Lattice simulation of two-color QCD with $N_f = 2$ at non-zero baryon density

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Abstract

We present the results of lattice simulation of $QCD_2$ with two flavors of staggered fermions and non-zero chemical potential. Low-temperature scan of the phase diagram has been performed. Dependencies of the chiral condensate, baryon number density and diquark condensate on $\mu_B$ were studied. We focus on raising of the baryon chemical potential leads to the chiral symmetry restoration. At small $\mu_B$ ($\mu_B < 0.45$ MeV) our results agree with ChPT predictions.

Introduction. Why do we need QCD in the low-temperature phase?

At the current moment there is no way in lattice QCD to perform direct simulations at finite $\mu_B$ (sign problem). Instead we can study simpler theory: QFT with $SU(2)$ gauge group and two degenerate flavors, in order to better understand qualitative features and test theoretical predictions.

Two-color QCD has the following peculiarities:

1. no sign problem, can be simulated on the lattice at finite quark chemical potential;
2. diquark condensate is colorless, we can directly study the superfluid phase;
3. phase diagram qualitatively resembles the phase diagram of QCD (see fig. 1).

![Figure 1: Schematic phase diagram of $QCD_2$ in the $(T, \mu_B)$ plane.](image)

Important note: we are not aiming to study hadron spectroscopy, we want to explore the properties of QGP with two colors.

Previous studies:

- $N_f = 1$ and $N_f = 3$ of staggered quarks in [1, 2, 3];
- $N_f = 2$ with Wilson fermions in [4, 5, 6].

But: Wilson fermions explicitly break the chiral symmetry, staggered fermions have the lattice version of it [7].

Lattice formulation and observables

We are working in Euclidean space, the QCD partition function for $N_f = 2$ has the following form:

$$Z = \int \mathcal{D}A \det(M(\mu_B)M(\mu_B)^*) e^{-S_0[A]}.$$  

where $S_0[A]$ is the pure gauge action, $\mathcal{D}A$ stands for the functional integration over the gluon fields and $M(\mu_B)$ is the Euclidean Dirac operator:

$$M(\mu_B) = \gamma_D p_B + m - i \gamma_5.$$  

The following relation holds in general case [7]:

$$\det[M(\mu_B)] = \det[M(-\mu_B)].$$

but $QCD_2$ has specific relation:

$$\det[M(\mu_B)] = \det[(\tau_3 C \gamma_5)^{-1} M(\mu_B) (\tau_3 C \gamma_5)] - \det[M(\mu_B)];$$

where $C = \gamma_5$.

So $\det[M(\mu_B)]$ is positive at real $\mu_B$.

Lattice formulation has the following form:

$$Z = \int D\psi \det[(\gamma_4 \psi^T)^{-1} M(\mu_B) (\gamma_4 \psi^T)];$$  

where

$$M_{\psi} = m_{\psi} + \frac{1}{2} \sum_{\nu} u_{\nu}(x) \left( \gamma_{\nu} \gamma_5 \gamma_{\nu} \psi_{\nu}(x) + \gamma_{\nu} \psi_{\nu}(x) \right) - \left( \gamma_{\nu} \psi_{\nu}(x) \right) \gamma_5.$$  

is the Dirac operator for the staggered fermions. Here $x$ is the lattice spacing, $m_\psi$ = quark mass, $\psi_\nu$ = diquark condensate $[1]$, and functions $u_{\nu}(x)$ are defined as follows: $u_{\nu}(x) = 1$, $u_{\nu}(x) = (-1)^{x_{\nu}^1}$, $u_{\nu}(x) = (-1)^{x_{\nu}^3}$, $u_{\nu}(x) = (-1)^{x_{\nu}^1} x_{\nu}^3$. Chemical potential $\mu_B$ is introduced in the lattice Dirac operator in the form of $\delta^{\nu\nu'}$ multipliers at temporal links to evade divergencies in the continuum limit [8]. Most of the power 4 from the fermionic determinant in (3) is due to specific nature of staggered fermions formulation, this root provides us $N_f = 2$ [7].

Wilson gauge action [9] was used in calculations:

$$S_G = \beta \sum_{x,\mu} \left( 1 - \frac{1}{2} U_{\mu,\mu} \right).$$

The following observables were considered:

- Polyakov loop:

$$\langle \Phi \rangle = \frac{1}{N_f^2} \sum_{x,\mu} \left( 1 - \frac{1}{2} U_{\mu,\mu} \right);$$

- chiral condensate:

$$\langle \langle \bar{\psi} \gamma_\mu \psi \rangle \rangle = \frac{1}{N_f^2} \left( \log Z \right);$$

- baryon number density:

$$\rho_B = \frac{1}{N_f^2} \left( \log Z \right);$$

- diquark condensate:

$$\langle \langle \bar{\psi} \gamma_\mu \gamma_5 \psi \rangle \rangle = \frac{1}{N_f^2} \left( \log Z \right);$$

Note, that in $QCD_2$ diquark condensate is colorless.

Lattice spacing and pion mass:

$$\beta = 1.5, \mu_B, MeV, \rho_B, MeV,$$

$$a = \frac{1}{0.1137[2]}, \frac{1}{MeV}.$$  

Figure 2: Chiral condensate as a function of $\mu_B$ in the low-temperature phase. The fit $(\gamma^2/\rho_B) \sim 1/\mu_B$ is shown, chiral condensate is plotted for comparison.

Conclusions

- increasing of the baryonic chemical potential leads to the chiral symmetry restoration;
- in the confinement phase the dependence of $\langle \langle \bar{\psi} \gamma_\mu \psi \rangle \rangle$ and diquark condensate on $\mu_B$ agrees with ChPT predictions [3, 10].

Now we are working on the phase diagram of $QCD_2$ in $(T, \mu_B)$ plane.

Numerical results

We have considered $\mu_B$ in the range of $[0, 0.25]$ MeV at fixed $T = 55$ MeV. For this temperature Polyakov loop is always zero within the errors. Result in the limit $\lambda = 0$ are shown below:

![Figure 3: Baryon number density as a function of $\mu_B$ in the low-temperature phase. Linear fit is shown.](image)

![Figure 4: Diquark condensate as a function of $\mu_B$ in the low-temperature phase. The fit $(\gamma^2/\rho_B) \sim 1/\mu_B$ is shown, chiral condensate is plotted for comparison.](image)

References