Stationary fluctuation theorem in high-energy nuclear collisions

Ryuichi Kurita[a,b,c] Collaborators: T. Hirano,[c] K. Murase,[a,b,c] K. Nagai[c]
[a] the Univ. of Tokyo, [b] RIKEN, [c] Sophia Univ.

Motivation

Relativistic hydrodynamics in heavy ion collisions

Ideal hydrodynamics(-2000)

↓

Viscous hydrodynamics(recently)

↓

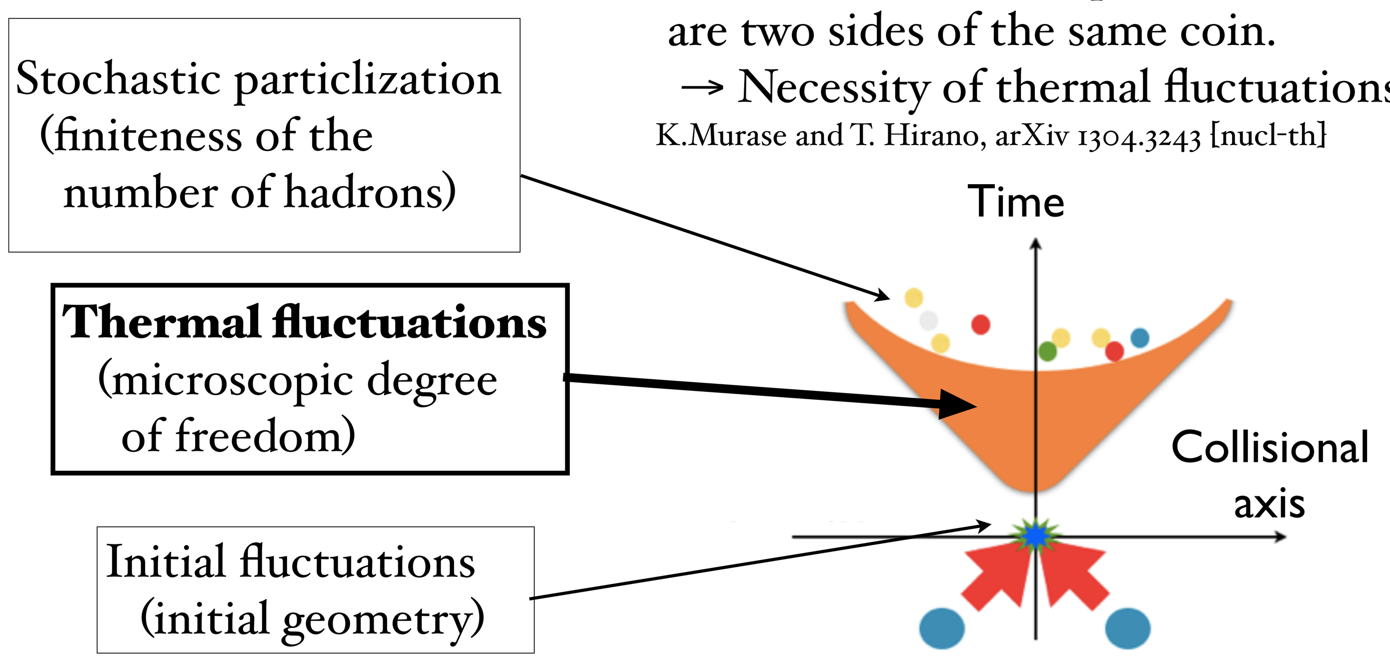
Event-by-event hydrodynamics(very recently)

- Initial fluctuations
- Thermal fluctuations

Various fluctuations

Fluctuation-dissipation relation

Fluctuations and dissipations are two sides of the same coin.
→ Necessity of thermal fluctuations
K.Murase and T. Hirano, arXiv 1304.3243 [nucl-th]



＊This study

- For **event-by-event precise calculation**, it is necessary to include **hydrodynamic(thermal) fluctuations**.
- General result for thermal fluctuations
  → **Stationary fluctuation theorem**

Fluctuating hydrodynamics

Bjorken expansion

Approximated by 1-dimensional expansion

4-dim. velocity $u^{\mu} = \left(\frac{t}{\tau}, 0, 0, \frac{z}{\tau}\right)$ $\quad \tau = \sqrt{t^2 - z^2}$: proper time

- Energy-momentum conservation

$$\frac{de}{d\tau} = -\frac{e + p(e)}{\tau}\left(1 - \frac{\pi - \Pi}{sT}\right)$$

e : energy density $\quad s$: entropy density $\quad \pi$: shear stress
p : static pressure $\quad T$: temperature $\quad \Pi$: bulk viscosity

- Constitutive equations

Shear : $\left(\tau_R \frac{d}{d\tau} + 1\right)\pi = \frac{4\eta}{3\tau} + \xi_\pi$

Bulk : $\left(\tau_R \frac{d}{d\tau} + 1\right)\Pi = -\frac{\zeta}{\tau} + \xi_\Pi$

τ_R : relaxation time
η : shear viscosity
ζ : bulk viscosity

ξ_π , ξ_Π : hydrodynamic(thermal) fluctuations

- **Stochastic properties of hydrodynamic fluctuations**

(in the special case of Bjorken expansion) $\quad \langle \dots \rangle$: ensemble average

- mean value, independence

$$\langle \xi_\pi \rangle = \langle \xi_\Pi \rangle = 0$$

$$\langle \xi_\pi(x) \xi_\Pi(x') \rangle = 0$$

- fluctuation-dissipation relation
  → two point correlations

$$\langle \xi_\pi(x) \xi_\pi(x') \rangle = \frac{8\eta T}{3}\delta^{(4)}(x - x')$$

$$\langle \xi_\Pi(x) \xi_\Pi(x') \rangle = 2\zeta T \delta^{(4)}(x - x')$$

Stationary fluctuation theorem

Stationary fluctuation theorem

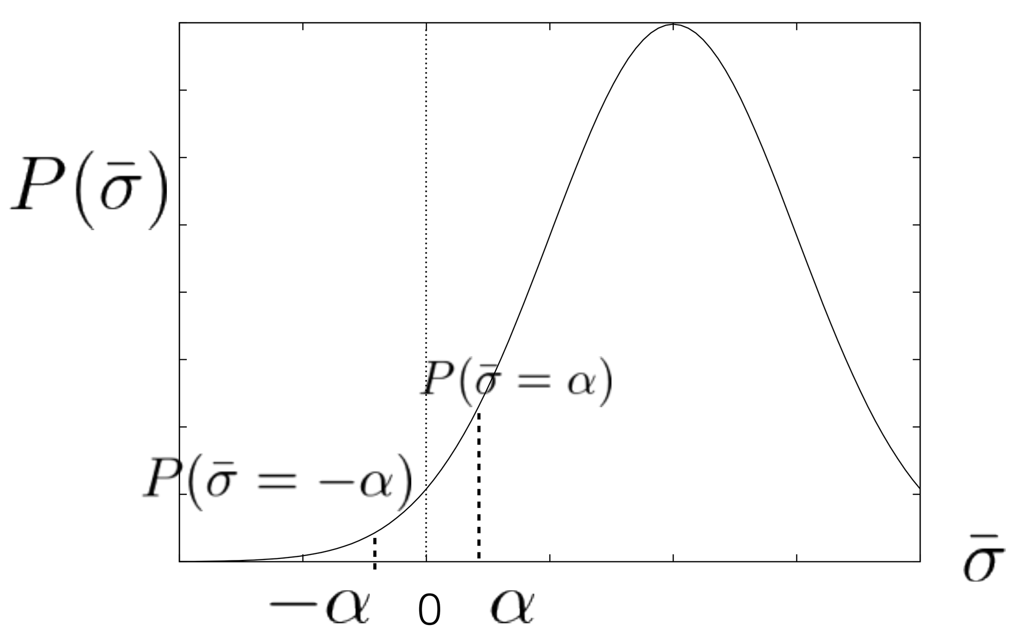
$$\boxed{\frac{P(\bar{\sigma} = \alpha)}{P(\bar{\sigma} = -\alpha)} = \mathrm{e}^{\alpha\tau} \dots (1)}$$

$\sigma = \frac{dS}{d\tau}$: entropy production

$\bar{\sigma} = \frac{1}{\tau}\int_0^\tau d\tau' \sigma(\tau')$: its time average

$P(\bar{\sigma})$: distribution of $\bar{\sigma}$

τ : observation time $\quad S$: entropy in one fluid element



Negative entropy production is quantified.

If the distribution is Gaussian

$$P(\bar{\sigma}) = \frac{1}{\sqrt{2\pi}a}\exp\left[-\frac{(\bar{\sigma} - \langle\bar{\sigma}\rangle)^2}{2a^2}\right]$$

$\langle\bar{\sigma}\rangle$: mean value $\quad a^2$: variance

$$(1) \Leftrightarrow \boxed{\frac{2\langle\bar{\sigma}\rangle}{a^2} = \tau \dots (2)}$$

In the limit of vanishing relaxation time($\tau_R \to 0$), this system satisfies stationary fluctuation theorem (2).

Proof of (2)

Formal solutions for constitutive equations

$$\pi(\tau) = \int_{\tau_i}^{\tau} d\tau' G(\tau - \tau')\frac{4\eta}{3\tau'} + \delta\pi(\tau)$$

$$\Pi(\tau) = -\int_{\tau_i}^{\tau} d\tau' G(\tau - \tau')\frac{\zeta}{\tau'} + \delta\Pi(\tau)$$

G : Green function

Integrated fluctuations

$$\delta\pi(\tau) = \int_{\tau_i}^{\tau} d\tau' G(\tau - \tau')\xi_\pi(\tau')$$

$$\delta\Pi(\tau) = \int_{\tau_i}^{\tau} d\tau' G(\tau - \tau')\xi_\Pi(\tau')$$

In the limit of vanishing relaxation time,

$$G(\tau - \tau') \to \delta(\tau - \tau'),$$

$$\delta\pi, \delta\Pi \to \xi_\pi, \xi_\Pi.$$

From thermodynamic relation,

$$\sigma = \frac{d(\tau s)}{d\tau}\Delta\eta_s \Delta x \Delta y = \frac{\pi - \Pi}{T}\Delta\eta_s \Delta x \Delta y.$$

$\tau \Delta\eta_s \Delta x \Delta y$: volume of one flid element

Then,

$$\langle\bar{\sigma}\rangle = \frac{\Delta\eta_s \Delta x \Delta y}{\tau - \tau'}\int_{\tau_i}^{\tau}\frac{d\tau'}{T}\left(\frac{4\eta}{3\tau'} + \frac{\zeta}{\tau'}\right)$$

$$a^2 = \left(\frac{\Delta\eta_s \Delta x \Delta y}{\tau - \tau_i}\right)^2 \int_{\tau_i}^{\tau} d\tau_1 \int_{\tau_i}^{\tau} d\tau_2 \times$$

$$\frac{\langle\delta\pi(\tau_1)\delta\pi(\tau_2)\rangle + \langle\delta\Pi(\tau_1)\delta\Pi(\tau_2)\rangle}{T(\tau_1)T(\tau_2)} = \frac{2\langle\bar{\sigma}\rangle}{\tau - \tau_i}.$$

Summary

- For event-by-event precise calculations, we consider hydrodynamic fluctuations.
- Proof of stationary fluctuation theorem in the limit of vanishing relaxation time
  → confirmation of equivalence between fluctuation-dissipation relation and stationary fluctuation theorem