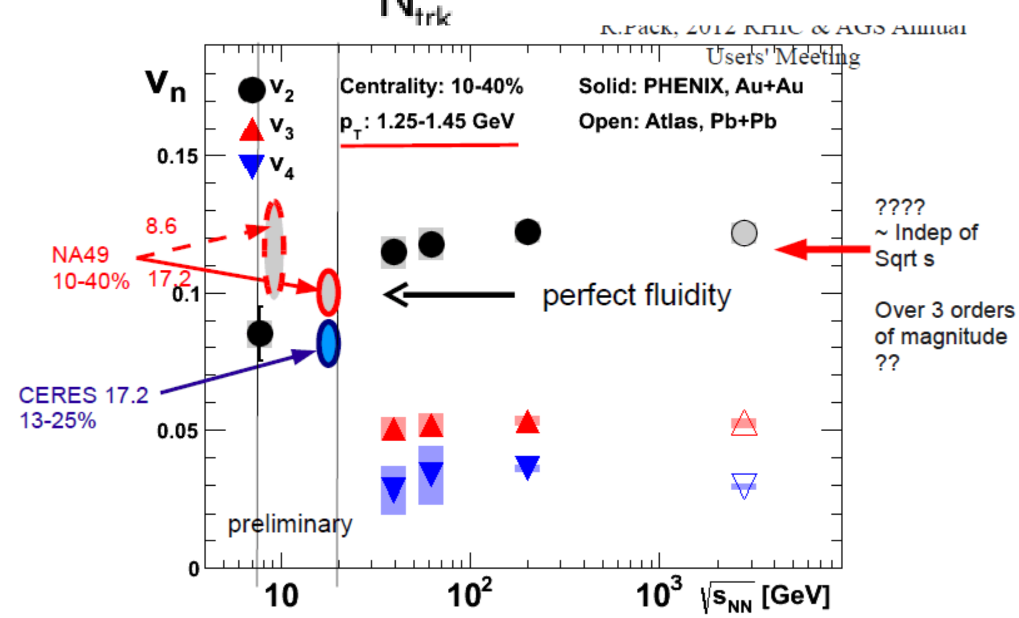

Fig. 8. First line of evidence: bulk collective flow is the barometric signature of QGP production. Left figure combines STAR [43–46] and PHENIX [47] measurements of the azimuthal elliptic flow ($v_2(p_T)$) of $\pi$, $K$, $p$, $\Lambda$ in Au + Au at 200 $A$ GeV. The predicted hydrodynamic flow pattern from [48–52] agrees well with observations in the bulk $p_T < 1$ GeV domain. Right figure from [41] shows $v_2$ scaled to the initial elliptic spatial anisotropy, $\epsilon$, as a function of the charge particle density per unit transverse area. The bulk hydrodynamic limit is only attained at RHIC.



# QM12 p(D) + A & BES surprises: v3(p+Pb) =v3(Pb+Pb) ; v2(D+Au)=v2(Au+Au) ; Ecm independ



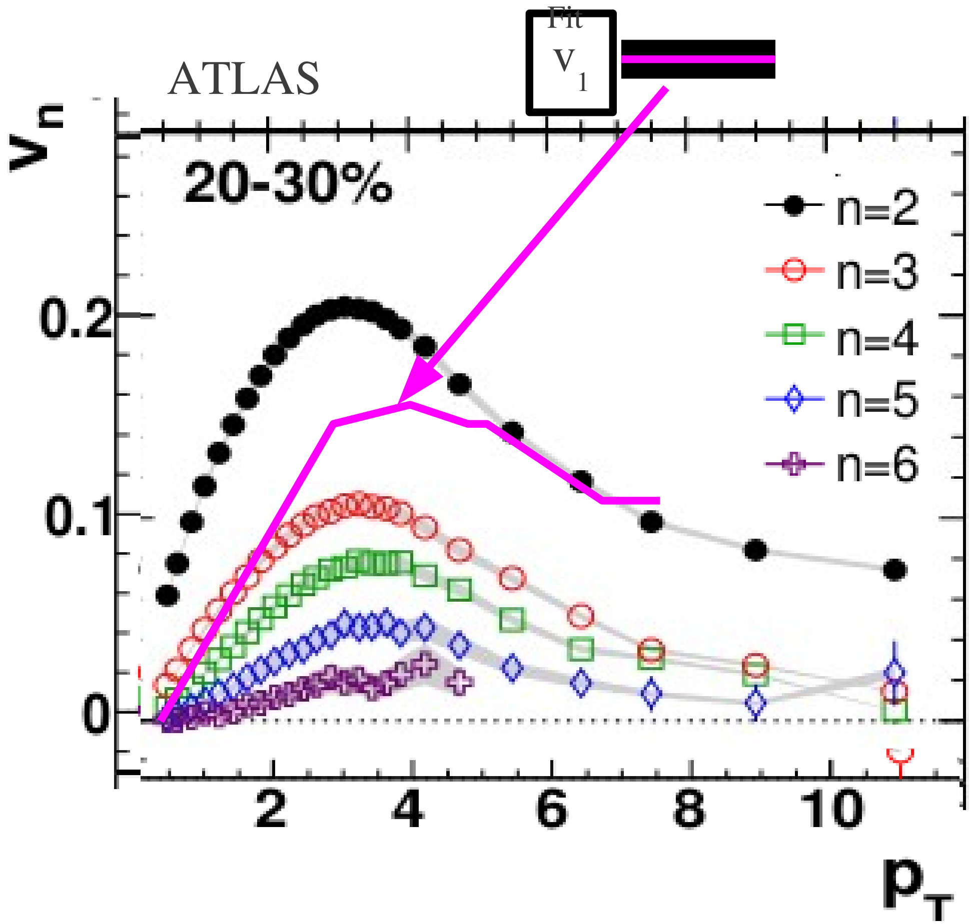




**BES vn independent of Ecm**

$$\chi^2 = \sum_{\mathrm{a,b}} \frac{\left(v_{1,1}(p_{\mathrm{T}}^{\mathrm{a}}, p_{\mathrm{T}}^{\mathrm{b}}) - [v_1^{\mathrm{Fit}}(p_{\mathrm{T}}^{\mathrm{a}}) v_1^{\mathrm{Fit}}(p_{\mathrm{T}}^{\mathrm{b}}) - c p_{\mathrm{T}}^{\mathrm{a}} p_{\mathrm{T}}^{\mathrm{b}}]\right)^2}{\left(\sigma_{\mathrm{a,b}}^{\mathrm{stat}}\right)^2 + \left(\sigma_{\mathrm{a,b}}^{\mathrm{sys,p2p}}\right)^2},$$

Rapidity-even, positive collective dipole <Cos[ φ1 -φ1]> ~ - (negative mom.conserv. term)




Is v1 an artifact of fit? See *

A hydro response ? See Schenke

or is it a fingerprint of basic

pQCD Color Scintillation?

*Beware v1 is notoriously tricky, see L. Csernai, H. Stoecker, JPG 2014

Below we show rapidity-even ATLAS v1 Ridge component natural from pQCD Bremsstrahlung

## II. FIRST ORDER IN OPACITY (GB) BREMSSTRAHLUNG AND AZIMUTHAL ASYMMETRIES $v_n$

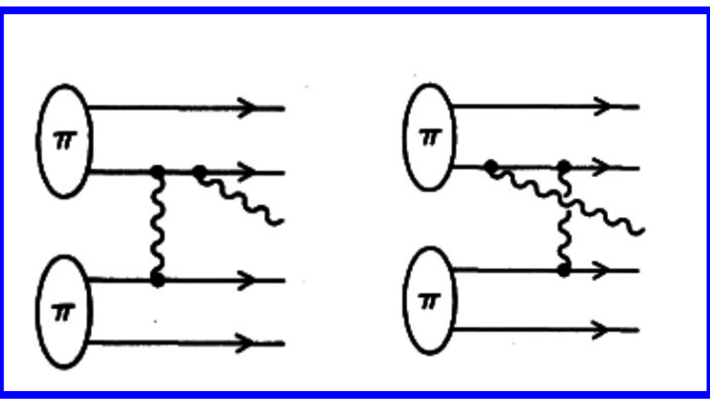
The above puzzles with BES and $D + Au$ at RHIC and with $p + Pb$ at LHC and models proposed so far motivate us to consider simpler more basic perturbative QCD sources of azimuthal asymmetries.The well known non-abelian bremstralung Gunion-Bertsch (GB) formula[29] for the soft gluon radiation single inclusive distribution is

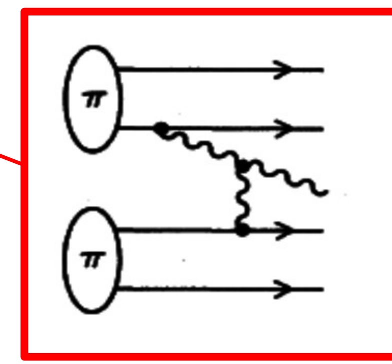
Color Dipole
Form factor

$$\frac{dN_g^{(1)}}{d\eta d^2\mathbf{k}d^2\mathbf{q}} \equiv f(\eta, \mathbf{k}, \mathbf{q})$$

$$= \frac{C_R\alpha_s}{\pi^2 k^2} \frac{\mu^2}{(\mathbf{q}^2+\mu^2)} \frac{P_\eta}{(\mathbf{k}-\mathbf{q})^2+\mu^2}$$

$$\equiv \frac{F \cdot P}{A_{kq} - \cos(\phi - \psi)}$$



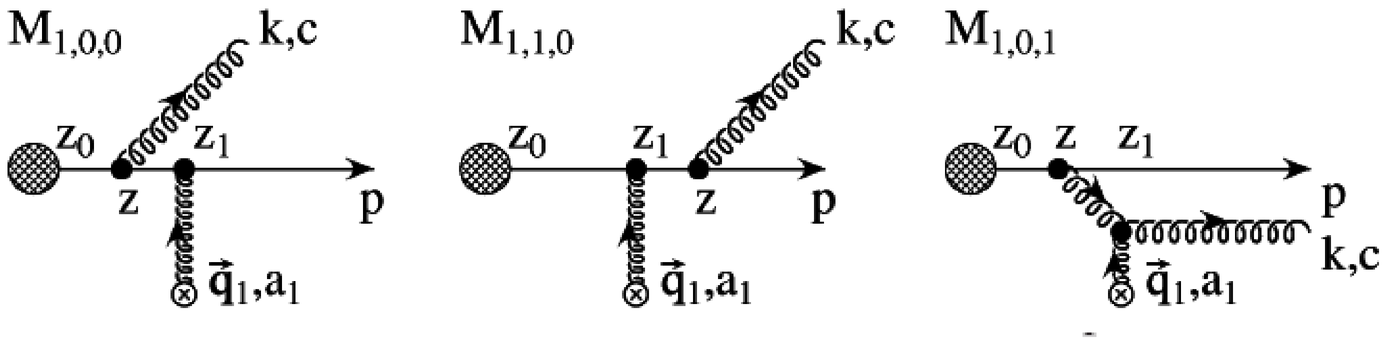


Gluon Bremsstrahlung peaks in transverse direction near net momentum transfer $\vec{Q} = (Q, \text{Psi})$ that also defined reaction Event Plane (EP)

Basic Non-Abelian feature: uniform *rapidity-even* distributed (unlike QED)

Of course also peaks in beam direction 1/k^2 (as in QED)

Basic lowest order pQCD Bremsstrahlung




$$M_{1,0,1} = J(p)e^{ipx_0}(-i)\int \frac{d^2\mathbf{q}_1}{(2\pi)^2} v(0,\mathbf{q}_1)e^{-i\mathbf{q}_1\cdot\mathbf{b}_1} \times 2ig_s \frac{\epsilon\cdot(\mathbf{k}-\mathbf{q}_1)}{(\mathbf{k}-\mathbf{q}_1)^2} e^{i(\omega_0-\omega_1)z_1}\left(e^{i\omega_1 z_1}-e^{i\omega_1 z_0}\right)[c,a_1]T_{a_1}$$

$$M_{1,0,0} + M_{1,1,0} = J(p)e^{ipx_0}(-i)\int \frac{d^2\mathbf{q}_1}{(2\pi)^2} v(0,\mathbf{q}_1)e^{-i\mathbf{q}_1\cdot\mathbf{b}_1} \times (-2ig_s)\frac{\epsilon\cdot\mathbf{k}}{\mathbf{k}^2} e^{i\omega_0 z}(ca_1T_{a_1} - a_1 c T_{a_1})$$

GLV, Nucl.Phys. B594 (2001) 371

M. Djordjevic, M. Gyulassy / Nuclear Physics A 733 (2004) 265–298

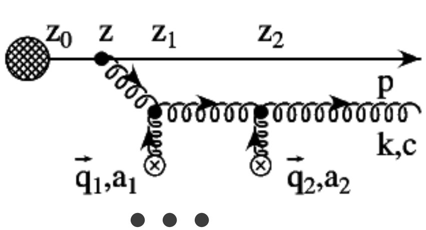
$$\frac{dE^{(1)}_{\rm ind}}{dx} = \frac{C_R\alpha_S}{\pi}\frac{L}{\lambda}E\int\frac{d\mathbf{k}^2}{\mathbf{k}^2+m_g^2+M^2x^2}\int\frac{d^2\mathbf{q}_1}{\pi}\frac{\mu^2}{(\mathbf{q}_1^2+\mu^2)^2}$$
$$\times 2\frac{\mathbf{k}\cdot\mathbf{q}_1(\mathbf{k}-\mathbf{q}_1)^2+(m_g^2+M^2x^2)\mathbf{q}_1\cdot(\mathbf{q}_1-\mathbf{k})}{(\frac{4Ex}{L})^2+((\mathbf{k}-\mathbf{q}_1)^2+M^2x^2+m_g^2)^2},$$

LPM
Formation
Coherence

Triple gluon vertex
Radiation Dipole focuser
Into reaction plane

"Dead Cone"
Massive quarks
HTL gluons

A higher order opacity amplitude




L/λ=1 L/λ=2 L/λ=M

k – q1 → k – (q1 + q2) → k – (q1 + …+ qM)

e-b-e In Plane focusing gets stronger with opacity

$\phi$ is the azimuthal angle of $\mathbf{k}$ and $\psi$ is the azimuthal angle of $\mathbf{q}$ and abreviations

$$A \equiv A_{kq} \equiv (k^2+q^2+\mu^2)/(2k\,q) \geq 1$$

$$F \equiv F_{kq} \equiv \frac{C_R\alpha_s}{\pi^2k^2}\frac{\mu^2}{(\mathbf{q}^2+\mu^2)}\frac{1}{2kq}P_\eta$$

Kinematic rapidity envelope

$$P_\eta \equiv (1-e^{Y_T-\eta})^{n_f}(1-e^{\eta-Y_P})^{n_f} \quad ,$$

$$v_n(k,q,\psi)f_0(k,q) \equiv \int\frac{d\phi}{2\pi}\cos(n\phi)f(\eta,k,\phi,q,\psi)$$

$$= F\int\frac{d\phi}{2\pi}\frac{\cos(n\phi)}{A-\cos(\phi-\psi)}$$

$$= \cos(n\psi)\,F\int\frac{d\phi}{2\pi}\frac{\cos(n\phi)}{A-\cos(\phi)} \quad .$$

$f_0 \equiv \int d\phi f = \int d\phi\ d^7N/d\eta dk^2 d\phi dq^2 d\psi$ is the $\phi$ integrated single gluon inclusive

$$dN/d\eta dk^2 = F_{kq}P_\eta/(A^2_{kq}-1)^{1/2}$$

A single GB color antennas has analytic vn:

$$A_{kq} \equiv (k^2 + q^2 + \mu^2)/(2k\, q) \geq 1$$

$$v_1^{GB}(k,q,\psi) = \cos[\psi](A_{kq} - \sqrt{A_{kq}^2 - 1})$$

1/ Dipole size

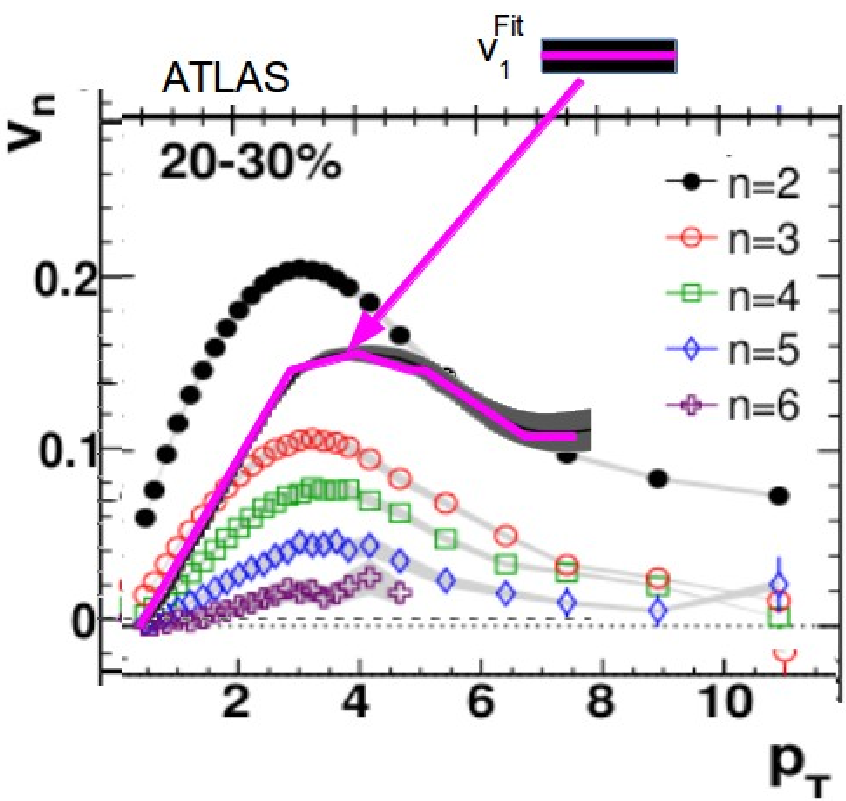
$$\lim_{\mu\to 0} v_1^{GB}(k,q,0) = (k/q)\,\theta(q-k)$$

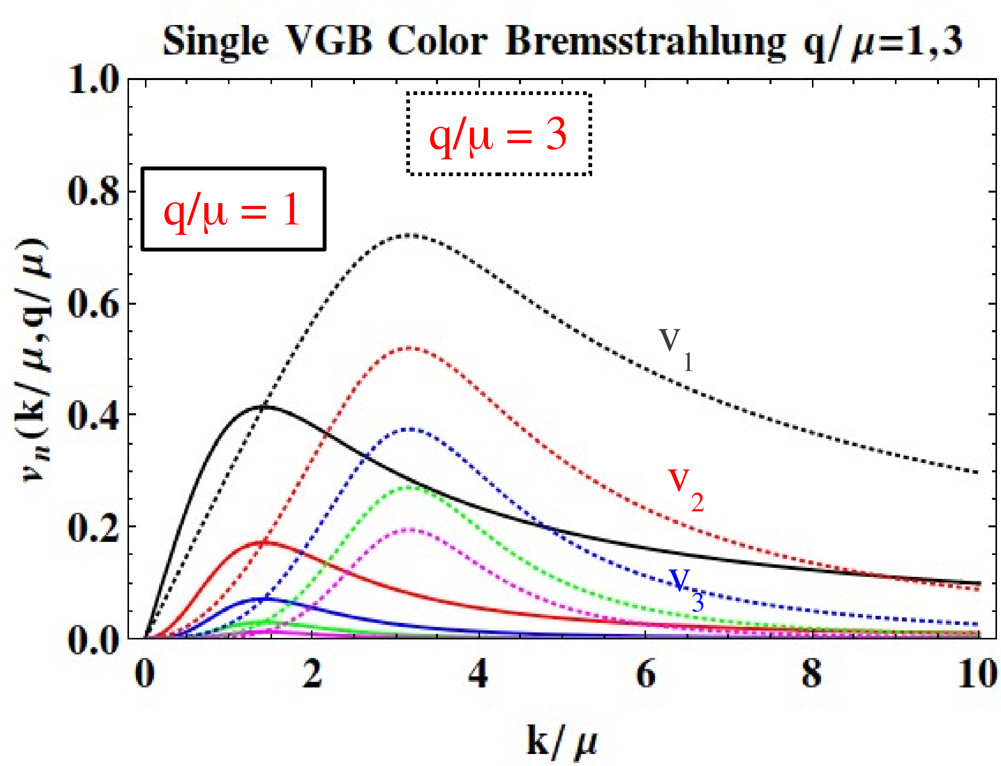
$$v_n^{GB}(k,q,\psi) = \cos[n\psi]\,(v_1^{GB}(k,q,0))^n$$

$$\lim_{\mu\to 0} v_n^{GB}(k,q,0) = (k/q)^n\,\theta(q-k)\ .$$

Perfect $v_n^{1/n} = v_1$ Scaling

Two particle vn from ATLAS

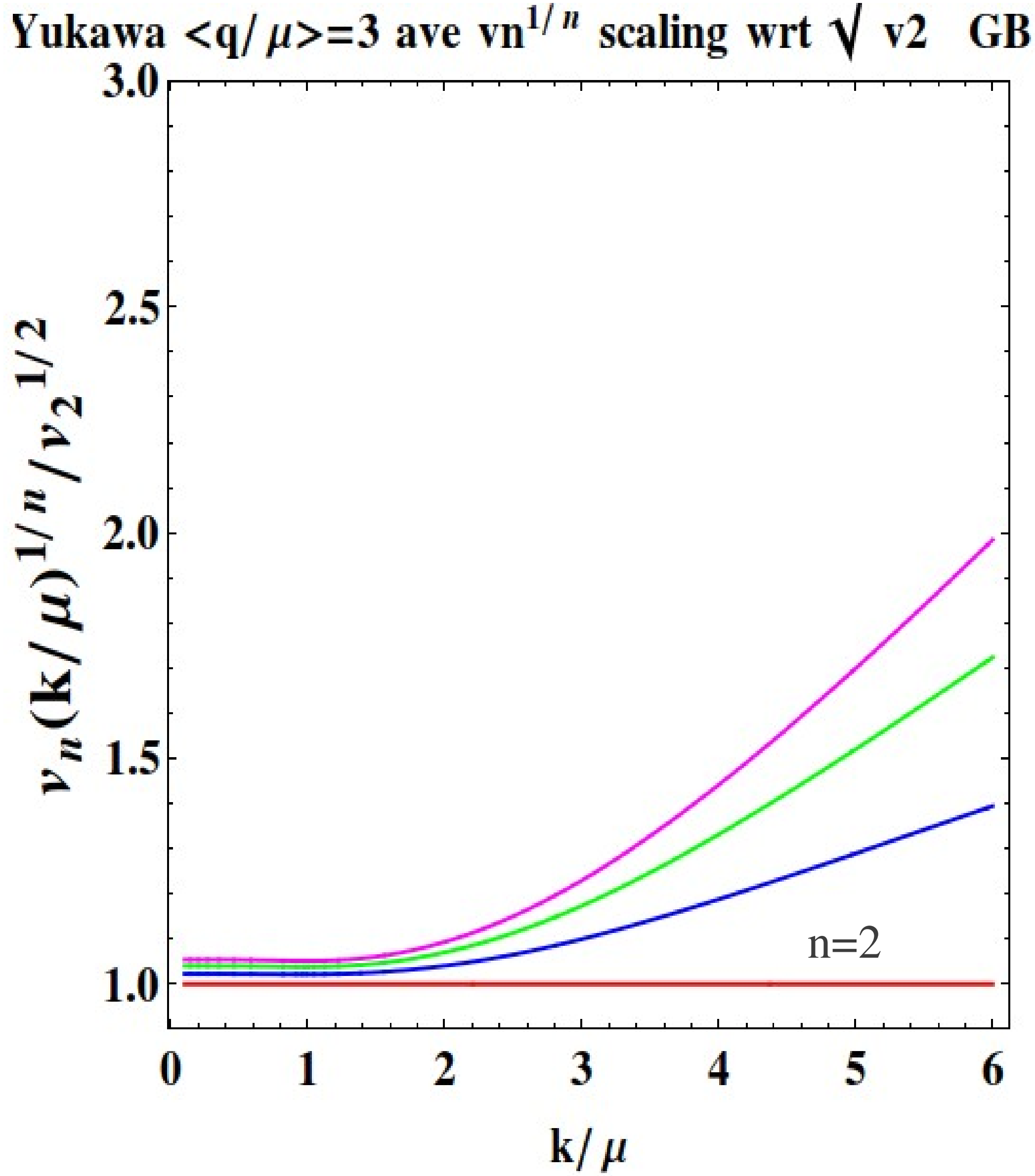


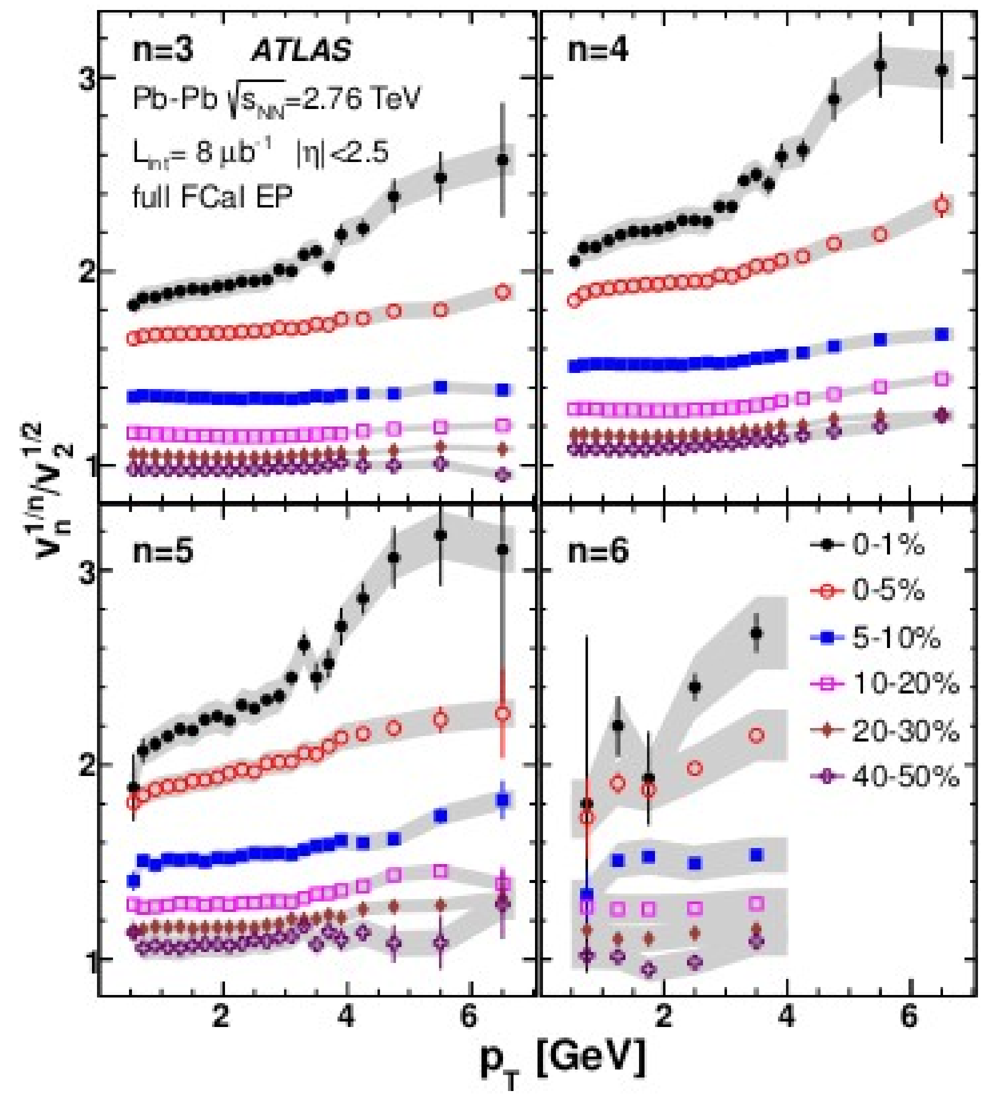



Fixed q GB pQCD Bremsstrahlung harmonics scale perfectly via 1/n power law

$$[v_n^{GB}(k,q,0)]^{1/n} \;=\; [v_m^{GB}(k,q,0)]^{1/m}$$

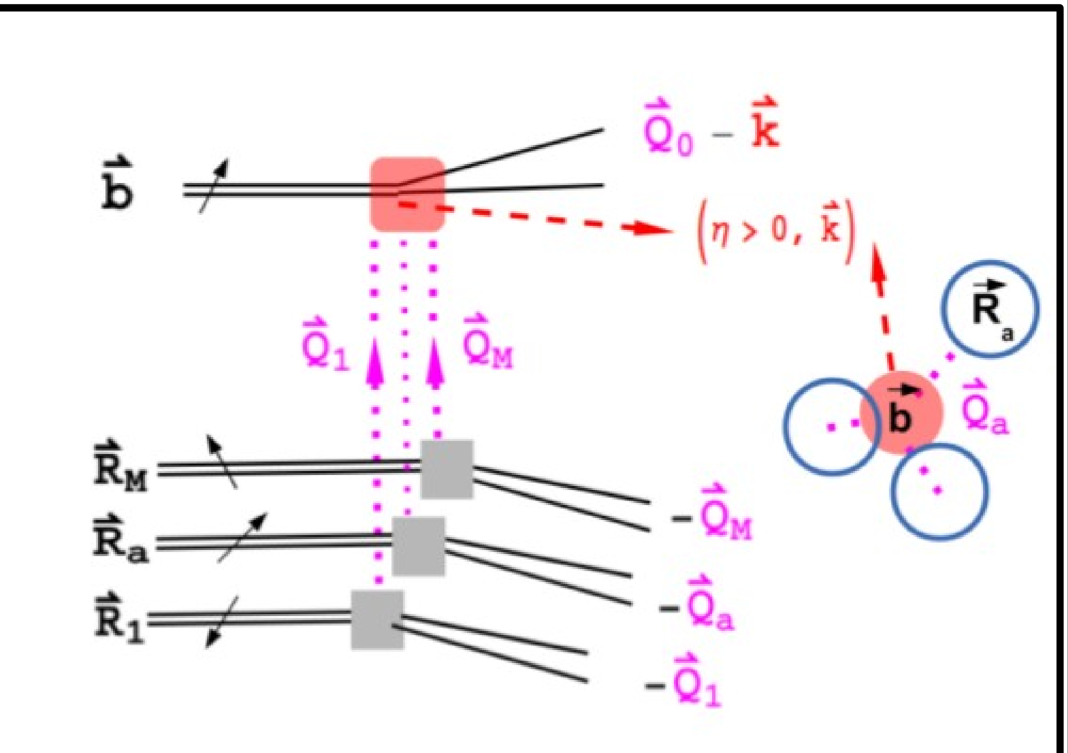
For Yukawa averaged <q/mu>=√M, GB 1/n scaling hold for k< √M and breaks down for k>√M



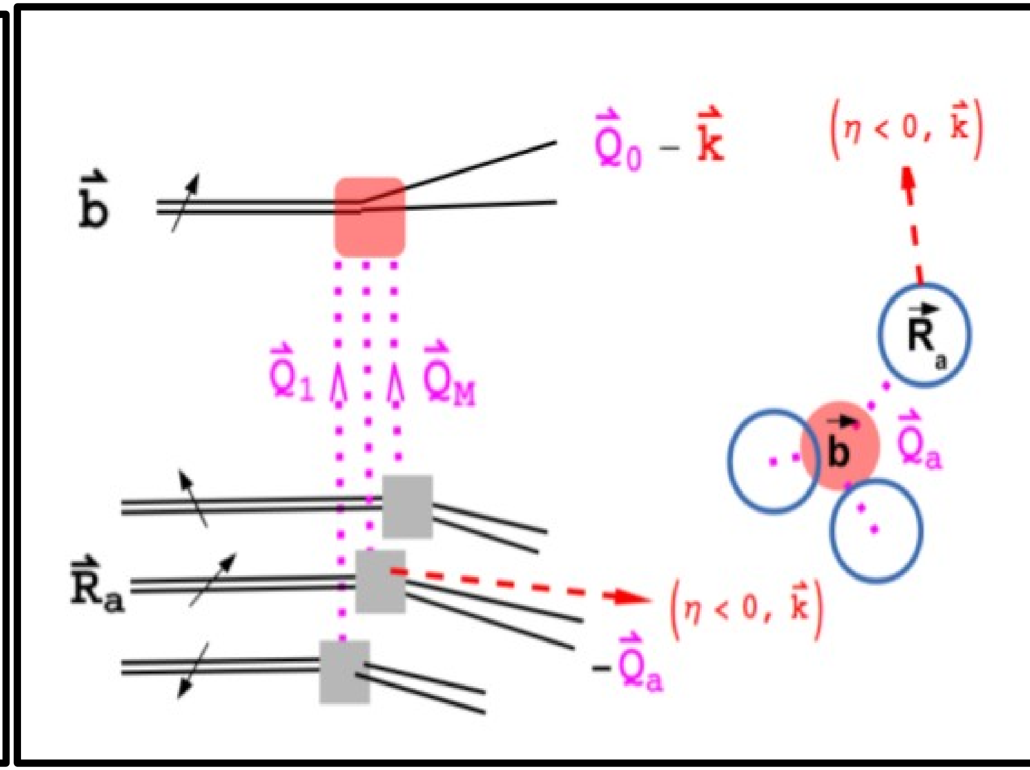



Combined projectile and target participants soft recoil Bemsstrahlung

Projectile Beam Jet Brems




Participant Recoil Target Beam Jets Brems




Target dipoles act
Coherently if transverse separation
cannot be resolved

$$R_{ij} \lesssim d(k) = \frac{c}{k}$$

1< M < N coherent target clusters defined by Resolution scale 1/k

If $i \in I_a$ and $j \in I_a$ as well as $j \in I_b$, then $j$ is added to $I_a$ if its $\langle d_{ij}\rangle_{i\in I_a} < \langle d_{ij}\rangle_{i\in I_b}$

Vitev all order in opacity multiple scattering generalization of GB Brems

$$dN^{VGB}_{coh}(\mathbf{k}) = \sum_{n=1}^{\infty} \int d^2\mathbf{Q}\, P^{el}_n(\mathbf{Q})\, dN^{GB}(\mathbf{k},\mathbf{Q})$$

$$P^{el}_n(\mathbf{Q}) \;=\; \exp[-\chi]\frac{\chi^n}{n!} \int \left\{ \prod_{j=1}^{n} \frac{d^2\mathbf{q}_j}{\sigma_{el}} \frac{d\sigma_{el}}{d^2\mathbf{q}_j} \right\} \times\delta^2(\mathbf{Q} - (\mathbf{q}_1 + \cdots + \mathbf{q}_n))$$

Cumulative momentum transfer from n coherent scatterings

At n=N th order in opacity with M coherent target clusters that can resolved by k
Projectile plus Target bremsstrahlung sums to

$$\begin{aligned} dN^{M,N} &= dN^{N}_{P}(\eta,\mathbf{k}_1;\mathbf{Q}_P) + dN^{M,N}_{T}(\eta,\mathbf{k}_1;\{\mathbf{Q}_a\}) \\ &= \sum_{a=0}^{M} \frac{B_{1a}}{(\mathbf{k}_1+\mathbf{Q}_a)^2+\mu_a^2} \;, \end{aligned}$$

$$B_{ia} \equiv F_{k_i,Q_a}\, P_a(\eta_i)$$

$$\mathbf{Q}_0 \equiv -\mathbf{Q}_P = -\textstyle\sum_a \mathbf{Q}_a$$

2 glue Brems in independent emission approx

$$dN^{N,M}_{2}(\mathbf{k}_1,\mathbf{k}_2) \;=\; \sum_{a=0}^{M}\sum_{b=0}^{M} \frac{B_{1a}}{A_{1a}-\cos(\phi_1+\psi_a)}\frac{B_{2b}}{A_{2b}-\cos(\phi_2+\psi_b)}$$

Two gluon relative Cos( n (φ1 – φ2)) analytic azimuthal harmonics VGA color antennas

$$f_n^{N,M}(k_1,k_2) \equiv \int_{-\pi}^{\pi} d\Phi \int_{-\pi}^{\pi} d\Delta\phi \cos(n\Delta\phi) dN_2^{N,M}(k_1,\Phi+\Delta\phi/2,k_2,\Phi-\Delta\phi/2)$$

$$= \sum_{a,b=0}^{M} B_{1a}B_{2b} \int_{-\pi}^{\pi} d\Phi' \frac{1}{A_{1a}-\cos(\Phi')} \int_{-\pi}^{\pi} d\Delta\phi \frac{\cos(n\Delta\phi)}{A_{2b}-\cos((\Phi'+\psi_b-\psi_a)-\Delta\phi)}$$

$$= \sum_{a,b=0}^{M} B_{1a}B_{2b}\, f_{0,1,a}\, f_{0,2,b} \boxed{(v_1^{GB}(k_1,Q_a)v_1^{GB}(k_2,Q_b))^n \cos(n(\psi_b-\psi_a))}$$

---

$$f_{n,1,a} = \int_{-\pi}^{\pi} d\Phi \frac{\cos(n\Phi)}{A_{1a}-\cos(\Phi)} = (v_1^{GB}(k_1,Q_a))^n f_{0,1,a} = \frac{\left(A_{k_1,Q_a}-\sqrt{A_{k_1,Q_a}^2-1}\right)^n}{\sqrt{A_{k_1,Q_a}^2-1)}}$$

---

$$\boxed{v_n^{M,N}\{2\}[k_1,k_2] \equiv= \langle\cos(n(\phi_1-\phi_2))\rangle_{k_1,k_2} = \frac{\langle f_n^{M,N}(k_1,k_2)\rangle}{\langle f_0^{M,N}(k_1,k_2)\rangle}}$$

$$\langle\cdots\rangle = \int \left\{\prod_{a=0}^{M} d\mathbf{Q}_a\right\} \delta(\sum_{a=0}^{M}\mathbf{Q}_a) \sum_{m_1,\cdots,m_M} \delta(N-\sum_{a=1}^{M} m_a)\, p_{\{m_j\}}^{M,N}\, P_{m_1}^{el}(\mathbf{Q}_1)\cdots P_{m_1}^{el}(\mathbf{Q}_M)$$

### A. Special $Z_n$ color antenna arrays

From Eq,(36,42) it is clear that a particularly simple special case arises if $M = n-1$ target clusters have similar number of members $m_a = N/M = N/(n-1)$ and transfer similar $Q_a^2 = N/M\mu^2$ to the projectile but only at special angles , $\{\psi_a\} = 2\pi a/n$, corresponding to the discrete unitary group of $n = M+1$ roots of unity,

$$Z_n = \{z_{a,n} = e^{i2\pi a/n} | a = 0, \cdots, n-1; \sum_{a=0}^{n-1} z_{a,n} = 0\} \quad . \tag{43}$$

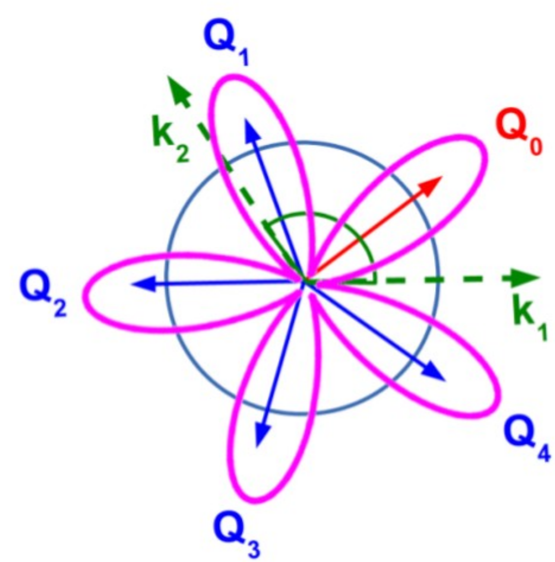
For this special geometry of projectile and target color dipole antennas the double sum over $a$ and $b$ decouple because

$$\cos(n(\psi_a - \psi_b)) = \cos(2\pi(a-b)) = 1 \quad , \tag{44}$$

$$v_n^{M,N}\{2\}(k_1,k_2) \xrightarrow{Z_n} \delta_{n,M+1} v_{M+1}^{GB}(k_1,Q_0) v_{M+1}^{GB}(k_2,Q_0)$$

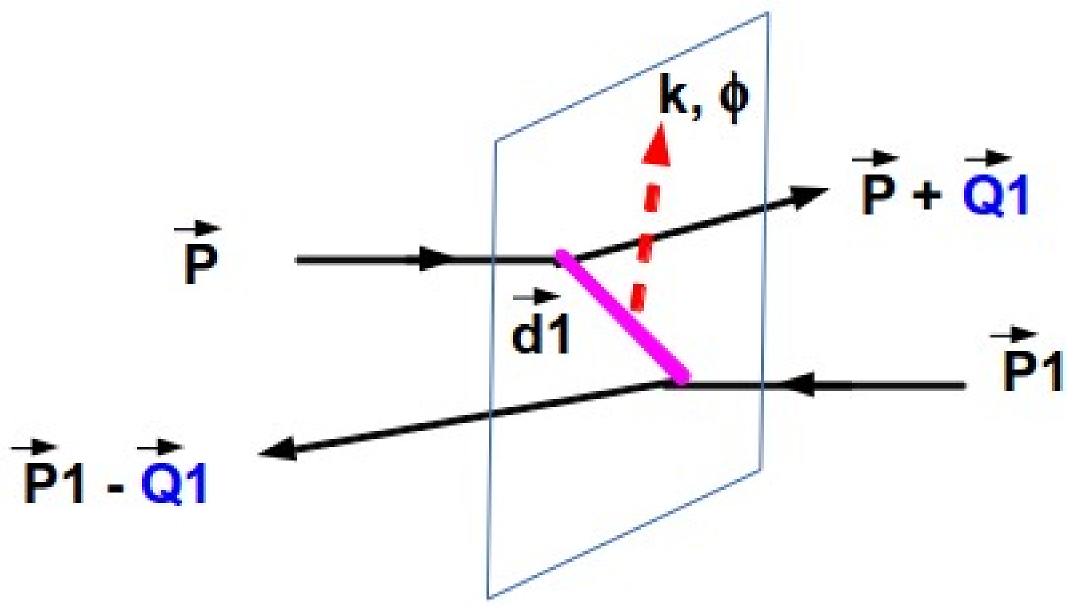
$$\frac{v_n^{M,N}\{2\}(k_1,k_2)}{v_{M+1}^{GB}(k_2,Q_0)} \xrightarrow{Z_n} \delta_{n,M+1} \; (v_1^{GB}(k_1,Q_0))^{M+1}$$

Zn color dipole antennas produce perfect pitch
Unique multipole vn azimuthal harmonics



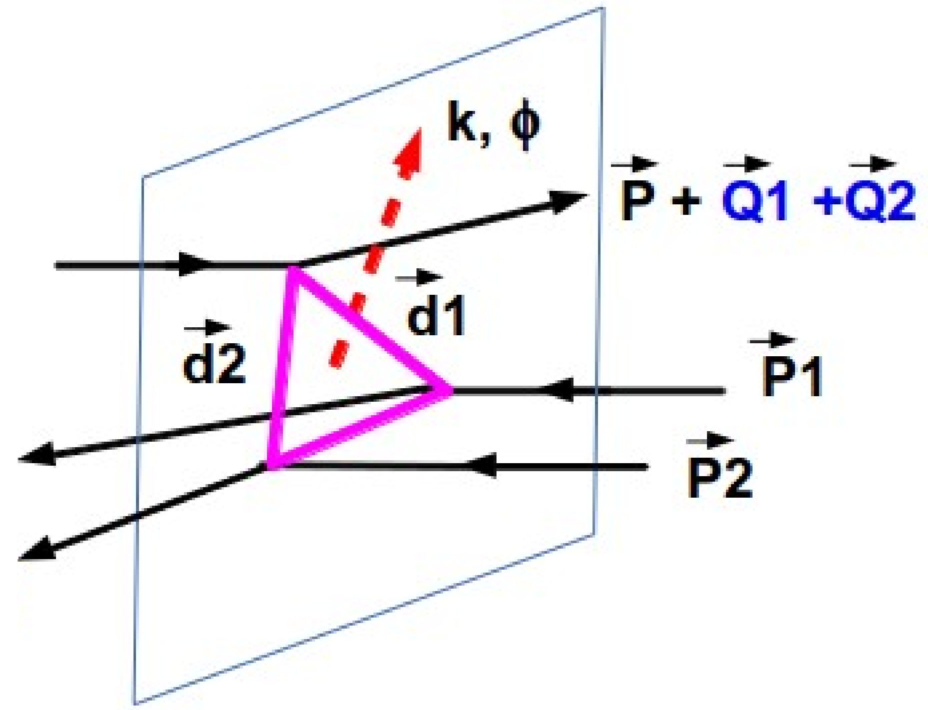

$Z_5$ "star fish" antenna array with $M = 4$ target clusters recoiling off projectile with $\mathbf{Q}_0 = -\sum_{a=1}^{M} \mathbf{Q}_a$

# Classical Color Field Produced by Interfering 3 dipole currents




Two BG dipole antenna array

Produce only n=2,4,6, ...




Three BG dipole antenna array

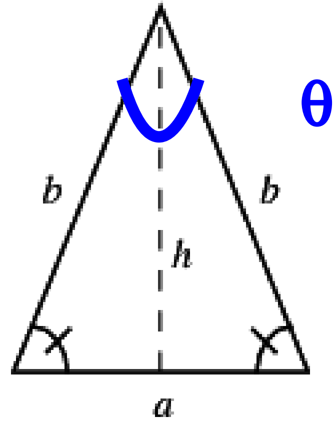
Produce all n=1, 2, 3, 4, ...

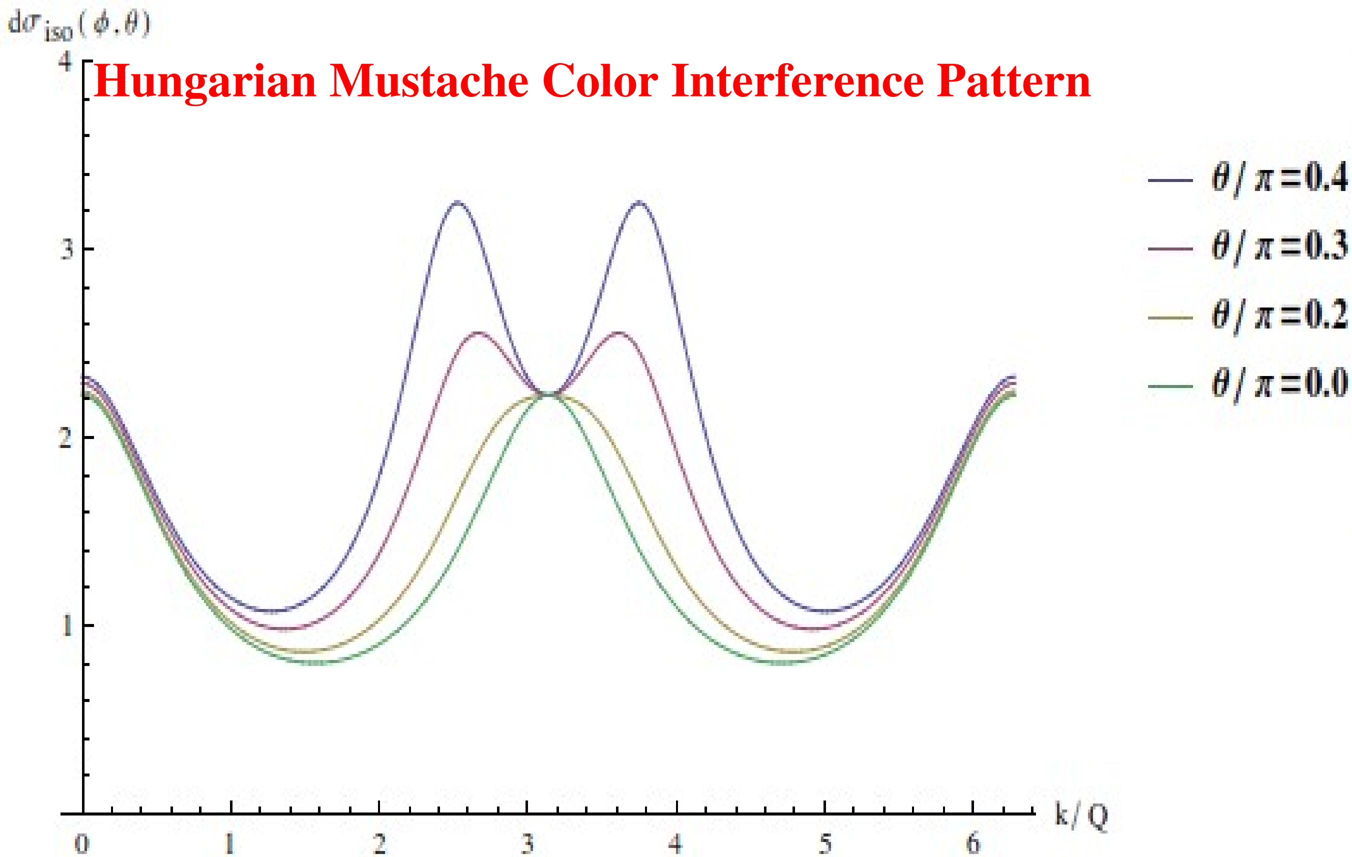
# Example: Color Scintillating Arrays can easily produce large v2, v3 Interferences

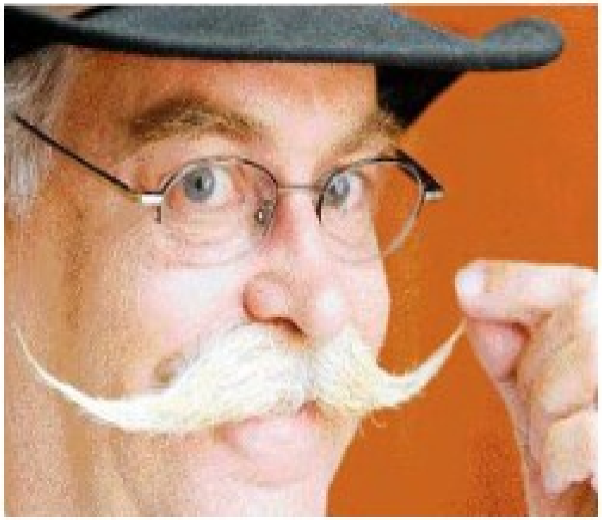
Isoscelese Triangle GB antenna (Magyar Bajusz) array

$$\frac{d\sigma_{iso}}{d\eta\, dk\, d\phi}[\{\vec{Q}_a\}] = \frac{C_A\,\alpha}{k^2}\sum_{a=1}^{3}\frac{Q_a^{\,2}}{k^2+Q_a^{\,2}-2\,k\,Q_a\,\mathrm{Cos}[\phi-\Theta_a]}$$

$$v_n(k/Q_1,\theta) = \int d\phi\ \mathrm{Cos}[n\ \phi]\ d\sigma_{iso}\ /\ \left(\int d\phi\ d\sigma_{iso}\right)$$



**M. Gyulassy, P. L., I. Vitev, and T. S. Biro, PRD (2014)**

### B. Random recoil color antenna arrays

Another simple limit is when the recoil azimuthal angles $\psi_a$ are in random $[0, 2\pi]$ and the $\mathbf{Q}_a$ are distributed with a Gaussian of same width squared $\langle Q_a^2 \rangle = Q_T^2 = (N/M)\mu^2$ for $a \in [1, \cdots, M]$ In this antenna array t, the projectile $\mathbf{Q}_0$ is also Gaussian distributed with zero mean but with an enhanced second moment,

$$\langle Q_0^2 \rangle = M Q_T^2 = N\mu^2 \quad . \tag{48}$$

Unlike for perfect $n^{th}$ harmonic antenna arrays with Eq,(49), in the random Gaussian distributed case

$$\cos(n(\psi_a - \psi_b)) = \delta_{a,b} \quad , \tag{49}$$

$$f_n^{N,M}(k,k) \overset{Gauss}{\rightarrow} \int d^2\mathbf{Q} \left\{ \frac{\exp[-Q^2/(2N\mu^2)]}{2\pi N\mu^2} + M \frac{\exp[-Q^2/(2(N/M)\mu^2)]}{2\pi(N/M)\mu^2} \right\} \{B_{kQ} f_{0,k,Q} v_n^{GB}(k,Q)\}^2$$

$$\sqrt{f_n^{N,M}(k,k)} \approx \left(\frac{C_R \alpha_s \mu^2}{\pi^2 k^2}\right) \left\{ \frac{P_P(\eta)}{(N+1)\mu^2} \frac{(v_1^{GB}(k, \sqrt{N}\mu))^n}{((k^2+(N+1)\mu^2)^2 - 4Nk^2\mu^2)^{1/2}} + \frac{M(\eta)}{(N/M(\eta)+1)\mu^2} \frac{(v_1^{GB}(k, \sqrt{N/M(\eta)}\mu))^n}{((k^2+(N/M(\eta)+1)\mu^2)^2 - 4(N/M(\eta))k^2\mu^2)^{1/2}} \right\}$$

Mixture of two scale GB harmonics with $Q^2 \sim N\mu^2$ from projectile beam jet
And M $Q^2 \sim (N/M)\mu^2$ recoil target beam jet clusters that depends of eta

**MGy: Work in progress implement correlated VGA brems into HIJING Monte Carlo generator
Replace phi ave ARIADNE in JETSET with VGA recoil correlated gluon Bremsstrahlung**

**Point 1: The Case for Perfect Fluidity and CGC/Glasma *Sufficiency* for Flow Harmonics in Non-central A+A as of QM12**

**Point 2 : Trouble for Fluidity since 2012**

**a) Beam Energy *independence***

**b) p(D)+A = A+A v2 and v3 Beam size *independence***

**c) large rapidity-even v1 Dipole**

**Point 3 : Is Perfect Fluidity *Necessary* ??**

**Are vn in B+A mostly Nonabelian Wave interference?**

**Could rapidity-even v odd Dipole,Triangular, etc harmonics be just fingerprints of basic pQCD beam jet anisotropic bremsstrahlung?**

**GLVB: all order in opacity pQCD beam jets vn**

**Aim: Bremsstrahlung in P+A via HIJING replacing ARIADNE with VGB**

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## Non-Abelian Bremsstralung and Azimuthal Asymmetries in High Energy p+A

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(Dated: May 13, 2014 v11)