Lattice Two-Color QCD with nonzero chiral density

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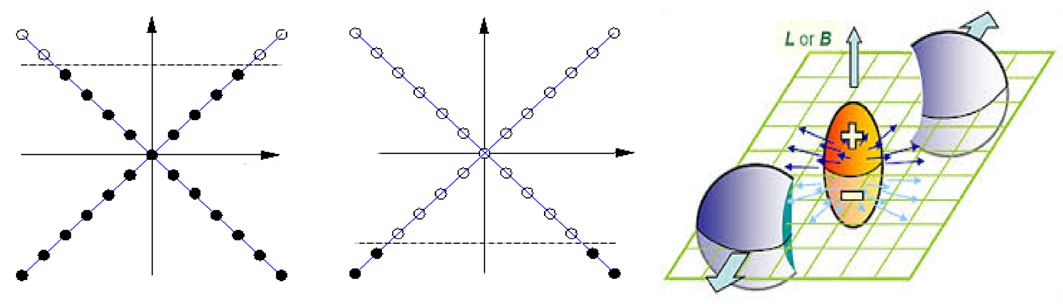
Abstract

The phase diagram of two-color QCD with non-zero chiral chemical potential is studied by means of lattice simulation. The main focus is on the influence of a chiral chemical potential on the confinement/deconfinement phase transition and the breaking/restoration of chiral symmetry. The simulation is carried out with dynamical staggered fermions. The dependences of the Polyakov loop, the chiral condensate and the corresponding susceptibilities on the chiral chemical potential and the temperature are presented. The critical temperature is observed to increase with increasing chiral chemical potential.

Introduction

The vacuum of QCD has nontrivial topological structure due to instantons, i.e. gluon configurations with nonzero topological charge(indirectly confirmed by the solution of the $U_A(1)$ problem). The existence of topologically nontrivial gluon configurations may be observed in heavy ion collisions, where the blobs of hot matter are assumed to be created(QGP). Topological sphaleron transitions and chiral anomaly $\Longrightarrow$ nonzero chiral density = different densities of left- and right-handed quarks.



External magnetic field $\Longrightarrow$ a nonzero electric current along the field - *chiral magnetic effect* (CME). But QCD matter must be in the deconfined chirally restored phase $\Longrightarrow$ theoretical study of the phase diagram of QCD with nonzero chiral density.
Nonzero chiral density is introduced by a chiral chemical potential μ_5.
No sign problem $\Longrightarrow$ standard Monte-Carlo algorithms!

Simulation details

Gauge group: $SU(2)$ with Wilson plaquette action:

$$S_g = \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{N_c} \mathrm{Tr}\, U_{\mu\nu}(x)\right) \quad (1)$$

Staggered fermionic action:

$$S_f = ma \sum_x \bar{\psi}_x \psi_x + \frac{1}{2} \sum_{x\mu} \eta_\mu(x)(\bar{\psi}_{x+\mu} U_\mu(x) \psi_x - c.c.) + \frac{1}{2}\mu_5 a \sum_x s(x)(\bar{\psi}_{x+\delta} \bar{U}_{x+\delta,x} \psi_x - c.c.), \quad (2)$$

where the $\eta_\mu(x)$ are the standard staggered phase factors: $\eta_1(x) = 1, \eta_\mu(x) = (-1)^{x_1+\ldots+x_{\mu-1}}$ for $\mu = 2,3,4$. Furthermore, a denotes the lattice spacing, m the bare fermion mass, and μ_5 the value of the chiral chemical potential. In the chirality breaking term $s(x) = (-1)^{x_2}$, $\delta = (1,1,1,0)$ represents a shift to a diagonally located site of a spatial elementary cube. The combination $\bar{U}_{x+\delta,x} = \frac{1}{6} \sum_{i,j,k=\mathrm{perm}(1,2,3)} U_i(x+e_j+e_k) U_j(x+e_k) U_k(x)$ is connecting sites x and $x+\delta$ symmetrized over the 6 shortest paths between these sites.
The continuum limit of a fermionic action:

$$S_f \to S_f^{(cont)} = \int d^4x \sum_{i=1}^{4} \bar{q}_i (\partial_\mu \gamma_\mu + igA_\mu \gamma_\mu + m + \mu_5 \gamma_5 \gamma_4) q_i. \quad (3)$$

Observables:

- Polyakov loop $\langle L \rangle$:

$$L = \frac{1}{N_\sigma^3} \sum_{n_1,n_2,n_3} \frac{1}{2} \mathrm{Tr} \left(\prod_{n_4=1}^{N_\tau} U_4(n_1,n_2,n_3,n_4) \right)$$

- Chiral condensate $a^3 \langle \bar{\psi}\psi \rangle$:

$$a^3 \langle \bar{\psi}\psi \rangle = -\frac{1}{N_\tau N_\sigma^3} \frac{1}{4} \frac{\partial}{\partial(ma)} \log(Z) = \frac{1}{N_\tau N_\sigma^3} \frac{1}{4} \langle \mathrm{Tr}\,(D+ma)^{-1} \rangle$$

Results

The chiral chemical potential: transition temperature grows!

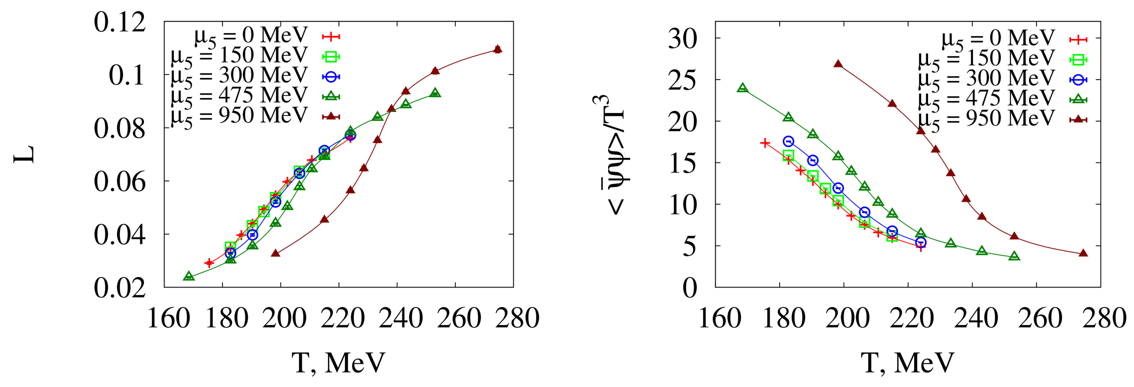


Figure 2: *Polyakov loop and chiral condensate versus T for five values of μ_5. Lattice size is 6×20^3, fermion mass is $m \approx 12$ MeV. Errors are smaller than the data point symbols.*

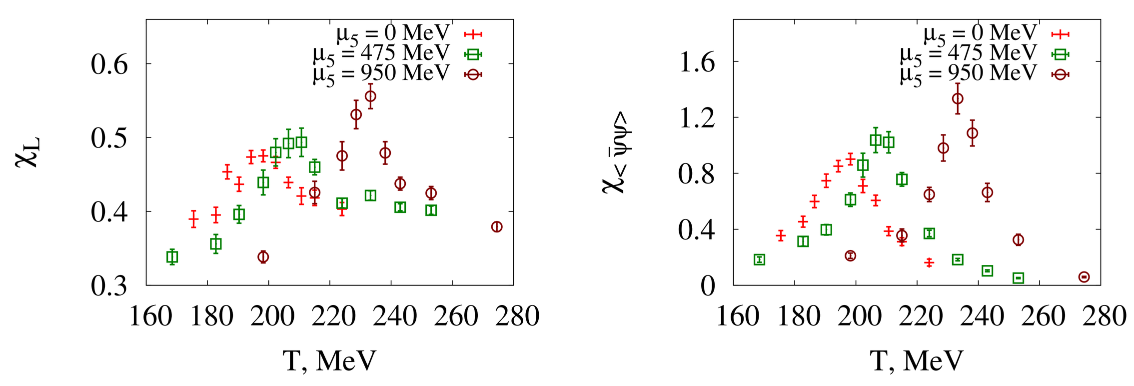


Figure 3: *Polyakov loop susceptibility and chiral susceptibility versus β for three values of $\mu_5 = 0, 475, 950$ MeV. Lattice size is 6×20^3, fermion mass is $m \approx 12$ MeV.*

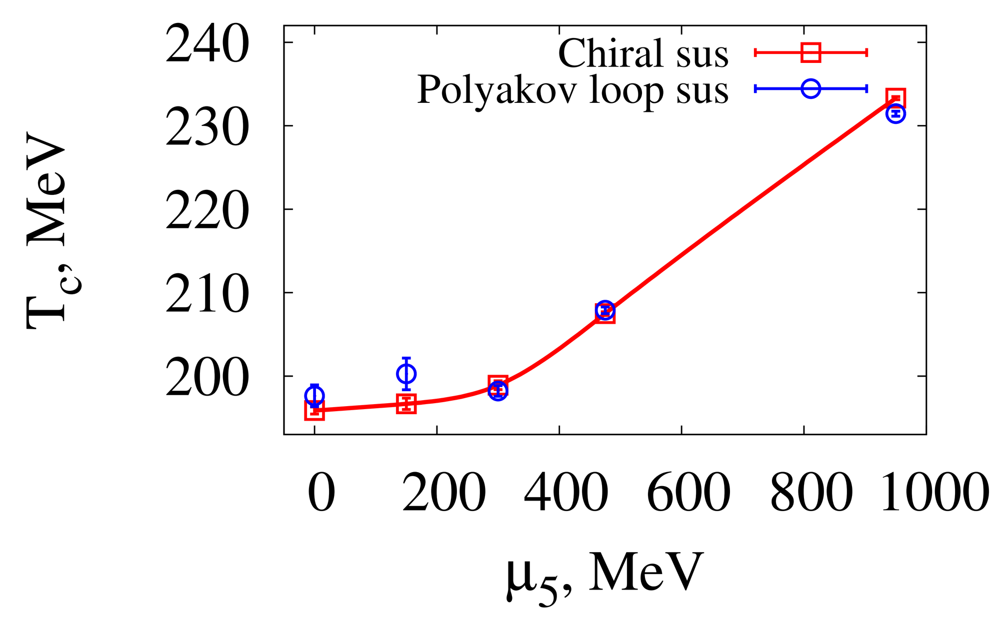


Figure 4: *The dependence of the critical temperature on the value of the chiral chemical potential. Lattice size is 6×20^3, fermion mass is $m \approx 12$ MeV.*

The results on larger lattice 10×28^3 match the results on the smaller lattice. The transition seems to become sharper.

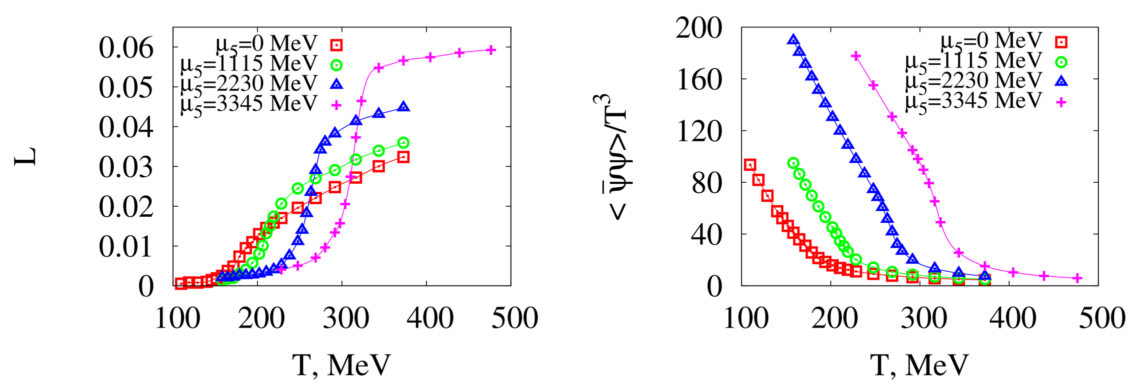


Figure 5: *Polyakov loop and chiral condensate versus T for three values of μ_5 and lattice size 10×28^3, fermion mass is $ma \approx 18.5$ MeV. Errors are smaller than the data point symbols.*

Observables L and $\langle \bar{\psi}\psi \rangle$ as functions of μ_5 for fixed temperature. The Polyakov loop drops down at large μ_5, the chiral condensate grows $\Longrightarrow$ the system goes into the confinement for large μ_5. The results are in agreement with the results above.

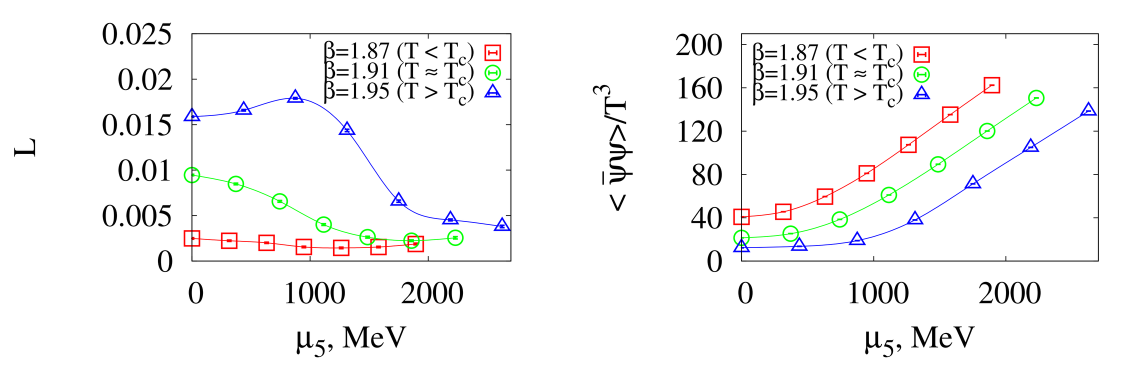


Figure 6: *Polyakov loop and chiral condensate versus μ_5 for three β values and lattice size 10×28^3, fermion mass is $ma \approx 18.5$ MeV. Errors are smaller than the data point symbols.*

Chiral limit

The chiral limit was studied for two temperatures: one in the confinement phase $T = 119$ MeV and one in the deconfinement phase $T = 268$ MeV.

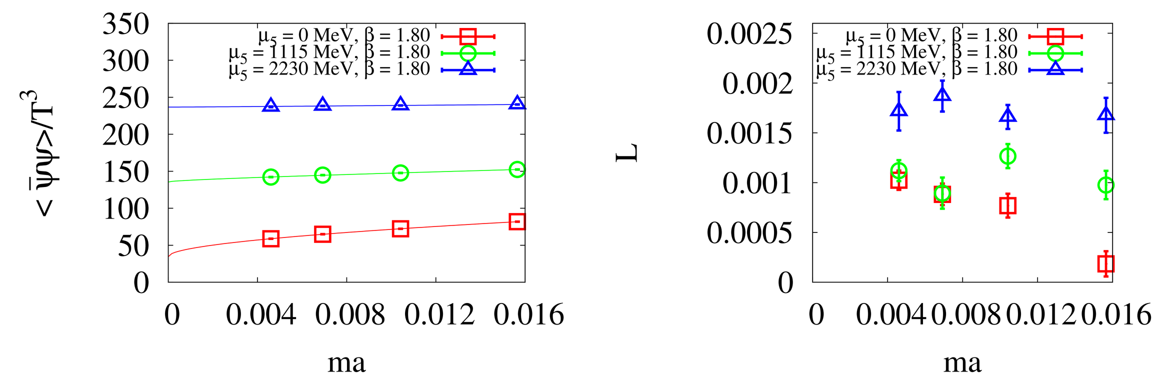


Figure 7: *Chiral condensate and Polyakov loop versus ma for three μ_5 values and lattice size 10×28^3. $\beta = 1.80$ corresponds to the confinement phase $T = 119$ MeV.*

At small $T = 119$ MeV the chiral condensate remains almost constant when the fermion mass changes. Extrapolation to the chiral limit.

$$f_1(ma) = a_0 + a_1 \sqrt{ma} + a_2 ma \quad (4)$$

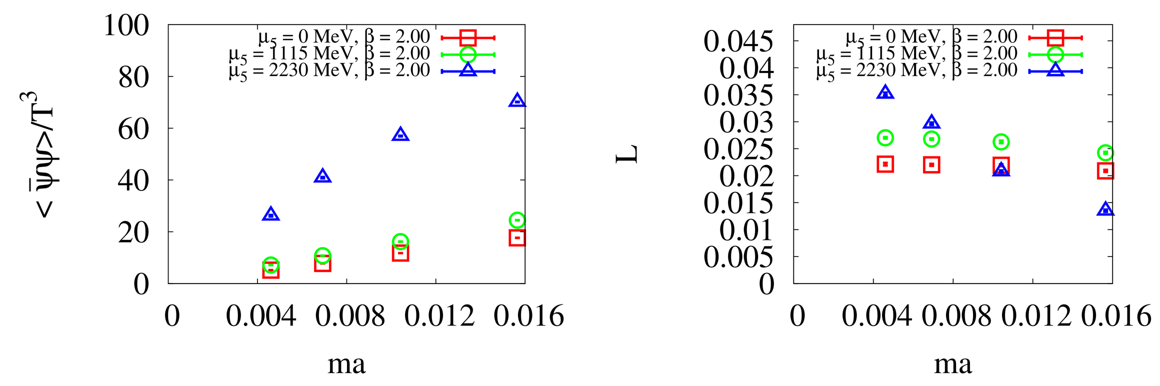


Figure 8: *Chiral condensate and Polyakov loop versus ma for three μ_5 values and lattice size 10×28^3. $\beta = 2.00$ corresponds to the deconfinement phase $T = 268$ MeV.*

At larger $T = 268$ MeV the system for all values of μ_5 is in the chirally restored phase. The results confirm that the critical temperature grows with μ_5!

First results for $N_f = 2$ SU(3) Wilson fermions

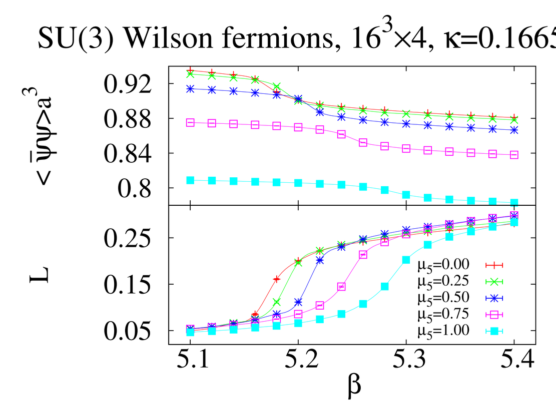


Figure 9: *Chiral condensate and Polyakov loop versus β for $N_f = 2$ SU(3) Wilson fermions, five μ_5 values and lattice size 4×16^3.*

Our first data for SU(3) confirm our results. The transition temperature grows with μ_5!

Conclusions and discussion

- The critical temperatures of the confinement/deconf. and of the χS breaking/restoration coincide;
- The transition shifts to larger temperatures as μ_5 increases;
- In the chiral limit the situation is the same;

Effective models of QCD [1, 2, 3] contradict our results - the critical temperature decreases as μ_5 increases and at $\mu > \mu_5^c$ the transition becomes of the first order.
But! Predictions in different effective models contradict each other(the chiral condensate in [2, 3, 4]).
Also analytical results corraborating our simulations:

1. Dyson-Schwinger equations[5]
2. Universality of QCD-like theories at large N_c. μ_5 is equivalent to μ_I, chiral condensate grows with μ_I [6].

References

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