Analysis of Bose-Einstein Correlations at fixed multiplicities in the quantum optical approach

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3. Formulation in the quantum optical approach
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1.Introduction

Semi-inclusive events in pp collisions

$$pp \to n\pi^- + X$$

Normalized two-particle BEC function

$$\tilde{C}_n^{(2)}(\Delta y, \Delta \boldsymbol{p}_T) = \frac{nP(n)}{n-1} C_n^{(2)}(\Delta y, \Delta \boldsymbol{p}_T),$$

$$C_n^{(2)}(\Delta y, \Delta \boldsymbol{p}_T) = \frac{\int\int\int \rho_n^{(2)}(y_1, \boldsymbol{p}_{1T}; y_1 + \Delta y, \boldsymbol{p}_{1T} + \Delta \boldsymbol{p}_T) dy_1\, d^2\boldsymbol{p}_{1T}}{\int\int\int \rho_n^{(1)}(y_1, \boldsymbol{p}_{1T}) \rho_n^{(1)}(y_1 + \Delta y, \boldsymbol{p}_{1T} + \Delta \boldsymbol{p}_T) dy_1\, d^2\boldsymbol{p}_{1T}}$$

- Multiplicity distribution $P(n)$
- Single-particle momentum density $\rho_n^{(1)}(p)$
- Two-particle momentum density $\rho_n^{(2)}(p_1, p_2)$

Parameters on the BEC (source radii) are included in the multiplicity distribution.

Data Analysis CMS pp at 7 TeV $|\eta| < 2.4$

- Multiplicity distribution $P(n)$
- Two-particle BEC $C_n^{(2)}(Q_{\text{inv}})$

2. Formulation in the quantum-optical approach

The n-particle momentum density in the QO approach

$$\rho_n(p_1,\ldots,p_n) = c_0 \left\langle |f(p_1)|^2 \cdots |f(p_n)|^2 \right\rangle_a ,$$

$$f(p) = \sum_{i=1} a_i \phi_i(p) + f_c(p)$$

$\phi_i(p)$: amplitude of the ith chaotic source

$f_c(p)$: amplitude of the coherent source

c_0 : normalization factor

a_i : Gaussian random variable

Parenthesis $\langle F \rangle_a$ denotes the average of F over the complex random number a_i with a Gaussian weight:

$$\langle F \rangle_a = \prod_{i=1} \Big(\frac{1}{\pi \lambda_i} \int \mathrm{exp}[-\frac{|a_i|^2}{\lambda_i}] d^2 a_i \Big) F.$$

$$\rho_1(p_1) = c_0[r(p_1,p_1) + c(p_1,p_1)],$$

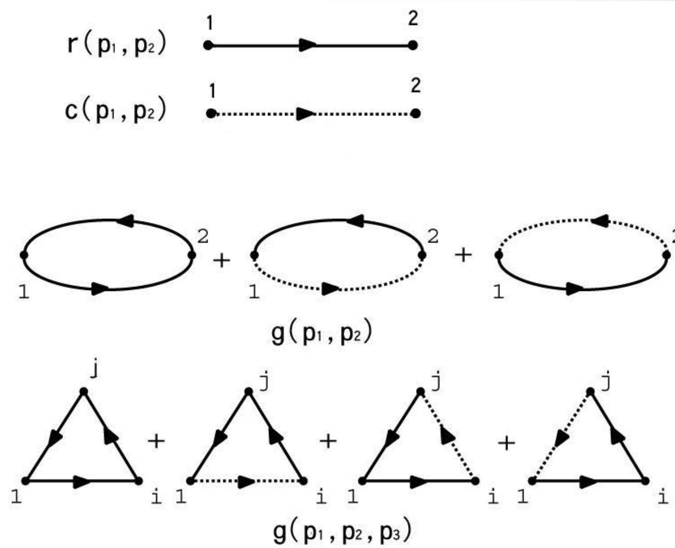
$$\rho_2(p_1,p_2) = c_0 \Big\{ \rho(p_1)\rho(p_2) + |r(p_1,p_2)|^2 + 2\mathrm{Re}[r(p_1,p_2)c(p_2,p_1)] \Big\}$$

$$r(p_1,p_2) = \sum_i \lambda_i \phi_i(p_1) \phi_i^*(p_2), \quad c(p_1,p_2) = f_c(p_1) f_c^*(p_2)$$

Cumulant expansion of momentum densities

$$\rho_1(p_1) = c_0 g_1(p_1),$$
$$\rho_n(p_1,\cdots,p_n) = g_1(p_1)\rho_{n-1}(p_2,\cdots,p_n)$$
$$\sum_{i=1}^{n-2}\sum g_{i+1}(p_1,p_{j_1},\cdots,p_{j_i})\rho_{n-i-1}(p_{j_{i+1}},\cdots,p_{j_{n-1}})$$
$$+c_0 g_n(p_1,\cdots,p_n), \quad n = 1,2,\cdots$$

$$g_1(p_1) = r(p_1,p_1) + c(p_1,p_1),$$
$$g_2(p_1,p_2) = r(p_1,p_2)r(p_2,p_1)$$
$$+c(p_1,p_2)r(p_2,p_1) + r(p_1,p_2)c(p_2,p_1)$$



$(i,j) = (2,3),(3,2)$

k-particle momentum density at fixed n ($n \geq k$)

$$\rho_n^{(k)}(p_1,\cdots,p_k) = \frac{1}{(n-k)!}$$
$$\int\cdots\int \rho_n(p_1,\cdots,p_k,p_{k+1},\cdots,p_n)\frac{d^3p_{k+1}}{E_{k+1}}\cdots\frac{d^3p_n}{E_n}$$

kth order cummulant ($n \geq k$)

$$g_n^{(k)}(p_1,\cdots,p_k) = \frac{1}{(n-k)!}$$
$$\int\cdots\int g_n(p_1,\cdots,p_k,p_{k+1},\cdots,p_n)\frac{d^3p_{k+1}}{E_{k+1}}\cdots\frac{d^3p_n}{E_n}$$

cf. N.S. and M.Biyajima, Prog. Theor. Phys., **88**(1992)609 ;
Phys. Rev. C **60**(1999)034903
T.Csörgő, et al., Eur. Phys. J. C **9**(1999)275

Multiplicity distribution

$$P(n) = \frac{1}{n}\sum_{j=1}^{n} j g_j^{(0)} P(n-j), \quad P(0) = c_0$$

One-particle momentum density at fixed n

$$\rho_n^{(1)}(p) = \sum_{j=1}^{n} j g_j^{(1)}(p) P(n-j)$$

Two-particle momentum density at fixed n

$$\rho_n^{(2)}(p_1, p_2) = \sum_{j=1}^{n-1} (n-j) g_j^{(1)}(p_1) \rho_{n-j}^{(1)}(p_2) + \sum_{j=2}^{n} g_j^{(2)}(p_1, p_2) P(n-j)$$

$$g_j^{(0)} = \frac{1}{j}[\Delta_j^{(R)} + j\Delta_{j-1}^{(T)}], \quad j = 1, 2, \ldots.$$

$$g_j^{(1)}(p_1) = R_j(p_1, p_1) + \sum_{l=0}^{j-1} T_{l,j-l-1}(p_1, p_1), \quad j = 1, 2, \ldots$$

$$g_j^{(2)}(p_1, p_2) = \sum_{l=1}^{j-1} R_l(p_1, p_2) R_{j-l}(p_2, p_1)$$
$$+ \sum_{l=0}^{j-2} \sum_{m=0}^{l} \{T_{m,l-m}(p_1, p_2) R_{j-l-1}(p_2, p_1) + R_{j-l-1}(p_1, p_2) T_{m,l-m}(p_2, p_1)\}, \quad j = 2, 3, \ldots,$$

$R_j(p_1, p_2)$ is made from the chaotic component only, and

$T_{i,j}(p_1, p_2)$ contains the coherent component $c(p_l, p_m)$:

$$R_j(p_1, p_2) = \int r(p_1, k) R_{j-1}(k, p_2) \frac{d^3k}{\omega}, \quad \Delta_j^{(R)} = \int R_j(k, k) \frac{d^3k}{\omega}, \quad j = 1, 2, \ldots$$
$$T_{j,l}(p_1, p_2) = \int R_j(p_1, k_1) c(k_1, k_2) R_l(k_2, p_2) \frac{d^3k_1}{\omega_1} \frac{d^3k_2}{\omega_2}, \quad \Delta_{j+l}^{(T)} = \int T_{j,l}(k, k) \frac{d^3k}{\omega}, j, l = 0, 1, \ldots,$$

with $R_0(k_1, k_2) = \omega_1 \delta^3(\boldsymbol{k}_1 - \boldsymbol{k}_2)$.

2-1.Parametrization

Chaotic component

$$r(y_1, \mathbf{p}_{1T}; y_2, \mathbf{p}_{2T}) \;=\; p_{\rm sm}\sqrt{\rho(y_1, \mathbf{p}_{1T})\rho(y_2, \mathbf{p}_{2T})}\, I(\Delta y, \Delta \mathbf{p}_{1T}),$$

Coherent component

$$c(y_1, \mathbf{p}_{1T}; y_2, \mathbf{p}_{2T}) \;=\; (1 - p_{\rm sm})\sqrt{\rho(y_1, \mathbf{p}_{1T})\rho(y_2, \mathbf{p}_{2T})},$$

$$\begin{aligned}
\rho(y_1, \mathbf{p}_{1T}) &= \langle n_0 \rangle \sqrt{\frac{\pi}{\alpha}\frac{\pi}{\beta}}\, \mathrm{exp}[-\alpha\, y_1^2 - \beta\, \mathbf{p}_{1T}^2], \\
I(\Delta y, \Delta \mathbf{p}_T) &= \mathrm{exp}[-\gamma_L (\Delta y)^2 - \gamma_T (\Delta \mathbf{p}_T)^2], \\
\Delta y &= y_2 - y_1, \quad \Delta \mathbf{p}_T = \mathbf{p}_{2T} - \mathbf{p}_{1T}.
\end{aligned}$$

$p_{\rm sm} = r(p_i, p_i)/\rho(p_i)$: chaoticity parameter in the semi-inclusive events ($p_{\rm sm} =$ const.)

6 parameters : $p_{\rm sm}$, $\langle n_0 \rangle$, α, β, γ_L and γ_T.

$\langle n_0 \rangle$ satisfys, $P(1) = \langle n_0 \rangle P(0)$.

$$R_1(y_1,\mathbf{p}_{1T};y_2,\mathbf{p}_{2T}) = r(y_1,\mathbf{p}_{1T};y_2,\mathbf{p}_{2T}),$$
$$T_{0,0}(y_1,\mathbf{p}_{1T};y_2,\mathbf{p}_{2T}) = c(y_1,\mathbf{p}_{1T};y_2,\mathbf{p}_{2T})$$

For $j,l = 1,2,\ldots$

$$R_j(y_1,\boldsymbol{p}_{1T},y_2,\boldsymbol{p}_{2T}) = N_j \exp[-A_j(y_1^2+y_2^2)+2C_jy_1y_2] \times \exp[-U_j(\boldsymbol{p}_{1T}^2+\boldsymbol{p}_{2T}^2)+2W_j\boldsymbol{p}_{1T}\boldsymbol{p}_{2T}]$$

$$R_{j+l}(y_1,\boldsymbol{p}_{1T},y_2,\boldsymbol{p}_{2T}) = \int R_j(y_1,\boldsymbol{p}_{1T},y,\boldsymbol{p}_T)R_l(y,\boldsymbol{p}_T,y_2,\boldsymbol{p}_{2T})dyd^2\boldsymbol{p}_T$$

Recurrence equations

$$N_{j+l} = \frac{\pi^{3/2}}{\sqrt{A_j+A_l}(U_j+U_l)}N_jN_l$$

$$A_{j+l} = A_j - \frac{C_j^2}{A_j+A_l} = A_l - \frac{C_l^2}{A_j+A_l}, \quad C_{j+l} = \frac{C_jC_l}{A_j+A_l}$$

$$U_{j+l} = U_j - \frac{W_j^2}{U_j+U_l} = U_l - \frac{W_l^2}{U_j+U_l}, \quad W_{j+l} = \frac{W_jW_l}{U_j+U_l}.$$

$$N_1 = p_{\mathrm{sm}}\langle n_0\rangle\frac{\alpha^{1/2}\beta}{\pi^{3/2}}, \quad A_1 = \frac{\alpha}{2}+\gamma_L, \quad C_1 = \gamma_L, \quad U_1 = \frac{\beta}{2}+\gamma_T, \quad W_1 = \gamma_T.$$

These recurrence equations can be solved.

$$A_j = \frac{r_2(1-u)}{2}\frac{1+u^j}{1-u^j}, \quad C_j = r_2(1-u)\frac{u^{j/2}}{1-u^j}, \quad j=1,2,\ldots,$$

$$U_j = \frac{t_2(1-v)}{2}\frac{1+v^j}{1-v^j}, \quad W_j = t_2(1-v)\frac{v^{j/2}}{1-v^j}, \quad j=1,2,\ldots.$$

$$N_j = \frac{\sqrt{r_2}\,t_2}{\pi^{3/2}}\xi^j\{\frac{1-u}{1-u^j}\}^{1/2}\frac{1-v}{1-v^j}, \quad j=1,2,\ldots,$$

$$r_1 = \alpha\frac{1+2h_L-\sqrt{1+4h_L}}{2}, \quad r_2 = \alpha\frac{1+2h_L+\sqrt{1+4h_L}}{2}, \quad h_L = \gamma_L/\alpha,$$

$$t_1 = \beta\frac{1+2h_T-\sqrt{1+4h_T}}{2}, \quad t_2 = \beta\frac{1+2h_T+\sqrt{1+4h_T}}{2}, \quad h_T = \gamma_T/\beta.$$

$$u = r_1/r_2 = \left(2h_L/(1+2h_L+\sqrt{1+4h_L}\,)\right)^2, \quad 0<u<1,$$

$$v = t_1/t_2 = \left(2h_T/(1+2h_T+\sqrt{1+4h_T}\,)\right)^2, \quad 0<v<1.$$

$$\xi = \frac{p_{\mathsf{sm}}\langle n_0\rangle\sqrt{\alpha}\beta}{\sqrt{r_2}\,t_2} = (1-\sqrt{u}\,)(1-\sqrt{v}\,)^2 p_{\mathsf{sm}}\langle n_0\rangle,$$

$$A_0 = \sqrt{1-u}\,(1-v)(1-p_{\mathsf{sm}})\langle n_0\rangle.$$

Cf. T.Csörgő and J.Zimanyi, Phys. Rev. Lett., **80**(1998)916,

N.S., M.Biyajima and T.Mizoguchi, Phys. Part. Nucl. Lett., **8**(2011)1007.

2-2.Multiplicity distribution in the QO approach

$$P(n) = \frac{1}{n}\sum_{j=1}^{n} j g_j^{(0)} P(n-j) = \frac{1}{n}\sum_{j=1}^{n}(\Delta_j^{(R)} + j\Delta_{j-1}^{(T)})P(n-j),$$

$$\Delta_j^{(R)} = \frac{\xi^j}{(1-u^{j/2})(1-v^{j/2})^2}, \quad \Delta_{j-1}^{(T)} = \frac{A_0\xi^{j-1}}{\sqrt{1-u^j}(1-v^j)}$$

The MD is approximately given by the Glauber-Lachs formula.

$$\Delta_j^{(R)} \approx \xi^j, \quad \Delta_{j-1}^{(S)} \approx A_0\xi^{j-1} \rightarrow \quad P(n) = (1-\xi)\xi^n \exp[-A_0/(1-\xi)]\, L_n(A_0/\xi)$$

- KNO scaling function

$$\phi(z) = \frac{1}{p_{\rm in}} \exp[-\frac{z+1-p_{\rm in}}{p_{\rm in}}] I_0(\frac{2}{p_{\rm in}}\sqrt{(1-p_{\rm in})z}).$$

$$\xi = (1-\sqrt{u})(1-\sqrt{v})^2 p_{\rm sm}\langle n_0\rangle = \frac{p_{\rm in}\langle n\rangle}{1+p_{\rm in}\langle n\rangle}, \qquad u = (\frac{2h_L}{1+2h_L+\sqrt{1+4h_L}})^2,$$

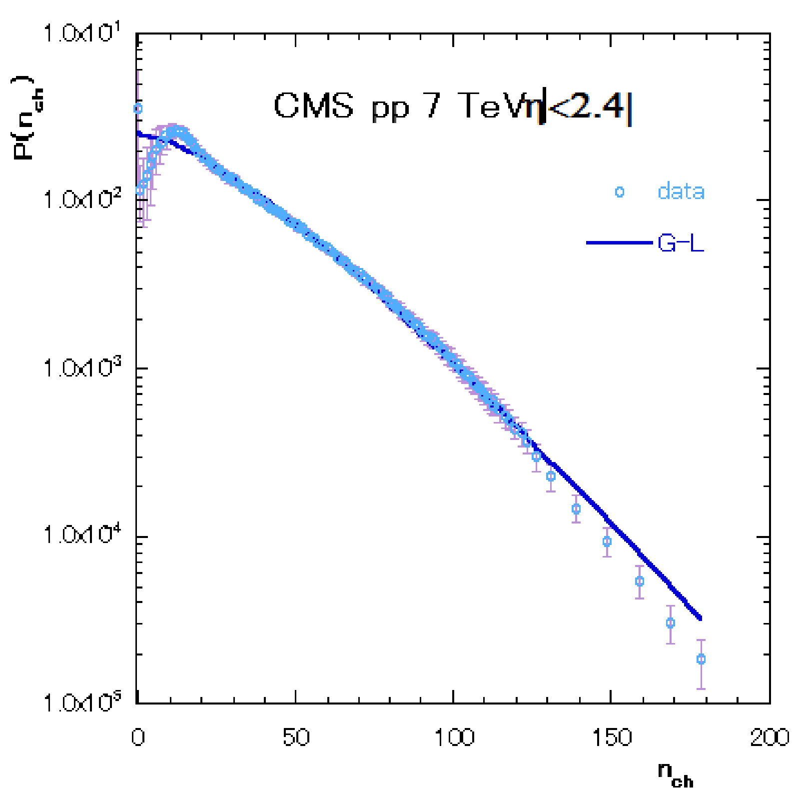
$$A_0 = \sqrt{1-u}\,(1-v)(1-p_{\rm sm})\langle n_0\rangle = \frac{(1-p_{\rm in})\langle n\rangle}{(1+p_{\rm in}\langle n\rangle)^2} \qquad v = (\frac{2h_T}{1+2h_T+\sqrt{1+4h_T}})^2$$

4 parameters, p_{sm}, $\langle n_0\rangle$, h_L and h_T, are contained in the MD.

cf. N.S., M. Biyajima and T. Mizoguchi, Phys. Part. Nucl. Lett., **8**(2011)1007

3.Data analysis

3-1. Multiplicity distribution



$p + p \to n_{ch} + X, \quad |\eta| < 2.4$

CMS Collaboration, V. Khachatryan, et al, J. High Energy Phys. **01**(2011)079

2 parameters, $p_{\rm in}$ and $\langle n \rangle$ $(= \frac{\langle n_{\rm ch} \rangle}{2})$ are contained in the KNO scaling function.

paremeters	estimated value
$p_{\rm in}$	0.514 ± 0.008
$\langle n_{\rm ch} \rangle$	30.48 ± 0.12
χ^2/N_{dof}	110.5/(126-2)

Estimated values of $p_{\rm in}$ and $\langle n \rangle$ give 2 constrants among 4 parameters, $p_{\rm sm}$, $\langle n_0 \rangle$, $h_L = \gamma_L/\alpha$ and $h_T = \gamma_T/\beta$.

3-2. Analysis of BEC

CMS collaboration JHEP05(2011)029

pp 7 TeV, $|\eta|<2.4$, $p_T>200$ MeV/c variable : Qinv

Range of k_T (GeV/c) : $k_T = (0.1, 0.3), (0.3, 0.5), (0.5, 1.0)$,

where $k_T = |\boldsymbol{k}_T|$, $\boldsymbol{k}_T = (\boldsymbol{p}_{1T} + \boldsymbol{p}_{2T})/2$

$$\tilde{C}_n^{(2)}(Q_{\text{inv}}, \boldsymbol{k}_T) = \frac{nP(n)}{n-1} \frac{\int \rho_n^{(2)}(y_1, \boldsymbol{p}_{1T}; y_1 + \Delta y, \boldsymbol{p}_{1T} + \Delta \boldsymbol{p}_T) dy_1 d^2 \Delta \boldsymbol{p}_T |_{k_T \approx c}}{\int \rho_n^{(1)}(y_1, \boldsymbol{p}_{1T}) \rho_n^{(1)}(y_1 + \Delta y, \boldsymbol{p}_{1T} + \Delta \boldsymbol{p}_T) dy_1 d^2 \Delta \boldsymbol{p}_T |_{k_T \approx c}}$$

(i) Change variable from Δy to Q_{inv}

$$Q_{\text{inv}}^2 = Q_{\text{L}}^2 + Q_{\text{T}}^2 \approx 2\langle m_{\text{T}}{}^2 \rangle (\cosh \Delta y - 1) + \Delta \boldsymbol{p}_{\text{T}}{}^2,$$
$$\Delta y \approx \ln(a + \sqrt{a^2 - 1})$$

$$a = \frac{Q_{\text{inv}}^2 - \Delta \boldsymbol{p}_{\text{T}}{}^2}{2\langle m_{\text{T}}{}^2 \rangle} + 1$$

If $\Delta \boldsymbol{p}_{\text{T}}{}^2 << 1$, we can aproximate

$$a_0 = \frac{Q_{\text{inv}}^2}{2\langle m_{\text{T}}{}^2 \rangle} + 1$$

$$\Delta y \approx \ln(a_0 + \sqrt{a_0{}^2 - 1}) - \frac{\Delta \boldsymbol{p}_{\text{T}}{}^2}{2\langle m_T{}^2 \rangle \sqrt{a_0{}^2 - 1}},$$

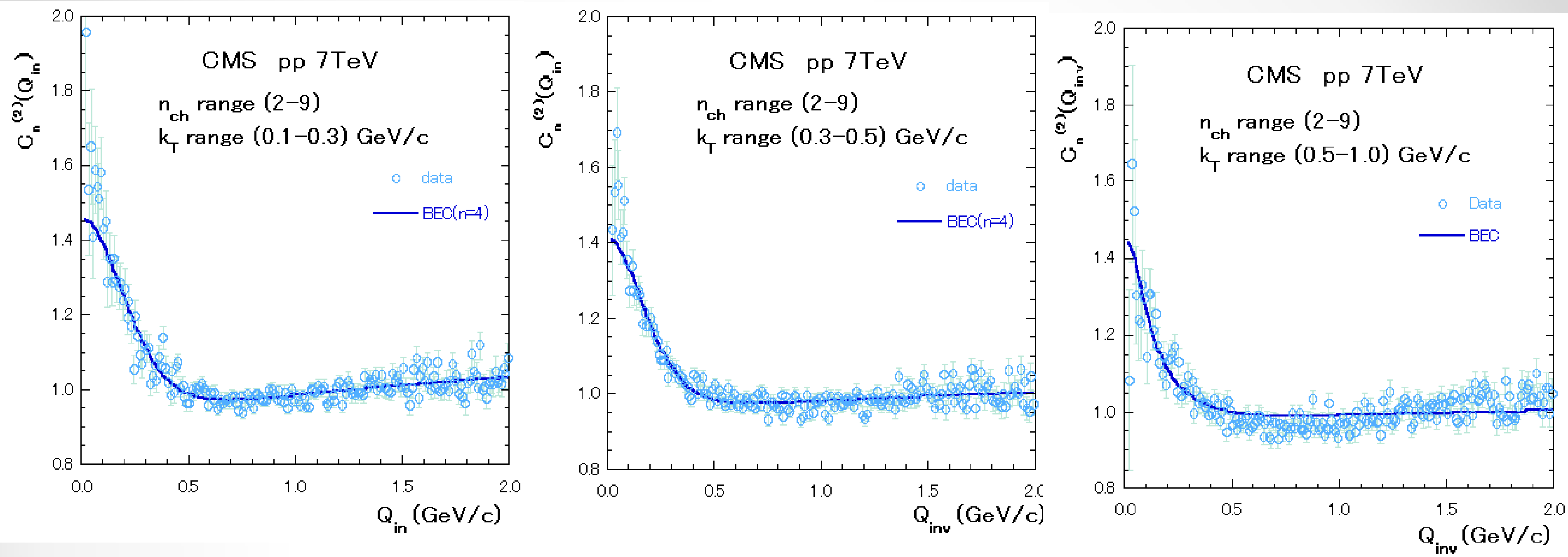
$$\langle m_{\text{T}}{}^2 \rangle = 0.14^2 + 0.3^2$$

$$(\Delta y)^2 \approx \left\{ \ln(a_0 + \sqrt{a_0{}^2 - 1}) \right\}^2 - \frac{\ln(a_0 + \sqrt{a_0{}^2 - 1})}{\langle m_T{}^2 \rangle \sqrt{a_0{}^2 - 1}} \Delta \boldsymbol{p}_{\text{T}}{}^2$$

(ii) Change variables from $\boldsymbol{p}_{1\text{T}}$, $\Delta \boldsymbol{p}_{\text{T}}$ to $\boldsymbol{k}_{\text{T}}$, $\Delta \boldsymbol{p}_{\text{T}}$, and integrate over $\Delta \boldsymbol{p}_{\text{T}}$.

$$\boldsymbol{p}_{1\text{T}} = \boldsymbol{k}_{\text{T}} - \Delta \boldsymbol{p}_{\text{T}}/2, \quad (\boldsymbol{p}_{2\text{T}} = \boldsymbol{k}_{\text{T}} + \Delta \boldsymbol{p}_{\text{T}}/2)$$

Nch range (2-9)

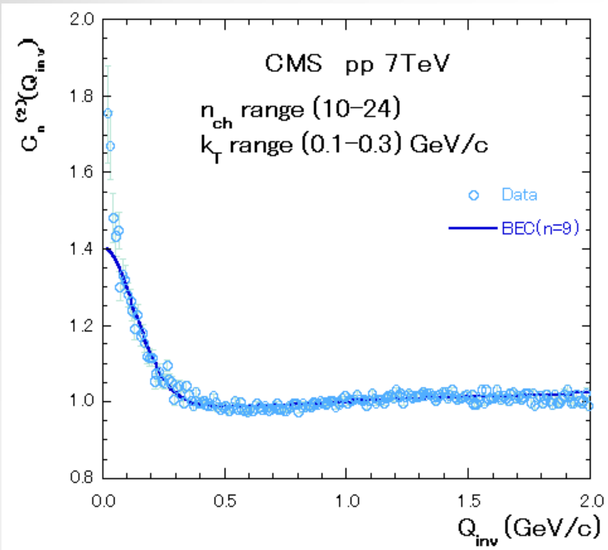


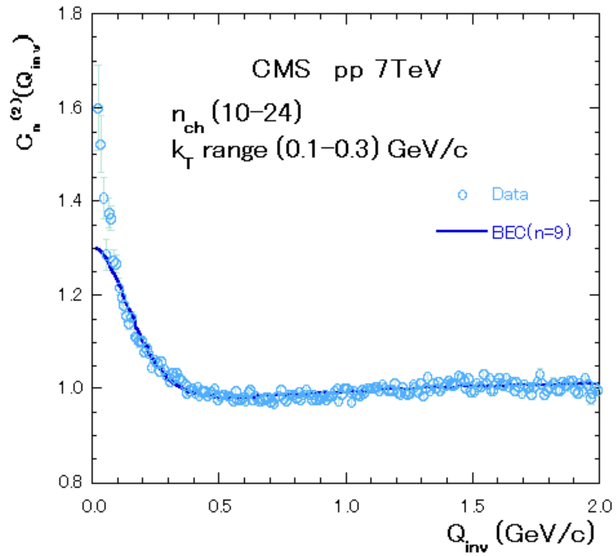
Estimated parameters from $C_n^{(2)}(Q_{\rm inv})$, $n_{\rm ch} = (2-9)$

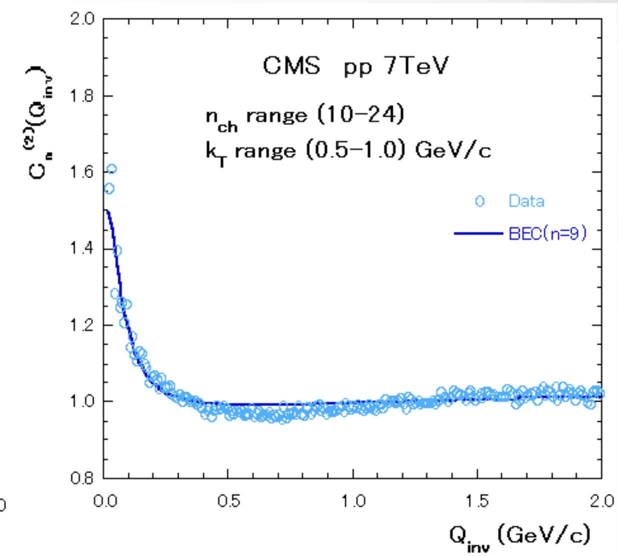
$k_{\rm T}$ (GeV/c)	0.10-0.30	0.30-0.50	0.50-1.00
$p_{\rm sm}$	0.965±0.002	0.978±0.002	0.980±0.004
$\langle n_0 \rangle$	3.45±0.17	6.21±0.50	7.31±2.05
α	0.334±0.028	0.303±0.038	0.203±0.037
β	33.1±1.6	5.05±0.34	2.11±0.21
h_L	2.47±0.32	2.38±0.40	2.19±0.94
h_T	0.427±0.025	1.43±0.13	1.94±0.73
χ^2/N_{dof}	322.5/(198-4)	529.1/(198-4)	323.5/(198-4)

$N_{\rm ch}$	2-9	10-24	25-79
$n(\pi^-)$	4	9	20

Nch (10-24)



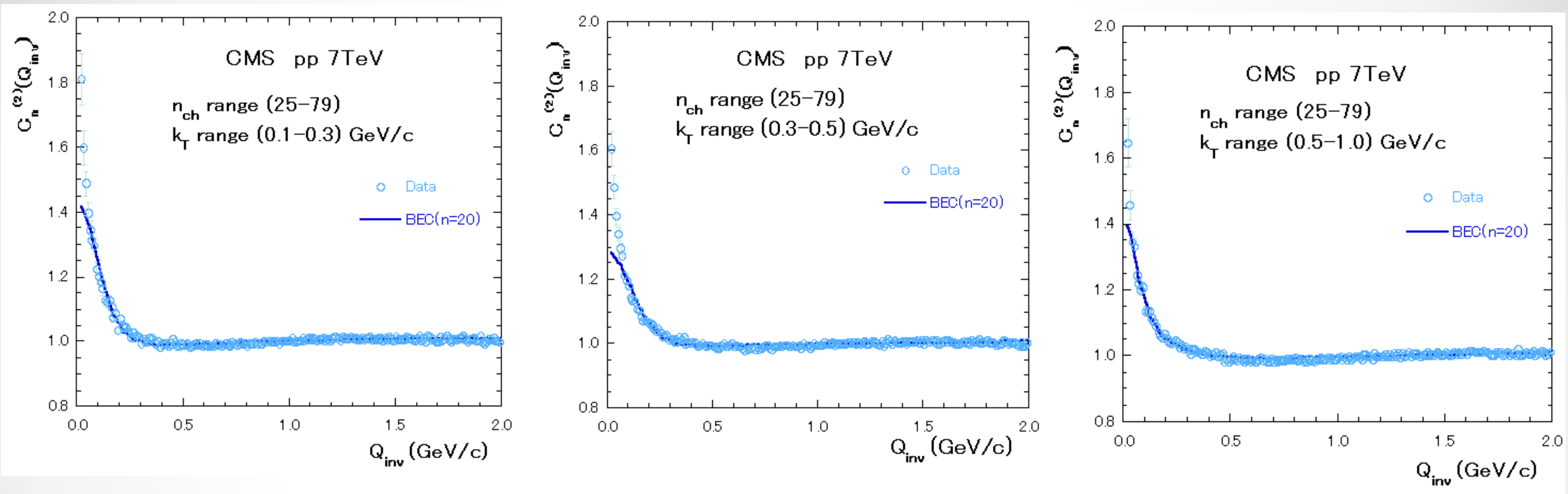




Estimated parameters from $C_n^{(2)}(Q_{\rm inv})$, $n_{\rm ch} = (10 - 24)$

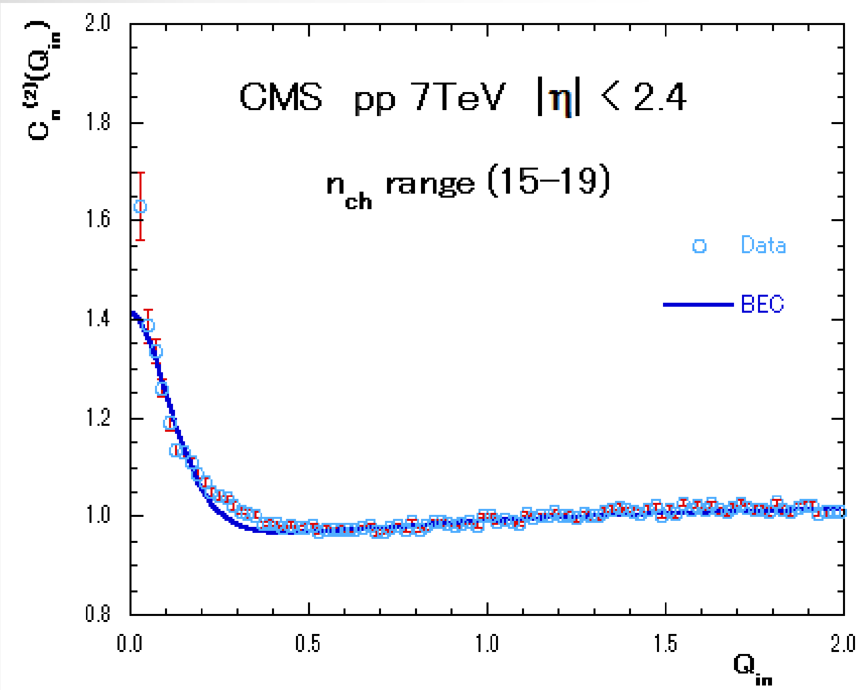
$k_{\rm T}$ (GeV/c)	0.10-0.30	0.30-0.50	0.50-1.00
$p_{\rm sm}$	0.979±0.001	0.982±0.001	0.981±0.001
$\langle n_0 \rangle$	6.99±0.25	8.92±0.32	8.07±0.41
α	0.294±0.018	0.363±0.018	0.225±0.007
β	27.9±1.1	5.57±0.19	2.25±0.06
h_L	6.22±0.48	3.24±0.23	2.37±0.10
h_T	0.942±0.032	2.13±0.08	2.16±0.09
χ^2/N_{dof}	322.5/(198-4)	529.1/(198-4)	323.5/(198-4)

nch (25-79)

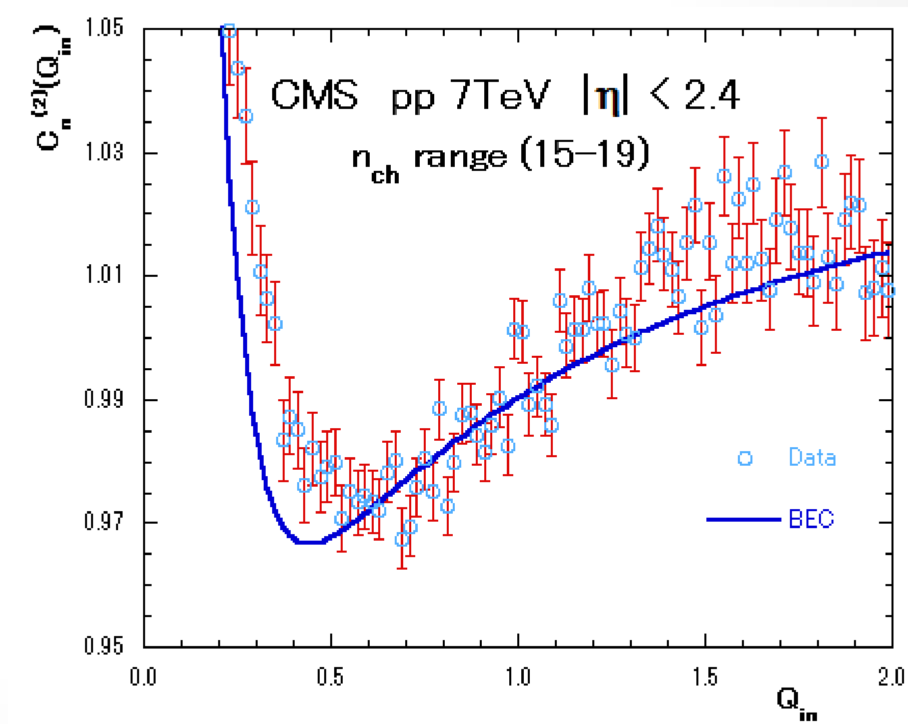


Estimated parameters from $C_n^{(2)}(Q_{\text{inv}})$, $n_{\text{ch}} = (25 - 79)$

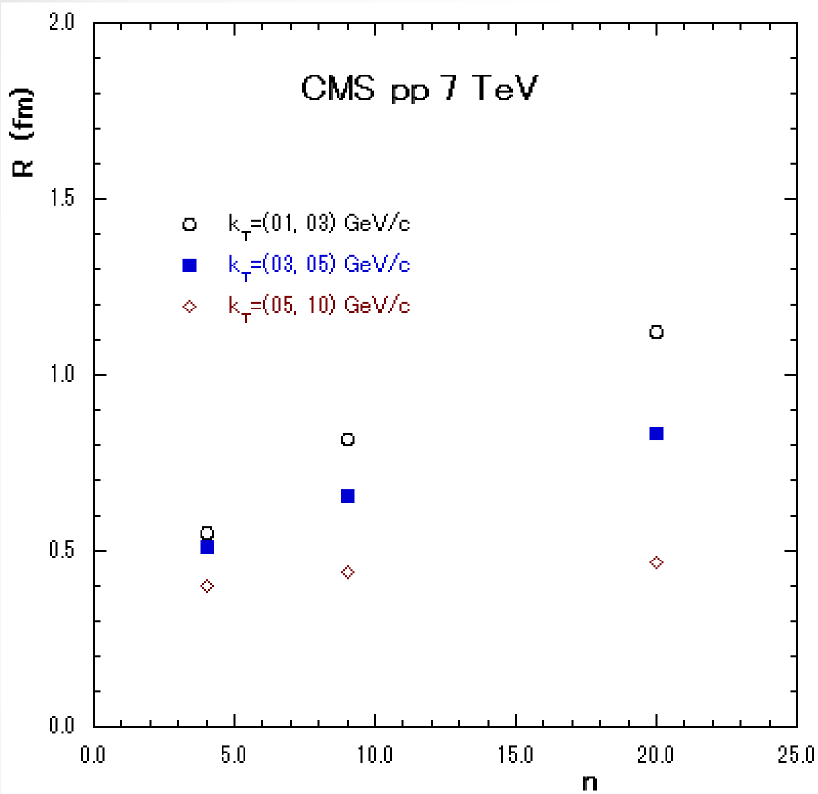
k_{T} (GeV/c)	0.10-0.30	0.30-0.50	0.50-1.00
p_{sm}	0.986±0.001	0.989±0.001	0.985±0.001
$\langle n_0 \rangle$	13.7±0.6	17.9±0.8	11.1±0.3
α	0.265±0.021	0.256±0.018	0.184±0.003
β	22.4±1.2	5.36±0.23	1.87±0.03
h_L	13.1±0.9	7.41±0.62	3.25±0.15
h_T	1.76±0.09	3.63±0.13	2.92±0.10
χ^2/N_{dof}	322.5/(198-4)	529.1/(198-4)	323.5/(198-4)



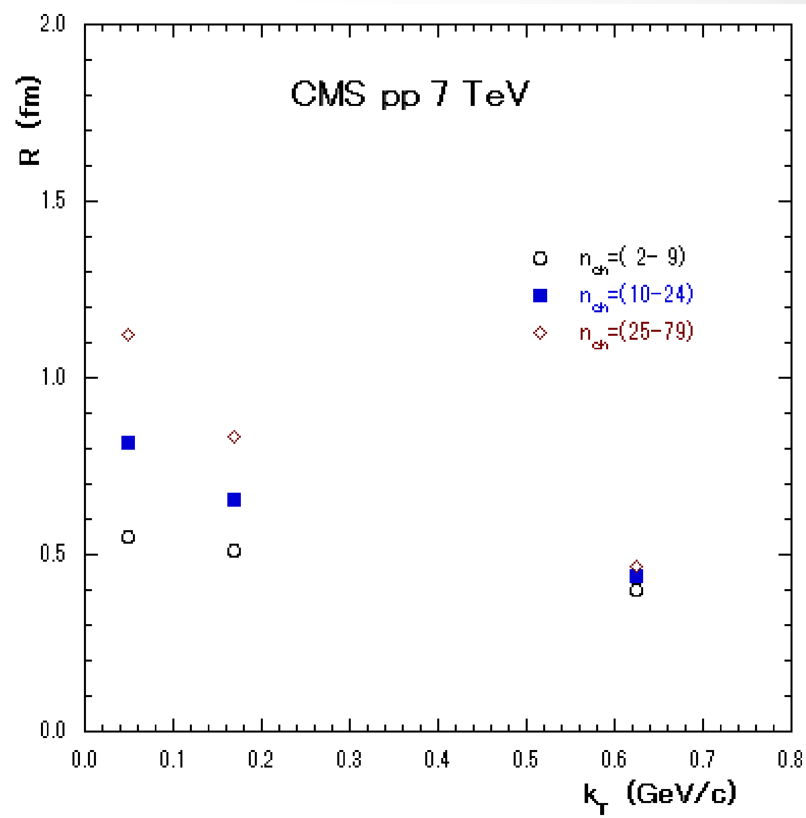
$p_{\rm sm}$	0.984
$\langle n_0 \rangle$	9.88
α	0.584
β	6.85
h_L	3.13
h_T	2.52
χ^2/N_{dof}	349.6/(100-4)



Multipicity dapendence of R



k_{T} dapendence of R



$$R = \sqrt{\gamma_L / \langle {m_T}^2 \rangle}, \ \gamma_L = \alpha \times h_L$$

$$\langle {m_T}^2 \rangle = 0.14^2 + 0.3^2 = 0.110 \quad (GeV/c)^2$$

Source radius R increases with multiplicity n, and decreases with k_{T}.

5. Summary

◇ The MD and $C_n^{(2)}(Q_{\rm inv})$ measured in pp collisons in $|\eta| < 2.4$ at 7 TeV by the CMS collaboration is analyzed by the model with 6 parameters, $p_{\rm sm}$, $\langle n_0 \rangle$, α, β, γ_L and $\gamma_{\rm T}$ in the QO approach.

◇ From the analysis, the chaoticity parameter p_{in} in the inclusive events and the average charged multiplicity $\langle n_{ch} \rangle$ are estimated.

◇ p_{in} and $\langle n_{ch} \rangle$ /2 gives two constraints among parameters, p_{sm}, $\langle n_0 \rangle$, $h_L = \gamma_L/\alpha$ and $h_T = \gamma_T/\beta$.

◇ Source raius R increases with multiplicity n, and decreaes with $k_{\rm T}$. Source radius R in fm is within the range (from 0.403 to 1.13).