The role of the sigma meson in thermal models

W. Broniowski¹,², F. Giacosa¹,³, V. Begun¹

¹ Institute of Physics, Jan Kochanowski University, PL-25406 Kielce, Poland
² The H. Niewodniczanski Institute of Nuclear Physics, Polish Academy of Sciences, PL-31342 Krakow, Poland
³ Institute for Theoretical Physics, J.W. Goethe University, Frankfurt am Main, Germany

Abstract
The by now well-established scalar-isoscalar resonance \( f_0(500) \) (the \( \sigma \) meson) seems potentially relevant for the evaluation of thermodynamic quantities of a hadronic gas, since its mass is low. In this poster, based on the recent work of Ref. [1], we show that its contribution to isospin-averaged observables is, to a surprising accuracy, canceled by the repulsion from the pion-pion scalar-isotensor channel. As a result, in practice one should not incorporate \( f_0(500) \) in standard hadronic resonance-gas models for studies of isospin averaged quantities. In our analysis we use the formalism of the virial expansion, which allows one to calculate the thermal properties of an interacting hadron gas in terms of derivatives of the scattering phase shifts, hence in a model-independent way directly from experimentally accessible quantities. A similar cancellation mechanism occurs for the scalar kaonic interactions between \( I = 1/2 \) channel (containing the alleged light \( k \) meson) and the \( I = 3/2 \) channel.

Introduction
The resonance \( f_0(500) \) is the lightest scalar-isoscalar state listed in PDG. It corresponds to a well-known pole in the complex energy plane. PDG2012 has updated, as a result of various studies such as the one in Ref. [2], the position of the pole. The uncertainty is now much smaller. Then, a natural question arises: is this resonance important for thermal models? At first sight yes, but a careful study gives rise to a different conclusion.

Short review of thermal models
The partition function for a gas of hadrons at temperature \( T \) is given by the following expressions [3,4].

\[
Z = \sum_k Z_k^{\text{stable}} + \sum_k Z_k^{\text{res}}
\]

\[
Z_k^{\text{stable}} = f_k V \int_0^\infty \frac{d^3 k}{(2\pi)^3} \ln \left( 1 + e^{-E_k/T} \right)^{I-1}
\]

\[
Z_k^{\text{res}} = f_k V \int_0^{\infty} \frac{d^3 k}{(2\pi)^3} \ln \left( 1 - e^{-E_k/T} \right)^{-I}
\]

One can directly relate the function \( d_k(M) \) to scattering data by using the following link to the phase shifts:

\[
d_k(M) = \frac{d_k(M)}{\pi M}
\]

Finally, for hadrons (in particular, mesons) we have:

\[
Z = Z + f_0 \int_0^{\infty} \frac{d^3 k}{(2\pi)^3} \ln \left( 1 - e^{-E_k/T} \right)^{-1} \frac{1}{(2I+1)}
\]

Experimental data on pion-pion phase shifts
The solid red line is the attraction induced by the \( p \) meson. The dashed blue line corresponds to the attraction due to \( f_0(500) \). The dotted black line is an isoscalar scalar repulsion.

Results
Spectral function of the p-meson: it has a Breit-Wigner shape.

Trace anomaly: the contribution of \( f_0(500) \) (dashed blue) is cancelled by the I=2 repulsion.

Conclusions
The \( f_0(500) \) is not relevant in isospin-averaged thermal observables. This is due to the repulsion in the isotensor channel, which ‘de facto’ cancels the effect of \( f_0(500) \) (which would be at the 5% level).

Summary for thermal-model builders: forget about the \( f_0(500) \) (or \( \sigma \)) and also about the isotensor repulsion. Similar conclusion (but not that accurate, see also [6]) holds for \( K^0(800) \).

However: when studying correlated \( m-n \) pair production, the cancellation does not occur and an effect of the \( f_0(500) \) is still present.

Literature