Interacting Ensemble of the Instanton-dyons and Deconfinement Phase Transition in the SU(2) Gauge Theory

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1: Motivation

- Use Instanton-dyons as an interacting gas to describe confinement-deconfinement phase transition in $SU(2)$ gauge theory
- Simulate 64 interacting dyons using Metropolis Monte-Carlo simulation
- Minimize free energy and find dominating configuration
- Observe how dominating configuration of dyons goes from deconfinement to confinement at low temperature

2: From Instanton to caloron to instanton-dyon

Instantons are **Classical Euclidian** Solutions, ie. local minima

$$\frac{\delta S}{\delta A_\mu} = 0 \qquad (1)$$

Temperature T=0: **Instantons** corresponds to tunneling in Minkowski space

$$\Downarrow$$

Finite T: **Instantons** become **Calorons** with periodic time boundaries

$$\Downarrow$$

Non-zero expectation value of color 3 component of A_4, ie. $< A_4^3 >$: $SU(2)$ **Calorons** are seen to be composed of 2 **Instanton-dyons** or dyons for short

3: Instanton-Dyons in $SU(2)$

The **dyons** have non-zero $< A_4^3 >$, which we define as **holonomy** ν

$$< A_4^3 > = 2\pi\nu T, \qquad 0 \le \nu \le 1 \qquad (2)$$

SU(2) have **two** Dyons, M and L, and two anti-dyons $\bar{M}$ and $\bar{L}$

M Dyon:

Electric and **magnetic** charge (+1)
Size scale as $1/\nu$
Action scale as ν

L Dyon:

Electric and **magnetic** charge (-1)
Size scale as $1/(1-\nu)$
Action scale as $(1-\nu)$

Small ν means large M dyon with small action
Anti-dyons, $\bar{M}$ and $\bar{L}$, have opposite magnetic charge of the dyons

4: Interactions

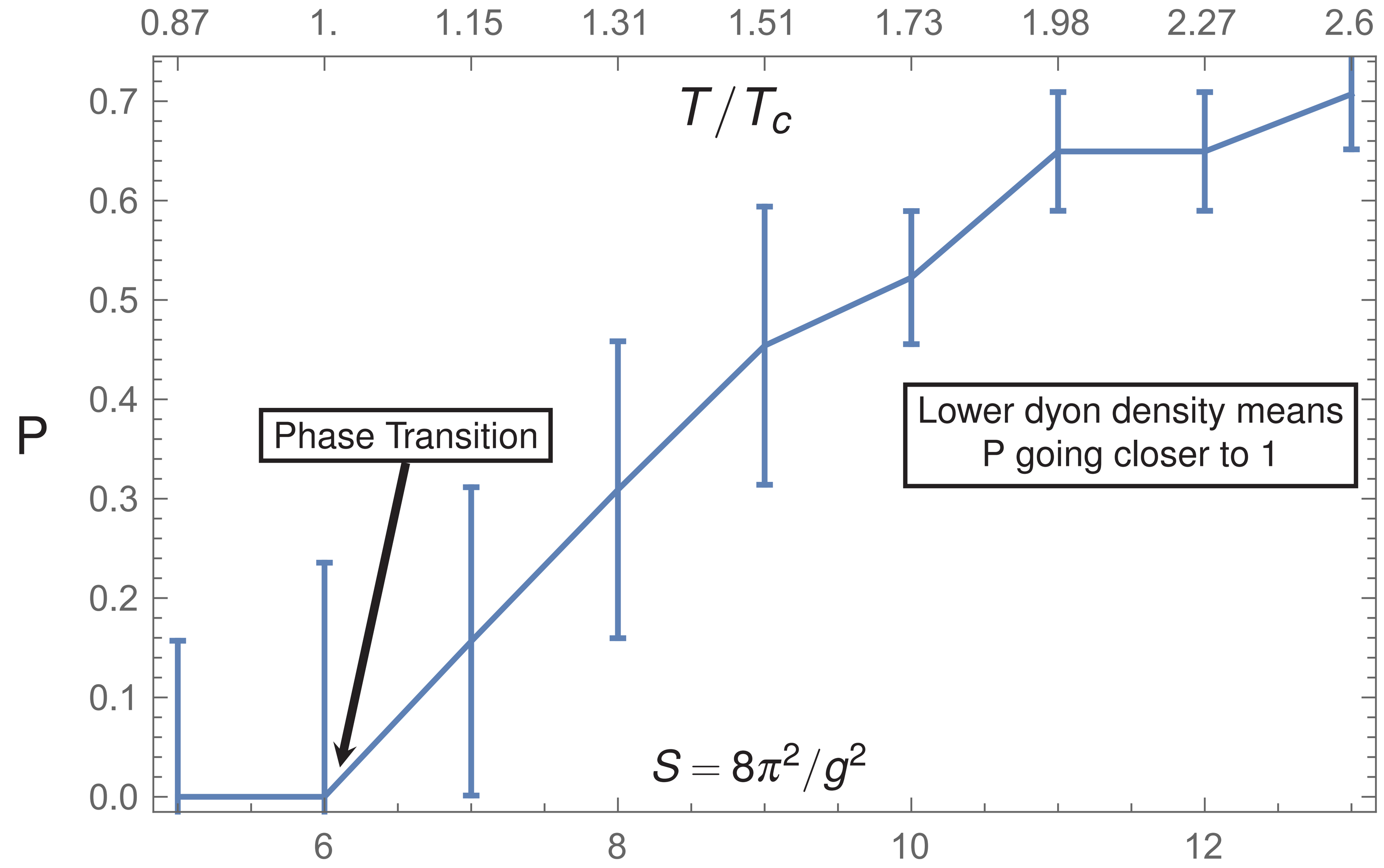
Long range **Coulomb like** interaction between dyons separated by a distance r

$$\Delta S = \frac{8\pi^2\nu}{g^2}\left(-e_1 e_2 \frac{1}{x} + m_1 m_2 \frac{1}{x}\right)$$
$$x = (2\pi\nu T) r \qquad (3)$$

plus a **hard core** below $x_0 = 2$ between dyons of same type, MM $L\bar{L}$ etc.
ν is replaced with $1-\nu$ for L type dyons, such that the core at $x_0 = 2$ scales correctly.

8 a: Results: Polyakov Loop

At high density, low temperature, the dyons force the **Polyakov loop** to **zero**



- The phase transition from deconfinement to confinement is observed
- The Polyakov loop gets closer to 0 as the increased density of the dyons forces the M type dyons to become smaller and smaller until they become the same size as L type dyons

10: References

- Interacting Ensemble of the Instanton-dyons and Deconfinement Phase Transition in the SU(2) Gauge Theory
  R. Larsen and E. Shuryak, arXiv:1504.03341 [hep-ph]

5: Confinement and the Polyakov Loop

Order parameter for confinement is the trace of the **Polyakov loop**

$$P = < \frac{1}{2} Re\left(Tr\left[Path-ordered\left(\exp(i\int_0^{1/T} d\tau A_4)\right)\right]\right) > \qquad (4)$$

Dyons live in the configurations of **non-trivial Polyakov loop**, ie. $P \neq 1$

$$P = < \cos(A_4^3/(2T)) > \qquad (5)$$
$$= \cos(\pi\nu) \qquad (6)$$

$P = 0$, $\nu = 0.5$ is confined phase and $P = 1$, $\nu = 0$ is deconfined phase

6: Free Energy Density

Dominating configuration **minimizes** the free energy density f

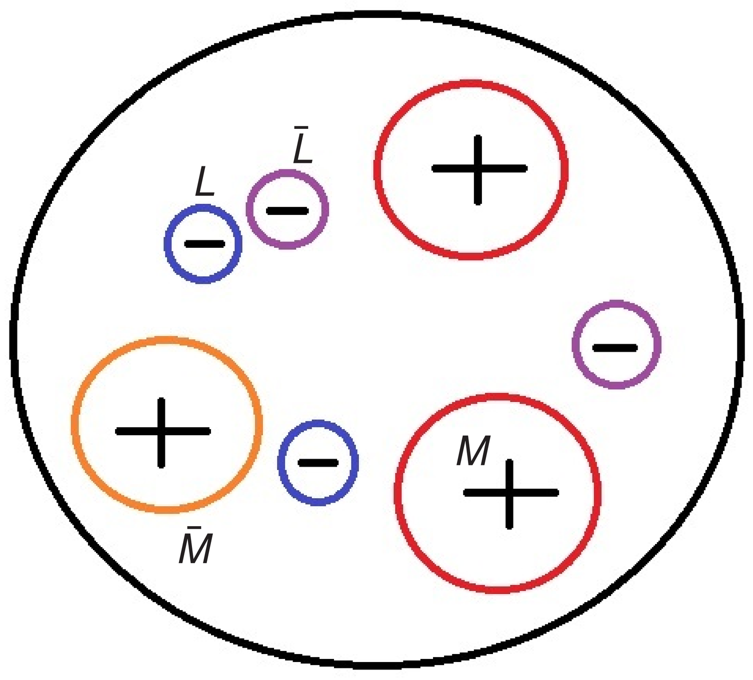
$$f = \underbrace{\frac{4\pi^2}{3}\nu^2(1-\nu)^2}_{\text{GPY potential}} - \underbrace{2n_M \ln\left[\frac{d_\nu e}{n_M}\right] - 2n_L \ln\left[\frac{d_{(1-\nu)} e}{n_L}\right]}_{\text{Entropy of M and L type dyons}} + \Delta f_{Interactions} \qquad (7)$$

- Free energy density contains **3** items
  - The GPY potential that prefer **trivial Holonomy** $\nu = 0$
  - The **entropy** due to the density of the dyons n_i
  - $(\Delta f_{Interactions})$ **Correction** to the energy due to the interactions of the dyons
- d_ν and $d_{(1-\nu)}$ **increase as Temperature decrease**

7: Metropolis Algorithm and Dyon Ensemble

- Dyon ensemble done on sphere in 4D, using the Metropolis algorithm.
- Position changed randomly and new configuration accepted based on the change in action

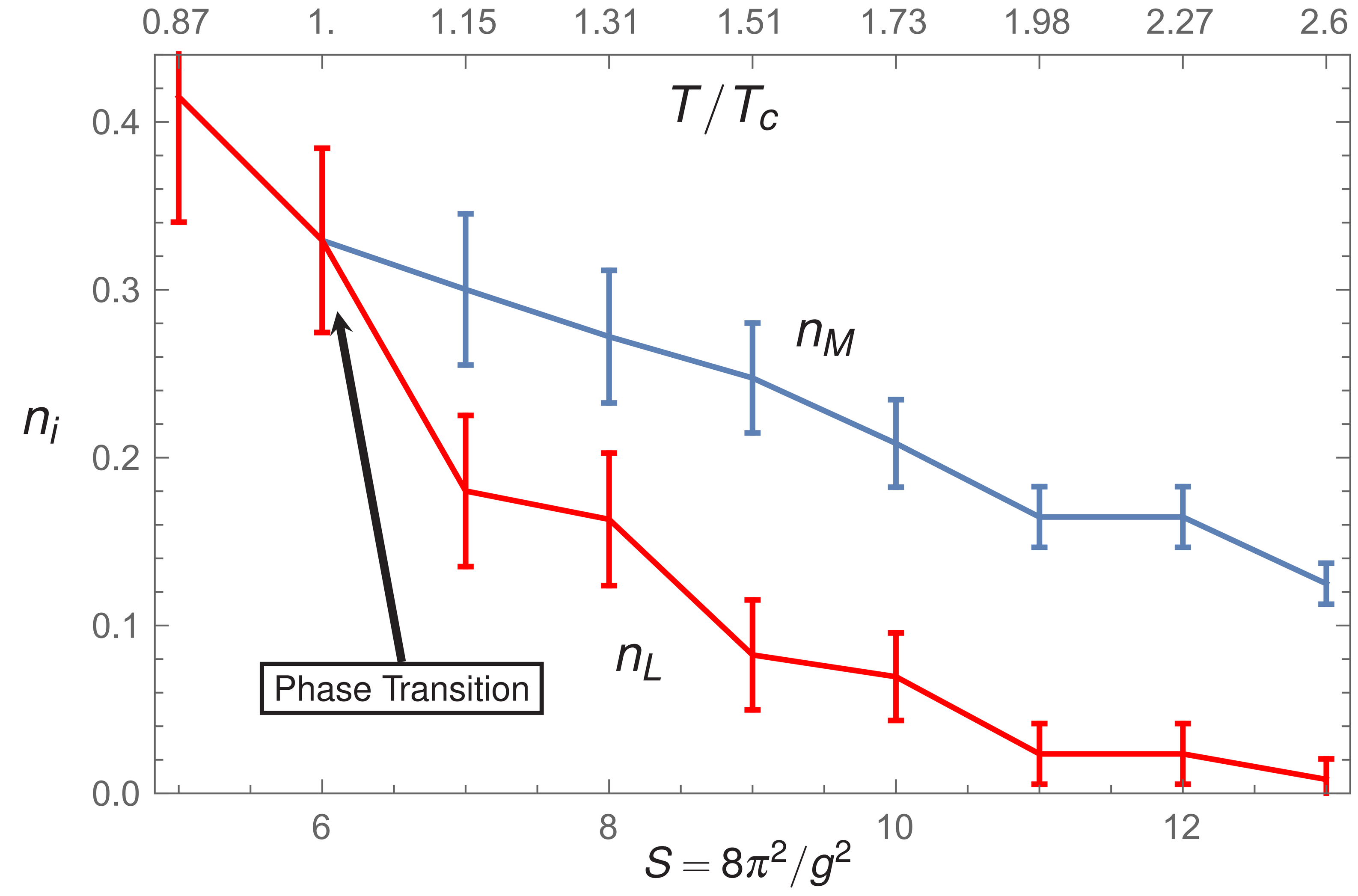
- The right picture shows a sketch of the setup
- Circle represents dyons in the sphere
- M type dyons are $+$
- L type dyons are $-$
- Size is given by $1/\nu$ and $1/(1-\nu)$
- Action is νS and $(1-\nu)S$
- $S = 8\pi^2/g^2$
- **Action and size are important because:**
  - Low action means more dyons
  - Big dyons means that you can fit fewer dyons into the sphere



- Small ν makes big M dyons
- **High density** of dyons force the M dyons to **become smaller**, drives toward confinement
- Density of dyons **increases** at low temperature T

8 b: Results: Dimensionless Dyon Density

- The confined phase is when the holonomy is $\nu = 0.5$
- When $\nu = 0.5$ the action and size of all the dyons are the same
- The confined-deconfined phase transition is a transition from **symmetric** dyon density, to **non-symmetric** dyon density



9: Conclusion

- Metropolis Monte-Carlo simulations have been done with as gas of dyons
- The phase transition from deconfinement to confinement is observed
- Increased density of dyons forces the holonomy to become larger
- At $\nu = 0.5$, $P = 0$, the holonomy stops. It has reached the confined phase
- The densities of dyons are all the same in the confined phase