An analytic hydrodynamical model of rotating 3D expansion

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- Rotation in high-energy heavy-ion collision
- A new exact solution of fireball hydrodynamics
- Observables, signatures of rotation

Nemzeti Kiválóság Program

TÁMOP 4.2.4.A

Introduction

Motivation:

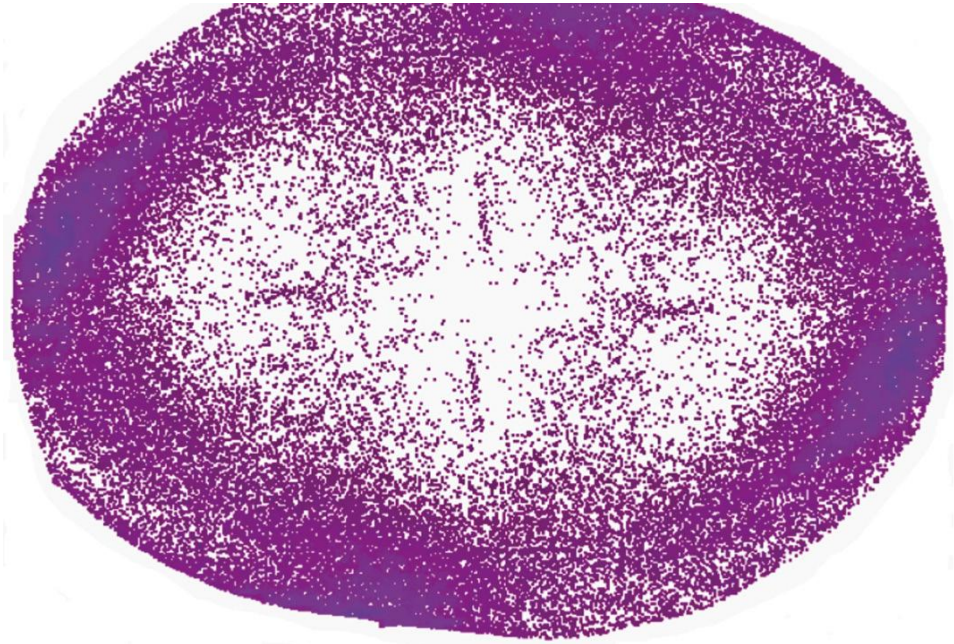
In non-central collisions:
initial angular momentum
leads to final state tilt of fireball

Exact hydrodynamical solutions:

insight into the dynamics
using simple analytic formulas

Determination of rotation:

important question,
theoretically, experimentally.



Example from EPN 43/22 (2012) 91 (L. Cifarelli, L.P. Csernai, H. Stöcker)

Looking for rotating hydro solutions to follow time evolution, calculate observables, tell if there is rotation

This work follows the footsteps of:

Csizmadia, Csörgő, Lukács, PLB443 (1998) 21
Csörgő, Acta Phys. Polon. B37 (2006) 483 & references therein
Csörgő, Akkelin, Hama, Lukács, Sinyukov, PRC67 (2003) 034094
Csörgő, Nagy, PRC89 (2014) 044901

Equations of hydrodynamics

Basic equations (non-relativistic, perfect fluid)

$$\partial_t n + \mathbf{v}\nabla n = -n\nabla \mathbf{v}$$ mass conservation

$$\partial_t \varepsilon + \mathbf{v}\nabla \varepsilon = -(\varepsilon + p)\nabla \mathbf{v}$$ energy conservation

$$\partial_t \mathbf{v} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{1}{nm_0}\nabla p$$ Euler equation

Follow only from local thermal equilibrium & local conservation laws.

Equation of State (EoS):

$$p = nT \qquad \varepsilon = \kappa(T)p$$

Frames: inertial K , co-rotating: K'

tilt angle (x-z plane): ϑ

angular velocity: $\omega = \begin{pmatrix} 0 \\ \dot{\vartheta} \\ 0 \end{pmatrix}$

Equations in rotating frame:

$$\left(\frac{\kappa}{T} + \frac{d\kappa}{dT}\right)(\partial_t + \mathbf{v}'\nabla')T + \nabla'\mathbf{v}' = 0$$

$$\partial_t n + \mathbf{v}'\nabla' n = -n\nabla'\mathbf{v}'$$

$$\partial_t \mathbf{v}' + (\mathbf{v}'\nabla')\mathbf{v}' = -\frac{1}{nm_0}\nabla' p + f'$$

$$f' = 2\mathbf{v}'\times\omega + \mathbf{r}'\times\dot{\omega} + \omega\times(\mathbf{r}'\times\omega)$$

A Hubble-like rotating solution

Hubble-type velocity:

$$\mathbf{v}' = \begin{pmatrix} \frac{\dot{X}}{X} r_x' + \dot{\psi} \frac{Z}{X} r_z' \\ \frac{\dot{Y}}{Y} r_y' \\ \frac{\dot{Z}}{Z} r_z' - \dot{\psi} \frac{X}{Z} r_x' \end{pmatrix}$$

scaling variable:
(rotating ellipsoids)

$$s = \frac{r_x'^2}{X^2} + \frac{r_y'^2}{Y^2} + \frac{r_z'^2}{Z^2}$$

„volume”:

$$V = XYZ$$

coordinate transformation:

$$r_x' = r_x \cos\vartheta - r_z \sin\vartheta$$

$$r_z' = r_x \sin\vartheta + r_z \cos\vartheta$$

$$r_y' = r_y$$

Solution of the continuity equations:

$$n(\mathbf{r},t) = n_0 \frac{V_0}{V} e^{-s/2}$$

for constant κ :

$$T(\mathbf{r},t) = T(t) = T_0 \left(\frac{V_0}{V(t)} \right)^{\frac{1}{\kappa}}$$

for non-constant κ:

$$T(\mathbf{r},t) = T(t)$$

$$\frac{V_0}{V} = \exp\left(\int_{T_0}^{T} \mathrm{d}\beta \left(\frac{d\kappa}{d\beta} + \frac{\kappa}{\beta} \right) \right)$$

Similar to & generalization of:

Csörgő, Acta Phys. Polon. B37 (2006) 483 & references therein

Csörgő, Akkelin, Hama, Lukács, Sinyukov, PRC67 (2003) 034094

Csörgő, Nagy, PRC89 (2014) 044901

Arriving at a parametric solution

Remaining functions:

Axes: $X(t)$, $Y(t)$, $Z(t)$

Auxiliary function: $\dot{\psi}(t)$

Angular velocity: $\dot{\vartheta}(t)$

„Motion" of axes:

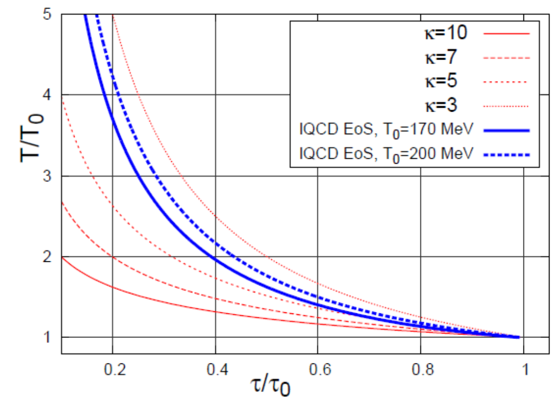
Governed by a Lagrangian!

Solution if

$$\dot{\psi}(t) = \frac{\chi_0}{(X+Z)^2}$$

$$\dot{\vartheta}(t) = \frac{\chi_0}{(X+Z)^2}$$

A slight (not quite physical) generalization is possible…



In the constant κ case:

$$L = \frac{m}{2}\left(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2\right) - \frac{1}{\kappa}\left(\frac{V_0}{V}\right)^{1/\kappa} - \frac{m\chi_0^2}{(X+Z)^2}$$

Equations of motion:

Ordinary p.d.e.-s

Solution is easy numerically

Conserved quantities: total particle number: $N_0 = n_0 V_0 (2\pi)^{3/2}$

total angular momentum (y component): $M_y = m N_0 \chi_0$

total energy: $E = \frac{m}{2}\left(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2\right) + \frac{1}{\kappa}\left(\frac{V_0}{V}\right)^{1/\kappa} + \frac{m\chi_0^2}{(X+Z)^2}$

Observables

Hydro evolution until freeze-out:

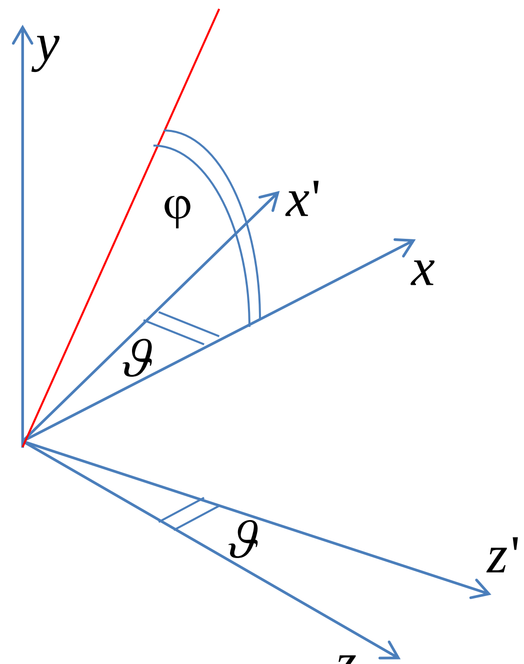
Dependent on EoS

Our freeze-out criterion:

Same freeze-out temperature

Same time everywhere

Analytic results!



Definitions & recipes:

Single-particle spectrum:

$$\frac{\mathrm{d}n}{\mathrm{d}^3\mathbf{p}} = \int \mathrm{d}^3\mathbf{r}\, S(\mathbf{r},\mathbf{p})$$

Bose-Einstein correlation:

$$C(\mathbf{K},\mathbf{q}) = 1 + \lambda \frac{\left|\tilde{S}(\mathbf{K},\mathbf{q})\right|^2}{\left|\tilde{S}(\mathbf{K},0)\right|^2}$$

Source function:

$$S(\mathbf{r},\mathbf{p}) = \frac{n(t_f,\mathbf{r})}{\left(2\pi m T_f\right)^{3/2}} \exp\left\{-\frac{(\mathbf{p}-m\mathbf{v}(t_f,\mathbf{r}))^2}{2mT_f}\right\}$$

Important auxiliary quantity („speed of rotation”):

$$U = \frac{\chi_0}{X_f + Z_f}$$

Observables: single-particle spectrum

Definition:

$$\frac{\mathrm{d}n}{\mathrm{d}^3\mathbf{p}} = \int \mathrm{d}^3\mathbf{r}\, S(\mathbf{r},\mathbf{p})$$

Result:

$$\frac{\mathrm{d}n}{\mathrm{d}^3\mathbf{p}} \propto \exp\left(-\frac{p_x^2}{2mT_x} - \frac{p_y^2}{2mT_y} - \frac{p_z^2}{2mT_z} - \frac{\beta_{xz}}{m} p_x p_z\right)$$

Auxiliary quantities:

$$T_y^* = T_f + m\dot{Y}^2$$

$$T_x^* = T_f + m\left(\dot{X}^2 + U^2\right), \quad T_z^* = T_f + m\left(\dot{Z}^2 + U^2\right), \quad \beta_* = mU(\dot{X} - \dot{Z})$$

Coefficients:

$$T'_x = T_x^* + \frac{\beta_*^2}{T_z^*}$$

$$T'_x = T_x^* + \frac{\beta_*^2}{T_z^*}$$

$$\beta'_{xz} = \frac{\beta_*}{T_x^* T_z^* - \beta_*^2}$$

In lab frame:

$$\frac{1}{T_x} = \frac{\cos^2 \vartheta_f}{T'_x} + \frac{\sin^2 \vartheta_f}{T'_z} + \beta'_{xz} \sin(2\vartheta_f)$$

$$\frac{1}{T_z} = \frac{\sin^2 \vartheta_f}{T'_x} + \frac{\cos^2 \vartheta_f}{T'_z} - \beta'_{xz} \sin(2\vartheta_f)$$

$$\beta_{xz} = \frac{\sin(2\vartheta_f)}{2}\left(\frac{1}{T'_z} - \frac{1}{T'_x}\right) + \beta'_{xz} \cos(2\vartheta_f)$$

Observables: average p_T-spectrum

Definition:

$$E\frac{\mathrm{d}n}{\mathrm{d}^3\mathbf{p}}=\frac{1}{2\pi p_T}\frac{\mathrm{d}n}{\mathrm{d}p_T\mathrm{d}y}\left\{1+2\sum_{n=1}^{\infty}v_n\cos\left[n\left(\varphi-\Psi_n\right)\right]\right\}$$

In this model:

All event planes coincide!

Result

(same as in *Csörgő et al., PRC67 (2003) 034094*)

Azimuthal average, (up to 2nd order in ν):

$$\frac{1}{2\pi p_T}\frac{\mathrm{d}n}{\mathrm{d}p_T\mathrm{d}y}\propto\exp\left(-\frac{p_z^2}{2mT_z}-\frac{p_T^2}{2mT_{\mathrm{eff}}}\right)\times$$

$$\times\left[I_0(w)+\frac{\nu^2}{4}\left(I_0(w)+I_1(w)\right)\right]$$

Auxiliary quantities:

$$w=\frac{p_T^2}{4m}\left(\frac{1}{T_y}-\frac{1}{T_x}\right)$$

$$T_{\mathrm{eff}}=\frac{1}{2}\left(\frac{1}{T_y}+\frac{1}{T_x}\right)$$

$$\nu=-\frac{\beta_{xz}}{m}p_z p_T$$

Observables: flow coefficients

Results: (same as in *Csörgő et al., PRC67 (2003) 034094*):

Flow coefficients (up to 2nd order in ν):

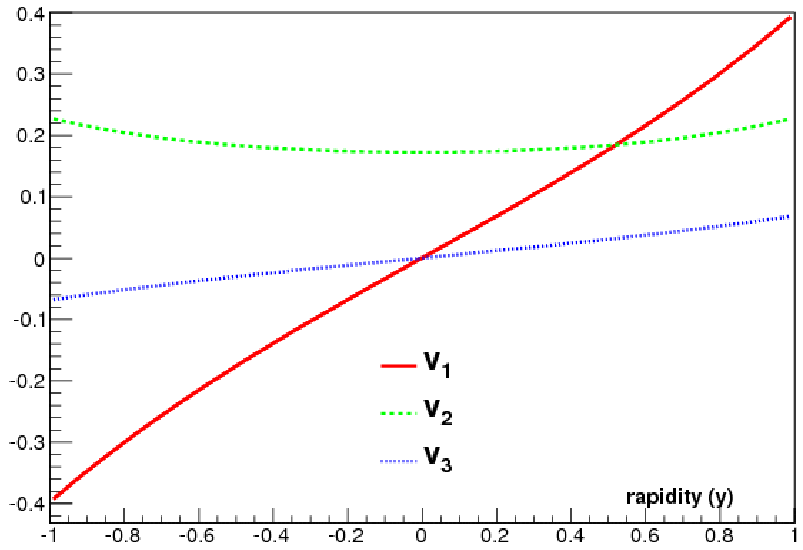
$$v_1 = \frac{\nu}{2}\left[1 + \frac{I_1(w)}{I_0(w)}\right]$$

$$v_2 = \frac{I_1(w)}{I_0(w)} + \frac{\nu^2}{8}\left[1 + \frac{I_2(w)}{I_0(w)} - 2\frac{I_1^2(w)}{I_0^2(w)}\right]$$

$$v_3 = \frac{\nu}{2}\left[\frac{I_1(w)}{I_0(w)} + \frac{I_2(w)}{I_0(w)}\right]$$

Modified Bessel function:

$$I_\nu(x) = \int_0^\pi \mathrm{d}\varphi \frac{\cos(\nu\varphi)}{\pi} e^{x\cos\varphi}$$



Universal scaling of v_2 is preserved
(*Csanád et al., Nucl.Phys. A742 (2004) 80*)

Directed flow: opposite in two directions!
Useful for rotation orientation determination

Observables: two-particle correlations

Bose-Einstein correlations (raw):

Definition:

$$C(\mathbf{K},\mathbf{q}) = 1 + \lambda \frac{\left|\tilde{S}(\mathbf{K},\mathbf{q})\right|^2}{\left|\tilde{S}(\mathbf{K},0)\right|^2}$$

$$\tilde{S}(\mathbf{K},\mathbf{q}) = \int \mathrm{d}^3\mathbf{r}\, e^{iq\mathbf{r}} S(\mathbf{r},\mathbf{K})$$

Result: Gaussian

$$C(\mathbf{K},\mathbf{q}) = 1 + \lambda \exp\left(-\sum_{ij} q_i q_j R_{ij}^2\right)$$

Radii in tilted frame:

$$R'^2_x = X^2 \frac{T_f}{T^*_x}$$

$$R'^2_y = Y^2 \frac{T_f}{T^*_y}$$

$$R'^2_z = Z^2 \frac{T_f}{T^*_z}$$

$$R'^2_{xz} = XZT_f \beta'_{xz}$$

In the lab frame:

$$R_x^2 = R'^2_x \cos^2\vartheta_f + R'^2_z \sin^2\vartheta_f + R'^2_{xz}\sin(2\vartheta_f)$$

$$R_y^2 = R'^2_y$$

$$R_x^2 = R'^2_x \sin^2\vartheta_f + R'^2_z \cos^2\vartheta_f - R'^2_{xz}\sin(2\vartheta_f)$$

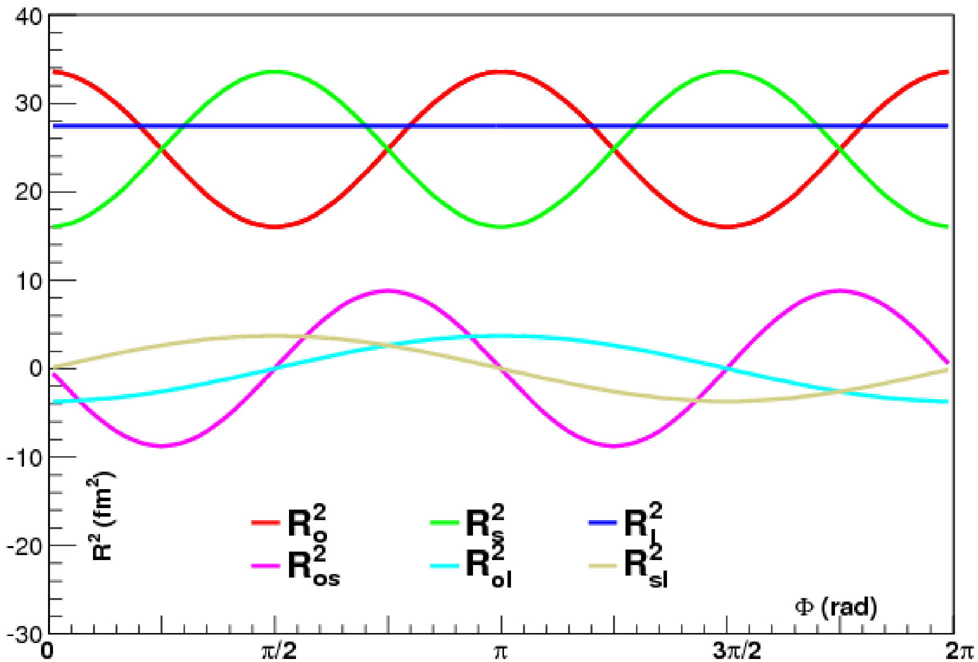
$$R_{xz}^2 = \frac{\sin(2\vartheta_f)}{2}\left(R'^2_x - R'^2_z\right) + \cos(2\vartheta_f) R'^2_{xz}$$

Coulomb correction treatment important!

Observables: two-particle correlations

Bertsch-Pratt parametrization:

$$C(\mathbf{K},\mathbf{q}) = 1 + \lambda \exp\left(-\sum_{i,j=o,s,l} q_i q_j R_{ij}^2 \right)$$



HBT Bertsch-Pratt radii:

$$R_o^2 = R_x^2 \cos^2\varphi + R_y^2 \sin^2\varphi$$

$$R_s^2 = R_x^2 \sin^2\varphi + R_y^2 \cos^2\varphi$$

$$R_l^2 = R_z^2$$

$$R_{os}^2 = \left(R_y^2 - R_x^2\right)\sin\varphi\cos\varphi$$

$$R_{ol}^2 = R_{xz}^2 \cos\varphi$$

$$R_{sl}^2 = -R_{xz}^2 \sin\varphi$$

Oscillations: s, o, os, ol, sl !

ol, sl values vanish for no tilt: $\vartheta = 0$

No 2nd order oscillations of ol, sl!

Signatures of rotation

Observables that appear because of tilt ($\vartheta = 0$):

Flow coefficients vs. rapidity: v_3 and v_1 have characteristic rapidity dependence!

HBT-radii: azimuthal oscillation vs. reaction plane:

out-side: known, measured multiple times (characteristic to ellipsoid-like sources)

- eg. STAR beam energy scan HBT paper: extensive oscillation measurements (*arXiv:1403.4972*)

out-long, side-long: not measured yet above AGS energies (as far as I know)

Interesting ***ol, sl oscillations*** depend on $\cos\varphi$ and $\sin\varphi$:

1th order event plane needed! (fluctuations may interplay...)

*Rotation detection with HBT measurements is possible***!**

Flow coefficients vs. rapidity: may help to establish 1th order event plane

***

Effect of physical rotation (not jus final tilt):

Eigen-frames of the rotated ***coordinate-space*** ellipsoid, of the ***single particle spectrum*** and that of the ***HBT correlation*** functions are ***three different*** frames!

Presented hydro solution allows a dynamical investigation of the rotation vs time!

Summary and outlook

Hydrodynamical models:

dynamically connect initial state with final state

Non-relativistic hydrodamical equations:

Extended the class of known parametric solutions

Found rotating, 3-axis ellipsoidal solutions, for arbitrary EoS

Calculated observables

Simplified treatment

Needs at least some relativistic generalization (eg. HBT radii now do not depend on transverse momentum) Relativistic generalization of solutions: hard task…

Signals of rotation:

Directed & third flow vs. Rapidity

1th order oscillations of out-long, side-long HBT radii parameters

Measurement of tilt angle:

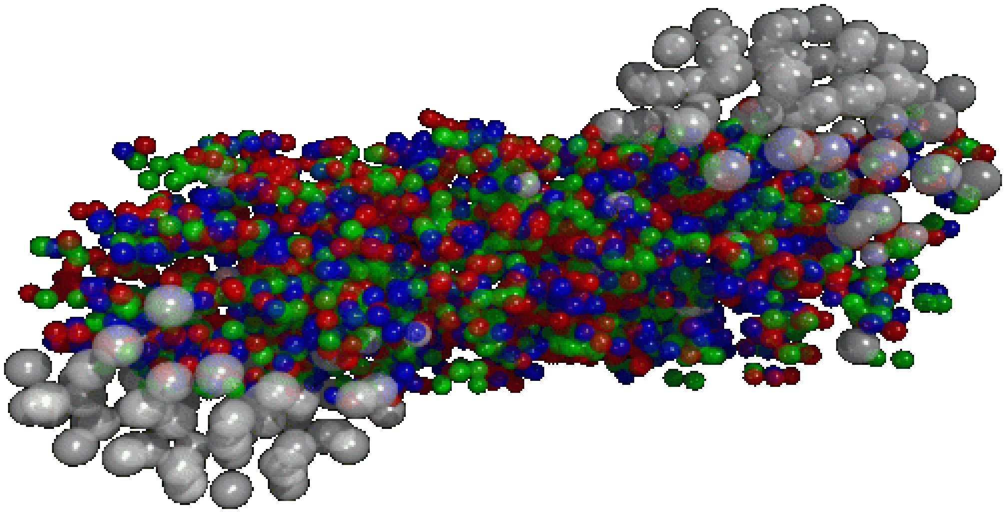
A final state variable; initial angle known to be zero?

Possible to investigate EoS!

Further work needed:

Extension of the Buda-Lund model to this rotating case: a semi-analytic relativistic generalization, *coming soon*…

Thank you for your attention!



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Spare slides coming…

(soon)