Recently, it becomes more and more attractive to study properties of QCD matter under strong magnetic field. In the laboratory, heavy ion collisions offer a unique platform to probe properties of QCD vacuum and hot/dense quark matter under strong magnetic fields.

Maxim Chernodub firstly proposed that charged p meson can form a supercondensate state when magnetic field is stronger than the critical magnetic field \( eB_c = 0.2\text{ GeV}^2 \). It is still under debate whether there exits a vacuum supercondensate. Different methods give different results. Moreover, for charged p condensation, the obtained results for the critical magnetic field are quite different in different calculations. In our last work, we have obtained the critical magnetic field \( eB_c = 0.2\text{ GeV}^2 \) in the NJL model by calculating the p meson polarization function to the leading order of \( 1/N_c \) expansion.

In this work, we are going to study the charged p mesons in a magnetic field at finite temperature and chemical potential region for the survival of charged p condensation.

**Model and formalism**

In our model, the Lagrangian density is given by

\[
\mathcal{L} = \bar{u}(\gamma^\mu D^\mu + m) u + G_\rho [\bar{u} \gamma^\rho u]^2 + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - i \bar{u} \gamma^\mu \gamma^5 \rho \Gamma_{\mu
u}(u,0)^\dagger \Gamma^{\mu
u}(0,u) \]

In the NJL model, mesons are constructed by the infinite sum of quark-loop chains by using random phase approximation. We calculate the p meson polarization function to the leading order of \( 1/N_c \) expansion. The one loop polarization

\[
P_{\mu\nu}(q^2) = -\int d^4x.e^{-i(q\cdot x)}T\{\bar{u}(\gamma^\mu \gamma^5 \rho \Gamma_{\mu\nu}(u,0)^\dagger \Gamma^{\mu\nu}(0,u)\}
\]

The propagator of the p meson \( D_{\rho}(q^2) \) can be obtained from the one quark loop polarization \( P_{\mu\nu}(q^2) \) via the Schwinger-Dyson equation,

\[
-2G_\rho \Delta_{\rho}(q^2) = -2G_\rho \Delta_{\rho}(q^2) + \text{I}_{\rho\rho}(q^2)\text{I}^*_{\rho\rho}(q^2)
\]

In the rest frame of p, the Lorentz and flavor structure of \( \text{I}_{\rho\rho}(q^2) \) allows the decomposition with different spin projections on the direction of the magnetic field,

\[
\text{I}_{\rho\rho}^{(\sigma)}(q^2) = \text{I}_{\rho\rho}^{(\sigma)}(p^2)\text{I}_{\rho\rho}^{(\sigma)}(q^2) \]

Obviously, the \( \text{I}_{\rho\rho}^{(\sigma)}(q^2) \) can be decomposed in the same form. At last, we get the gap equations \( 1 + 2G_\rho \Pi_{\rho\rho}(q^2) = 0 \) to determine the masses of p meson with different spin projections. Moreover, we calculate the \( \Pi_{\rho\rho}^{(\sigma)}(q^2) \) in the rest frame of p meson, we can get the matrix as following:

\[
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

Combining the results above, we can easily find the relation for charged p meson

\[
\Pi_{\rho\rho}^{(\sigma)} = \begin{pmatrix}
(a + b) & 0 & 0 & 0 \\
0 & b - a & 0 & 0 \\
0 & 0 & -c & 0 \\
0 & 0 & 0 & -c \\
\end{pmatrix}
\]

**Numerical results and discussions**

For numerical calculations, we use the soft cut-off function

\[
f_A = \sqrt{\frac{\Lambda^2}{\Lambda^2 + (q^2)^2}}
\]

\[
f_{\rho\rho} = \frac{\Lambda^2}{\Lambda^2 + (q^2)^2 + 2q^2B^2}
\]

for zero and nonzero magnetic field, respectively. At finite magnetic field, we sum up to 20 Landau levels, and the results are saturated. We obtain \( \Lambda = 582 \text{ MeV}, G_\rho \Lambda^2 = 2.388, \Gamma_{\mu\nu} = 1.73 \) and \( m_0 = 5 \text{ MeV} \) by choosing \( f_A = 95 \text{ MeV}, m_0 = 140 \text{ MeV}, M_p = 768 \text{ MeV} \) in the vacuum and the vacuum quark mass \( M = 458 \text{ MeV} \).

In our calculations, the quark mass \( M \) is solved self consistently and we solve the gap equations for charged p meson at finite temperature and chemical potential. Fig. 1 shows the mass of charged p as the function of the magnetic field \( eB \) at zero temperature for different chemical potentials. It is observed that at zero temperature, when \( \mu \) is smaller than the vacuum constituent quark mass \( M_0^2 = 458 \text{ MeV} \), the behavior of magnetic field dependence of the charged p mass almost does not change for different chemical potentials, and the critical magnetic field remains as \( eB_c = 0.2\text{ GeV}^2 \). This indicates that the charged p condensation can survive at high baryon density and might occur inside compact stars.

At last, in Fig. 3, we show the critical magnetic field \( eB_c \) as a function of the temperature for \( \mu = 0, 0.3, 0.46 \text{ GeV} \), respectively. It is worthy of mentioning that at zero chemical potential, when the temperature is below \( T = 250 \text{ MeV} \), which is almost the critical temperature for the chiral phase transition, we can read that the critical magnetic field does not change so much. However, when the temperature is higher than \( T = 250 \text{ MeV} \), it is found that the critical magnetic field increases linearly with the temperature, and in the temperature range \( 200 \sim 500 \text{ MeV} \), the critical magnetic field for charged p condensation is in the range of \( 0.2 \sim 0.6 \text{ GeV}^2 \), which is just located inside the range of the magnetic field generated in the non-central heavy ion collisions of LHC. It means that high temperature superconductor might be produced at LHC.

**Conclusions**

We have investigated the mass behavior of charged p mesons in the background without p condensation under external magnetic field at finite temperature and density by using the NJL model. When the charged p meson mass drops to zero or becomes negative, it indicates that this background is not stable, and a new charged p condensation would be developed. The mesons are constructed as in the vacuum by summing up infinite quark-loop chains by using the random phase approximation. In this paper, we calculate the p meson polarization tensor to the leading order of \( 1/N_c \) expansion. In this process, the constituent quark mass is solved self consistently with magnetic field at finite temperature and density. It is noticed that in our current framework, there is no inverse magnetic catalysis effect.

The mass of charged p meson depending on the magnetic field is calculated at finite temperature and density. It is found that at fixed temperature and density, the polarized charged p meson becomes lighter with the increase of magnetic fields and drops to massless at the critical magnetic field \( eB_c \), which indicates that there would appear charged p meson condensation when the strength of the magnetic field is greater than the \( eB_c \). At zero density, our results show that in the temperature region \( 200 \text{ MeV} < T < 500 \text{ MeV} \), the critical magnetic field \( eB_c \) is in the range of \( 0.2 \sim 0.6 \text{ GeV}^2 \), which indicates that the high temperature superconductor could be created in the early stage of LHC.

However, we have to mention that in our current framework, there is no inverse magnetic catalysis for the quark mass. In the next step, we will investigate how the inverse magnetic catalysis will affect our results on the charged p condensation, especially at high temperatures.

**References**