Rotation in Fluid Dynamics, Instabilities, and their Observation

Zimányi School 2014,
Budapest, Dec. 1-4, 2014

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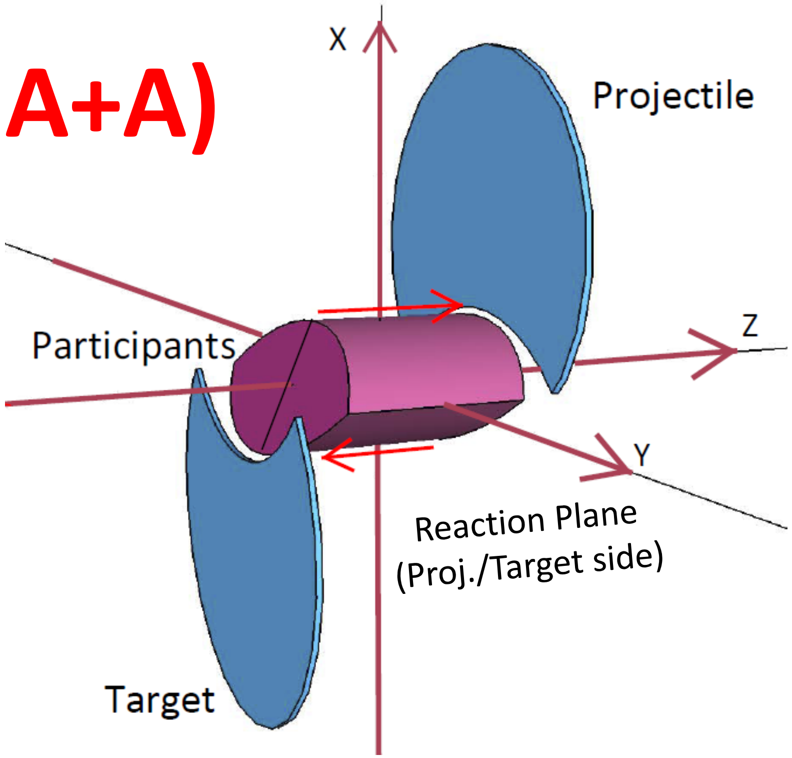
Peripheral Collisions (A+A)



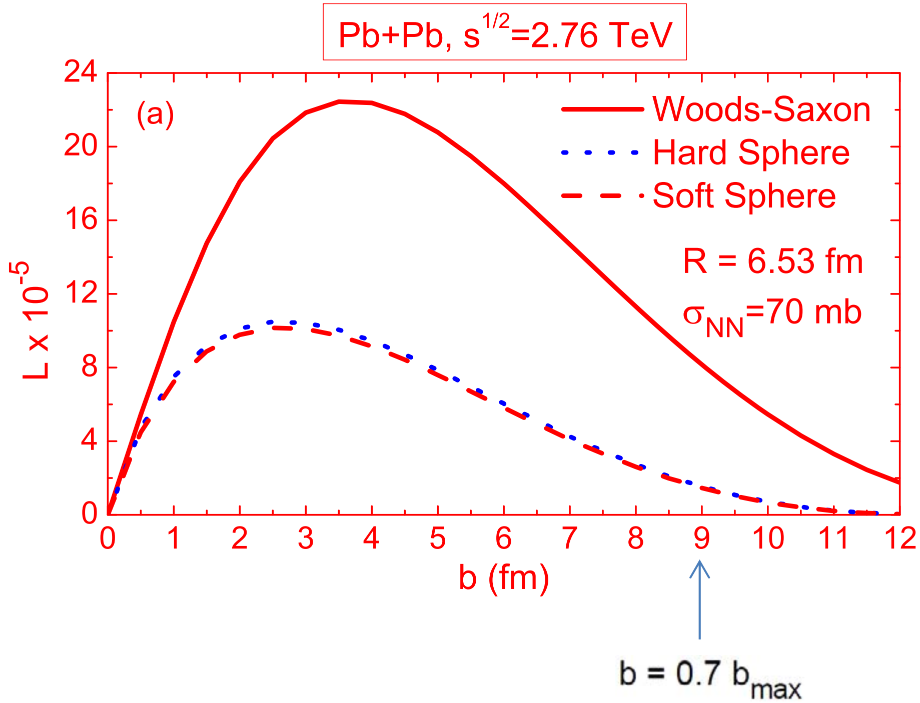
- Global Symmetries
- Symmetry axes in the global CM-frame:
  - (y ←→ -y)
  - (x,z ←→ -x,-z)
  - Azimuthal symmetry: ϕ-even (cos nϕ)
  - Longitudinal z-odd, (rap.-odd) for $v_{\_odd}$
  - Spherical or ellipsoidal flow, expansion

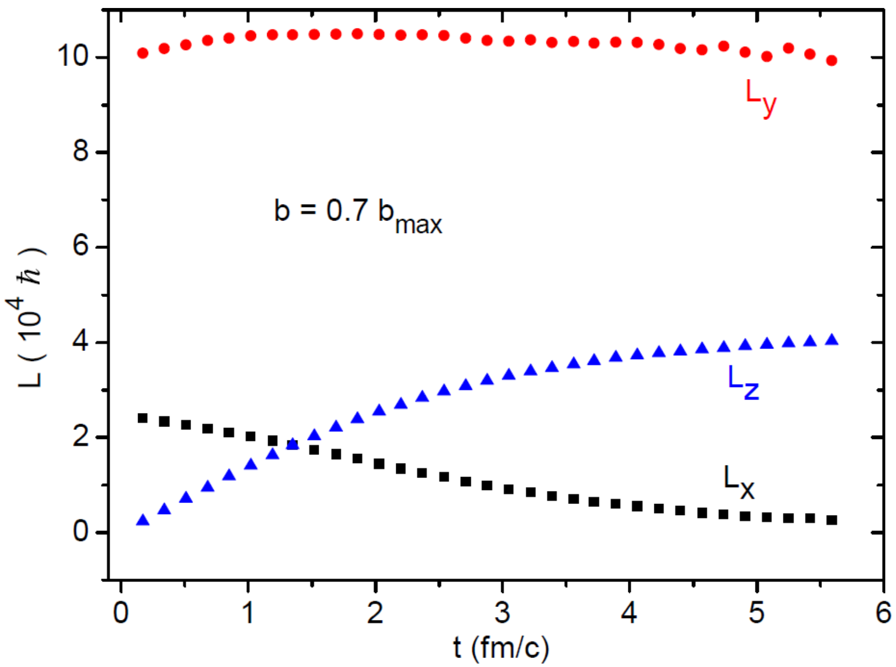
$$\frac{d^3N}{dy dp_t d\phi} = \frac{1}{2\pi}\frac{d^2N}{dy dp_t}\left[1 + 2v_1(y,p_t)\cos(\phi) + 2v_2(y,p_t)\cos(2\phi) + \cdots\right]$$

$$\frac{d^3N}{dy dp_t d\phi} = \frac{1}{2\pi}\frac{d^2N}{dy dp_t}\left[1 + 2v_1(y-y_{CM},p_t)\cos(\phi-\Psi_{RP}) + 2v_2(y-y_{CM},p_t)\cos(2(\phi-\Psi_{RP})) + \cdots\right]$$

- Fluctuations
- Global flow and Fluctuations are simultaneously present → ∃ interference
  - Azimuth - Global: even harmonics - Fluctuations : odd & even harmonics
  - Longitudinal – Global: v1, v3 y-odd - Fluctuations : odd & even harmonics
  - The separation of Global & Fluctuating flow is a must !! (not done yet)

Collective dynamics - rotation

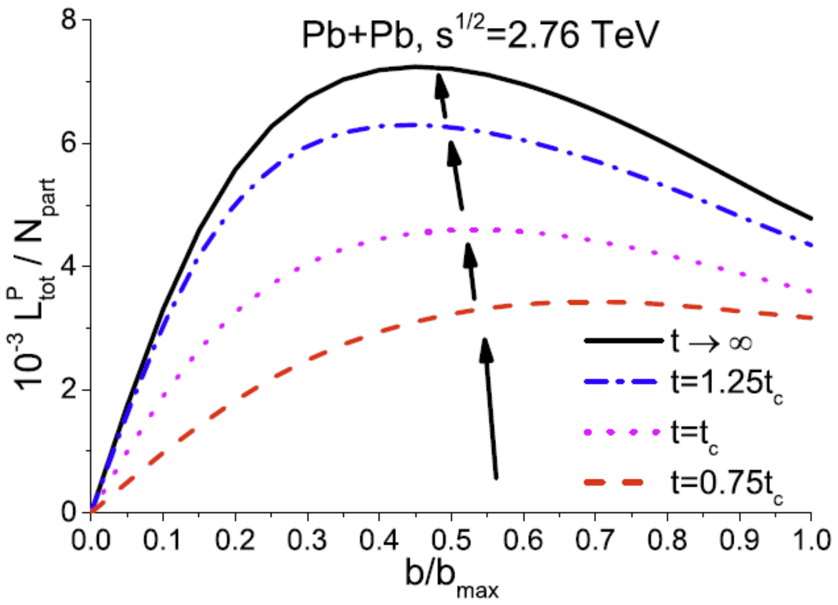


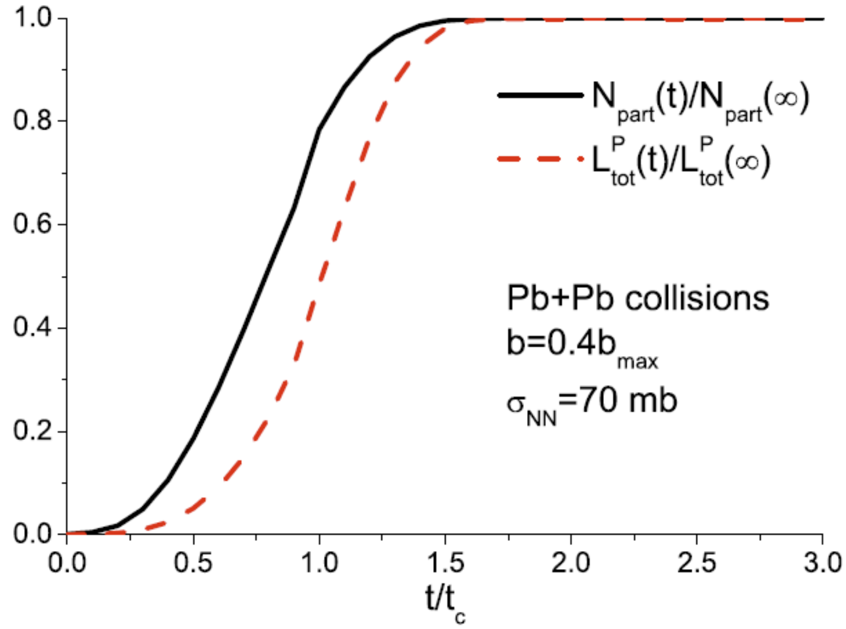


V. Vovchenko, D. Anchishkin, and L.P. Csernai,
Phys. Rev. C **88**, 014901 (2013)

J. H. Gao, S. W. Chen, W. T. Deng, Z. T. Liang, Q. Wang and X. N. Wang, Phys. Rev. C 77, 044902 (2008).

F. Becattini, F. Piccinini, J. Rizzo, Phys. Rev. C 77, 024906 (2008).





V. Vovchenko[1,2,3], D. Anchishkin[4], L.P. Csernai[5] [PHYSICAL REVIEW C **90**, 044907 (2014)]

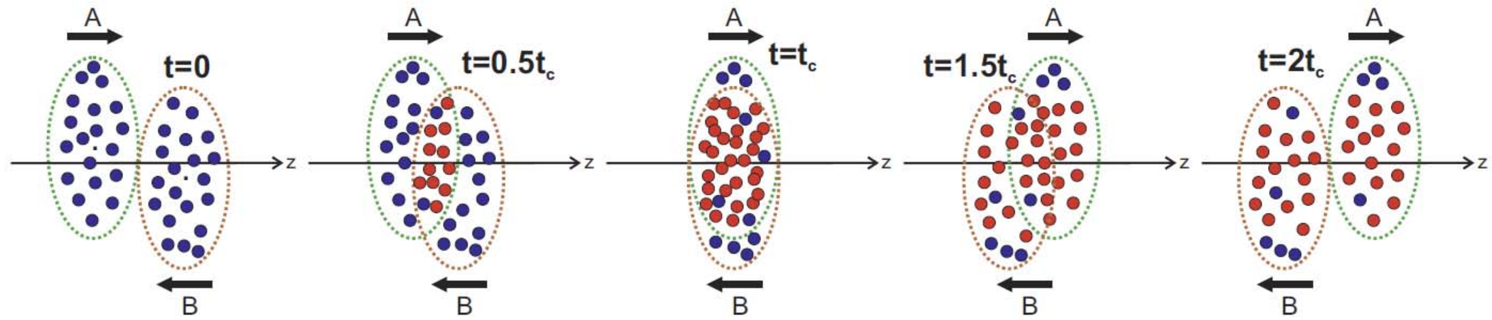
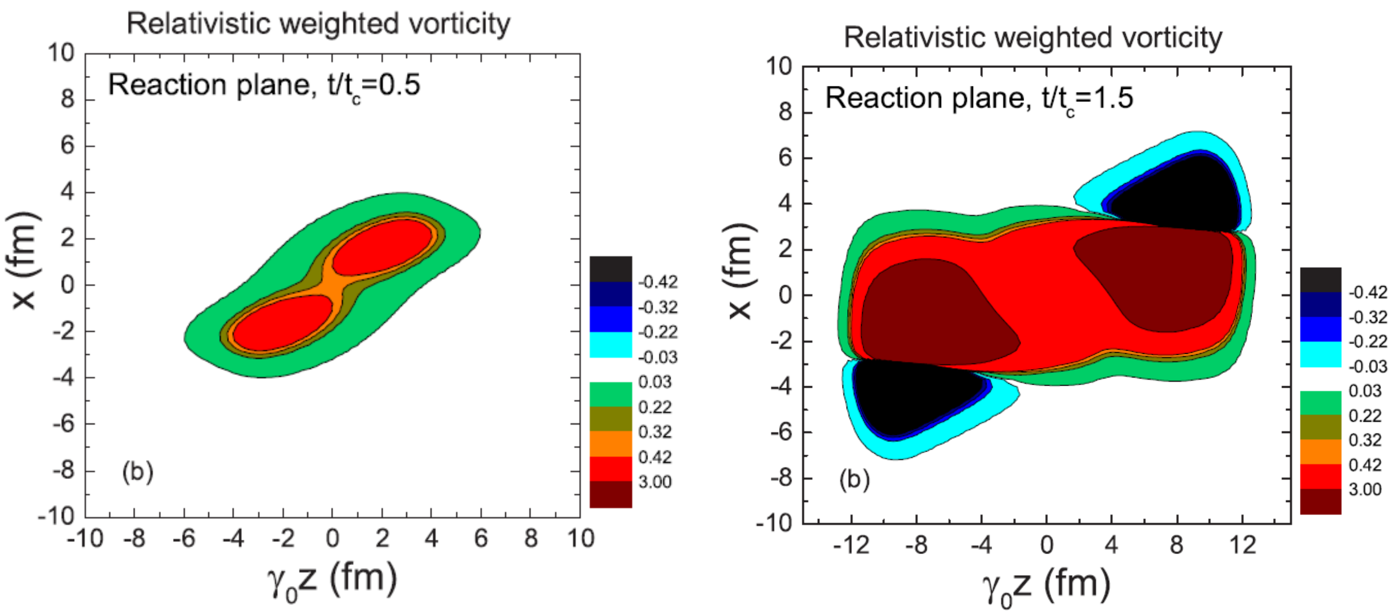


FIG. 1. Schematic drawing of the system evolution in the presented model. Blue points indicate nucleons which have not interacted before present time moment while red points indicate nucleons which already have interacted.

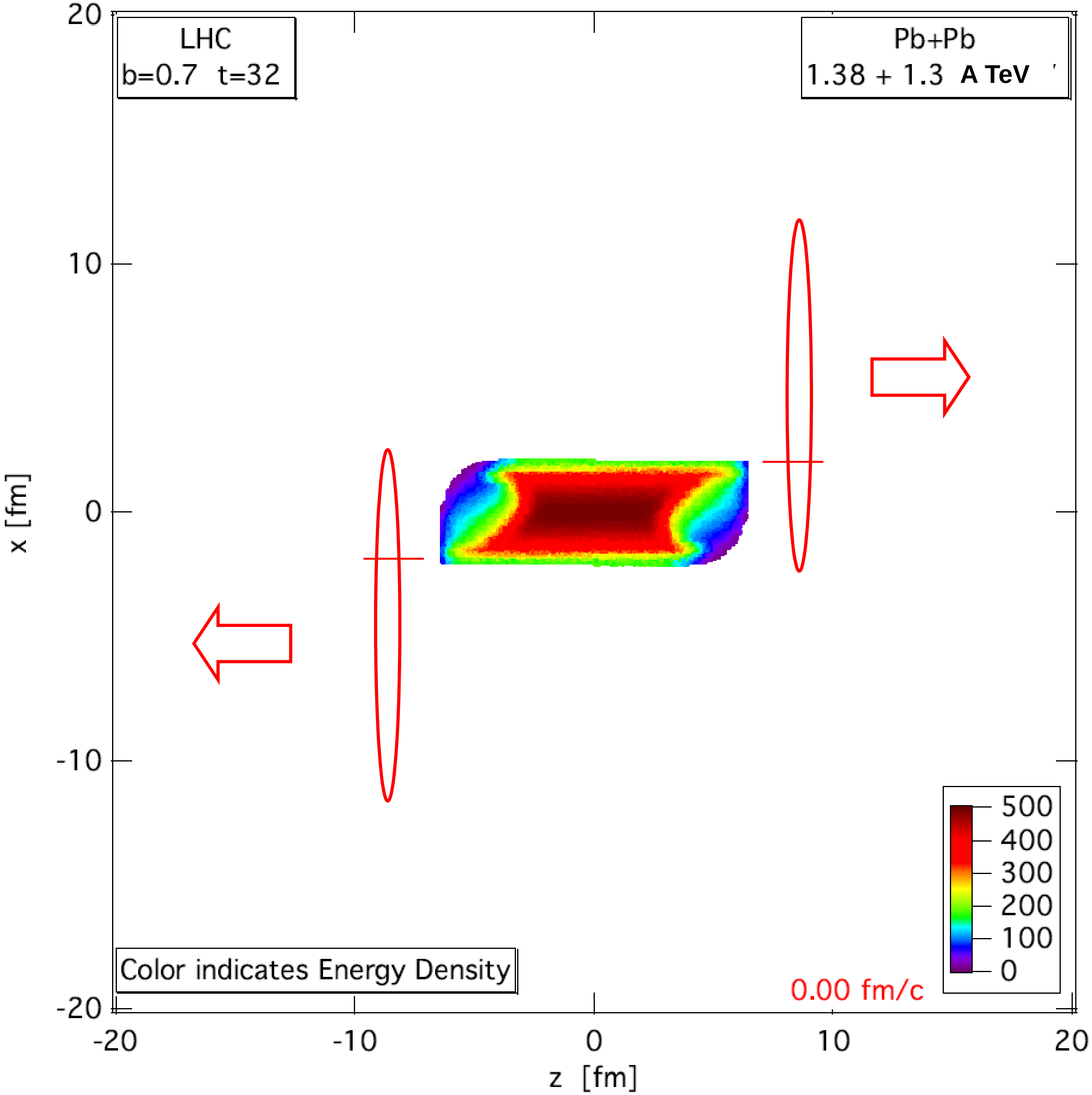
V. Vovchenko et al., [PHYSICAL REVIEW C **90**, 044907 (2014)]

Pb+Pb $\sqrt{s_{NN}} = 2.76$ TeV and $b = 0.7 b_{\max}$



Semi-analytic cascade model (~ uRQMD) - -
more transparency → larger longitudinal extent
less collective pressure → less transverse expansion
But: significant Angular Momentum, Shear and Vorticity

D. Anchiskin , later today !!!



PIC-hydro

Pb+Pb 1.38+1.38 A TeV, b= 70 % of b_max

Lagrangian fluid cells, moving, ~ 5 mill.

MIT Bag m. EoS

FO at T ~ 200 MeV, but calculated much longer, until pressure is zero for 90% of the cells.

Structure and asymmetries of init. state are maintained in nearly perfect expansion.

\zz-Movies\LHC-Ec-1h-b7-A.mov

[Csernai L P, Magas V K, Stoecker H, Strottman D D, *Phys. Rev*. C **84** (2011) 024914]

Kelvin-Helmholtz instability in high-energy heavy-ion collisions

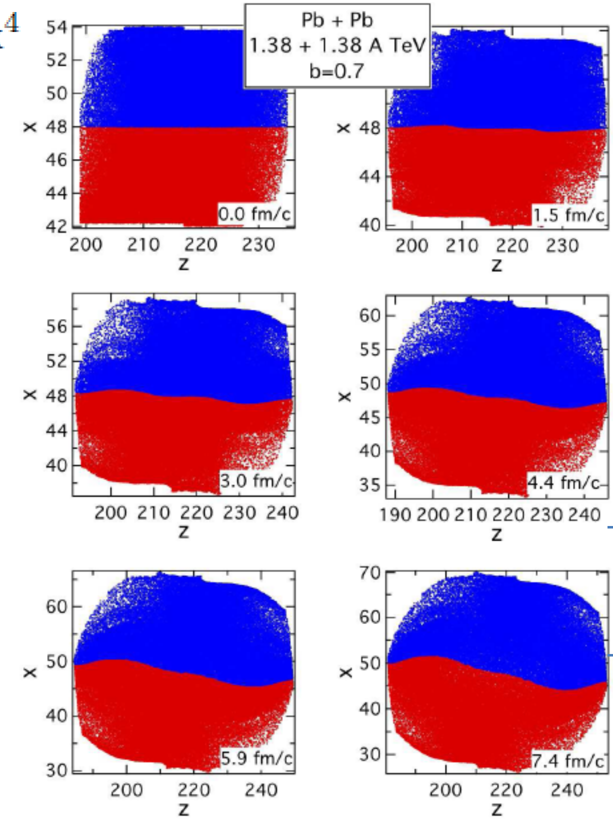
L.P. Csernai[1,2,3], D.D. Strottman[2,3], and Cs. Anderlik[4]

PHYSICAL REVIEW C **85**, 054901 (2012)

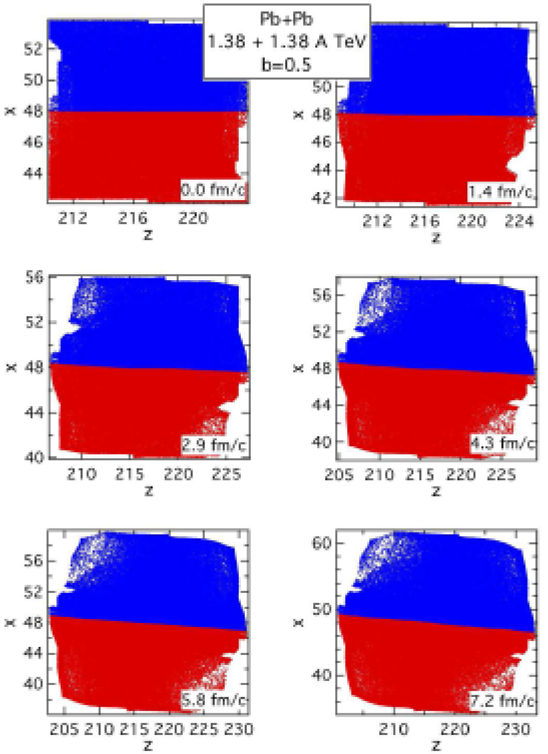
arXiv:1112.4287v3 [nucl-th]

PIC method !!!

KHI →



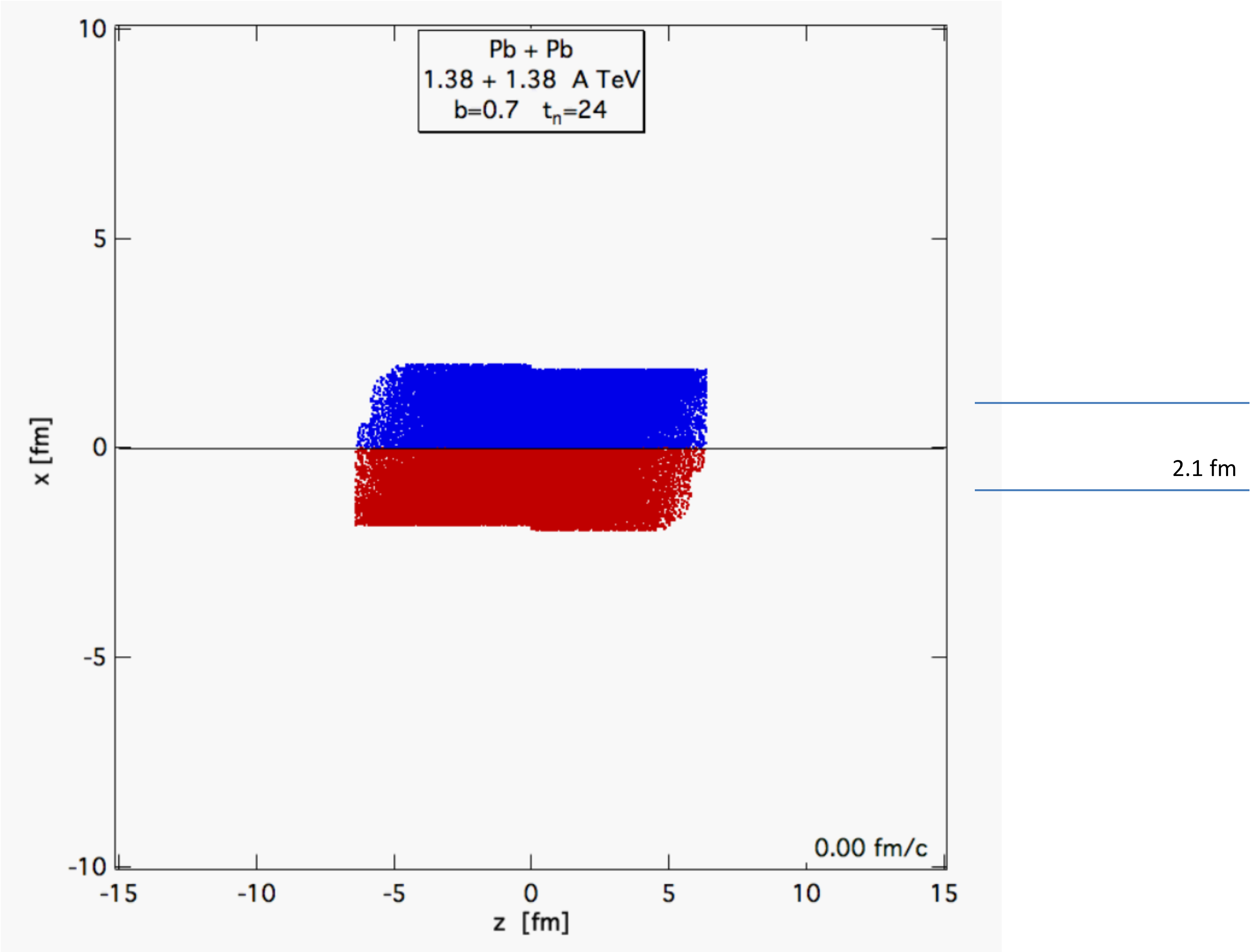
ROTATION



KHI

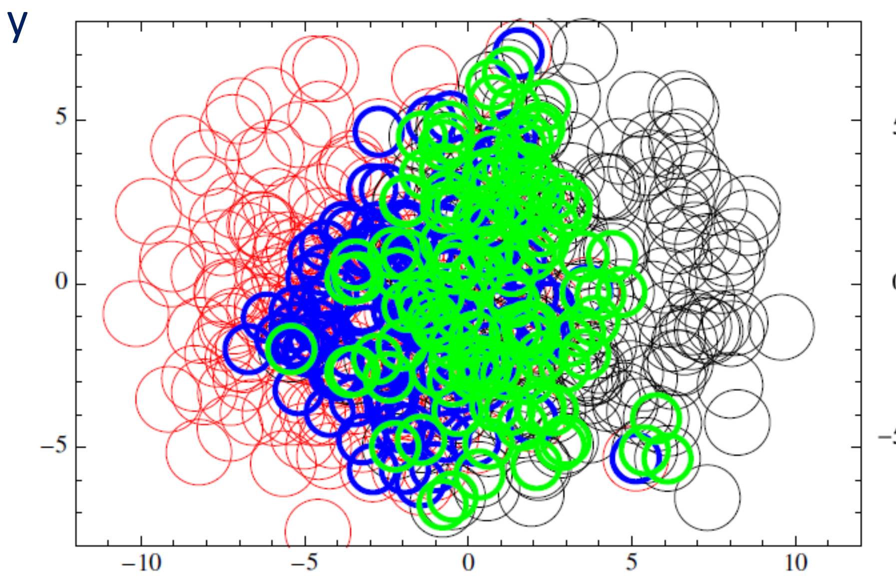
2.4 fm

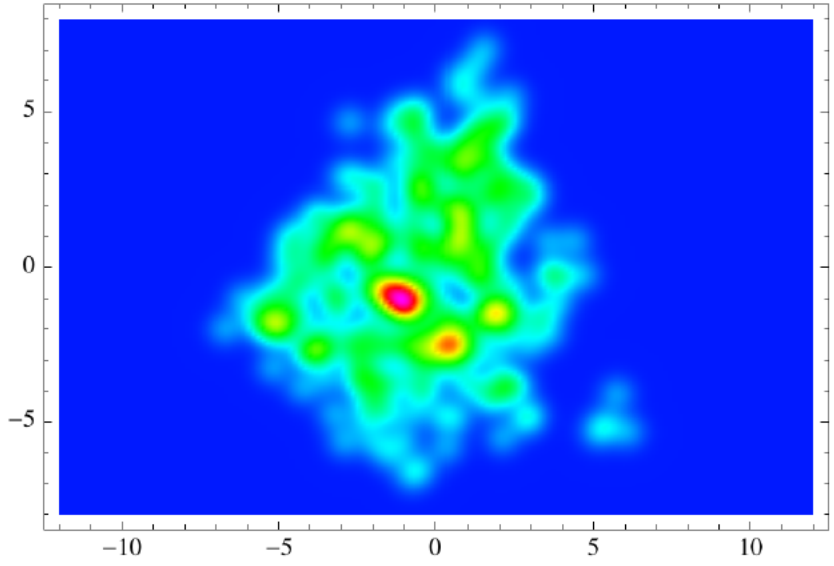
FIG. 1: (color online) Growth of the initial stage of Kelvin-Helmholtz instability in a 1.38*A* + 1.38*A* TeV peripheral, $b = 0.7 b_{\max}$, Pb+Pb collision in a relativistic CFD simulation using the PIC-method. We see the positions of the marker particles (Lagrangian markers with fixed baryon number content) in the reaction plane. The calculation cells are $dx = dy = dz = 0.4375$fm and the time-step is 0.04233 fm/c The number of randomly placed marker particles in each fluid cell is 8^3. The axis-labels indicate the cell numbers in the x and z (beam) direction. The initial development of a KH type instability is visible from $t = 1.5$ up to $t = 7.41$ fm/c corresponding from 35 to 175 calculation time steps).


Pb + Pb
1.38 + 1.38 A TeV
b=0.7 t_n=24
x [fm]
z [fm]
0.00 fm/c
10
5
0
-5
-10
-15
-10
-5
0
5
10
15
2.1 fm

Onset of turbulence around the Bjorken flow

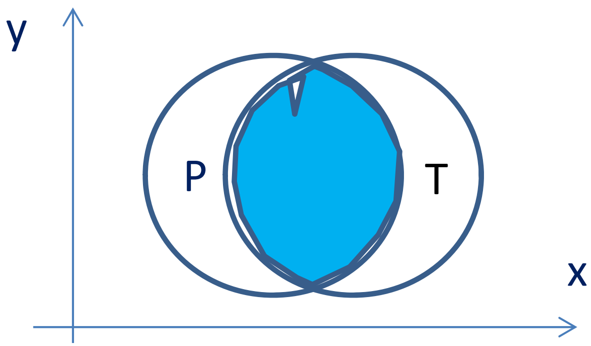
S. Floerchinger & U. A. Wiedemann, JHEP 1111:100, 2011; arXiv: 1108.5535v1





nucleons x [fm] energy density x [fm]

- Transverse plane [x,y] of a Pb+Pb HI collision at $\sqrt{s_{NN}}$=2.76TeV at b=6fm impact parameter
- Longitudinally [z]: **uniform** Bjorken flow, (expansion to infinity), depending on τ only.

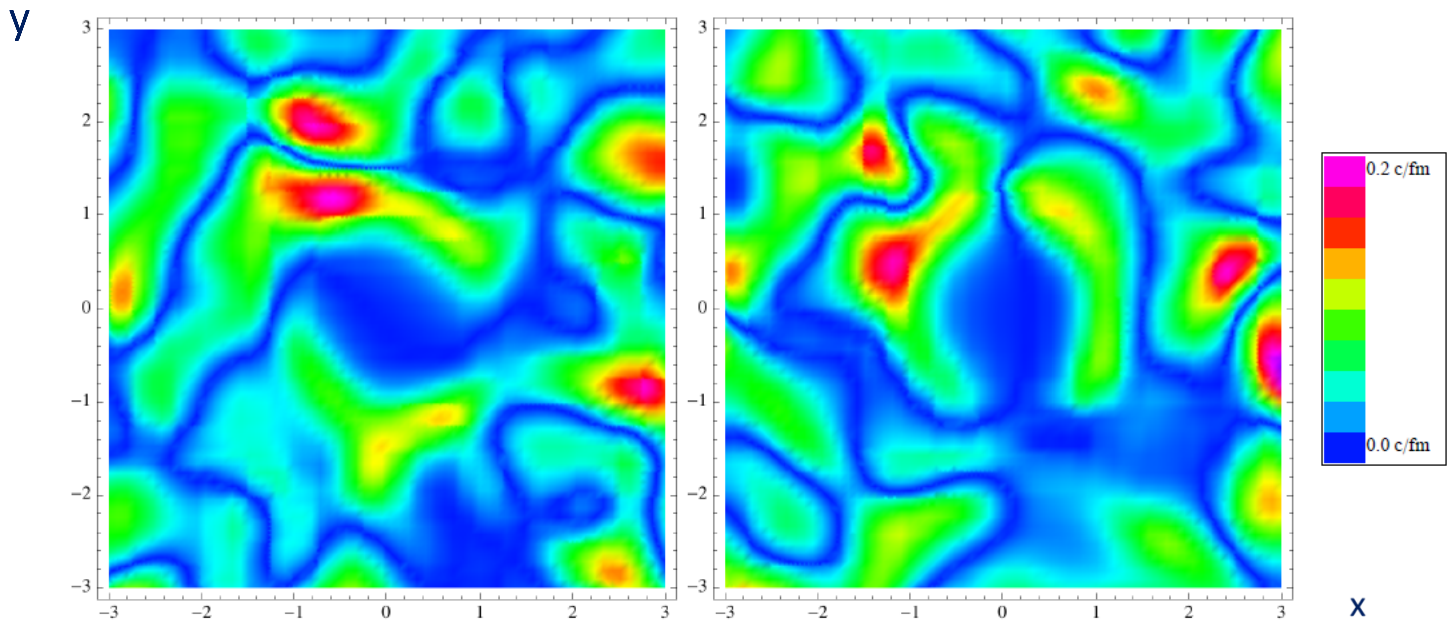


Green and **blue** have the same longitudinal speed (!) in this model. Longitudinal shear flow is omitted.

Onset of turbulence around the Bjorken flow

S. Floerchinger & U. A. Wiedemann, JHEP 1111:100, 2011; arXiv: 1108.5535v1

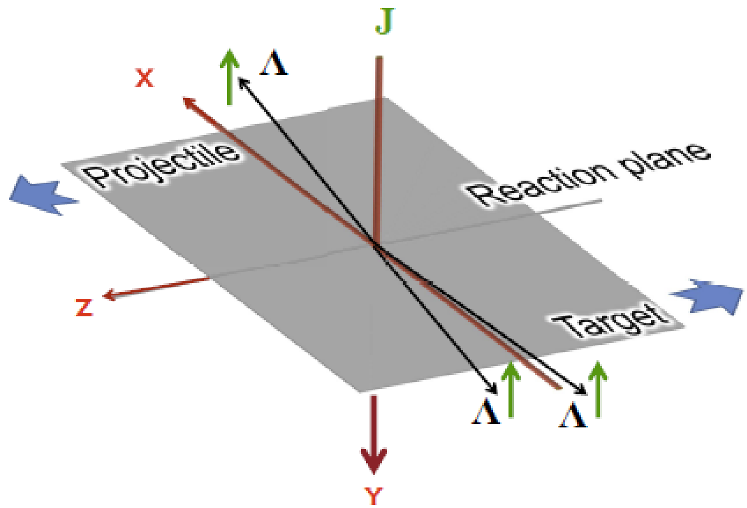
Max = 0.2 c/fm



absolute value of vorticity $|\partial_1 u^2 - \partial_2 u^1|$ divergence $|\partial_1 u^1 + \partial_2 u^2|$

- Initial state Event by Event vorticity and divergence fluctuations.
- Amplitude of random vorticity and divergence fluctuations are the same
- In dynamical development viscous corrections are negligible (→ no damping)
- Initial transverse expansion in the middle (±3fm) is neglected (→ no damping)
- High frequency, high wave number fluctuations **may feed** lower wave numbers

Detecting rotation: Lambda polarization

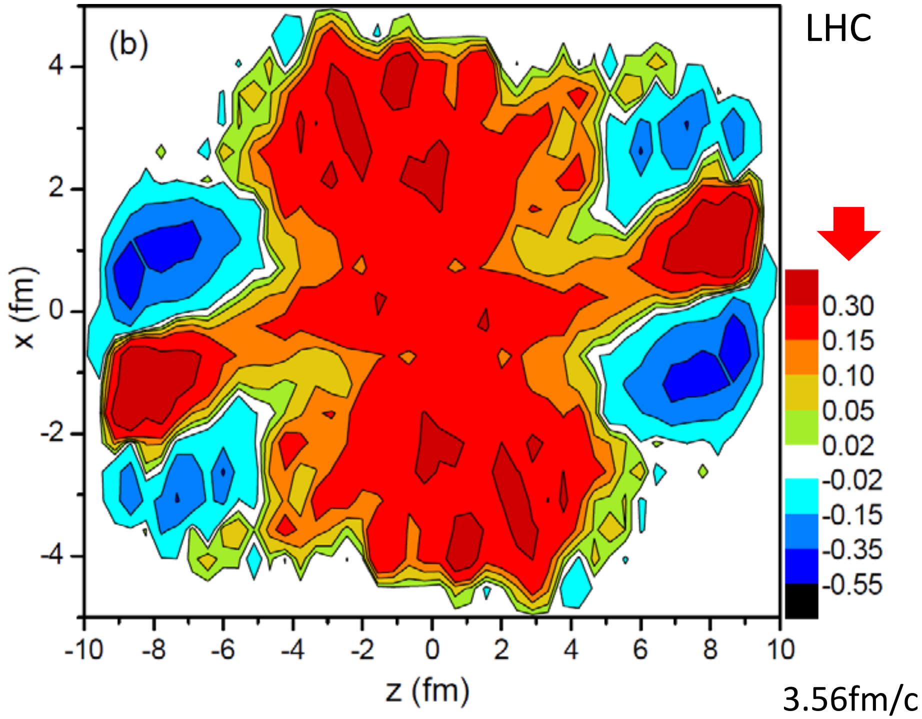


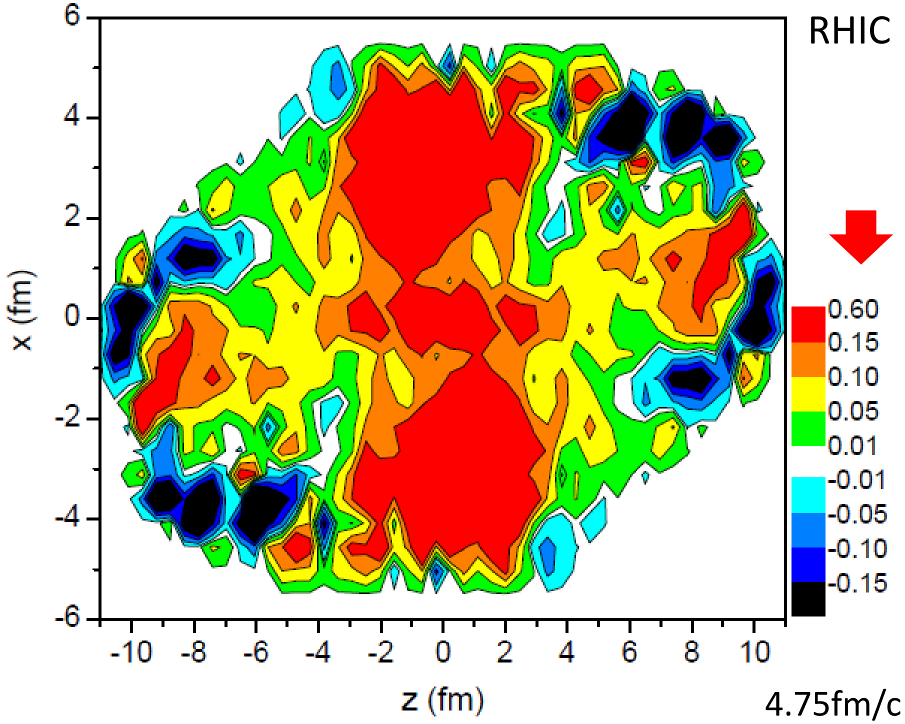
$$\mathbf{\Pi}(p) = \frac{\hbar\varepsilon}{8m}\frac{\int \mathrm{d}V\, n_F\, (\nabla \times \beta)}{\int \mathrm{d}V\, n_F}$$

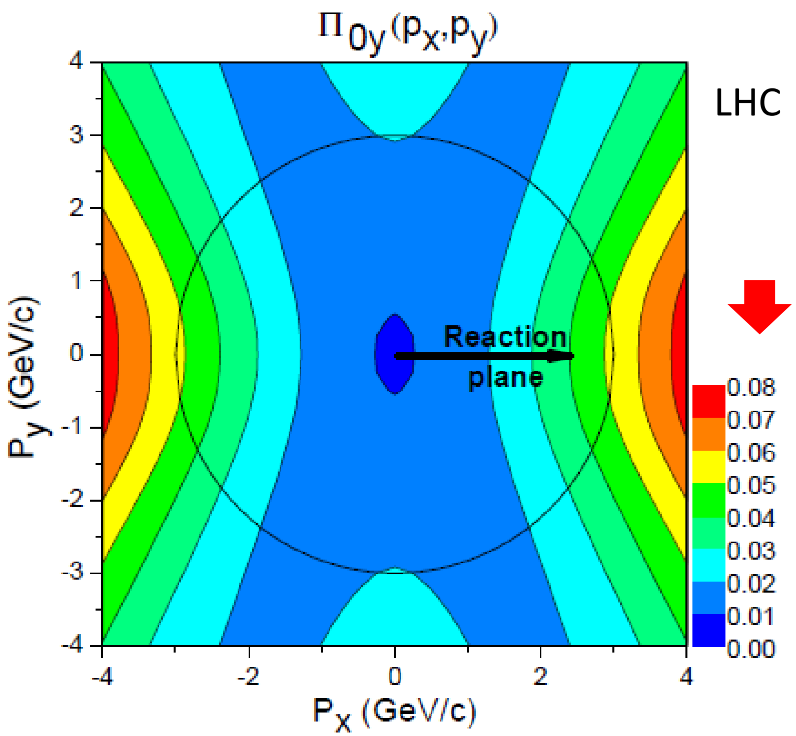
$$\beta^\mu(x) = \underline{(1/T(x))u^\mu(x)}$$ ← From hydro

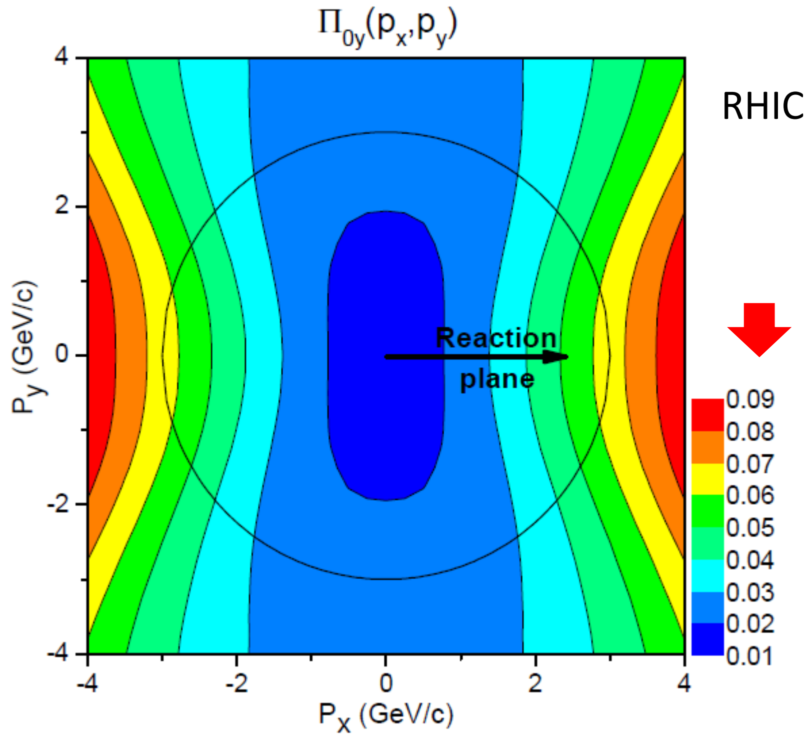
$$\mathbf{\Pi}_0(p) = \mathbf{\Pi}(p) - \frac{\mathbf{p}}{\varepsilon(\varepsilon+m)}\mathbf{\Pi}(p)\cdot\mathbf{p}$$

[F. Becattini, L.P. Csernai, D.J. Wang, Phys. Rev. C **88**, 034905 (2013)]









- The **POLARIZATION** of Λ and $\overline{\Lambda}$ due to thermal equipartition with local vorticity is slightly stronger at RHIC than at LHC due to the much higher temperatures at LHC.
- Although early measurements at RHIC were negative, these were averaged over azimuth! We propose selective measurement in the reaction plane (in the +/- x direction) in the EbE c.m. frame. Statistical error is much reduced now, so significant effect is expected at $p_x \geq 3$ GeV/c.

The Differential HBT method

The method uses two particle correlations: $$C(p_1, p_2) = \frac{P_2(p_1, p_2)}{P_1(p_1) P_1(p_2)},$$

with k= $(p_1+p_2)/2$ and q=p_1-p_2 : $$C(k,q) = 1 + \frac{R(k,q)}{|\int d^4x\ S(x,k)|^2},$$

where $$R(k,q) = \int d^4x_1\, d^4x_2\, \cos[q(x_1 - x_2)] \times S(x_1, k + q/2)\, S(x_2, k - q/2).$$

and S(k,q) is the space-time source or emission function, while R(k,q) can be calculated with, $\boldsymbol{k} = \boldsymbol{p}/\hbar$, & the help of a function J(k,q): $$J(k,q) = \int d^4x\ S(x, k + q/2) \exp(iqx),$$

which leads to: $$R(k,q) = Re\,[J(k,q)\, J(k, -q)].$$

This is one of the standard method used for many years. The crucial is the function **S(k,q).**

The space-time source function, S(k,q)

- Let us start from the pion phase space distribution function in the Jüttner approximation,
  with $n_\pi(x) \propto n(x)\,\sigma(x)$

$$f^J(x,p) = \frac{n(x)\sigma(x)}{C_\pi} \exp\left(-\frac{p^\mu u_\mu(x)}{T(x)}\right),$$

$$\int d^4x\; S(x,p) = \int d^4x\; f^J(x,p)\; P(x,p)$$

$$= \int d^4x\; f^J(x,p)\; \delta(x - x_{\mathrm{FO}})\; p^\mu \hat{\sigma}_\mu$$

- Then

$$J(k,q) = \int d^4x\; S(x,k) \exp\left[-\frac{q \cdot u(x)}{2T(x)}\right] \exp(iqx).$$

- and

$$R(k,q) = \int d^4x_1 d^4x_2\; S(x_1,k)S(x_2,k)\cos[q(x_1 - x_2)]$$

$$\times \exp\left[-\frac{q}{2} \cdot \left(\frac{u(x_1)}{T(x_1)} - \frac{u(x_2)}{T(x_2)}\right)\right],$$

The space-time source function, S(k,q)

- Let us now consider the emission probability in the direction of ***k***, for sources *s* :

$$\int d^4x\, S(x,k) = \sum_s \int_s d^3x_s\, dt_s\, S(x_s,k)$$

$$= (2\pi R^2)^{3/2} \sum_s \frac{\gamma_s n_s(x)\big(k_\mu\, \hat{\sigma}_s^\mu\big)}{C_s} \times \exp\left[-\frac{k\cdot u_s}{T_s}\right],$$

- In this case the *J*-function becomes:

$$J(k,q) = \sum_s e^{-\frac{q}{2}\cdot\frac{u_s}{T_s}}\, e^{iqx_s} \int_s d^4x\; S_s(x,k)\, e^{iqx}.$$

- We perform summations through pairs reflected across the c.m.: $\{i,j,k\}$- $\{i^*,j^*,k^*\}$

$$\int d^4x S(x,k) \;\; = (2\pi R^2)^{3/2} \sum_s P_s\left[w_s \exp\left(\frac{\boldsymbol{k}\boldsymbol{u}_s}{T_s}\right) + w_s^* \exp\left(\frac{\boldsymbol{k}\boldsymbol{u}_s^*}{T_s}\right)\right]$$

$$J(k,q) = Q_c \sum_s P_s\left[Q_s^{(q)} w_s \exp\left[\left(\boldsymbol{k}+\frac{\boldsymbol{q}}{2}\right)\frac{\boldsymbol{u}_s}{T_s}\right] e^{iqx_s} + Q_s^{(q)} w_s^* \exp\left[\left(\boldsymbol{k}+\frac{\boldsymbol{q}}{2}\right)\frac{\boldsymbol{u}_s^*}{T_s}\right] e^{iqx_s^*}\right]$$

where

$$Q_c = (2\pi R^2)^{3/2} \exp\left[-\frac{R^2 q^2}{2}\right],\quad Q_s^{(q)} = \exp\left[-\frac{q_0\, u_s^0}{2T_s}\right],\quad P_s = \frac{\gamma_s n_s}{C_s} \exp\left[-\frac{k_0\, u_s^0}{T_s}\right],\quad w_s = \left(k_\mu\, \hat{\sigma}_s^\mu\right) \exp\left[-\frac{\Theta_s^2}{2} q_0^2\right],$$

Results

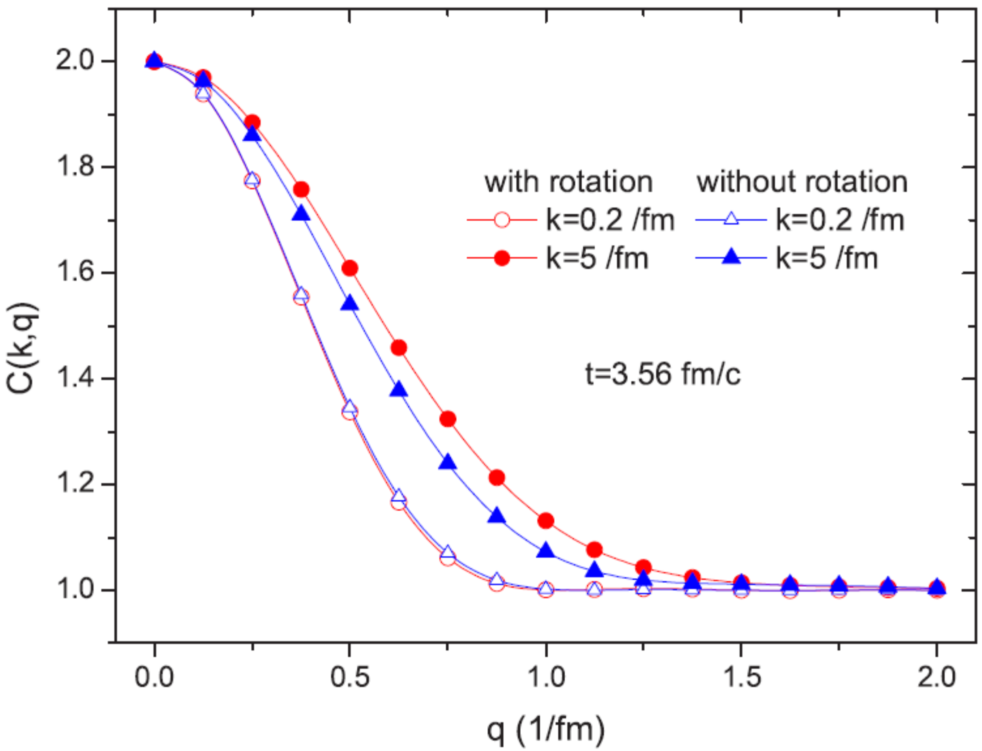


FIG. 1. (Color online) Dependence of the standard correlation function in the $\boldsymbol{k}_+$ direction from the collective flow, at the final time, 3.56 fm/c after reaching local equilibrium and 8.06 fm/c from the first touch, including the initial longitudinal expansion Yang-Mills field dynamics [19].

$b = 0.7 b_{\max}$,

KHI occurred

$L \approx 10^6 \hbar$ [7].

The correlation function depends on the direction and size of **k** , and on rotation.

→ we introduce two vectors $\boldsymbol{k}_+$, $\boldsymbol{k}_-$ symmetrically and define the Differential c.f. (DCF):

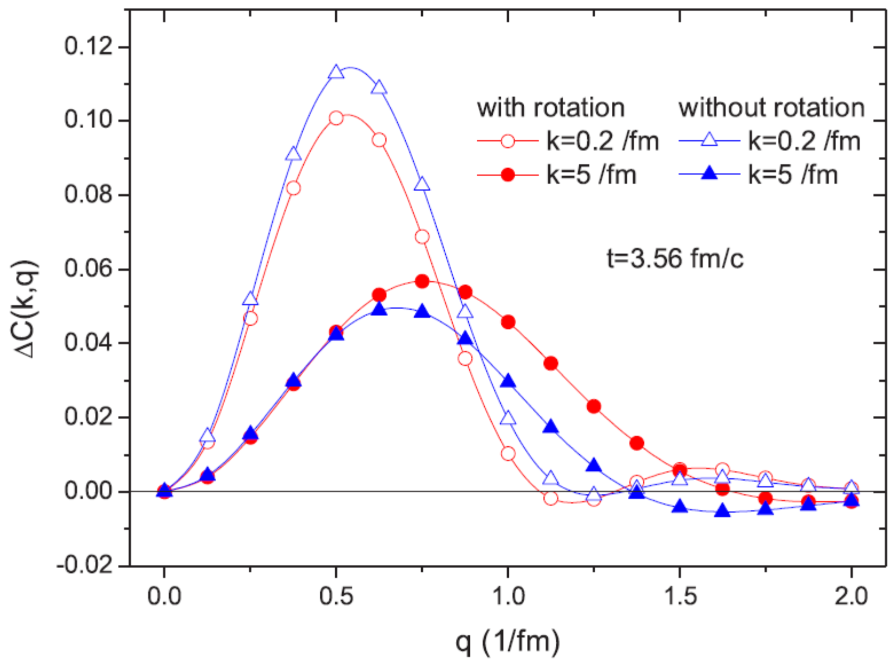
$$\Delta C(k,q) \equiv C(k_+,q_{\text{out}}) - C(k_-,q_{\text{out}}).$$

The DCF would vanish for symmetric sources (e.g. spherical and non-rotation sources)

Results

PHYSICAL REVIEW C **89**, 034916 (2014)

FIG. 2. (Color online) Differential correlation function, $\Delta C(k,q)$, at the final time with and without rotation.



We can rotate the frame of reference:

$$\boldsymbol{k}'(\alpha) = \begin{Bmatrix} k_{x'} \\ k_{z'} \end{Bmatrix} = \begin{Bmatrix} k_x \cos\alpha - k_z \sin\alpha \\ k_z \cos\alpha + k_x \sin\alpha \end{Bmatrix}.$$

$$\rightarrow \quad \Delta C_\alpha(\boldsymbol{k}', \boldsymbol{q}'),$$

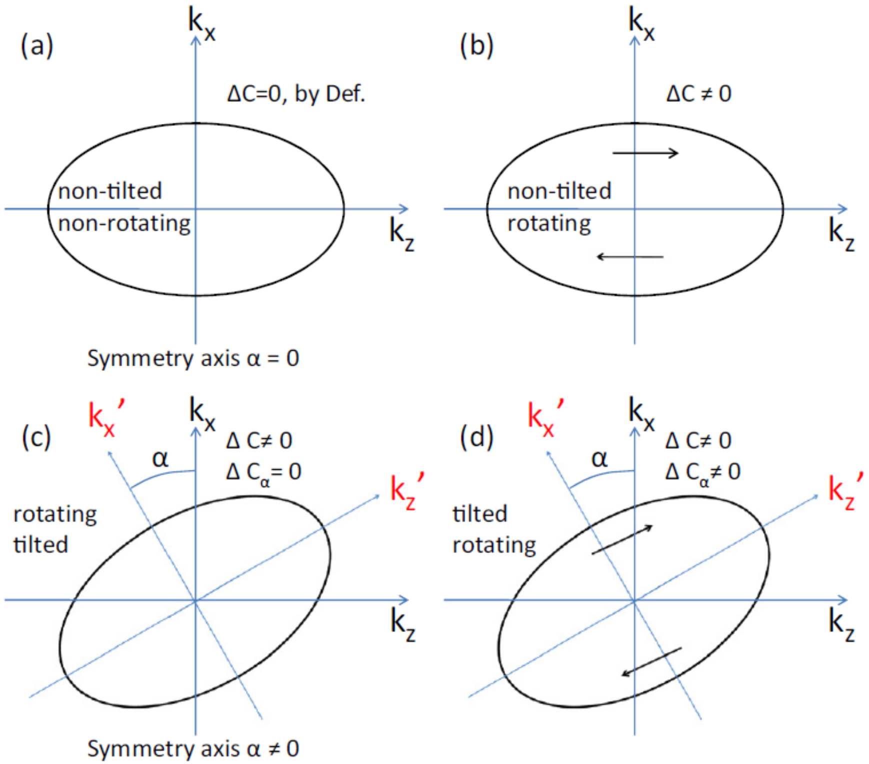
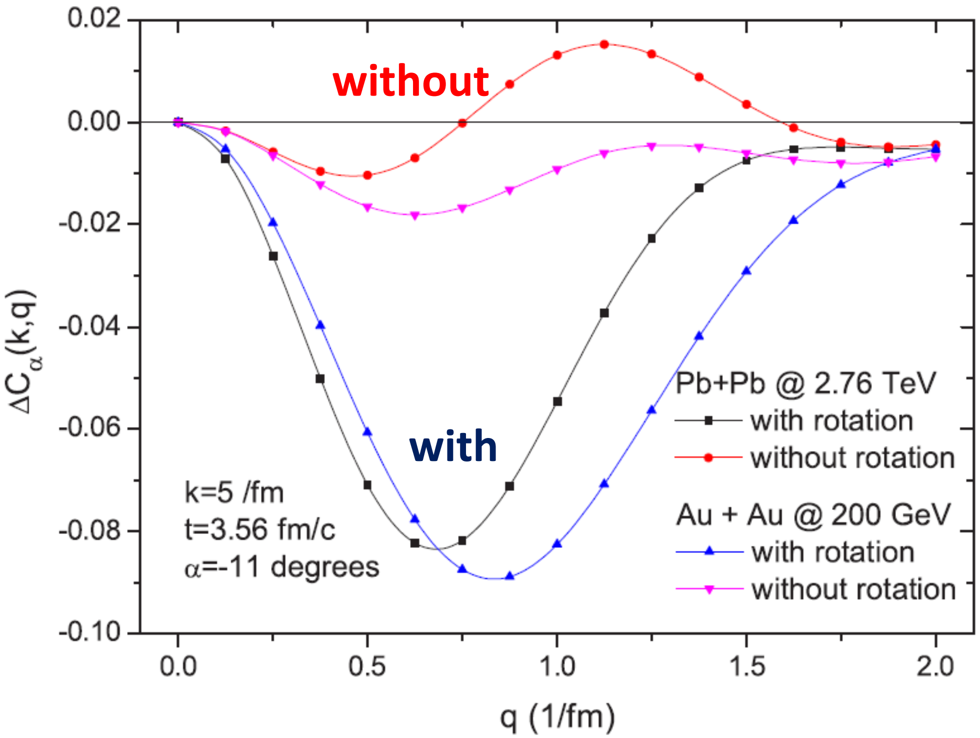


FIG. 3. (Color online) Sketch of the configuration in different reference frames, with and without rotation of the flow. The nonrotating configurations may have radial flow velocity components only. The DCF, $\Delta C_\alpha(k,q)$, is evaluated in a K' reference frame rotated by an angle α in the x,z, reaction plane. We search for the angle α, where the nonrotating configuration is "symmetric," so that it has a "minimal" DCF as shown in Fig. 4.

Results

FIG. 5. (Color online) The DCF with and without rotation in the reference frames, deflected by the angle α, where the rotationless DCF is vanishing or minimal. In this frame the DCF of the original, rotating configuration indicates the effect of the rotation only. The amplitude of the DCF of the original rotating configuration doubles for the higher energy (higher angular momentum) collision.

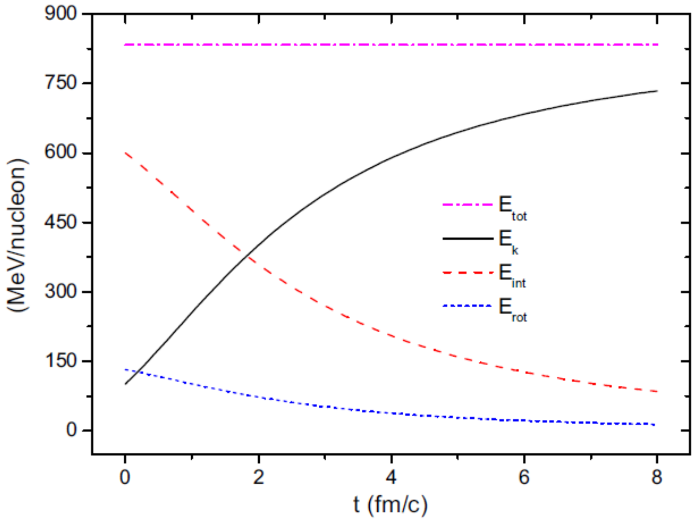


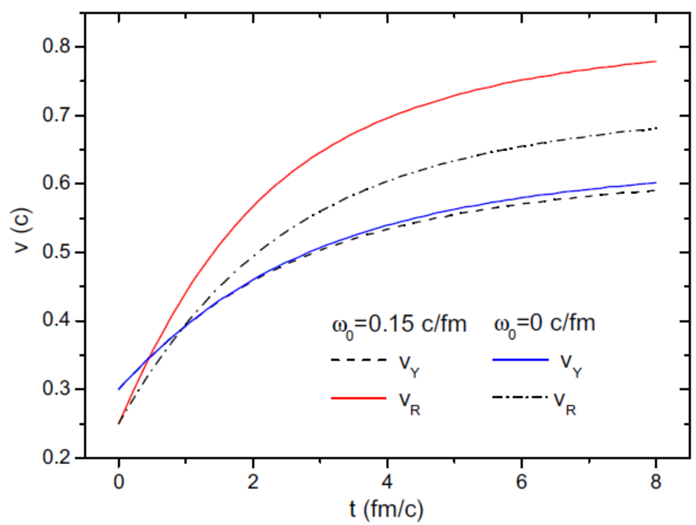
To perform the analysis in the rotationless symmetry frame one can find the symmetry axis the best with the azimuthal HBT method, which provides even the transverse momentum dependence of this axis [20]. It is also important to determine the precise event-by-event c.m. position of the participants [21] and minimize the effect of fluctuations to be able to measure the emission angles accurately, which is crucial in the present $\Delta C(k,q)$ studies.

L. P. Csernai, D. J. Wang and T. Csörgő, PHYSICAL REVIEW C **90**, 024901 (2014)

Rotation in an exact hydrodynamical model

based on T.Csorgo & M.I.Nagy, Phys.Rev.C**89**, 044901 (2014).



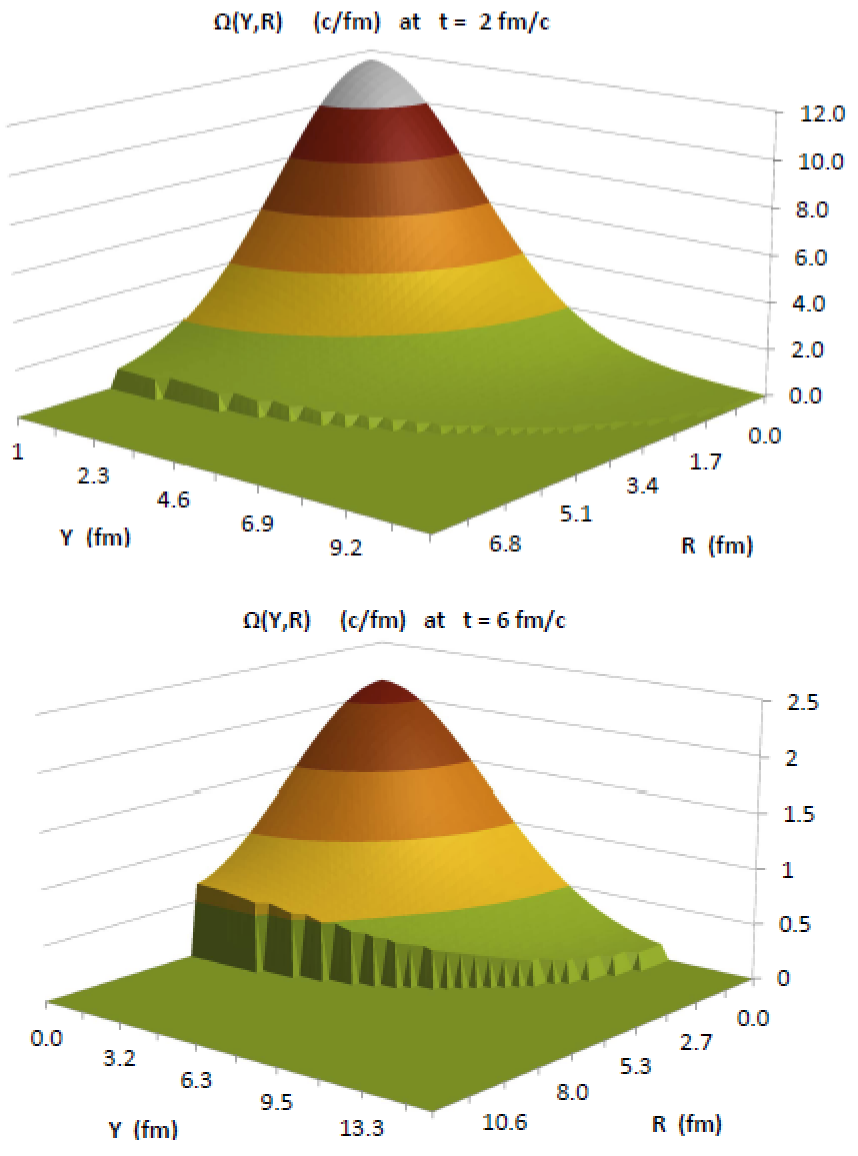


Two alternative boundary configurations:
infinite w/ Gaussian tails
or finite cylindrical shape →

The model is nonrelativistic, has a Hubble flow profile → tail behavior may be unrealistic

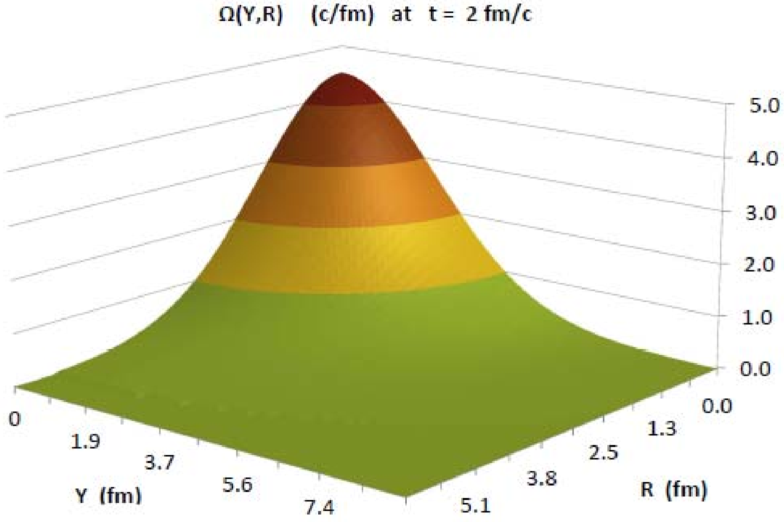
L.P. Csernai, J.H. Inderhaug, in preparation: Vorticity in the exact model:

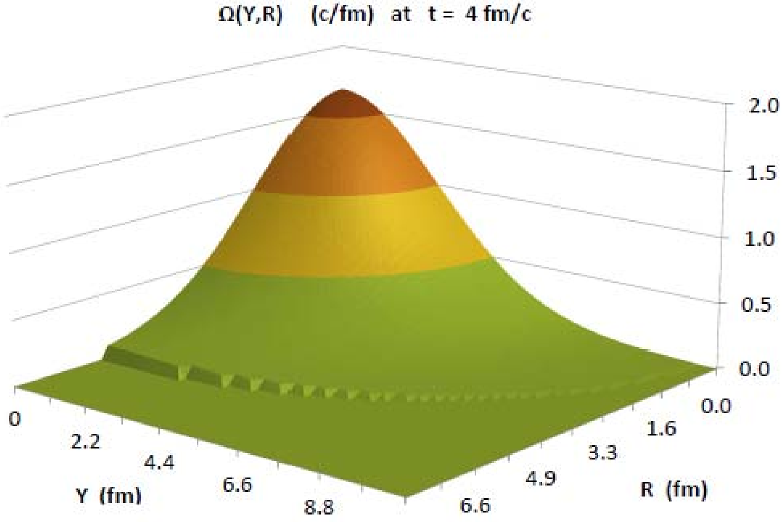
R_0=3.5 fm, Y_0 = 5 fm, v_R=0.25 c, v_Y=0.3c, ω_0=0.1c/fm, T_0=400 MeV

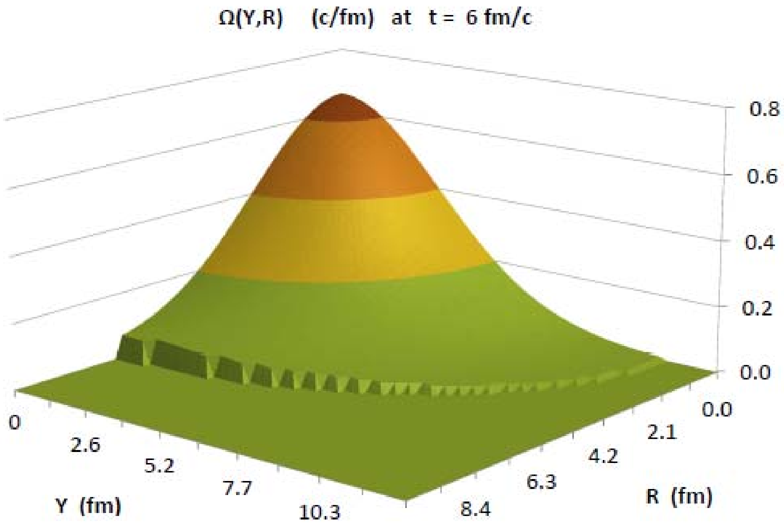


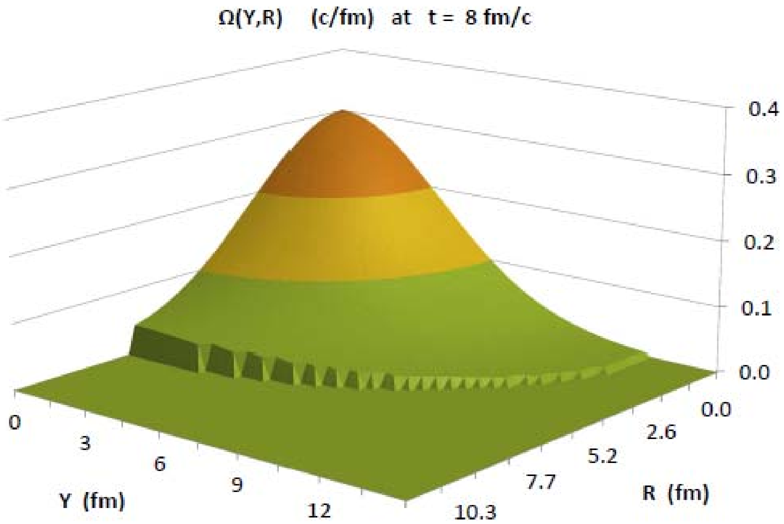
L.P. Csernai, J.H. Inderhaug, in preparation: Vorticity in the exact model:

R_0=3.16 fm, Y_0 = 4.64 fm, v_R=0.43 c, v_Y=0.34c, ω_0=0.06c/fm, T_0=300 MeV



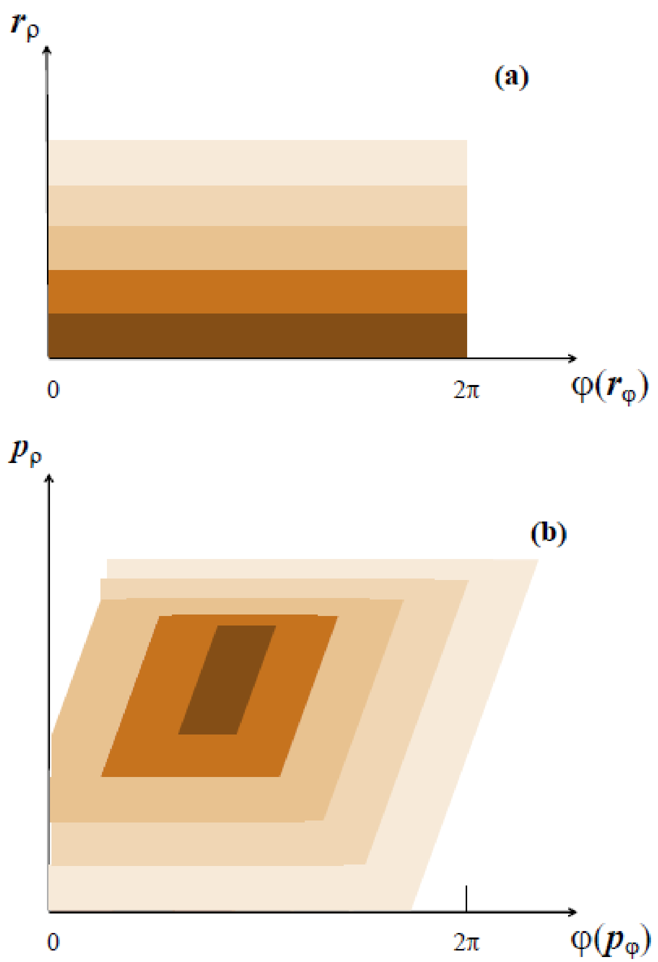


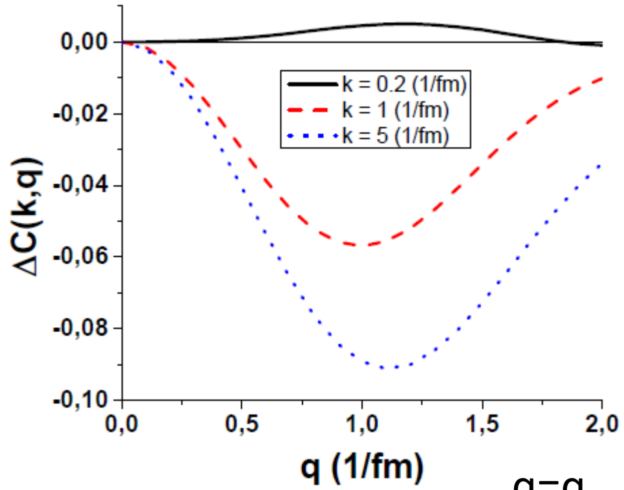




S. Velle, L.P. Csernai, in preparation: Differential HBT for an exact hydro model with rotation

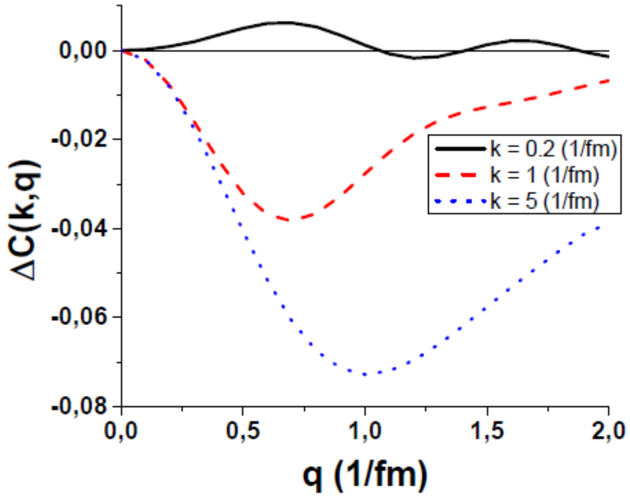
The model is azimuthally symmetric in the configuration space but not in the momentum space:





t=0 fm/c

$q=q_{out}$



t=3 fm/c

$q=q_{out}$

There seems to be a paradox btwn C(k,q) & DC(k,q)
but … $C(k,q_{out})$ or $C(k,q_{long})$ or $C(k,q_z)$ are …. !

Summary

- HIC models: Initial State + **EoS** + Freeze out & Hadronization
- In peripheral A+A the interactions are causing collective flow with shear and rotation
- Rotation can be enhanced by KHI at low viscosity
- Experimental outcome must be observable both by
  - **Polarization** ← from **L** equipartition of **roatation**&**spin**
  - **Diff. HBT** ← from ***azimuthal asymmetry in phase space***
  - V_1 is small – difficult to observe
  - **C.M.** and **RP$_{(P/T)}$** determination EbE is of utmost importance!

Thank you