Lattice simulation of two-color QCD with $N_f = 2$ at non-zero baryon density

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Abstract

We present the results of lattice simulation of QC_2D with two flavors of staggered fermions and non-zero chemical potential. Low-temperature scan of the phase diagram has been performed. Dependencies of the chiral condensate, baryon number density and diquark condensate on μ_q were studied. We found, that raising of the baryon chemical potential leads to the chiral symmetry restoration. At small μ_q ($\mu_q < 400$ MeV) our results agree with ChPT predictions.

Introduction. Why do we need QC_2D?

At the current moment there is no way in lattice QCD to perform direct simulations at finite μ_B (sign problem). Instead we can study simpler theory: QFT with $SU(2)$ gauge group and two degenerate flavors, in order to better understand qualitative features and test theoretical predictions.

Two-color QCD has the following peculiarities:

1. no sign problem, can be simulated on the lattice at finite quark chemical potential;
2. diquark condensate is colorless, we can directly study the superfluid phase;
3. phase diagram qualitatively resembles the phase diagram of QCD (see fig. 1).

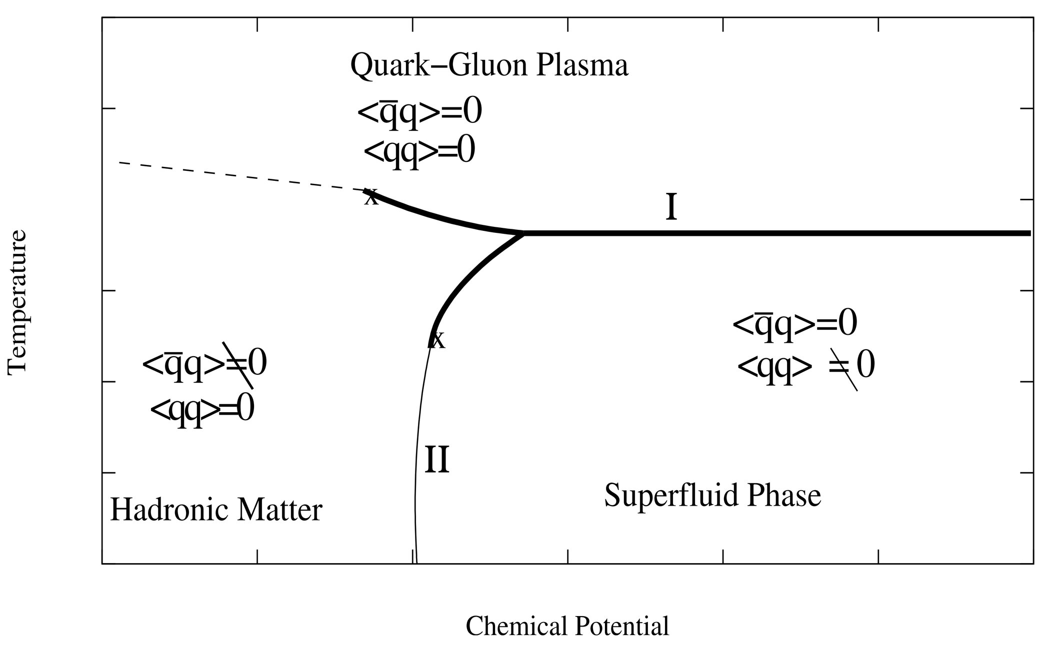

Figure 1: *Schematic phase diagram of QC_2D in the (T, μ) plane [1].*

Important note: we are not aiming to study hadron spectroscopy, we want to explore the properties of QGP with two colors.

Previous studies:

- $N_f = 4$ and $N_f = 8$ of staggered quarks in [1, 2, 3];
- $N_f = 2$ with Wilson fermions in [4, 5, 6].

But: Wilson fermions explicitly break the chiral symmetry, staggered fermions have the lattice version of it [7].

Lattice formulation and observables

We are working in Euclidean space, the QCD partition function for $N_f = 2$ has the following form:

$$Z = \int DA \, det\left[M^\dagger(\mu_q)M(\mu_q)\right] e^{-S_G[A]}, \qquad (1)$$

where $S_G[A]$ is the pure gauge action, DA stands for the functional integration over the gluon fields and $M(\mu_q)$ is the Euclidean Dirac operator:

$$M(\mu_q) = \gamma_\mu D_\mu + m_q - \mu_q \gamma_4. \qquad (2)$$

The following relation holds in general case [7]:

$$det\left[M(\mu_q)\right]^* = det\left[M(-\mu_q^*)\right],$$

but QC_2D has specific relation:

$$det\left[M(\mu_q)\right] = det\left[(\tau_2 C\gamma_5)^{-1} M(\mu_q)\,(\tau_2 C\gamma_5)\right] = det\left[M(\mu_q^*)\right]^*,$$

where $C = \gamma_2\gamma_4$.

So $det\left[M(\mu_q)\right]$ is positive at real μ_q.

Lattice formulation has the following form:

$$Z = \int DU \, det\left[M^\dagger(\mu_q)M(\mu_q) + \lambda^2\right]^{\frac{1}{4}} e^{-S_G[U]}, \qquad (3)$$

where

$$M_{xy} = ma\delta_{xy} + \frac{1}{2}\sum_{\mu=1}^{4}\eta_\mu(x)\Big[U_{x,\mu}\delta_{x+\hat\mu,y}e^{\mu_q a\delta_{\mu,4}} - U^\dagger_{x-\hat\mu,\mu}\delta_{x-\hat\mu,y}e^{-\mu_q a\delta_{\mu,4}}\Big], \qquad (4)$$

is the Dirac operator for the staggered fermions. Here a is the lattice spacing, m — quark mass, λ — diquark source [1], and functions $\eta_\mu(x)$ are defined as follows: $\eta_1(x) = 1$, $\eta_2(x) = (-1)^{x_1}$, $\eta_3(x) = (-1)^{x_1+x_2}$, $\eta_4(x) = (-1)^{x_1+x_2+x_3}$. Chemical potential μ_q is introduced in the lattice Dirac operator (4) in the form of $e^{\pm\mu_q a}$ multipliers at temporal links to evade divergencies in the continuum limit [8]. Root of the power 4 from the fermionic determinant in (3) is due to specific nature of staggered fermions formulation, this root provides us $N_f = 2$ [7].
Wilson gauge action [9] was used in calculations:

$$S_G = \beta \sum_x \sum_{\mu<\nu=1}^{4}\left(1 - \frac{1}{2}\mathrm{Tr}\, U_{x,\mu\nu}\right), \qquad (5)$$

where $\beta = \frac{4}{g^2}$ and $U_{x,\mu\nu} = U_{x,\mu}U_{x+\hat\mu,\nu}U^\dagger_{x+\hat\nu,\mu}U^\dagger_{x,\nu}$.

The following observables were considered:

- Polyakov loop:

$$\langle L \rangle = \frac{1}{N_s^3}\sum_{x_1,x_2,x_3=0}^{N_s-1}\frac{1}{2}\left\langle \mathrm{Tr}\prod_{x_4=0}^{N_\tau-1}U_{x,4}\right\rangle;$$

- chiral condensate:

$$a^3\left\langle\bar\psi\psi\right\rangle = -\frac{1}{N_s^3 N_\tau}\frac{1}{8}\frac{\partial(log\,Z)}{\partial(ma)};$$

- baryon number density:

$$a^3 n_B = \frac{1}{N_\tau N_s^3}\frac{1}{16}\frac{\partial(log\,Z)}{\partial(\mu_q a)};$$

- diquark condensate:

$$a^3\left\langle\psi^T\tau_2\psi\right\rangle = -\frac{1}{N_s^3 N_\tau}\frac{1}{8}\frac{\partial(log\,Z)}{\partial\lambda}.$$

Note, that in QC_2D diquark condensate is colorless.

Lattice spacing and pion mass:

β	a, fm	M_π, MeV
2.15	0.113(3)	355(8)

Numerical results

We have considered μ_q in the range of $[0.0; 525]$ MeV at fixed $T \approx 55$ MeV. For this temperature Polyakov loop is always zero within the errors.
Result in the limit $\lambda \to 0$ are shown below:

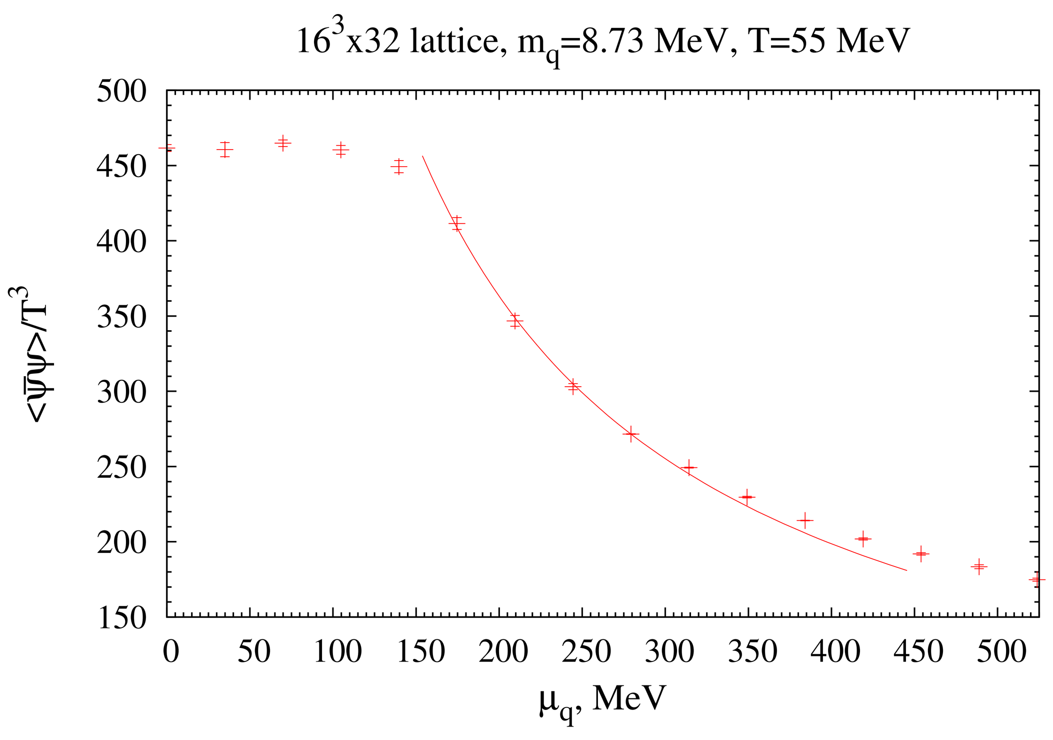

Figure 2: *Chiral condensate as a function of μ_q in the low-temperature phase. The fit $\langle\bar\psi\psi\rangle \sim 1/\mu_q$ is shown.*

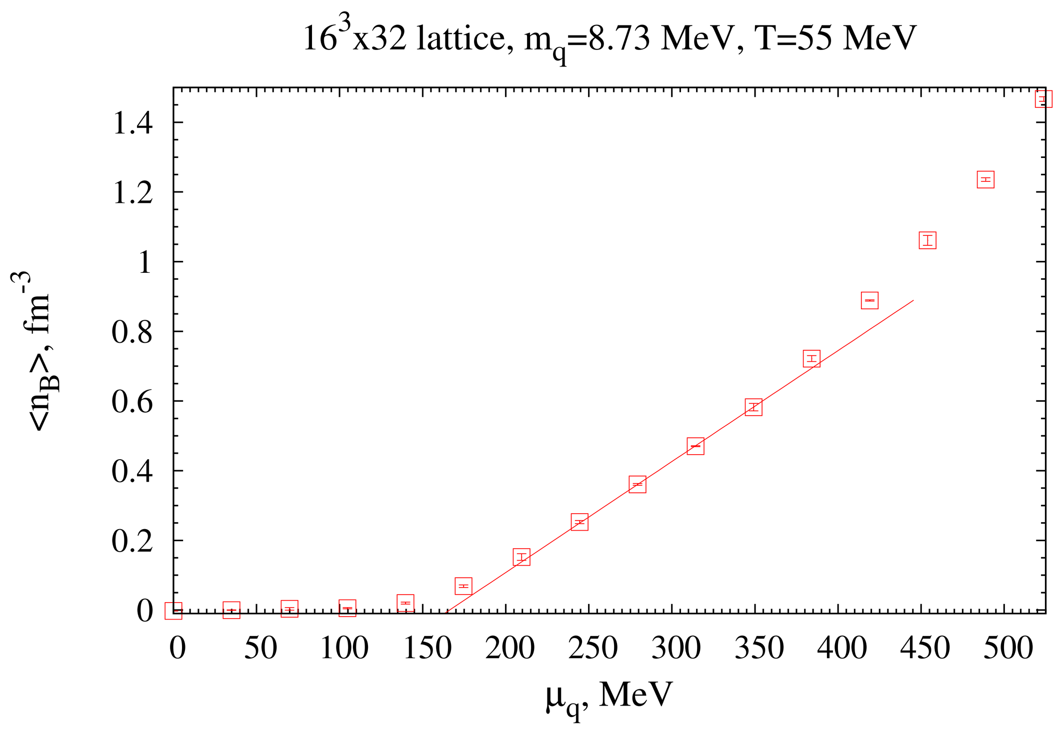

Figure 3: *Baryon number density as a function of μ_q in the low-temperature phase. Linear fit is shown.*

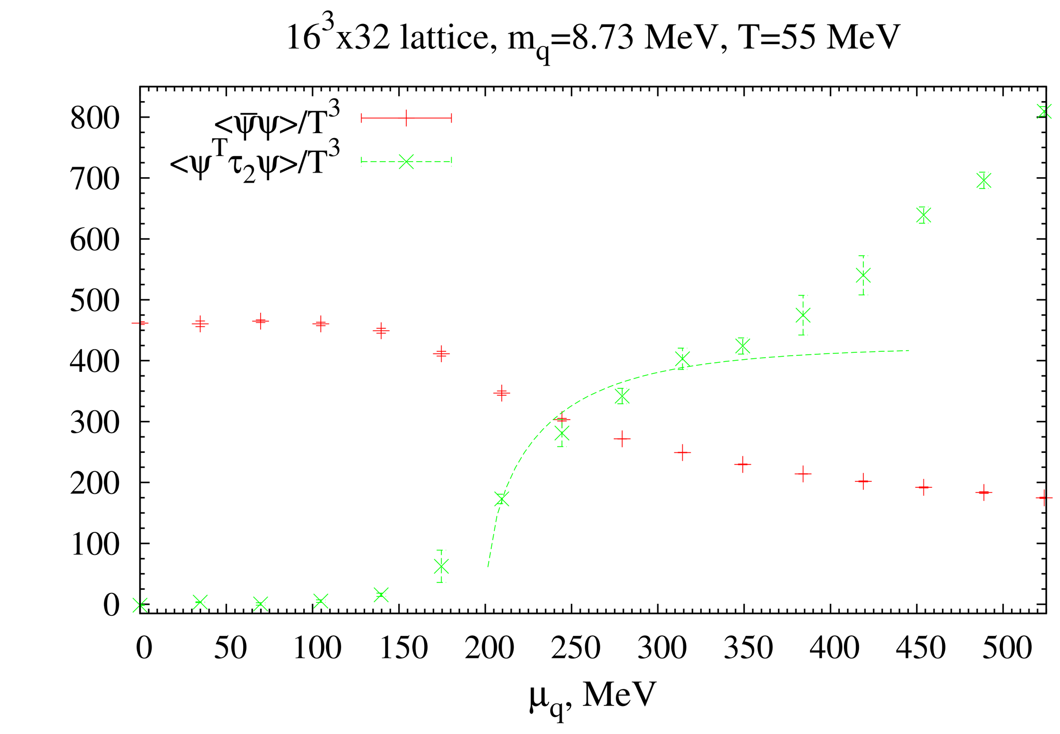

Figure 4: *Diquark condensate as a function of μ_q in the low-temperature phase. The fit $\left\langle\psi^T\tau_2\psi\right\rangle \sim \sqrt{1-\gamma/\mu_q^4}$ is shown, chiral condensate is plotted for comparison.*

Conclusions

- Increasing of the baryonic chemical potential leads to the chiral symmetry restoration;
- In the confinement phase the dependence of $\langle n_B\rangle$ and diquark condensate on μ_q agrees with ChPT predictions [3, 10].

Now we are working on the phase diagram of QC_2D in (T, μ_q) plane.

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