How does viscosity and the speed of sound effect spatial asymmetries in heavy ion collisions?

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Motivation

- how some simple effects influence time evolution of asymmetries
- effects which can't be discussed analytically
- not real initial condition (e.g. from Monte-Carlo simulation): real simulation mixes effects
- initial condition close to exact solution but more realistic

Contents

Equations of hydrodynamics

- Nonrelativistic hydrodynamics:

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \rho \mathbf{v} = 0 \tag{1}$$

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\boldsymbol{\nabla})\mathbf{v}\right) = -\boldsymbol{\nabla} p + \mu \Delta \mathbf{v} + \left(\zeta + \frac{\mu}{3}\right)\boldsymbol{\nabla}(\boldsymbol{\nabla}\mathbf{v}) + \mathbf{f} \tag{2}$$

$$\frac{\partial \varepsilon}{\partial t} + \boldsymbol{\nabla} \varepsilon \mathbf{v} = -p\boldsymbol{\nabla}\mathbf{v} + \boldsymbol{\nabla}(\boldsymbol{\sigma}\mathbf{v}) \tag{3}$$

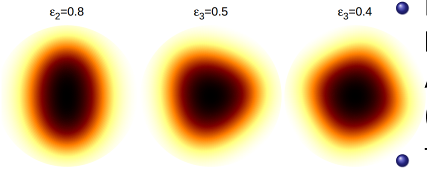
- We need an EoS:

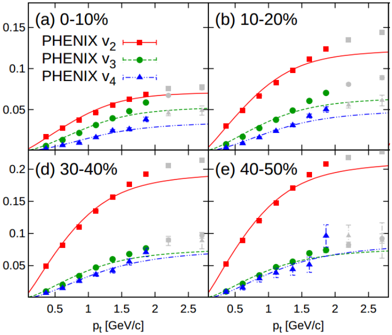
$$\varepsilon = \kappa(T)p \tag{4}$$

- Relativistic case:

$$T^{\mu\nu} = (\varepsilon + p)\frac{u^\mu u^\nu}{c^2} - pg^{\mu\nu}, \quad \partial_\mu T^{\mu\nu} = 0 \tag{5}$$

Multipole solution





- New exact solution of relativistic hydrodynamics by Máté Csanád and András Szabó, published Phys.Rev. C90 (2014) 054911
- The solution in cylindrical coordinates:
  $u^\mu = \frac{x^\mu}{\tau}$, $n = n_f \left(\frac{\tau_f}{\tau}\right)^3 \nu(s)$,
  $p = p_f \left(\frac{\tau_f}{\tau}\right)^{3+3/\kappa}$
- Where τ is the coordinate-proper time, τ_f is the freeze-out proper time
- The s scale variable with any asymmetries
  $$s = \frac{r^N}{R^N}\left(1 + \epsilon_N \cos N\phi\right) + \frac{z^N}{R^N}$$
- That's a solution if $R = u_t t$

Numerical scheme

- At mid-rapidity distributions have local maximum and are constant in its environment, so enough to solve hydro in 2+1 dimension
- Transform equations to advection from:
  $\partial_t Q_i + \partial_x F_i(Q) + \partial_y G_i(Q) = 0$
- Solve numerically: discretization
- Finite volume method: average of quantities in control volume, that contains the grid point
- Problem: we have to evaluate fluxes between grid points, exactly not possible
- Instability: we can add to real solution a wave solution which is null at grid points $\rightarrow$ Courant–Friedrichs–Lewy condition (e.g. $C = u\Delta t/\Delta x < 1$)
- 2 spatial dimension difficult $\rightarrow$ operator splitting
- Viscosity: ideal substep + step only with viscous fluxes

Numerical scheme: MUSTA method

- This method was published by E. F. Toro et al, 2006, J. Comp. Phys
- The l^{th} predicted values: $Q^{(l)}_{i/(i+1)}$, $F^{(l)}_{i/(i+1)} \equiv F\left(Q^{(l)}_{i/(i+1)}\right)$
- Initially: $Q^{(0)}_i \equiv Q^n_i$, $Q^{(0)}_{i+1} \equiv Q^n_{i+1}$
- Intermediate value and flux:

$$Q^{(l)}_{i+\frac{1}{2}} = \frac{1}{2}\left[Q^{(l)}_i + Q^{(l)}_{i+1}\right] - \frac{1}{2}\frac{\Delta t}{\Delta x}\left[F^{(l)}_{i+1} - F^{(l)}_i\right], \quad F^{(l)}_M \equiv F\left(Q^{(l)}_{i+\frac{1}{2}}\right) \quad (6)$$

- Corrected flux:

$$F^{(l)}_{i+\frac{1}{2}} = \frac{1}{4}\left[F^{(l)}_{i+1} + 2F^{(l)}_M + F^{(l)}_i - \frac{\Delta x}{\Delta t}\left(Q^{(l)}_{i+1} - Q^{(l)}_i\right)\right] \quad (7)$$

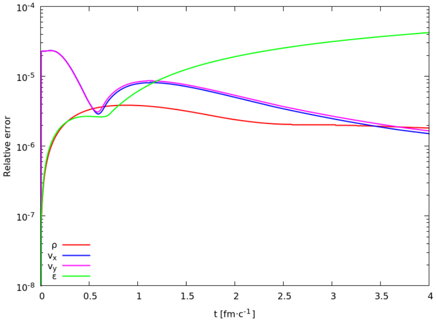
- Next prediction to compute corrected flux:

$$Q^{(l+1)}_i = Q^{(l)}_i - \frac{\Delta t}{\Delta x}\left[F^{(l)}_{i+\frac{1}{2}} - F^{(l)}_i\right] \quad (8)$$

$$Q^{(l+1)}_{i+1} = Q^{(l)}_{i+1} - \frac{\Delta t}{\Delta x}\left[F^{(l)}_{i+1} - F^{(l)}_{i+\frac{1}{2}}\right] \quad (9)$$

Code testing

- We tested our code with exact solutions (from PRC67 (2003)):
  $s = \frac{x^2}{X^2(t)} + \frac{y^2}{Y^2(t)}$, $\rho = \rho_0 \frac{V_0}{V} e^{-s}$, $p = p_0 \left(\frac{V_0}{V}\right)^{1+\frac{1}{\kappa}}$,
  $\mathbf{v}(t, \mathbf{r}) = \left(\frac{\dot{X}}{X}x, \frac{\dot{Y}}{Y}y\right)$, $\ddot{X}X = \ddot{Y}Y = \frac{T_i}{m}\left(\frac{V_0}{V}\right)^{\frac{1}{\kappa}}$, $V = X(t)Y(t)$
- Relative difference between exact and numerical solution ($X = Y$ and $X \neq Y$ case):



Description of asymmetries

- We defined as asymmetry parameters: $\varepsilon_n = \langle \cos(n\phi) \rangle_{\rho/w/p}$
- The $w = \exp\left(-v_x^2 - v_y^2\right)$ is defined to calculate the asymmetry of speed distribution
- This ε_n not equal with the ϵ in s scale variable $(\rho,\ p \propto \exp\left(-s\right))$
- Initially we can approximate ε_n with ϵ_n using Taylor-series:

$$\varepsilon_1 = \frac{(\epsilon_2 + \epsilon_4)\epsilon_3}{2 + \sum_{n=2}^{4} \epsilon_n^2} \tag{10}$$

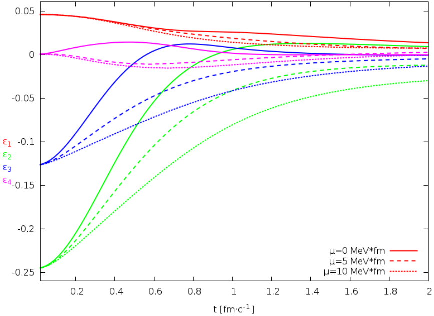
$$\varepsilon_2 = \frac{-\epsilon_2 + \epsilon_2\epsilon_4}{2 + \sum_{n=2}^{4} \epsilon_n^2} \tag{11}$$

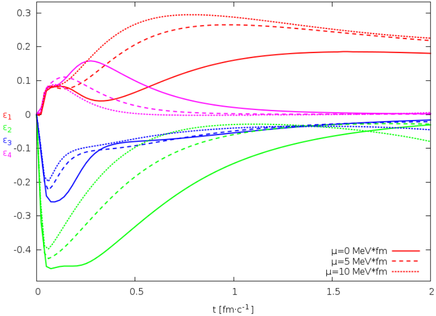
$$\varepsilon_3 = \frac{-\epsilon_3}{2 + \sum_{n=2}^{4} \epsilon_n^2} \tag{12}$$

$$\varepsilon_4 = \frac{-\epsilon_4 + \frac{1}{2}\epsilon_2^2}{2 + \sum_{n=2}^{4} \epsilon_n^2} \tag{13}$$

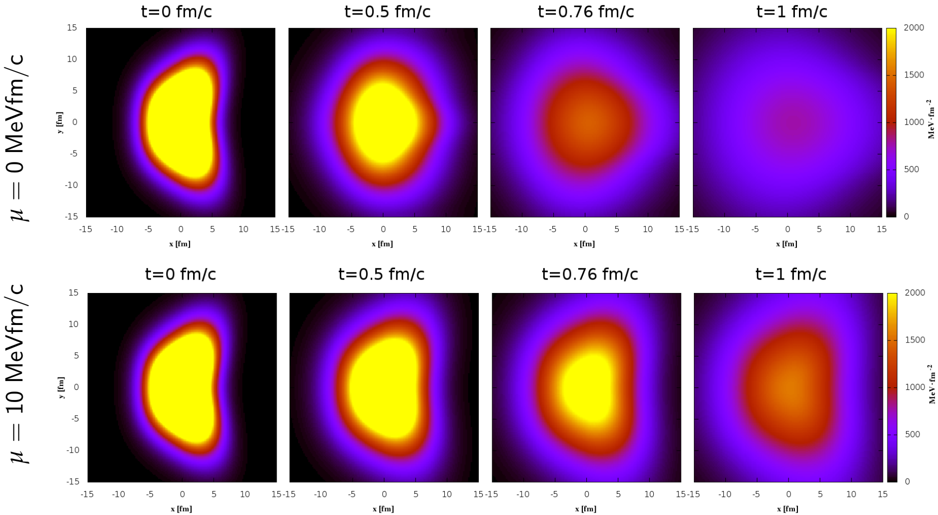
Effect of viscosity

- In energy and mass density the viscosity makes slower the disappearance
- In speed distribution makes faster

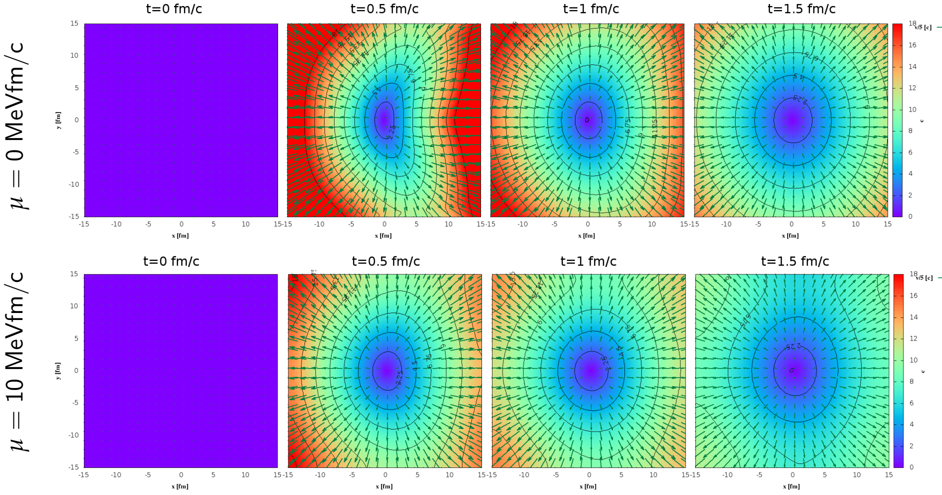




Effect of viscosity: The time evolution of energy density

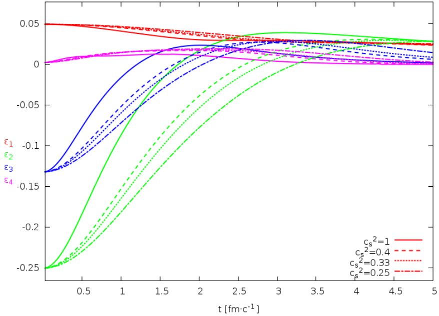


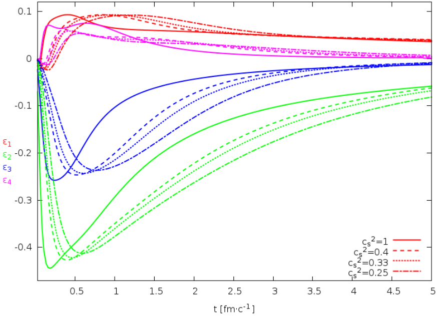
Effect of viscosity: The time evolution of speed distribution



Effect of speed of sound

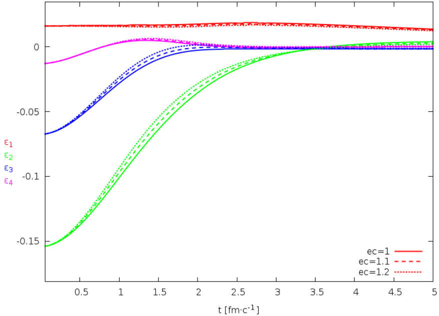
- As we expected the reduction of speed of sound makes the time evolution of asymmetries slower

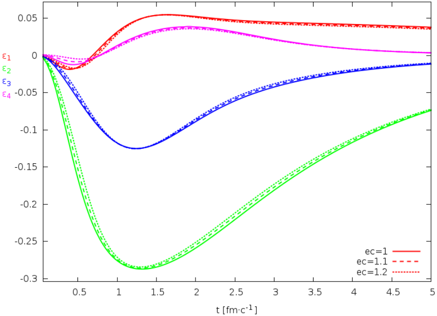




Effect of pressure gradient

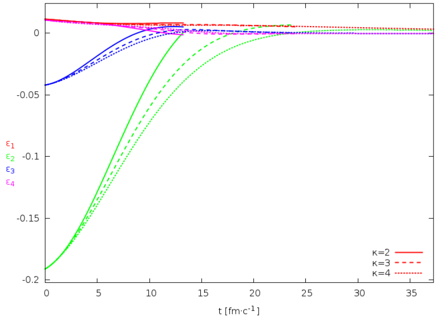
- The increase gradient of pressure makes the flow faster, so the asymmetries disappear faster

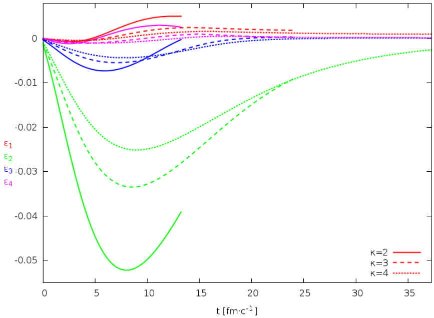




Effect of speed of sound

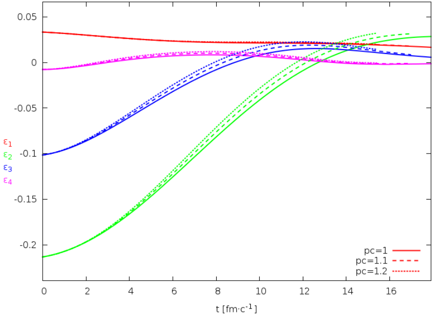
- Same as nonrelativistic case
- Different time to hadronization

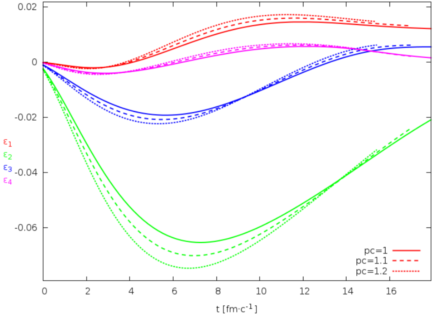




Effect of pressure gradient

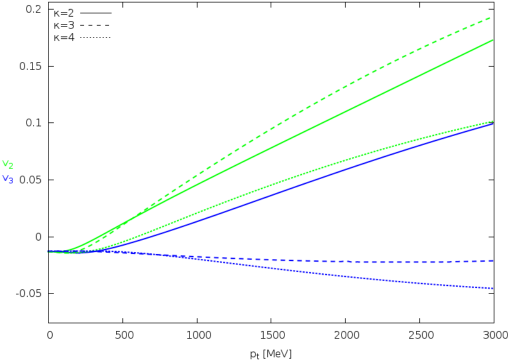
- Same as nonrelativistic case





Hadronization

- Maxwell-Jüttner type source function:
  $$S(x,p)d^4x = \mathcal{N} n(x) \exp\left(-\frac{p_\mu u^\mu}{T(x)}\right) H(\tau) p_\mu d^3 \frac{u_\mu d^3x}{u^0} d\tau$$
- With source function we can simply calculate the measurable quantities: $v_n(p_t) = \langle \cos(n\varphi) \rangle_N = \frac{1}{N(p_t)} \int_0^{2\pi} N(p_t,\varphi)\cos(n\varphi)d\varphi$



Summary

- Motivation: how some simple effects affect time evolution of asymmetries
- No much chance for analytic discussion so we used numerical methods
- Initial condition was very close to the existing exact solutions, but more realistic
- Viscosity makes slower the disappearance of asymmetries in energy density and faster in speed distribution
- Smaller speed of sound makes slower time evolution in all distribution, more time to hadronization