Bjorken Flow in Semi-Holographic QCD

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Abstract

A semi-holographic approach combining the Boltzmann equation and the gauge/gravity duality is applied to study a gluon plasma which consists of hard and soft degrees of freedom, where the former is at the saturation scale of QCD and the latter is around the temperature of the quark gluon plasma in relativistic heavy ion collisions. The model aims at analyzing gluonic transport in the phase after the thermalization of soft degrees of freedom. The hard sector is presumed to be weakly interacting, where its dynamics is characterized by kinetic theory with the collisional term incorporating the scattering of hard and soft gluons. In contrast, the soft sector is strongly interacting, which could be described by an infrared conformal field theory as the gauge theory dual of a supergravity theory in thermal equilibrium. In general, the dynamics of the soft gluons could be captured by hydrodynamics and the interaction with the hard sector then appears as a driving force thorough the conservation equation, while the input of holography is solely to obtain the transport coefficients. The corrections on both sectors could be solved order by order in terms of the marginal coupling of two sectors on the boundary. In early times, the hard gluons could be characterized by the classical Yang-Mills theory with large occupation numbers. In the longitudinal expanding system, the energy density of soft gluons receives non-hydrodynamic correction, which depends on the dynamics of the hard gluons captured by the non-thermal attractor solution. Given a particular choice of the scaling exponents, the correction could become hydrodynamic-like and increases the effective shear viscosity of the soft sector. In late times, we may expect the thermalization of the hard sector, which then gives rise to at most 2nd-order hydrodynamic correction on the soft sector.

The Semi-Holographic Model

We follow the semi-holographic model introduced in [1], which was initially developed to study the thermalization process in early times. In relativistic heavy ion collisions, the partons (mostly gluons) generated in the early stage are dictated by the saturation scale Q_s. Such gluon fields at high density could be characterized by the classical Yang-Mills theory with color sources in colliding nuclei, known as the color glass condensate (CGC) [2,3,4]. They may radiate soft gluons leading to the thermalization of the medium.

hard gluons (weakly coupled): $g(Q_s) \ll 1$ — non-perturbative $A_\mu \sim 1/g(Q_s)$ → high density $f \sim 1/\alpha_s(Q_s) \gg 1$

radiated soft gluons (strongly coupled): $\alpha_s(T) \gg 1$ (after thermalization) ← described by holographic (even before thermal equilibrium)

The semi-holographic model delineates the mutual interaction between the hard and soft sectors trough the marginal coupling on the boundary in gauge/gravity duality. In such a framework, the hard gluons from CGC yield a driving force in the conservation equations governing the dynamics of soft gluons, while the classical Yang-Mills equations are modified by the soft gluons as well.

effective action :
$$S_{\rm glasma} = -\int {\rm d}^4x \,\frac{1}{4N_c}{\rm Tr}(F_{\alpha\beta}F^{\alpha\beta}) - \frac{\beta}{Q_s^4}\int {\rm d}^4x\, h\,\mathcal{H} - \frac{\gamma}{Q_s^4}\int {\rm d}^4x\, t_{\mu\nu}\mathcal{T}^{\mu\nu}$$

hard gluons :
$$D_\mu F^{\mu\nu} = -\frac{\beta}{Q_s^4}D_\mu(F^{\mu\nu}\mathcal{H}) + \frac{\gamma}{Q_s^4}D_\mu(F^{\mu\nu}\mathcal{T}^{\alpha\beta}\eta_{\alpha\beta}) - \frac{2\gamma}{Q_s^4}D_\mu(\mathcal{T}^{\mu\alpha}F_\alpha{}^\nu + F^\mu{}_\alpha\mathcal{T}^{\alpha\nu})$$
$$t_{\mu\nu}(x) = \frac{1}{N_c}{\rm Tr}\left(F_{\mu\alpha}F_\nu{}^\alpha - \frac{1}{4}\eta_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right)$$
$$h(x) = \frac{1}{4N_c}{\rm Tr}(F_{\alpha\beta}F^{\alpha\beta})$$

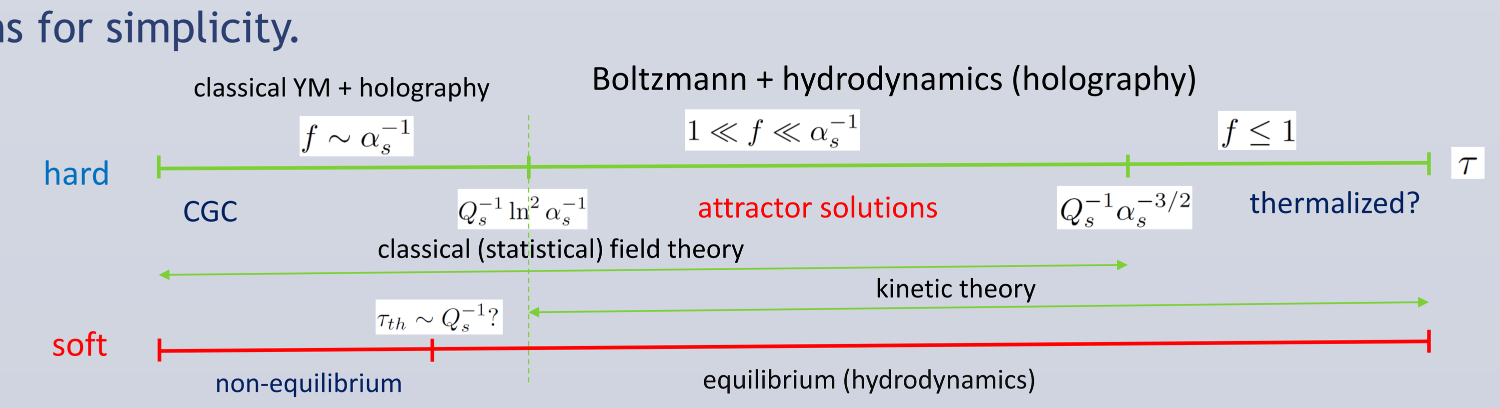
The conservation equations can be further written in terms of an effective metric :

soft gluons :
$$\nabla_\mu\mathcal{T}^{\mu\nu}(x) = \frac{\beta}{Q_s^4}\mathcal{H}(x)\nabla^\nu h(x),$$
$$g_{(0)\mu\nu}(x) = \eta_{\mu\nu} + \frac{\gamma}{Q_s^4}t_{\mu\nu}(x)$$
$$\partial_\mu\mathcal{T}^{\mu\nu} = \frac{\beta}{Q_s^4}\mathcal{H}\,g^{\mu\nu}_{(0)}[t]\,\partial_\mu h - \Gamma^\gamma_{\alpha\gamma}[t]\mathcal{T}^{\alpha\nu} - \Gamma^\nu_{\alpha\beta}[t]\mathcal{T}^{\alpha\beta}$$

The coupled equations of the hard and soft sectors have to be solved numerically with a proper initial condition. Although it is technically difficult to solve the early-time dynamics, it is enlightening to study the late-time behavior after the formation of the QGP, where we assume the thermalization of the soft degrees of freedom.

Combination with the Boltzmann Equation

In late times ($\tau \gg 1/Q_s$), the occupation number drops and the classical approximation for hard gluons may become invalid. Nonetheless, one may apply the kinetic theory to hard gluons for sufficiently small occupancies $f \ll 1/\alpha_s$. Particularly, it could be compatible with the classical-statistical field description for occupancies greater than unity $1 \ll f \ll 1/\alpha_s$. The emergence of universal attractor solutions governing the non-equilibrium evolution have been found [5,6,7]. Also, the dynamics of hard gluons could be modified by the scattering with soft gluons from the Boltzmann equation. However, it is found that such a correction is sub-leading in terms of the effective coupling γ (we set $\beta = 0$ for simplicity) from collisions. Considering only the leading correction $O(\gamma)$, we focus on the modification on the dynamics of soft gluons led by hard gluons characterized by attractor solutions for simplicity.



hard gluons : (with scattering)
$$\frac{p^\mu}{E_p}\partial_\mu f(p,x) = \frac{\gamma^2}{E_p}\int\frac{d^3\mathbf{q}}{(2\pi)^3E_{p-q}}\frac{\epsilon(q_0)}{Q^8}\tilde{t}^{\mu\nu\rho\sigma}(q,p){\rm Im}[\Pi^R_{\mu\nu,\rho\sigma}(q)]D(q,p) + \mathcal{C}[p;f]$$
$$t^{\mu\nu} = \int\frac{d^3p}{(2\pi)^3 2E_p}p^\mu p^\nu f(p,x)$$
$$\Pi^R_{\mu\nu,\rho\sigma}(q) = -i\int d^4x e^{-iq\cdot x}\theta(t)\langle[\mathcal{T}_{\mu\nu}(x),\mathcal{T}_{\rho\sigma}(0)]\rangle$$

Attractor Solutions and Non-Hydrodynamic Corrections

The classical statistical theory effectively describes non-equilibrium systems and reveals the universal behaviors characterized by non-thermal fixed points. The single-particle distribution function at fixed points is captured by the attractor solutions [6,7], which give rise to non-thermal energy-momentum tensors of hard gluons in our approach.

$$f(p_T,p_z,\tau) = (Q_s\tau)^\alpha f_S\left((Q_s\tau)^\lambda p_T, (Q_s\tau)^\kappa p_z\right) \Longrightarrow t^{\rm YM}_{\mu\nu} = {\rm diag}\left(2\frac{\Delta}{(Q_s\tau)^p} + \mathcal{O}((Q_s\tau)^{-q}), \frac{\Delta}{(Q_s\tau)^p}, \frac{\Delta}{(Q_s\tau)^p}, \mathcal{O}((Q_s\tau)^{-q})\right)$$
(assuming q>p by anisotropy)

For simplicity, we consider the Bjorken scenario for an expanding system. We assume that the soft gluons have reached thermal equilibrium in the temporal region where the kinetic-theory description of the hard gluons is valid. The dynamics of the soft part is now described by a holographic fluid living in the effective background metric, which follows the conservation equation.

$$\nabla_{({\rm b})\mu}\mathcal{T}^{\mu\nu} = 0, \qquad g_{({\rm b})\mu\nu} = {\rm diag}\left(-1 + 2\frac{\gamma}{Q_s^4}\frac{\Delta}{(Q_s\tau)^p}, 1 + \frac{\gamma}{Q_s^4}\frac{\Delta}{(Q_s\tau)^p}, 1 + \frac{\gamma}{Q_s^4}\frac{\Delta}{(Q_s\tau)^p}, \tau^2\right)$$
in Rindler coordinates (τ, x, y, ξ)

We consider the 1st order hydrodynamics for the soft gluons:

$$\mathcal{T}^{\mu\nu} = \epsilon\left(u^\mu u^\nu + \frac{1}{3}\Delta^{\mu\nu}\right) - \eta_0\epsilon^{\frac{3}{4}}\sigma^{\mu\nu} + \mathcal{O}(\nabla^2),$$
$$\Delta^{\mu\nu} = u^\mu u^\nu + g^{\mu\nu}_{({\rm b})},$$
$$\sigma^{\mu\nu} = \frac{1}{2}\Delta^{\mu\rho}\Delta^{\nu\sigma}\left(\nabla_{({\rm b})\rho}u_\sigma + \nabla_{({\rm b})\sigma}u_\rho\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\rho\sigma}\nabla_{({\rm b})\rho}u_\sigma .$$
η_0 is associated with the shear viscosity

We work under the co-moving frame of the effective metric: $u^\mu = \left(\frac{1}{\sqrt{1 - 2\frac{\gamma}{Q_s^4}\frac{\Delta}{(Q_s\tau)^p}}}, 0, 0, 0\right), \quad \epsilon = \epsilon(\tau)$

Now, $t^{YM}{}_{\mu\nu}$ will modify the late-time evolution of the soft-gluon fluid. We then determine the value of the scaling exponent p from the attractor solution found in the classical YM simulation in 2+1 dimensions [6,7]. Interestingly, the exponents $(\alpha, \lambda, \kappa)$ from the simulations agree with the bottom-up thermalization scenario previously proposed by Baier, Mueller, Schiff, and Son (BMSS) [8]. In general, the exponents could be determined by the small-angle scattering, number conservation, and energy conservation.

small-angle scattering : $2\alpha - 2\lambda + \kappa + 1 = 0$
number conservation: $\alpha - 2\lambda - \kappa + 1 = 0$
energy conservation: $\alpha - 3\lambda - \kappa + 1 = 0$
⟹ classical attractor sol. : $\alpha = -\frac{2}{3}, \lambda = 0, \kappa = \frac{1}{3}$

The classical attractor solution then leads to $p = 1, q = 5/3$.

$$t^{zz} = \tau^2 t^{\xi\xi} = P_L = 2\mathcal{N}_g\int\frac{d^2p_T dp_z}{(2\pi)^3 2E_p}\frac{p_z^2}{(Q_s\tau)^{2/3}}f_S(p_T,(Q_s\tau)^{1/3}p_z) \approx \frac{P_{L0}}{(Q_s\tau)^{5/3}},$$
$$t^{xx} = t^{yy} = P_T = \mathcal{N}_g\int\frac{d^2p_T dp_z}{(2\pi)^3 2E_p}\frac{p_T^2}{(Q_s\tau)^{2/3}}f_S(p_T,(Q_s\tau)^{1/3}p_z) \approx \frac{P_{T0}}{Q_s\tau},$$
$$t^{\tau\tau} = \mathcal{E} = 2P_T + P_L.$$

Solving the conservation equation of soft gluons driven by an external force of the hard gluons characterized by the classical attractor solution, we find the non-hydrodynamic correction for the late-time evolution of soft gluons. In addition, for an observer in flat space, the conformal symmetry of the soft part is now broken by the hard scale Q_s.

$$\epsilon = T_0^4\left(\frac{\tau_0}{\tau}\right)^{\frac{4}{3}} - \eta_0 T_0^3\frac{\tau_0}{\tau^2} - \frac{4}{3}\gamma T_0^4\frac{\Delta\tau_0^{\frac{4}{3}}}{Q_s^5\tau^{\frac{7}{3}}} + \mathcal{O}\left(\gamma^2, \tau^{-\frac{8}{3}}\right) \qquad \eta_{\mu\nu}\mathcal{T}^{\mu\nu} = -\frac{8}{3}\gamma\frac{\Delta T_0^4\tau_0^{\frac{4}{3}}}{Q_s^5\tau^{\frac{7}{3}}} + \mathcal{O}\left(\gamma^2,\tau^{-\frac{8}{3}}\right) \neq 0.$$
for $p = 1$

hydrodynamic Bjorken expansion : $\tau^{-(4+2n)/3}$, $n = 0,1,2 \ldots$.

non-hydrodynamic corrections : $\tau^{-(7+2n)/3}$, $n = 0,1,2 \ldots$.

On the other hand, we may propose an ad-hoc solution giving rise to $p = 2/3$, which yields the correction with the same power law in time as the hydrodynamic corrections. The leading-order correction in such a scenario increases the "effective shear viscosity". Nonetheless, such an attractor solution will violate the number and energy conservation, which might be attributed to nontrivial collisional terms.

$$\epsilon = \frac{T_0^4\tau_0^{\frac{4}{3}}}{\tau^{\frac{4}{3}}} - \left(\eta_0 T_0^3\tau_0 + \frac{4}{3}\gamma T_0^4\frac{\Delta\tau_0^{4/3}}{Q_s^{14/3}}\right)\frac{1}{\tau^2} + \mathcal{O}\left(\gamma^2,\tau^{-\frac{8}{3}}\right) \qquad \eta_{\mu\nu}\mathcal{T}^{\mu\nu} = -\frac{8}{3}\gamma\frac{\Delta T_0^4\tau_0^{4/3}}{Q_s^{14/3}}\frac{1}{\tau^2} + \mathcal{O}\left(\gamma^2,\tau^{-\frac{8}{3}}\right).$$
for $p = 2/3$

Increase of the effective shear viscosity

Finally, in very late times ($\tau > {Q_s}^{-1}{\alpha_s}^{-3/2}$), the dynamics of hard gluons should be solely dictated by the kinetic theory. It is also possible that hard gluons may be thermalized (isotropolized) in the end. However, when the hard gluons also follow the Bjorken expansion, the corrections on the soft part will be hydrodynamic-like at higher orders (starting from 2nd order $\delta\epsilon \sim \gamma\tau^{-8/3}$), which become negligible. In phenomenology, given that the hard gluons follow non-hydrodynamic evolution in late times, the γ corrections on the soft gluons may cause nontrivial modification of the particle production on the freeze-out surface.

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