Chiral Kinetic Theory

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PRL 109(2012)162001, 113(2014)182302, 115(2015)021601 and 1508.02396
with Yi Yin, Jingyuan Chen, Dam Son, Ho-Ung Yee.

Chiral Magnetic Effect and Anomaly

Consider free right-handed (Weyl) fermion:

- Lowest Landau level is chiral

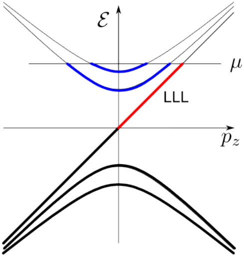
$$\boldsymbol{J} = \mu \frac{\boldsymbol{B}}{4\pi^2}$$

– nondissipative current

- Nielsen-Ninomia (1983):

$$\boldsymbol{E} \cdot \boldsymbol{J} = \mu \frac{\boldsymbol{E} \cdot \boldsymbol{B}}{4\pi^2} = \mu \frac{dn}{dt}$$

work = energy change



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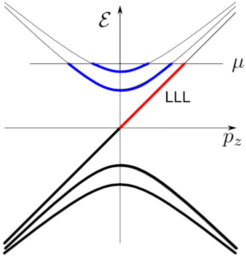

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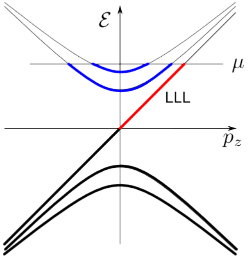


Kharzeev-McLerran-Warringa (2007):

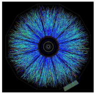
Strong magnetic field

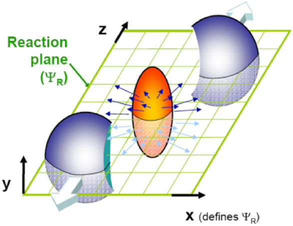
+

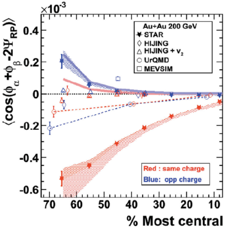
topological / net chirality fluctuations

=

fluctuations of charge asymmetry wrt reaction plane.

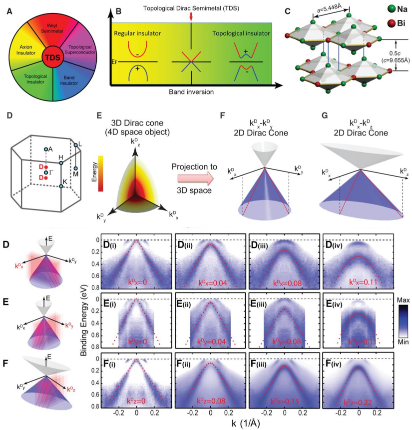




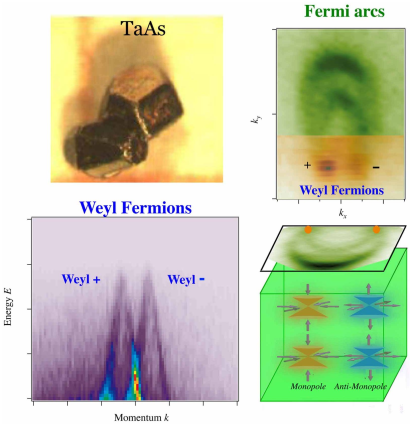


$$\boldsymbol{J} \sim (N_R - N_L)\boldsymbol{B}$$

Dirac and Weyl semimetals discovered



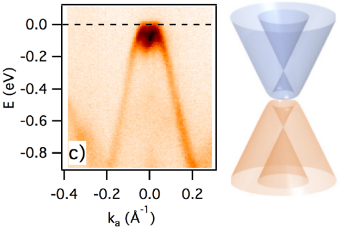
Liu *et al*, Science 343 (2014) 864

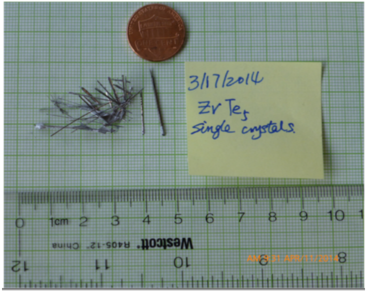


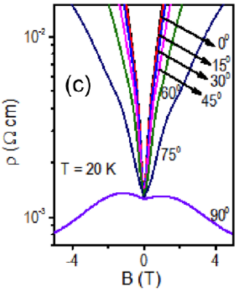
Xu *et al*, Science 349 (2015) 614

CME in $ZrTe_5$

BNL – Stony Brook – Princeton – Berkeley (arxiv:1412.6543)







90°: CME leads to negative magnetoresistance for $\boldsymbol{B} \parallel \boldsymbol{E}$.

$$J_{\rm CME} = \sigma_{\rm CME} E \sim n_5 B; \qquad \frac{n_5}{\tau} \sim EB;$$

$$\sigma_{\rm CME} \sim \tau B^2 > 0,\ \rho = \frac{1}{\sigma_0 + \sigma_{\rm CME}} < \rho_0.$$

CME and CVE in Relativistic Hydrodynamics

- Hydrodynamics with anomalous current (Son-Surowka 2009):

$$\partial_\alpha T^{\alpha\beta} = F^{\beta\gamma} J_\gamma; \qquad \partial_\alpha J^\alpha = C E \cdot B$$

$$\boldsymbol{J} = \text{normal} + C\mu \boldsymbol{B} + (C\mu^2 + AT^2)\boldsymbol{\omega}$$

  - Chiral Vortical Effect: $\boldsymbol{J} \sim \boldsymbol{\omega}$ (Vilenkin 1980, Erdmenger *et al* 2009)

- CME/CVE currents do not exert drag (Rajagopal-Sadofyev 2015),

  but, unlike superconducting currents, *can* carry entropy in the "no-drag" frame (MS-Yee 2015)

$$\boldsymbol{S} = \text{normal} + AT\boldsymbol{B} + A\mu T\boldsymbol{\omega}$$

Motivation

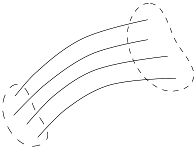
- Interesting applications of CME/CVE in non-equilibrium conditions – such as heavy-ion collisions, or high-frequency response – beyond hydro.
- Kinetic theory: a non-equilibrium description
- Important for understanding the mechanism of CME/CVE.

Puzzle 1: Anomaly

- Kin. regime: collisions are rare enough that motion is classical.

Each particle follows classical trajectory $\boldsymbol{x}(t)$, $\boldsymbol{p}(t)$. A “cloud” $f(\boldsymbol{x},\boldsymbol{p})$ evolves with time. In a comoving 6-volume, the number of particles can only be changed by collisions:

$$\frac{df}{dt} \equiv \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \boldsymbol{x}}\dot{\boldsymbol{x}} + \frac{\partial f}{\partial \boldsymbol{p}}\dot{\boldsymbol{p}} = C[f].$$



Ignore collisions for now.

- The number of particles in the phase space *cannot change*?
- How can *classical* equation account for *quantum* anomaly?

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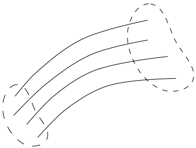

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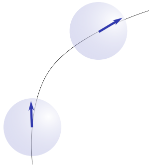
Action, Berry phase and e.o.m.s

The answer — Berry phase.

Change of momentum direction requires rotation in (quantum) spin space, which adds a phase to the action:



$$\mathcal{I} = \int (\boldsymbol{p} + \boldsymbol{A}) \cdot d\boldsymbol{x} - (\mathcal{E} + \Phi)dt \underbrace{- \ \boldsymbol{a_p} \cdot d\boldsymbol{p}}_{\text{Berry phase } \mathcal{O}(\hbar)}$$

Equations of motion

$$\dot{\boldsymbol{x}} - \boldsymbol{v} - \overbrace{\dot{\boldsymbol{p}} \times \boldsymbol{b}}^{\text{anom. velocity}} = 0;$$

$$\dot{\boldsymbol{p}} - \boldsymbol{E} - \dot{\boldsymbol{x}} \times \boldsymbol{B} = 0;$$

$$\boldsymbol{v} = \partial\mathcal{E}/\partial\boldsymbol{p}, \ \mathcal{E} \equiv |\boldsymbol{p}| - \frac{\hat{\boldsymbol{p}} \cdot \boldsymbol{B}}{2|\boldsymbol{p}|}.$$

Berry curvature:

$$\boldsymbol{b} \equiv \nabla_{\boldsymbol{p}} \times \boldsymbol{a_p} = \frac{\hat{\boldsymbol{p}}}{2|\boldsymbol{p}|^2}.$$

The invariant measure now is $\frac{d^3\boldsymbol{x}\, d^3\boldsymbol{p}}{(2\pi)^3}\sqrt{G}$, with $\sqrt{G} = 1 + \boldsymbol{b} \cdot \boldsymbol{B}$.

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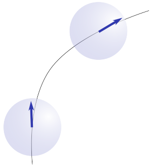

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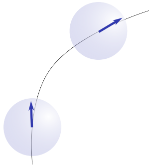

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Chiral anomaly

Liouville equation is now anomalous at $\boldsymbol{p}=0$:

$$\frac{\partial}{\partial t}\sqrt{G}+\frac{\partial}{\partial \boldsymbol{x}}(\sqrt{G}\dot{\boldsymbol{x}})+\frac{\partial}{\partial \boldsymbol{p}}(\sqrt{G}\dot{\boldsymbol{p}})=(\boldsymbol{E}\cdot\boldsymbol{B})\underbrace{(\boldsymbol{\nabla}_{\boldsymbol{p}}\cdot\boldsymbol{b})}_{2\pi\delta^3(\boldsymbol{p})},$$

Thus current is not conserved:

$$\frac{\partial n}{\partial t}+\boldsymbol{\nabla}\cdot\boldsymbol{J}=\frac{1}{4\pi^2}\boldsymbol{E}\cdot\boldsymbol{B}\, f|_{\boldsymbol{p}=0}\,,$$

Berry “monopole” at $\boldsymbol{p}=0$ acts as source/sink
of particle number current.
Region $p\lesssim\sqrt{B}$ is quantum.
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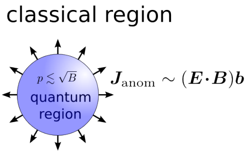

CME and CVE

- Solving eoms for $\dot{\boldsymbol{x}}$:

$$\boldsymbol{J} = \int_{\boldsymbol{p}} \sqrt{G} f\, \dot{\boldsymbol{x}} = \underbrace{\int_{\boldsymbol{p}} f \boldsymbol{v}}_{\text{normal current}} + \underbrace{\boldsymbol{E} \times \int_{\boldsymbol{p}} f \boldsymbol{b}}_{\text{anom. Hall current}} + \underbrace{\boldsymbol{B} \int_{\boldsymbol{p}} f (\boldsymbol{v} \cdot \boldsymbol{b})}_{\text{CME}\to \frac{1}{4\pi^2}\mu \boldsymbol{B}}$$

- CVE: Coriolis force (Yin-MS 2012)

$$\dot{\boldsymbol{p}} = 2\mathcal{E}\, \dot{\boldsymbol{x}} \times \boldsymbol{\omega} \qquad \text{i.e., } \boxed{\boldsymbol{B} \to 2\mathcal{E}\boldsymbol{\omega}}.$$

- Then CME $\longrightarrow$ CVE

$$J_{\text{CME}} = \boldsymbol{B} \int_{\boldsymbol{p}} f \hat{\boldsymbol{p}} \cdot \boldsymbol{b} \quad \longrightarrow \quad J_{\text{CVE}} = \boldsymbol{\omega} \int_{\boldsymbol{p}} 2\mathcal{E} f \hat{\boldsymbol{p}} \cdot \boldsymbol{b} \to \frac{1}{4\pi^2}\mu^2 \boldsymbol{\omega}$$

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$$J_{\text{CME}} = B \int_{p} f \hat{p} \cdot b \quad \longrightarrow \quad J_{\text{CVE}} = \omega \int_{p} 2\mathcal{E} f \hat{p} \cdot b \rightarrow \frac{1}{4\pi^2} \mu^2 \omega$$

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Puzzle 2: Where is Lorentz invariance?

$$\mathcal{I} = \int (\boldsymbol{p} + \boldsymbol{A}) \cdot d\boldsymbol{x} - (|\boldsymbol{p}| + \Phi) dt - \boldsymbol{a_p} \cdot d\boldsymbol{p} + \frac{\hat{\boldsymbol{p}} \cdot \boldsymbol{B}}{2|\boldsymbol{p}|} dt$$

Modified Lorentz transfromation: PRL 113(2014)182302

$$\delta \boldsymbol{x} = \boldsymbol{\beta} t + \frac{\boldsymbol{\beta} \times \hat{\boldsymbol{p}}}{2|\boldsymbol{p}|}, \quad \delta \boldsymbol{p} = \boldsymbol{\beta} \mathcal{E} + \frac{\boldsymbol{\beta} \times \hat{\boldsymbol{p}}}{2|\boldsymbol{p}|} \times \boldsymbol{B}, \quad \delta t = \boldsymbol{\beta} \cdot \boldsymbol{x}.$$

- Side jump.
- Magnetic moment ($\boldsymbol{m} = \frac{\hat{\boldsymbol{p}}}{2|\boldsymbol{p}|}$) needed by Lorentz invariance

(Son-Yamamoto)

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Magnetization current

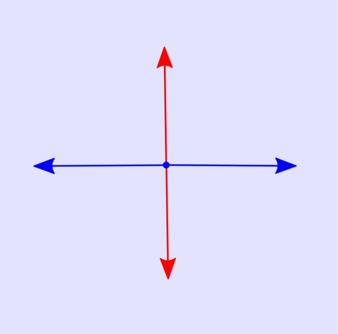
Noether current: $\mathcal{I} = \ldots + \int \boldsymbol{A} \cdot d\boldsymbol{x} + \frac{\hat{\boldsymbol{p}}}{2|\boldsymbol{p}|} \cdot \boldsymbol{B} dt$

$$\boldsymbol{J} \equiv \int_{\boldsymbol{p}} \sqrt{G} f \frac{\delta \mathcal{I}}{\delta \boldsymbol{A}} = \underbrace{\int_{\boldsymbol{p}} \sqrt{G} f \dot{\boldsymbol{x}}}_{\text{normal/minimal current}} + \underbrace{\boldsymbol{\nabla} \times \int_{\boldsymbol{p}} \sqrt{G} f \frac{\hat{\boldsymbol{p}}}{2|\boldsymbol{p}|}}_{\boldsymbol{\nabla} \times \boldsymbol{M}}$$

magnetization current

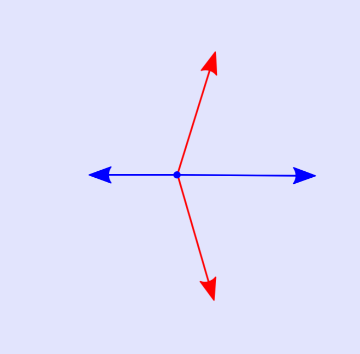
This current is Lorentz covariant

Boost, side jump and angular momentum conservation



+ boost =

$\longrightarrow \boldsymbol{\beta}$



$$\boldsymbol{P}_{\rm in} = \boldsymbol{P}_{\rm out} = 0$$
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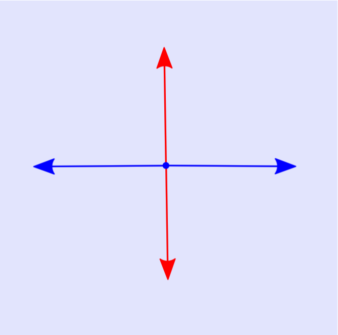
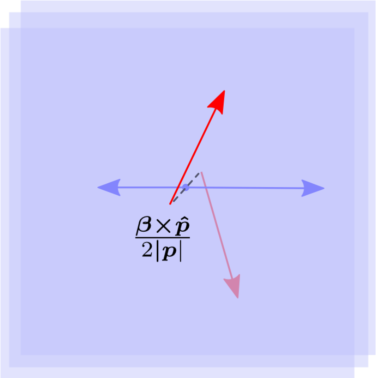
$$\boldsymbol{P}_{\rm in} = \boldsymbol{P}_{\rm out}$$
$$\boldsymbol{S}_{\rm in} = 0, \qquad \boldsymbol{S}_{\rm out} = \rightarrow \mathcal{O}(\hbar)$$
$$\boldsymbol{L}_{\rm in} = 0, \qquad \boldsymbol{L}_{\rm out} = 0???$$

Boost, side jump and angular momentum conservation





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"Side jump"

Collision kernel nonlocal

Collisions and Lorentz invariance

- No spin – no problem. Lorentz invariant form:

$$\partial_\mu j^\mu = \int_{BCD} \underbrace{C_{ABCD}}_{W_{CD\to AB} - W_{AB\to CD}}$$

$$j^\mu = p^\mu f$$

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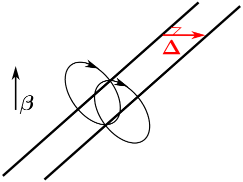
- Problems with spin:
- f is not a Lorentz scalar. Particle positions are frame-dependent.
- Side jump during collisions.

Position and spin in relativistic mechanics

- Center of mass (center of rotation) is frame dependent already in *classical* relativity.

  The side jump in frame n' relative to n:

$$\Delta^{\mu}_{nn'} = \lambda \, \frac{\epsilon^{\mu\alpha\beta\gamma} p_{\alpha} n_{\beta} n'_{\gamma}}{(p \cdot n)(p \cdot n')} \, .$$



  this is finite generalization of side jump (PRL 115(2015)021601)

- Correspondingly: $f_n(x) = f_{n'}(x + \Delta_{nn'})$.

Collisionfull CKT

Chen-Son-MS, PRL 115(2015)021601

$$\partial_\mu j^\mu = \int_{BCD} C_{ABCD}[\bar{f}]$$

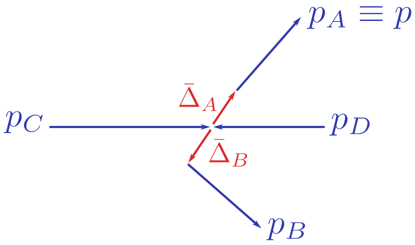
$$j^\mu = \underbrace{p^\mu f}_{\text{normal current}} + \underbrace{S^{\mu\nu}\partial_\nu f}_{\text{magnetization current}} + \underbrace{\int_{BCD} C_{ABCD}\,\bar{\Delta}^\mu_A}_{\text{jump current}}$$

($\bar{n}$ is the “no-jump” frame)

f, $S^{\mu\nu}$ and $\bar{\Delta} \equiv \Delta_{\bar{n}n}$ depend on n

but

j^μ is frame-independent!



Collisionfull CKT

Chen-Son-MS, PRL 115(2015)021601

$$\partial_\mu j^\mu = \int_{BCD} C_{ABCD}[\bar{f}]$$

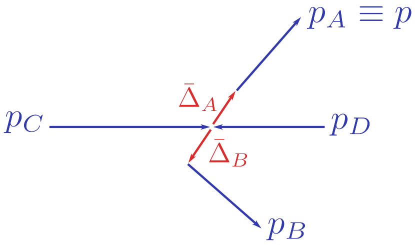
$$j^\mu = \underbrace{p^\mu f}_{\text{normal current}} + \underbrace{S^{\mu\nu}\partial_\nu f}_{\text{magnetization current}} + \underbrace{\int_{BCD} C_{ABCD}\,\bar{\Delta}^\mu_A}_{\text{jump current}}$$

($\bar{n}$ is the "no-jump" frame)

f, $S^{\mu\nu}$ and $\bar{\Delta} \equiv \Delta_{\bar{n}n}$ depend on n

but

j^μ is frame-independent!



H-theorem

- Covariant current:

$$\mathcal{H}^\mu = p^\mu \mathcal{H} + S^{\mu\nu}\partial_\nu \mathcal{H} + \int_{BCD} C_{ABCD}\,\bar{\Delta}^\mu \frac{\partial \mathcal{H}}{\partial f}$$

$$\mathcal{H} = f \ln \frac{1}{f} + (1-f)\ln\frac{1}{1-f} \quad \Leftrightarrow \quad \boxed{\partial_\mu \mathcal{H}^\mu \geq 0}$$

- Equilibrium solution ($\partial_\mu \mathcal{H}^\mu = 0$) is a rotating FD distribution.

$$J_{\text{CVE}} = \frac{\mu^2}{4\pi^2}\Big(\underbrace{\frac{1}{3}}_{\text{normal}} + \underbrace{\frac{2}{3}}_{\text{mag. current}}\Big)\omega$$

- No drag on a static impurity, but $S_{\text{CVE}} = \frac{\mu T}{6}\omega$ is not zero.

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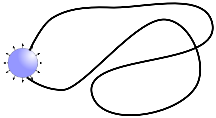
Summary/Conclusions

- Spin adds $\mathcal{O}(\hbar)$ terms to EOMs: Berry curvature and magnetic mom.

$$\dot{\boldsymbol{x}} = \frac{\partial \mathcal{E}}{\partial \boldsymbol{p}} + \dot{\boldsymbol{p}} \times \boldsymbol{b}$$

- Berry monopole accounts for CME and anomaly (source/sink at $\boldsymbol{p} = \boldsymbol{0}$).



- CVE from CKT in two ways: rotating frame or rotating distribution

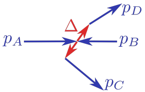
$$\boldsymbol{B} \to 2\mathcal{E}\boldsymbol{\omega}$$

$$\boldsymbol{J} = \underset{1/3}{\langle \dot{\boldsymbol{x}} \rangle} + \underset{2/3}{\boldsymbol{\nabla} \times \boldsymbol{M}}$$

- Nontrivial Lorentz invariance: side jump to conserve $\boldsymbol{L} + \boldsymbol{S}$; and requires $\Delta\mathcal{E} = -\boldsymbol{m} \cdot \boldsymbol{B}$.

$$\delta \boldsymbol{x} = \boldsymbol{\beta} t + \frac{\boldsymbol{\beta} \times \hat{\boldsymbol{p}}}{2|\boldsymbol{p}|}$$

- Lorentz invariance requires side jumps in collision kernel and jump currents



More . . .

Antiparticles

CPT

Same equations with opposite signs of $\boldsymbol{B}$, $\boldsymbol{E}$ and $\boldsymbol{b}$.

The charge current is $\boldsymbol{j} = \boldsymbol{j}_+ - \boldsymbol{j}_-$. The anomaly

$$\partial_\mu j^\mu = (\boldsymbol{E}\cdot\boldsymbol{B})\int_{\boldsymbol{p}} f_+ \boldsymbol{\nabla}_{\boldsymbol{p}}\boldsymbol{b} - \text{(a/p)} = \frac{1}{4\pi^2}\boldsymbol{E}\cdot\boldsymbol{B}\ \underbrace{(f_+ + f_-)_{\boldsymbol{p}=\boldsymbol{0}}}_{=1 \text{ for all } T \text{ and } \mu.}\ .$$